A Computational Test of the Information-Theory Based Entropy Theory of Perception: Does It Actually Generate the Stevens and Weber-Fechner Laws of Sensation?

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***Abstract*—K.H. Norwich et al. used Shannon Information Theory to derive their Entropy Theory of Perception (1975-present). The Entropy Theory produces the Entropy Equation, which relates the strength of sensation (represented by magnitude estimates) to the intensity of the sensory stimulus. At "high" intensities, the relation is approximately logarithmic, which Norwich et al. dubbed "the Weber-Fechner Law". At "low" intensities, the relation is approximately a power function, dubbed "Stevens' Law". Unfortunately, the Entropy Equation has three unknowns, so that what constitutes "high" and "low" can only be established through curve-fitting. Remarkably, the latter was never done. Establishing parameter values is especially important because one of the unknowns is a power exponent (the "Entropy Exponent", here denoted y) said to be identical in value to "Stevens' exponent" (here denoted x). The identity y=x was crucial to the numerous published applications of the Entropy Theory to psychophysical and neurophysiological phenomena. Curve-fitting of the Entropy Equation to magnitude estimates would therefore establish the ranges of the "Weber-Fechner" and "Stevens" laws and reveal whether y=x. The present author did the curve-fitting, following the custom in the literature: logarithmic forms of the Entropy Equation and Stevens' Law were fitted by least-squares regression to logarithm(magnitude-estimate) vs. logarithm(stimulus-strength) taken from 64 published curves of magnitude estimates. The resulting relation of y to x was broadly scattered; 62/64 times, y exceeded x. In theory, the fitted Entropy Equation allows calculation of the information transmitted in perception. Hence the regressions were re-run conditional to an information transmitted of 2.5 bits/stimulus, the mean value in the literature. y≈1.7x under the constrained regression. Altogether, the purported equality of the Entropy Exponent and Stevens' exponent was not confirmed. Further, neither the "Weber-Fechner Law" nor the "Stevens' Law" derived from any fitted Entropy Equation described the entire range of the respective magnitude estimation curve, contrary to the formal use of those laws. Norwich's later quantification of sensation growth by "physical entropy" makes identical mistakes. All of this emphasizes that the Entropy Theory does not derive rules of sensory perception from information theory, and it is recommended that further attempts to do so should be discouraged.**

Manuscript received March 25, 2009. Work supported by the author.
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***Index Terms*—entropy, information theory, sensation, Stevens' Law, Weber-Fechner Law.**

I. Introduction: the Entropy Theory of Perception

The Entropy Theory advanced by K.H. Norwich and colleagues [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14] proposes that the neuronal *or* psychophysical response F is proportional to the entropy (the stimulus equivocation), called E_S or H, defined as follows. The information associated with an event of known probability p_i is $-\log(p_i)$. Observing the outcome of an event from a set of n events of probabilities $\{p_i, i = 1, ..., n\}$ yields the average information I_S,

$$I_S = -\sum_{i=1}^{n} p_i \log p_i \tag{1}$$

(after [15]; see [11], [16]). Any base can be chosen for the logarithm; usually base 2 is used, giving information in binary units ("bits') per event. For a set of transmitted symbols "k", (1) becomes

$$I_S = -\sum_{k} p(k) \log p(k). \tag{2}$$

For simplicity, Norwich et al. always assumed that the number of symbols transmitted and received was identical. Let $p_j(k)$ be the probability of transmission of symbol k, given that symbol j has been received. The stimulus equivocation E_S is

$$E_S = -\sum_{j}\sum_{k} p_j(k) \log p_j(k). \tag{3}$$

The Garner-Hake information transmitted I_t [17] is

$$I_t = I_S - E_S = -\sum_{k} p(k) \log p(k) + \sum_{j}\sum_{k} p_j(k) \log p_j(k). \tag{4}$$

