

# Modeling of Banking Profit via Return-on-Assets and Return-on-Equity

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*Abstract*—In our contribution, we model bank profitability via return-on-assets (ROA) and return-on-equity (ROE) in a stochastic setting. We recall that the ROA is an indication of the operational efficiency of the bank while the ROE is a measure of equity holder returns and the potential growth on their investment. As regards the ROE, banks hold capital in order to prevent bank failure and meet bank capital requirements set by the regulatory authorities. However, they do not want to hold too much capital because by doing so they will lower the returns to equity holders. In order to model the dynamics of the ROA and ROE, we derive stochastic differential equations driven by Lévy processes that contain information about the value processes of net profit after tax, equity capital and total assets. In particular, we are able to compare Merton and Black-Scholes type models and provide simulations for the aforementioned profitability indicators.

*Keywords:* Stochastic Modeling; Lévy Process; Stochastic Differential Equations

## 1 Introduction

One of the biggest economic considerations in the 21st century is the maintenance of a profitable banking system. The main sources of bank profits originate from transaction fees on financial services and the interest spread on resources that are held in trust for clients who, in turn, pay interest on the asset (see, for instance, [7]). In our discussion, we derive dynamic models for bank profitability via Lévy processes (see, for instance, [2], [6] and [13]) appearing in a Merton-type model (see [10] and [4]). Lévy processes are characterized by (almost surely (a.s.)) right-continuous paths with their increments being independent and time-homogeneous. Such processes have an advantage over Brownian motion in that they are able to reflect the non-continuous nature of the dynamics of the components of bank profit. In the related Black-Scholes model, the markets are complete but some risks cannot be hedged. In addition, the motivation for using Lévy processes is their flexible (infinitely divisible) distribution which takes short-term skewness and excess kurtosis into account.

There are two main measures of the bank's profitability (see [11]). Let  $A^r = (A_t^r, t \geq 0)$  be the process representing the return-on-assets (ROA). In this regard, the net profit after taxes

per unit of assets may be represented by

$$\text{ROA } (A^r) = \frac{\text{Net Profit After Taxes}}{\text{Assets}} \quad (1)$$

The ROA provides information about how much profits are generated on average by each unit of assets. Therefore the ROA is an indicator on how efficiently a bank is being run. Let  $E^r = (E_t^r, t \geq 0)$  be the process representing the return-on-equity (ROE). Then the net profit after taxes per unit of equity capital may be given by

$$\text{ROE } (E^r) = \frac{\text{Net Profit After Taxes}}{\text{Equity Capital}} \quad (2)$$

From this relationship it follows that the lower the equity capital, the higher the ROE, therefore the owners of the bank (equity holders) may not want to hold too much equity capital. However, the equity capital cannot be too low, because the level of bank capital funds is subjected to capital adequacy regulation. Currently, this regulation takes the form of the Basel II Capital Accord (see [1]) that was implemented in 2007 on a worldwide basis. Also, from [9] and [8], it follows that there are other measures of the profitability.

The main problems addressed in this paper can be formulated as follows.

**Problem 1.1 (Modeling of Return-on-Assets):** *Can we deduce a Lévy process-driven model for the dynamics of the ROA?* (Proposition 3.1 in Section 3).

**Problem 1.2 (Modeling of Return-on-Equity):** *Can we deduce a Lévy process-driven model for the dynamics of the ROE?* (Proposition 3.3 in Section 3).

The paper is structured in the following way. In Section 2 we present a brief description of the stochastic banking model that we will consider. In the third section, we describe the dynamics of two measures of bank profitability, viz., the ROA and ROE. Section 4 contains numerical examples where we compare a Lévy-process driven model with a model driven by a Brownian motion. In Section 5, we provide concluding remarks and we point out further research problems that may be addressed.

