The Ignition Problem for the Simplified Magnetogasdynamics System

Yujin Liu

***Abstract*—We investigate the ignition problem of the Magnetogasdynamics system, and obtain constructively the unique perturbed solution in the (x,t) plane. There are a lot of interesting burning phenomena. Especially special, the transition between deflagration and detonation is also shown.**

***Index Terms*—Ignition problem, Riemann initial problem, Detonation, Deflagration, Magnetogasdynamics.**

I. Introduction

MAGNETOGASDYNAMICS is important in the study of engineering physics combustion [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]. It's hard to investigate the MHD system. There are less results than the usual model.

In [4], the authors found uniquely the Riemann solution of

$$\begin{cases} \tau_t - u_x = 0, \\ u_t + (p + \frac{B^2}{2\mu})_x = 0, \\ (E + \frac{B^2\tau}{2\mu})_t + (pu + \frac{B^2 u}{2\mu})_x = 0, \end{cases} \tag{1}$$

here $B = k\rho$ and $E = e + \frac{u^2}{2}$.

In [6], the Riemann problems and wave interactions for the reduced Magnetogasdynamics model were investigated.

In [11] they studied

$$\begin{cases} u_t + p_x = 0, \\ \tau_t - u_x = 0, \\ E_t + (up)_x = 0, \end{cases} \tag{2}$$

with

$$q(x,t) = \begin{cases} 0, & \text{if } \sup\limits_{0\le\varsigma\le t} T(x,\varsigma) > T_i; \\ q(x,0), & \text{otherwise}, \end{cases} \tag{3}$$

and the authors obtained the unique Riemann solution.

In [12], we modified the GEC and got the unique perturbed solution of (2) and (3).

In [13], we studied the unique solutions of (1), (3) with

$$(\tau,p,u,q)(x,0) = \begin{cases} (\tau_r,p_r,u_r,q_r), & x>0; \\ (\tau_l,p_l,u_l,q_l), & x\le 0, \end{cases} \tag{4}$$

and

$$q^{\pm} = \begin{cases} 0, & \text{if } T^{\pm} > T_i, \\ 0 \text{ or } q_0, & \text{if } T^{\pm} \le T_i, \end{cases}$$

and $q_0 > 0$ is the constant.

Here, we will study the the ignition problem of (1) with

$$(u,p,\tau,q)(x,0) = \begin{cases} (\tau_l,p_l,u_l,q_l), & x\in(-\infty,-\epsilon), \\ (\hat\tau,\hat p,\hat u,0), & x\in(-\epsilon,\epsilon), \\ (\tau_r,p_r,u_r,q_r), & x\in(\epsilon,+\infty), \end{cases} \tag{5}$$

where $(\tau_-,p_-,u_-) = (\tau_+,p_+,u_+) := (\tau_0,p_0,u_0)$, $q_- = q_+ := q_0 > 0$ and $\varepsilon > 0$ is small enough. $(\hat\tau,\hat p,\hat u,0)$ is viewed as the ignited state by means of the small energy input. The solution of (1), (3) and (5) can be constructed uniquely. Furthermore, we will find some important combustion phenomena.

The general structure of the article is as follows. In Section II, we list the main discussions of (1), (3) with (4). In Section III, we consider (1), (3) and (5). The results of the main discussions are presented in Section IV.

II. Preliminaries

We give as preparation of (1), (3) with (4). Please refer to the detailed derivation process to [4], [13].

Due to $\mu_1 = -(\frac{p-e_p\frac{BB_\tau}{\mu}+e_\tau}{e_p})^{\frac12}$, $\mu_2 = 0$ and $\mu_3 = (\frac{p-e_p\frac{BB_\tau}{\mu}+e_\tau}{e_p})^{\frac12}$, we have

$$\begin{cases} \eta \mathrm{d}\tau = -\mathrm{d}u, \\ \eta \mathrm{d}u = \mathrm{d}(p + \frac{B^2u}{2\mu}), \\ \eta \mathrm{d}(E + \frac{B^2u}{2\mu}\tau) = \mathrm{d}(up + \frac{B^2u}{2\mu}u). \end{cases} \tag{6}$$

$\overleftrightarrow{R}$ are

$$\begin{cases} p\tau^\gamma = p_1\tau_1^\gamma, \\ u = u_1 \pm \int_{p_1}^{p} \frac{\sqrt{\gamma p\tau + \frac{B^2\tau}{\mu}}}{\gamma p}\mathrm{d}p. \end{cases} \tag{7}$$

J at $\eta = \omega$ are

$$\begin{cases} \omega[\tau] = -[u], \\ \omega[u] = [p + \frac{B^2u}{2\mu}], \\ \omega[E + \frac{B^2u}{2\mu}\tau] = [up + \frac{B^2u}{2\mu}u], \end{cases} \tag{8}$$

and J is

$$[u] = [p + \frac{B^2}{2\mu}] = 0. \tag{9}$$

J is the curve in (τ,p,u). $\overset{<}{J}$ if $p_l < p_r,\ \tau_l < \tau_r$, and $\overset{>}{J}$ is similar.