Hypothetically, these quantities are computed by the microscopic sensory receptor. The receptor is hypothetically in a state of uncertainty about the microscopically mean level of a macroscopically constant stimulus [7], [8]. That uncertainty is reduced through repeated measurement of the stimulus [7], [8]. At the microscopic level, the intensity of that stimulus fluctuates, thermodynamically, from instant to instant [2], [3], [18]. Those microscopically discrete stimulus intensities were replaced for mathematical purposes by an intensity continuum having a standard deviation σ_S. There was also a stochastic "reference noise" with variance N^2. That led to a "variation of a Shannon entropy function" [4, p. 536]. Using logarithms to base "e" for mathematical convenience, and replacing E_S by the symbol H (after Norwich), gave

$$H\left(base\ e\right)=\frac{1}{2}\ \ell n\left(1+\frac{\sigma_S^2}{mN^2}\right),\ where\ m\geq 1. \quad (5)$$

Now from statistical physics the mean density of a particle population is sometimes a power function of the population variance [8]:

$$\sigma^2=\lambda\mu^n \quad where\ unknowns\ \lambda,n>0 \quad (6)$$

and where n is a constant for each kind of particle [10], [18]. Further, sensation magnitude F and stimulus equivocation H were hypothesized to obey

$$F\propto H,\ hence\ F=kH,\ k>0. \quad (7)$$

Thus altogether

$$\begin{aligned} F=kH&=\frac{k}{2}\ \ell n\left(1+\frac{\lambda\mu^n}{mN^2}\right)\\ &=\frac{k}{2}\ \ell n\left(1+A\mu^n\right),\ A=\frac{\lambda}{mN^2}\end{aligned} \quad (8)$$

where μ is stimulus strength, e.g. average photon flux, or molarity of a solution. Finally, the maximum transmitted information is $I_{t,max}= H_{max} - H_{min}$ units of information, thus

$$I_{t,max}=H_{max}-H_{min}=\frac{F_{max}-F_{min}}{k} \quad (9)$$

[4], [6], [16], [19], [20], [21].

II. The Entropy Theory Derivations of the "Weber-Fechner Law" and "Stevens' Law"

For large intensities μ or small noise variance N^2, then $\lambda\mu^n/mN^2 >> 1$, and (8) becomes

$$F=kH=\frac{k}{2}\ \ell n\left(A\mu^n\right)=\frac{nk}{2}\ell n\ \mu+\frac{k}{2}\ \ell n A \quad (10)$$

dubbed the "Weber-Fechner Law" [3], [4], [5], [8], [10], [18], [19], [21], [22], [23], [24], [25]. For the opposing case of small μ or large N^2,

$$F=kH=\frac{k}{2}\ \ell n\left(1+z\right)=\frac{k}{2}\left(z-\frac{1}{2}z^2+\dots\right)\cong\frac{kz}{2} \quad for\ z=\frac{\lambda\mu^n}{mN^2},$$

$$so\ that\ F=\frac{kA}{2}\mu^n\ \ for\ A=\frac{\lambda}{mN^2} \quad (11)$$

which Norwich and co-authors dubbed "Stevens' Law". As Norwich et al. explain, "It transpires that the exponent, n, is precisely the Stevens exponent that appears in the law of sensation [equation for Stevens' law, Norwich et al., 1989, Equation 12]" [22, p. 353] and "The parameter, n, has been shown in previous work to be equivalent to the power function exponent thoroughly examined experimentally by Stevens and others" [2, p. 169]. This equivalence is alleged throughout the Entropy Theory [3], [4], [5], [6], [8], [9], [10], [13], [16], [18], [21], [22], [23], [24], [25], [26], [27].

Returning to (8), the Entropy Theory provides no way of obtaining A = λ/mN^2 , as generally, λ, m, and N^2 are unknown. In fact, all of k, A, and n can only be obtained through regression of (8) on curves of magnitude estimates vs. intensity. Without that regression, the intensity limits within which (10) and (11) separately apply are unknown. Further, Norwich et al. never actually proved that the Entropy Exponent equals the empirical Stevens' exponent, an equivalence that was the basis for many applications of the Entropy Theory. The purported equivalence can only be established through curve-fitting of the Entropy Equation and of Stevens' Law and comparison of the resulting exponents.

III. Testing the Purported Equivalence of the Entropy and Stevens' Exponents

The alleged equality of the Entropy exponent and Stevens' exponent was tested here for 64 examples of published magnitude estimation curves - more than used in all of Norwich et al.'s collected Entropy publications. When possible, the actual data points were obtained from the original authors. When this was not possible, the published plots were digitized. Digitizing error was several percent and was estimated to be less than the error originally made in the authors' formatting of their illustrations. The data used was chosen for its apparent quality, based on the rigorousness of the training of the subjects, the number of repetitions of the stimulus, and so on. To avoid too much emphasis on any one laboratory or paper or sensory modality, the data were taken from 21 papers on taste, olfaction, audition, and vision.