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## 2 The Banking Model

In our model, we consider the filtered probability space  $(\Omega, \mathbf{F}, (\mathcal{F}_t)_{0 \leq t \leq \tau}, \mathbf{P})$ . As usual, we assume that  $\mathbf{P}$  is a real probability measure,  $\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq \tau}$  is the natural filtration,  $\mathcal{F}_0$  is trivial and  $\mathcal{F}_\tau = \mathbf{F}$ . The jump process  $\Delta L = (\Delta L_t, t \geq 0)$  associated with a Lévy process,  $L$ , is defined by  $\Delta L_t = L_t - L_{t-}$ , for each  $t \geq 0$ , where  $L_{t-} = \lim_{s \uparrow t} L_s$  is the left limit at  $t$ . Let  $L = (L_t)_{0 \leq t \leq \tau}$  with  $L_0 = 0$  a.s. be the càdlàg version of a Lévy process. Also, we assume that the Lévy measure  $\nu$  satisfies

$$\int_{|x| < 1} |x|^2 \nu(dx) < \infty, \quad \int_{|x| \geq 1} \nu(dx) < \infty. \quad (3)$$

Furthermore, the following definition of the Lévy-Itô decomposition is important.

**Definition 2.1 (Lévy-Itô Decomposition (see [4]))** Let  $(L_t)$  be a Lévy process and  $\nu$  its Lévy measure, given by equation (3). Then there exist a vector  $\gamma$  and a Brownian motion  $(B_t)$  such that

$$L_t = \gamma t + B_t + L_t^f + \lim_{\epsilon \downarrow 0} L_t^\epsilon,$$

where  $L_t^f$  is a compound Poisson process with a finite number of terms and  $L_t^\epsilon$  is also a compound Poisson process. However, there can be infinitely many small jumps.

An implication of the Lévy-Itô decomposition (see [4]) is that every Lévy process is a combination of a Brownian motion and a sum of independent compound Poisson processes. This implies that every Lévy process can be approximated a jump-diffusion process, that is by the sum of a Brownian motion with drift and a compound Poisson process. In this paper, we will consider Merton's jump-diffusion model (see [10] and [4]) of  $L$ . Thus

$$L_t = at + \tilde{s}B_t + \sum_{i=1}^{N_t} Y_i, \quad 0 \leq t \leq \tau, \quad (4)$$

where  $(B_t)_{0 \leq t \leq \tau}$  is a Brownian motion with standard deviation  $\tilde{s} > 0$ ,  $a = \mathbf{E}(L_1)$ ,  $(N_t)_{t \geq 0}$  is a Poisson process counting the jumps of  $L_t$  with jump intensity  $\lambda$ . The  $Y_i$  (i.i.d. variables) are jump sizes, the distribution of the jump sizes is Gaussian with  $\mu$  the mean jump size and  $\delta$  the standard deviation of  $Y_i$ .

A typical bank balance sheet identity consists of assets (uses of funds) and liabilities (sources of funds), that are balanced by bank capital (see, for instance [5]), according to the well-known relation

$$\begin{aligned} \text{Value of Assets (A)} &= \text{Value of Liabilities (}\Gamma) \\ &+ \text{Value of Bank Capital (K)}. \end{aligned} \quad (5)$$

### 2.1 Assets

In this subsection, we discuss bank asset price processes. The bank's investment portfolio is constituted by  $m + 1$  assets including loans, advances and intangible assets (all risky assets) and Treasuries (riskless asset). We pick the first asset to be the riskless Treasuries,  $T$ , that earns a constant, continuously-compounded interest rate of  $r^T$ . Profit maximizing banks set their rates of return on assets as a sum of the risk-free Treasuries rate,  $r^T \bar{1}$ , risk premium,  $\mu_r$ , and the default premium,  $\mathbf{E}(d)$ . Here the unitary vector and risk premium are given by

$$\bar{1} = (1, 1, \dots, 1)^T \text{ and } \mu_r = (\mu_1, \mu_2, \dots, \mu_m)^T,$$

respectively. Also, we have that the default premium is defined by

$$\mathbf{E}(d) = (\mathbf{E}(d_1), \mathbf{E}(d_2), \dots, \mathbf{E}(d_m))^T, \quad \begin{cases} \mathbf{E}(d_i) \neq 0 & i\text{-th asset is a loan,} \\ \mathbf{E}(d_i) = 0 & i\text{-th asset is not a loan.} \end{cases}$$