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If $[q] = 0$ in (8), we get $\overleftrightarrow{S}$ passing through the point (τ_1, p_1, u_1)

$$\begin{cases} (p+\theta^2 p_1+\theta^2(\frac{3B^2}{2\mu}+\frac{B_1^2}{2\mu}))\tau \\ =(p_1+\theta^2 p+\theta^2(\frac{3B_1^2}{2\mu}+\frac{B^2}{2\mu}))\tau_1, \\ u=u_1 \pm (p+\frac{B^2}{2\mu}-p_1-\frac{B_1^2}{2\mu})\sqrt{\frac{-(\tau-\tau_1)}{p+\frac{B^2}{2\mu}-p_1-\frac{B_1^2}{2\mu}}}, \end{cases} \tag{10}$$

where $\theta^2 = \frac{\gamma-1}{\gamma+1}$ and $B_1 = \frac{k}{\tau_1}$.

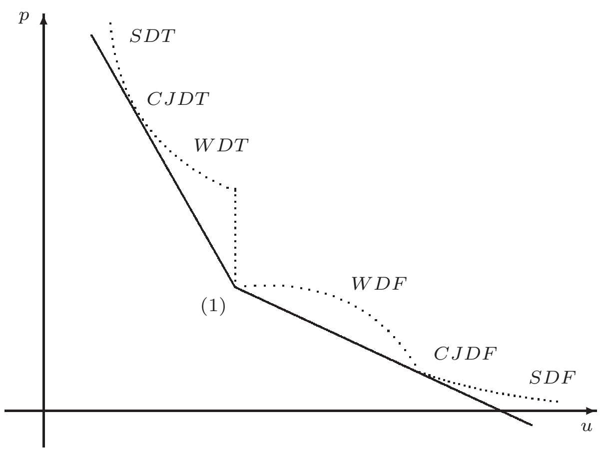


Fig. 2.1. $\overleftarrow{D}(1)$ in (u,p).

$\overleftarrow{D}(1)$ is (Fig. 2.1.)

$$u = u_1 - \sqrt{(p+\frac{B^2}{2\mu}-p_1-\frac{B_1^2}{2\mu})} \cdot \sqrt{\frac{(1-\theta^2)\tau_1(p-p_1)+\frac{\theta^2\tau_1}{\mu}(B^2-B_1^2)-2\theta^2 q_0}{p+\theta^2(p_1+\frac{B_1^2}{2\mu}+\frac{3B^2}{2\mu})}}. \tag{11}$$

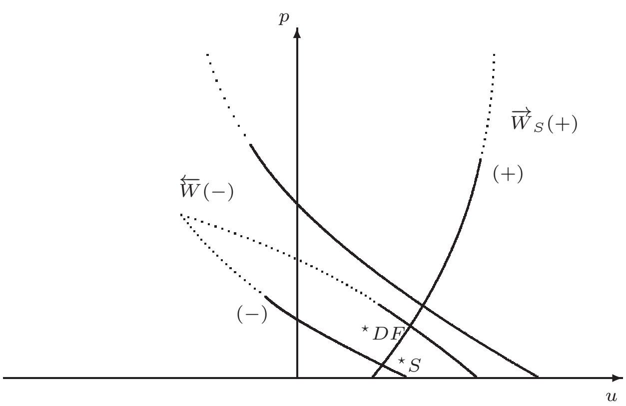


Fig. 2.2. $q_- > 0,\ q_+ = 0$ and there are there interaction points.

In (τ, p),

$$R_u(-): \quad p\tau^\gamma = p_-\tau_-^\gamma, \quad (0<p<p_-), \tag{12}$$

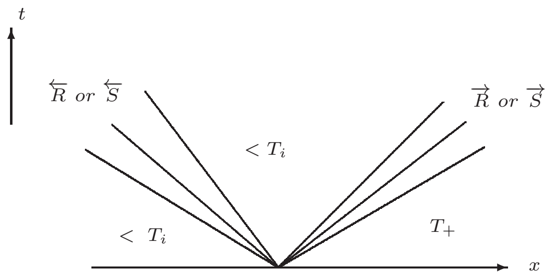
$$S_u(-): \quad (\tau-\theta^2\tau_-)(p+\theta^2(p_-+\frac{B_-^2}{2\mu}+\frac{3B^2}{2\mu})) \\ = (1-\theta^4)\tau_-p_- + \frac{\theta^2\tau_-}{\mu}[B_-^2(3-\theta^2)+B^2(1-3\theta^2)], \\ (p>p_-), \tag{13}$$

$$SDT(-): \quad (\tau-\theta^2\tau_-)(p+\theta^2(p_-+\frac{B_-^2}{2\mu}+\frac{3B^2}{2\mu})) \\ = (1-\theta^4)\tau_-p_- + \frac{\theta^2\tau_-}{\mu}[B_-^2(3-\theta^2)+B^2(1-3\theta^2)] \\ +2\theta^2 q_0, \quad (p>p_A), \tag{14}$$