Importantly, only data curves that are gently concave downward when plotted in logarithmic-by-logarithmic scales can be fitted by the Entropy Equation. Curves that run concave upwards, or that follow straight lines in log-log scales (power functions), drive one or more of k, A, and n to either ∞ or 0. This limitation always went unmentioned.

A. *Unconstrained regressions*

Magnitude estimates and stimulus intensities can both cover several orders of magnitude in a given experiment. Hence, magnitude estimates have traditionally been displayed as a function of stimulus intensity using coordinates of log(magnitude estimate) versus log(intensity). Equations fitted to the data have first to be transformed for logarithmic coordinates; those transformations reduce weighting bias in least-squares regression of the equation to the data. Here, natural logarithms (base *e*) were employed, producing plotting coordinates of ln F versus ln μ . When the power function $F=D\mu^{\eta}$ [28] is appropriately transformed it becomes a straight line, $\ln F = \ln D + \eta \ln \mu$. The Entropy Equation transforms to $\ln F=\ln (k/2 \ln(1+A(exp(\ln \mu))^{n}))$.

Fitting Stevens' Law to the data using logarithmic coordinates is typical of the literature. Hence fitted Stevens exponents already exist in the literature, to which the present fitted Stevens exponents could be compared. The present fitted Stevens exponents were found to differ by at most 10% from the published ones, confirming that the present study successfully emulated traditional curve-fitting methods.

B. *Constrained regressions*

If desired, the Entropy Equation regression can be stopped when information transmitted (given by (9)) reaches 2.5 bits/stimulus, the literature average for absolute identifications. This sum-of-squares-of-residuals (SSR) thus assumes an unnaturally high value, such that the resulting "constrained" Entropy Equation always fits less well (i.e. produces a higher SSR) than the "unconstrained" entropy function.

C. *Entropy Exponent vs. Stevens' exponent: results*

Figs. 1 and 2 show the regression-derived Entropy Exponents, for convenience called "y", plotted vs. the corresponding Stevens' exponents, for convenience called "x". The unconstrained regressions (Fig. 1) result in a broad scatter of {x,y} pairs, with only 2 of 64 Entropy exponents being smaller than the corresponding Stevens' exponents. The constrained regressions (Fig. 2) result in a streamlined relation of y to x; an unweighted regression of the equation y=Cx yielded y=1.7x. All but 2 points fall between the lines y=2.5x+0.2 and y=1.44x-0.2. Another figure, Fig.3, was made only for 1 kHz tones, the stimulus for which the most data was used in Figs. 1 and 2. For those cases in which the Entropy Exponent was approximately equal both with and without constraint, $y \approx 1.42x$.

D. *Origin of the observed discrepancy*

To understand the origin of the discrepancy observed between n and η, the reader must inspect the Taylor series (11) in $z = \lambda\mu^{n}/mN^{2}$. For the taste of sucrose, k=15.94, A=19.31 and n=1.77 (data of [29]). For μ =0.12 moles/litre, $A\mu^{n}$ =0.453, $½(A\mu^{n})^{2}$=0.103, and $1/3(A\mu^{n})^{3}$=0.031, i.e. higher-order terms in the series expansion in (11) cannot be ignored. For the next higher stimulus, μ =0.24 M, giving $A\mu^{n}$=1.54. Thus $A\mu^{n}$ >1 and a series in $A\mu^{n}$ is not justified; the low-intensity approximation to F (11) applies, at best, only to