The sum  $r^T \bar{1} + \mu_r$  covers, for instance, the cost of monitoring and screening of loans and cost of capital. The  $\mathbf{E}(d)$  component corresponds to the amount of provisioning that is needed to match the average expected losses faced by the loans. The  $m$  assets besides Treasuries are risky and their price process,  $S$  (reinvested dividends included), follows a geometric Lévy process with drift vector,  $r^T \bar{1} + \mu_r + \mathbf{E}(d)$  and diffusion matrix,  $\sigma_a$ , as in

$$S_t = S_0 + \int_0^t I_s^s \left( r^T \bar{1} + \mu_r + \mathbf{E}(d) \right) ds + \int_0^t I_s^s \sigma_a dL_s + \sum_{0 < s \leq t} \Delta S_s 1_{\{|\Delta S_s| \geq 1\}}, \quad (6)$$

where  $I_t^S$  denotes the  $m \times m$  diagonal matrix with entries  $S_t$  and  $L$  is an  $m$ -dimensional Lévy process. Also,  $\Delta S_s$  is the jump of the process  $S$  at time  $t > 0$  and  $1_{\{|\Delta S_s| \geq 1\}}$  is the indicator function of  $\{|\Delta S_s| \geq 1\}$ . We suppose, without loss of generality, that  $\text{rank}(\sigma_a) = m$  and the bank is allowed to engage in continuous frictionless trading over the planning horizon,  $[0, T]$ . Next, we suppose that  $\rho$  is the  $m$ -dimensional stochastic process that represents the current value of risky assets. Put  $\mu_a = r^T + \rho^T \left( \mu_r + \mathbf{E}(d) \right)$ ,  $\sigma^A = \tilde{s} \sigma_a$  and  $\mu^A = \mu_a + a \sigma_a$ . In this case, the dynamics of the current value of the bank's entire asset portfolio,  $A$ , over any reporting period may be given by

$$\begin{aligned}
 dA_t &= A_t \mu_a dt + A_t \sigma_a \left[ a dt + \tilde{s} dB_t^A + d \left[ \sum_{i=1}^{N_t} Y_i \right] \right] - r^T D_t dt \\
 &= A_t \left[ \mu^A dt + \sigma^A dB_t^A + \sigma_a d \left[ \sum_{i=1}^{N_t} Y_i \right] \right] - r^T D_t dt \quad (7)
 \end{aligned}$$

where the face value of the deposits,  $D$ , is described in the usual way, and  $r^T D_t dt$  represents the interest paid to depositors.

## 2.2 Capital

The total value of the bank capital,  $K = (K_t, t \geq 0)$ , can be expressed as

$$K_t = K_{t1} + K_{t2} + K_{t3}, \quad (8)$$

where  $K_{t1}$ ,  $K_{t2}$  and  $K_{t3}$  are Tier 1, Tier 2 and Tier 3 capital, respectively. *Tier 1 (T1) capital* is the book value of the bank's equity,  $E = (E_t, t \geq 0)$ , plus retained earnings,  $E^R = (E_t^R, t \geq 0)$ . Tier 2 (T2) and Tier 3 (T3) capital (collectively known as *supplementary capital*) is, in our case, the sum of subordinate debt,  $O$  where  $O_t = \exp\{rt\}$  and loan-loss reserves,  $R^L$ . However, for sake of argument, we suppose that

$$K_t = E_t + E_t^R + O_t. \quad (9)$$

For  $\sigma^E = \tilde{s}\sigma_e$  and  $\mu^E = (\mu_e + a\sigma_e)$  we describe the evolution of  $O$  and  $E$  as

$$dO_t = r \exp\{rt\} dt, \quad O_0 > 0 \quad (10)$$

and

$$dE_t = E_t \left[ \mu^E dt + \sigma^E dB_t^E + \sigma_e d \left[ \sum_{i=1}^{N_t} Y_i \right] \right] \quad (11)$$

respectively. Where  $\sigma_e$ ,  $\mu_e$  and  $B_t^E$  are the volatility of  $E$ , the total expected returns on  $E$ , and the standard Brownian motion, respectively.