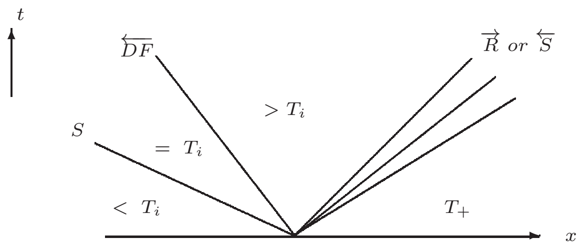
$$WDF(i): \quad (\tau-\theta^2\tau_i)(p+\theta^2(p_i+\frac{B_i^2}{2\mu}+\frac{3B^2}{2\mu})) \\ = (1-\theta^4)\tau_i p_i + \frac{\theta^2\tau_i}{\mu}[B_i^2(3-\theta^2)+B^2(1-3\theta^2)] \\ +2\theta^2 q_0, \quad ((p_D)_i<p<p_i), \tag{15}$$

and

$$\overleftarrow{W}_{DF}(-) := \overleftarrow{WDF}(i) \cup \overleftarrow{CJDF}(i) \cup \overleftarrow{R}(\overleftarrow{CJDF}(i)),$$
$$\overleftarrow{W}_{DT}(-) := \overleftarrow{SDT}(-) \cup \overleftarrow{CJDT}(-) \cup \overleftarrow{R}(\overleftarrow{CJDT}(-)).$$



(i) noncombustion wave.



(ii) *DF* solution.

Fig. 2.3. Solutions corresponding to Subcase 2.1.1.

If $\overleftarrow{W}(-)$ intersects with $\overrightarrow{W}(-)$ not uniquely, we get the unique solution in the following order by the *(MGEC) [13]:*

A. the lower the speed of DT or DF, the better;

B. the smaller of T_δ, the better, and T_δ is the oscillation temperature frequency factor between $\{\zeta \in R^1 : T(\zeta) \le T_i\}$ and $\{\zeta \in R^1 : T(\zeta) > T_i\}$;

C. DT or DF is as many as possible.

Case 2.1. $q_- > 0,\ q_+ = 0$. If $\overleftarrow{W}(-)$ intersects with $\overrightarrow{W}(+)$ only, we obtain $\overleftarrow{DT} + \overrightarrow{R}$ *or* $\overrightarrow{S}$ if $p_-\tau_-^\gamma = p_+\tau_+^\gamma$, or $\overleftarrow{DT} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$ under the condition of $p_-\tau_-^\gamma \ne p_+\tau_+^\gamma$.

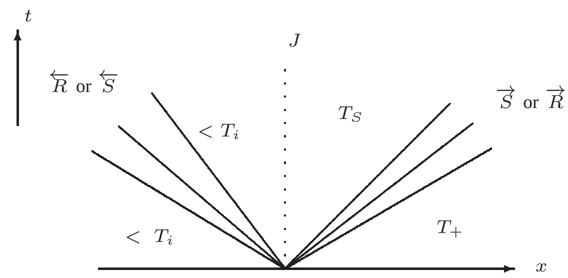
If not, we see the situation such as (Fig. 2.2.).
Suppose $p_-\tau_-^\gamma = p_+\tau_+^\gamma$, if $T_+ \leq T_i$, we get $\overleftarrow{R}$ *or* $\overleftarrow{S} + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.3. (i)). If $T_+ > T_i$, we know $\overleftarrow{DF} + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.3. (ii)). Otherwise,

(1) $T_+ > T_i$, $T_{DF} > T_i$, then $T_\delta(\star_S) = 1$, $T_\delta(\star_{DF}) = 1$, we have $\overleftarrow{DF} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.4. (ii)).

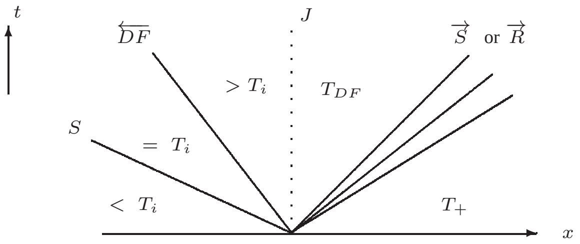
(2) $T_+ > T_i$, $T_{DF} \leq T_i$ (thus $T_S \leq T_i$), we obtain $\overleftarrow{R}$ *or* $\overleftarrow{S} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.4. (i)).