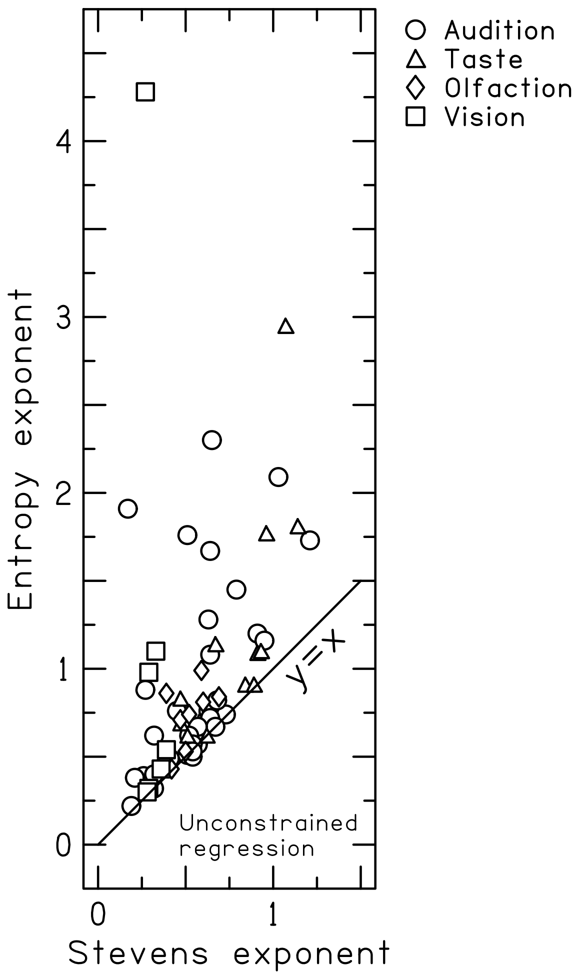

Fig. 1. The Entropy exponent plotted vs. Stevens' exponent. The Entropy exponent was obtained by unconstrained regressions (see text) on 64 sets of published magnitude estimates from audition, taste, olfaction, and vision. Four of the data points are based on papers of S.S. Stevens. The line y=x indicates equality of the two kinds of exponents. Data sources: [34] NaCl (Fig. 1), MgCl (Fig. 1), Na_2SO_4 (Fig. 1); [35] white noise (geometric means of magnitude estimates: series 1-3); [36] amyl acetate (Table 1: high standard, medium standard, low standard); [31] N-butanol (ratio scaling); [37] 1 kHz tone (Figs. 2, 3, 6, 7, 8, and 10); [38] 0.1 kHz tone (Fig. 2, crosses; Fig. 2, circles), 0.250 kHz tone (Fig. 3, geometric means of circles); [39] N-butanol (group means, median magnitude estimates); [40] Table 1: heptane, benzene, octane; [41] sucrose (group data, geometric means: Table 2, Table 3); [30] 1 kHz tone (subjects #8, 9, 10, 11, 12, 13); [42] white light (Fig. 3, top curve); [43] 1 kHz tone (curves 1-7); [29] sucrose ("after normalization" geometric means of plotted points); [44] NaCl (Table 1: subjects MK, LK, and KK); [32] white light (monocular group data, medians of magnitude estimates), blue light, red light, green light; [45] NaCl (subjects HT, GY, and BGN); [46] 0.550 kHz tone (subject AWS), 0.765 kHz tone (subjects EWB, RSM); [47] white noise (Fig. 2: binaural, magnitude production; binaural, magnitude estimation; monaural, magnitude production; monaural, magnitude estimation); [48] 1 kHz tone (Fig. 1, circles; Fig. 1, squares), white noise (Fig. 7, circles and crosses); [49] white light (monocular group data, geometric means, means of circles and squares; and Fig. 4, top curve); [50] binaural 1 kHz tone (cross-modality-matching, high range day 2; low range day 2).