## 2.3 Profit

Let  $\Pi^n$  be the bank's net profit after tax which is used to meet obligations such as dividend payments on bank equity,  $\delta_e$ , and interest and principal payments on subordinate debt,  $(1+r)O$ . Put  $\delta_s = (1+r)$ . In this case, we may compute the *retained earnings*,  $E^R = (E_t^R, t \geq 0)$ , as

$$E_t^R = \Pi_t^n - \delta_e E_t - \delta_s O_t \quad (12)$$

We assume that the retained earnings remain constant during the planning period so that  $dE^R = 0$ . Therefore, the dynamics of the net profit after tax may be expressed as

$$\begin{aligned}
 d\Pi_t^n &= \delta_e E_t \left[ \mu^E dt + \sigma^E dB_t^E + \sigma_e d \left[ \sum_{i=1}^{N_t} Y_i \right] \right] + \\
 &\delta_s r \exp\{rt\} dt. \quad (13)
 \end{aligned}$$

## 3 Dynamics of ROA and ROE

In this section, we derive stochastic differential equations for the dynamics of two measures of bank profitability, viz., the ROA and ROE. The procedure that we use to obtain the said equations is related to Ito's general formula (see [13]). An important observation about our foregoing description of the assets and liabilities of a commercial bank, is that it is suggestive of a simple procedure for obtaining a stochastic model for the dynamics of the profitability of such a depository institution. The solution of the SDE (13) is

$$\begin{aligned}
 \Pi_t^n &= E_t^R + \delta_e E_0 \exp \left\{ \sigma^E B_t^E + \left( \mu^E - \frac{1}{2} (\sigma^E)^2 \right) t + \right. \\
 &\left. \sigma_e \left[ \sum_{i=1}^{N_t} Y_i \right] \right\} + \delta_s \exp\{rt\}. \quad (14)
 \end{aligned}$$

### 3.1 Return-on-Assets (ROA)

The dynamics of the ROA (see equation (1)) may be calculated by considering the nonlinear dynamics of the value of total assets represented by (7) and the dynamics of the net profit after tax given by (13). One can easily check how efficiently a bank has been managed over a certain past time period by monitoring the fluctuations of the ROA. A stochastic system for the dynamics of the ROA for a commercial bank is given below.

**Proposition 3.1 (Dynamics of ROA using Merton's Model):** *Suppose that the dynamics of the value of total assets and the net profit after tax are represented by (7) and (13), respectively. Then a stochastic system for the ROA of a bank may be expressed as*

$$\begin{aligned}
 dA_t^r = & A_t^r \left[ \left( \delta_e E_t (\sigma^E)^2 \{ (\sigma^A)^2 \sigma_a^2 dB_t^A - \sigma_a^2 \} + \sigma_a^2 \right. \right. \\
 & \left. \left. + (\sigma^A)^2 - \mu^A + [\Pi_t^n]^{-1} \{ \delta_e \mu^E E_t + \delta_e r O_t \} \right) dt \right. \\
 & + \left( d \left[ \sum_{i=1}^{N_t} Y_i \right] \delta_e E_t \sigma^E \{ \sigma^A \sigma_a dB_t^A - \sigma_a \} \right. \\
 & \left. + [\Pi_t^n]^{-1} \delta_e \sigma^E E_t \right) dB_t^E \\
 & + \left( [\Pi_t^n]^{-1} \sigma^E \delta_e E_t + \sigma^A \sigma_a dB_t^A - \sigma_a \right. \\
 & \left. + \delta_e E_t \sigma^E [\Pi_t^n]^{-1} dB_t^E \{ \sigma^A \sigma_a dB_t^A - \sigma_a \} \right. \\
 & \left. - \delta_e E_t [\Pi_t^n]^{-1} \sigma^E \sigma^A dB_t^A \right) d \left[ \sum_{i=1}^{N_t} Y_i \right] \\
 & \left. - \sigma_a dB_t^A \right]. \tag{15}
 \end{aligned}$$

Next, we consider the special case where  $L_t = B_t$ , i.e.,  $\sum_{i=1}^{N_t} Y_i + at = 0$ .