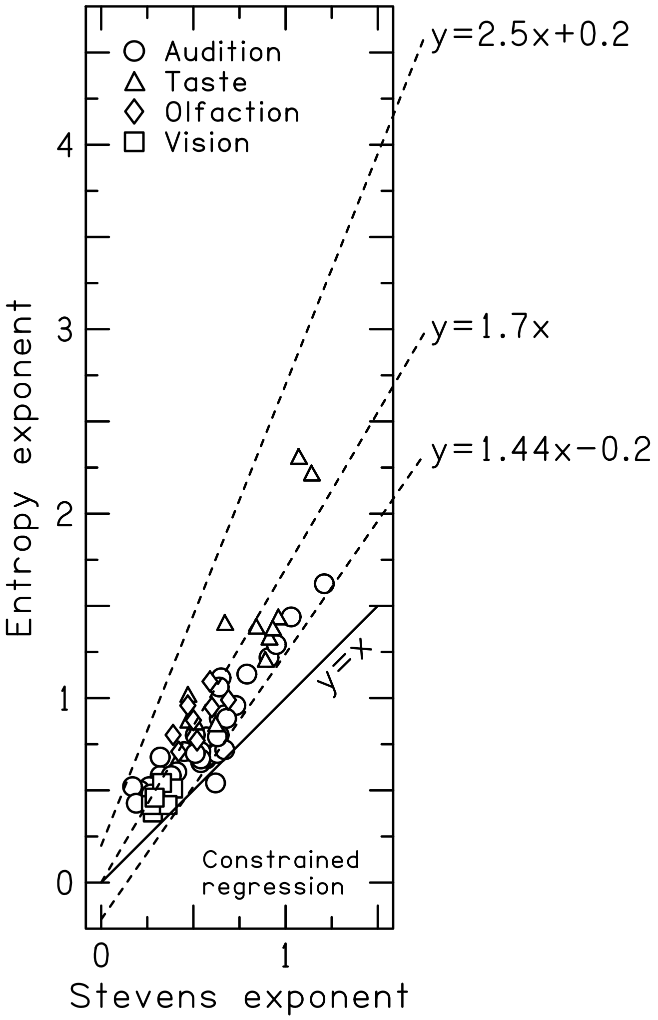

Fig. 2 The Entropy Exponent plotted vs. Stevens' exponent, from *constrained* regressions; information transmitted is constrained to 2.5 bits/stimulus (same magnitude estimates as used in Fig. 1). The lines y=2.5x+0.2 and y=1.44x-0.2 bracket most of the points.

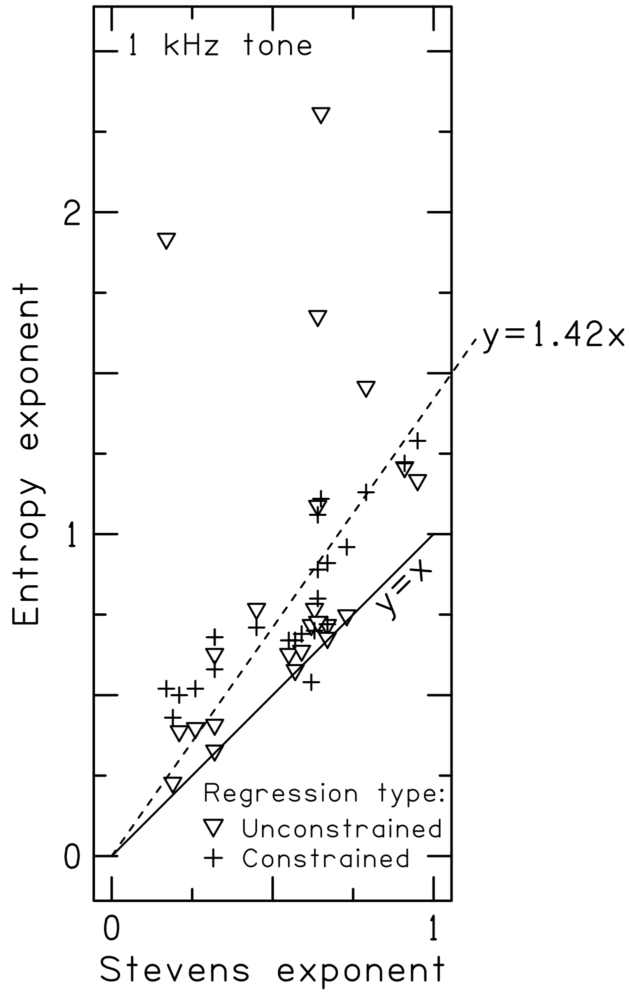

Fig. 3. The Entropy Exponent plotted vs. Stevens' exponent, from both unconstrained and *constrained* regressions, for the same data for 1 kHz tones as used in Fig. 1. For some data, the information transmitted was approximately equal in both the constrained and unconstrained regressions (line y=1.42 x).

the first two intensities. Similarly, the logarithmic range starts where $A\mu^n$ is sufficiently greater than unity that the "1" in (8) can be safely ignored. For Moskowitz [29], only the last 2 points are within the logarithmic range. Fig. 4 (top) shows the data, the entropy function, and its different regimes.