**Corollary 3.2 (Dynamics of ROA using Black-Scholes Model):** Suppose that  $L_t = B_t$  in equations (7) and (13). Then a stochastic system for the ROA (using Black-Scholes model) of a bank may be expressed as

$$\begin{aligned}
 dA_t^r = & A_t^r \left[ \{ \sigma_a^2 - \mu_r + [\Pi_t^n]^{-1} (\delta_e \mu_e E_t + \delta_s r O_t) \} dt \right. \\
 & \left. + [\Pi_t^n]^{-1} \delta_e \sigma_e E_t dB_t^E - \sigma_a dB_t^A \right]. \tag{16}
 \end{aligned}$$

### 3.2 Return-on-Equity (ROE)

The dynamics of the ROE (see equation (2)) may be calculated by considering the equation for the dynamics of the equity capital given by (11) and the net profit after tax represented by (13) and using Ito's formula (see [13]). A stochastic system for the dynamics of the return on equities for a commercial bank is given below.

**Proposition 3.3 (Dynamics of ROE using Merton's Model):** Suppose that the dynamics of the value of total assets and the net profit after tax are represented by (11) and (13), respectively. Then a stochastic system for the ROE of a bank may be expressed as

$$\begin{aligned}
 dE_t^r = & E_t^r \left[ \left( [\Pi_t^n]^{-1} \left\{ \delta_e E_t \mu^E + \delta_s r O_t \right. \right. \right. \\
 & \left. \left. + \delta_e E_t (\sigma^E)^2 \{ 2(\sigma^E)^2 + \sigma_e^2 dB_t^E - \sigma_e^2 \} \right. \right. \\
 & \left. \left. - \delta_e E_t (\sigma^E)^2 \right\} + [\sigma^E]^2 - \mu_e + \sigma_e^2 \right) dt \\
 & + \left( [\Pi_t^n]^{-1} \delta_e \sigma^E E_t - \sigma_e \right) dB_t^E \\
 & + \left( [\Pi_t^n]^{-1} \sigma^E \delta_e E_t - \sigma_e + 2\sigma^E \sigma_e dB_t^E \right) d \left[ \sum_{i=1}^{N_t} Y_i \right] \\
 & + \left( \delta_e E_t \sigma^E \{ 2\sigma^E \sigma_e dB_t^E - \sigma_e \} \right. \\
 & \left. - \delta_e E_t (\sigma^E)^2 \right) dB_t^E d \left[ \sum_{i=1}^{N_t} Y_i \right]. \tag{17}
 \end{aligned}$$

Next, we consider the special case where  $L_t = B_t$  i.e.  $\sum_{i=1}^{N_t} Y_i + at = 0$ .

**Corollary 3.4 (Dynamics of ROE using Black-Scholes Model):** Suppose that  $L_t = B_t$  in equations (11) and (13). Then a stochastic system for the ROE (using Black-Scholes model) of a bank may be expressed as

$$\begin{aligned}
 dE_t^r = & E_t^r \left[ \left( [\sigma_e]^2 - \mu_e + [\Pi_t^n]^{-1} \left\{ \delta_s r O_t + \delta_e E_t \mu_e \right. \right. \right. \\
 & \left. \left. - \delta_e E_t (\sigma_e)^2 \right\} \right) dt \\
 & \left. + \left( [\Pi_t^n]^{-1} \sigma_e \delta_e E_t - \sigma_e \right) dB_t^E \right]. \tag{18}
 \end{aligned}$$

## 4 Numerical Examples

In this section, we simulate the ROA and the ROE of the SA Reserve Bank (see [12]) over a two year period. There are a few methods for simulating stochastic differential equation (SDE). First we assume that the ROA and the ROE do have jumps. We therefore simulate the stochastic differential equations (15) and (17) by using Merton's model. Note that in Merton's model the driving Lévy process is a compounded Poisson process.

Although Lévy based models are structurally superior, the estimation procedures are complicated. For comparative purposes (see [14]), we compute the average absolute error (APE) as a percentage of the mean ROA (or ROE) as

$$\begin{aligned}
 APE = & \frac{1}{\text{mean ROA (or ROE) value}} \\
 & * \sum_{i=1}^{24} \frac{|Data\ value - Model\ value|}{\text{number of ROA (or ROE) values}} \tag{19}
 \end{aligned}$$

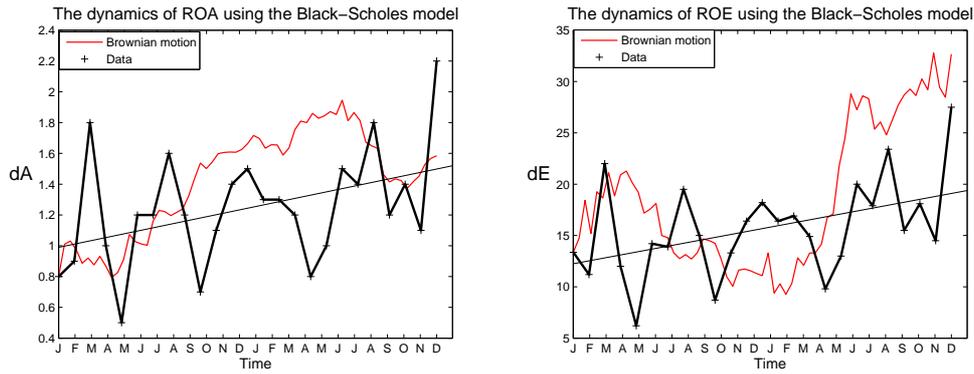


Figure 1: A solution of the SDEs (16) and (18) with a fitted linear trend line

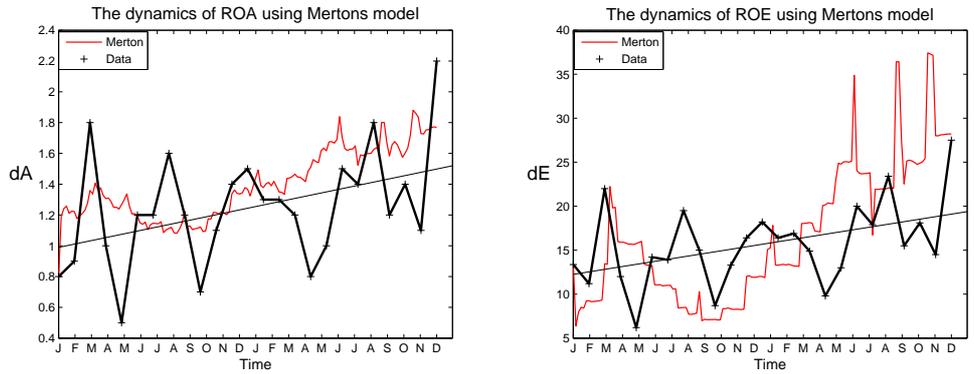


Figure 2: A solution of the SDEs (15) and (17) with a fitted linear trend line

Another measure which also gives an estimate of the goodness or quality of fit is the root-mean-square error (RMSE) given by

$$RMSE = \sqrt{\sum_{i=1}^{24} \frac{(Data\ value - Model\ value)^2}{number\ of\ ROA\ (or\ ROE)\ values}} \quad (20)$$

We estimate the model parameters by minimizing the APE and the RMSE errors. In Table 2 we give the relevant values of APE and RMSE. The calibrated Lévy model is very sensitive to the numerical starting point in the minimization algorithm or small changes in the input data. In our case, we use Merton's model with intensity  $\lambda = 2$  (ROA case) or  $\lambda = 16$  (ROE case) and the average jump size as  $\mu = 0.06$  (ROA) or  $\mu = 0.01$  (ROE). For another intensity the results of the minimization will be different.

Secondly, we assume that the ROA and the ROE do not have jumps. In this case our SDEs (16) and (18) are driven by Brownian motions. We apply Euler-Maruyama Method to simulate these SDEs over  $[0, T]$  discretized Brownian path using time steps of size  $Dt = R * dt$  for some positive integer  $R$  and  $dt = \frac{T}{2^8}$ . For a SDE of the form

$$dA_t = f(A_t)dt + g(A_t)dB_t, \quad 0 \leq t \leq T,$$

the Euler-Maruyama method takes the form

$$A_j = A_{j-1} + f(A_{j-1})\Delta t + g(A_{j-1})(B(t_j) - B(t_{j-1})),$$

where  $j = 1, 2, \dots, \frac{2^8}{R}$ .