Similar results appear for the other senses. Fig. 4 (bottom) shows the Entropy Equation fit for the loudness of a 1 kHz tone (data of [30]). The power function applies only to the first 3 of 20 points, and only the last 3 points are within the logarithmic range. Fig. 5 (top) shows the Entropy Equation fit for the odor of n-butanol (data of [31, Fig. 6]). The last 3 points of 9 are not in the power range, and there is, in fact, no logarithmic range at all. Fig. 5 (bottom) shows the Entropy Equation fit for the brightness of green light (white light through Wratten filter; data of [32, Fig. 1]). Only the first 2 of 10 points are in the power range, and only the last 5 points are within the logarithmic range. All the cases of Figs. 1 and 2 have been evaluated; all follow such patterns.

IV. Discussion and Conclusions

Commonly, Stevens' Law refers to a power function fitted over a large portion of the entire accessible perceptual range [33], not a range of lower stimulus intensities. Stevens' Law is not predicted by the Entropy Theory; it has been shown here that rarely will the exponent of Stevens' Law equal that of the Entropy Equation for regressions made on the same data. Similarly, the Weber-Fechner Law refers to a logarithmic equation fitted over a large portion of the entire accessible perceptual range [33], not a range of moderate or higher stimulus intensities. Hence the Weber-Fechner Law is not predicted by the Entropy Theory.

The eventual derivation of (11) from $\sigma_S{}^2=\lambda\mu^n$ is circular logic. A power function is brought into F only through the assumption of (6), $\sigma_S{}^2=\lambda\mu^n$ [2], [3], [6], [10], [18], [19]. There is no derivational link between n and Stevens' Law, or between Stevens' exponent and any theorized microscopic stimulus variance $\sigma_S{}^2$. The Stevens' and Weber-Fechner

Laws only exist as “laws” when applied to a broad intensity range, *viz* their original use; to refer to them as limiting cases of anything, as done by Norwich et al., is simply wrong. All of this emphasizes that the Entropy Theory does not derive rules of sensory perception from information theory, and it is recommended that further attempts to do so should be discouraged.