The following data on the ROA and ROE from the SA Reserve Bank was used in our simulation.

	ROA/ROE		ROA/ROE
Jan-2005	0.9/11.2	Jan-2006	1.3/16.4
Feb-2005	1.8/22.0	Feb-2006	1.3/16.9
Mrt-2005	1.0/12.0	Mrt-2006	1.2/14.9
Apr-2005	0.5/6.2	Apr-2006	0.8/9.8
May-2005	1.2/14.2	May-2006	1.0/13.0
Jun-2005	1.2/13.9	Jun-2006	1.5/20
Jul-2005	1.6/19.5	Jul-2006	1.4/17.9
Aug-2005	1.2/15.0	Aug-2006	1.8/23.4
Sep-2005	0.7/8.7	Sep-2006	1.2/15.5
Oct-2005	1.1/13.3	Oct-2006	1.4/18.1
Nov-2005	1.4/16.4	Nov-2006	1.1/14.5
Dec-2005	1.5/18.2	Dec-2006	2.2/27.5

Table 1: Source SA Reserve Bank

Using SA Reserve Bank's data we get the following parameter choices  $\sigma_e = 0.69$ ,  $\mu_e = 0.06$ ,  $\sigma_a = 0.01$ ,  $\mu_r = 0.003$ .

Also, for the Euler-Maruyama method we chose the value of net profit after tax as  $\Pi_t^p = 16878$ , the dividend payments on  $E$  as  $\delta_e = 0.05$ , the interest and principal payments on  $O$  as  $\delta_s = 1.06$ , the interest rate as  $r = 0.06$ , the subordinate debt  $O = 135$  and the bank equity  $E = 1164$ .

Model	APE(%)	RMSE
Black-Scholes (ROA)	27.58	0.3472
Black-Scholes (ROE)	38.7145	6.2475
Merton (ROA)	1.2588	0.06
Merton (ROE)	0.6124	0.3687

Table 2: Lévy models: APE and RMSE

In Figure 1, we plotted the actual ROA (or ROE) values versus the Black-Scholes model values for the two years 2005 and 2006. From Figure 1 and Table 2 where we used the model without jumps it follows that the root-mean-square error for the ROA is 0.3472 and for the ROE it is 6.2475.

In Figure 2, we plotted the actual ROA (or ROE) values versus the Merton's model values for the two years 2005 and 2006. From Figure 2 and Table 2 where we used the model with jumps it follows that the RMSE for the ROA is 0.06 and 0.3687 for the ROE.

Note that the APE (%) decreases from 27.58 % to 1.2588 % and the RMSE value decreases from 0.3472 to 0.06 for the ROA. Furthermore, in the ROE case the APE (%) decreases from 38.71 % to 0.6124 % and the RMSE value decreases from 6.2475 to 0.3687. We therefore conclude that in the ROA case and even more for the ROE case the Black-Scholes model performs worse than Merton's model. However, we still observe a significant difference from the data values. Note that calibrations to other datasets can favor the Black-Scholes model more.

## 5 Conclusions and Future Directions

Although the Black-Scholes model is powerful and simple to use, most profit indicators exhibit jumps rather than continuous changes. Therefore, we have constructed asset-liability models in a stochastic framework driven by a Lévy process for two measures of commercial bank profitability. In this regard, the ROA, that is intended to measure the operational efficiency of the bank and the ROE that involves the consideration of the bank owner's returns on their investment was central to our discussion. These stochastic models arose from a consideration of the bank's balance sheet and income statements associated with off-balance sheet items.

Discussions on the profitability and solvency of banking systems are intimately related (see, for instance, [3]). In particular, asset-liability management by banks cannot be separated from the decision about how much equity the bank owner

should invest. This means that banking decisions and equity policy have to be simultaneously addressed by bank managers. Further investigations will include descriptions of the dynamics of the other measures of bank probability.

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