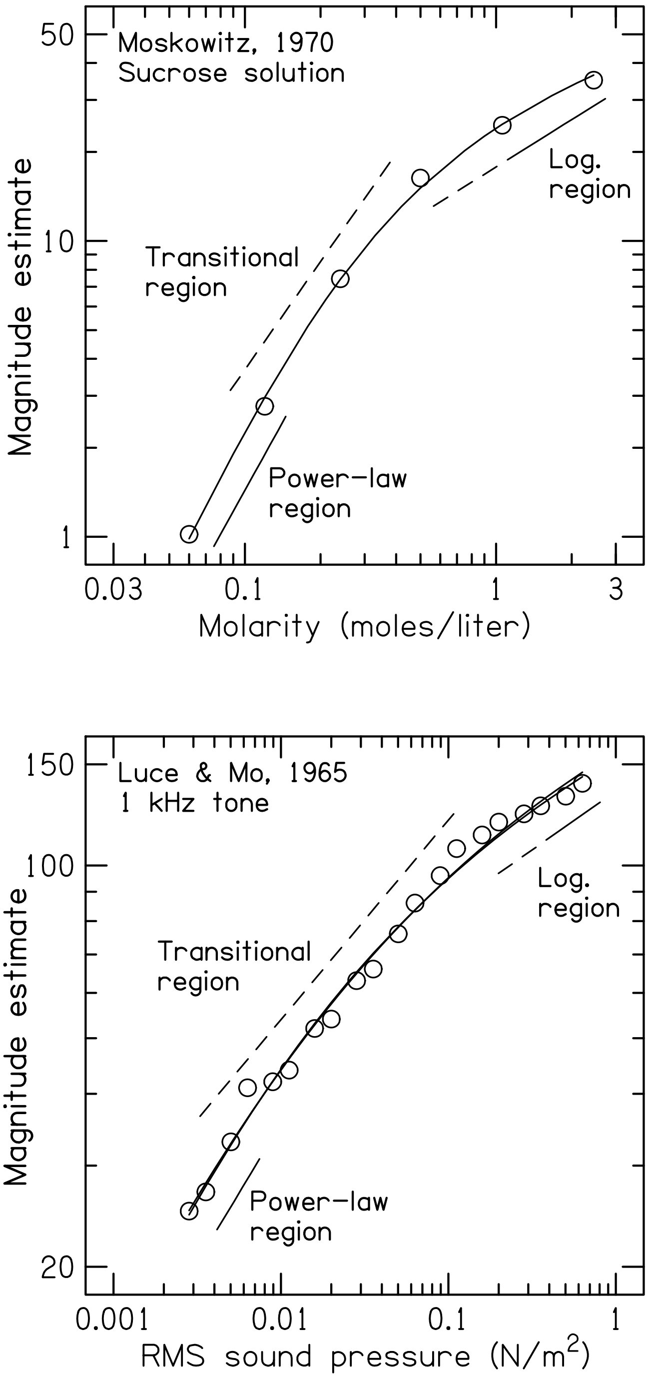

Fig. 4. (Top) The (unconstrained) fit of the Entropy Equation to the subjective intensity of a sucrose solution [29]. (Bottom) The (unconstrained) fit of the Entropy Equation to the loudness of a 1 kHz tone (subject #9, [30]).

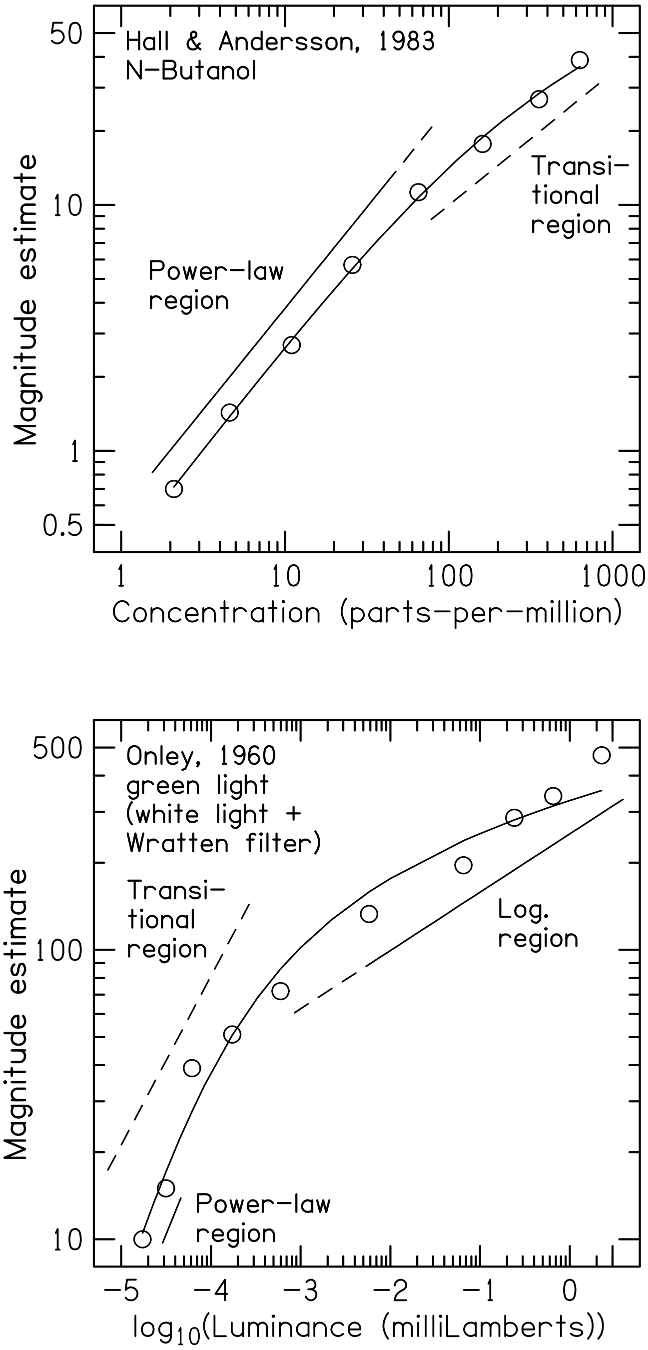

Fig. 5. (Top) The (unconstrained) fit of the Entropy Equation to the odorousness of N-butanol [31]. (Bottom) The (unconstrained) fit of the Entropy Equation to the subjective brightness of green light [32].

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