(3) $T_+ \leq T_i$, $T_S \leq T_i$, it follows that $\overleftarrow{R}$ *or* $\overleftarrow{S} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.4. (i)).

(4) $T_+ \leq T_i$, $T_S > T_i$ (therefore $T_{DF} > T_i$), it reveals that $\overleftarrow{DF} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.4. (ii)).



(i) noncombustion wave



(ii) combustion wave *DF*

Fig. 2.4. Solutions corresponding to Subcase 2.1.2.

Case 2.2. $q_- > 0,\ q_+ = 0$ (Fig. 2.5.).

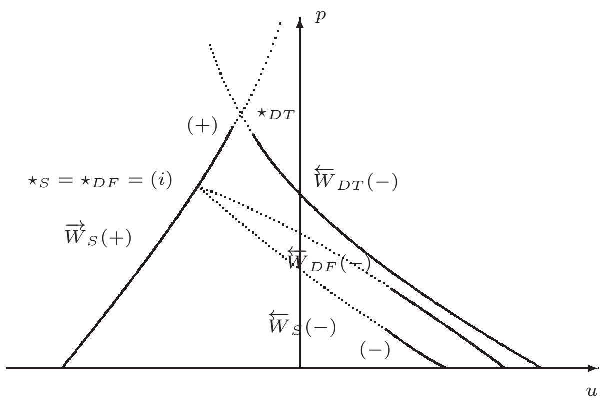
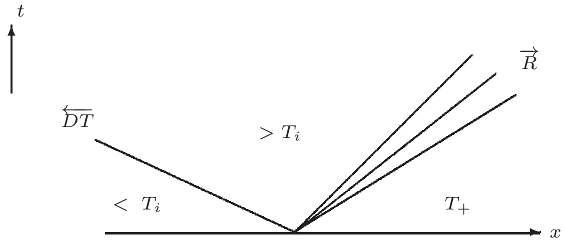
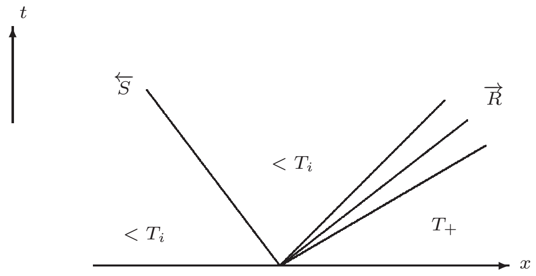


Fig. 2.5. $q_- > 0,\ q_+ = 0$ and there are two interaction points.

Suppose $p_-\tau_-^\gamma = p_+\tau_+^\gamma$, $T_+ > T_i$, then it is $\overleftarrow{DT} + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.6. (i)). $T_+ \leq T_i$, then we get $\overleftarrow{S} + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.6. (ii)).



(i) combustion wave *DT*



(ii) noncombustion wave

Fig. 2.6. Solutions corresponding to Subcase 2.2.1.

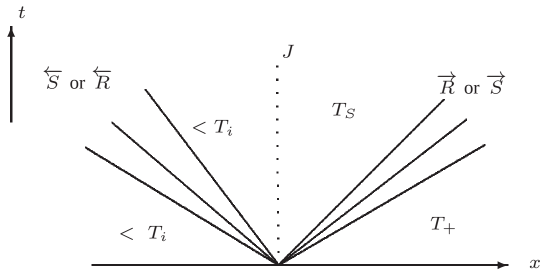
Otherwise,

(1) $T_+ > T_i$, $T_{DF} > T_i$, it follows $\overleftarrow{DF} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.7. (ii)).

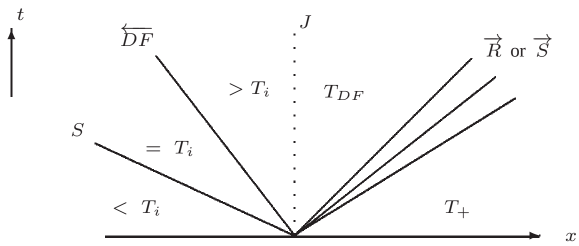
(2) $T_+ > T_i$, $T_{DF} \leq T_i$, it reveals $\overleftarrow{S}$ *or* $\overleftarrow{R} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.7. (i)).

(3) $T_+ \leq T_i$, $T_S \leq T_i$, we get $\overleftarrow{S}$ *or* $\overleftarrow{R} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.7. (i)).

(4) $T_+ \leq T_i$, $T_S > T_i$, we have $\overleftarrow{DF} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.7. (ii)).



(i) noncombustion wave



(ii) combustion wave *DF*

Fig. 2.7. Solutions corresponding to Subcase 2.2.2.

Case 2.3. $q_- > 0$, $q_+ > 0$. In this case ,we know that $\overleftarrow{W}(-) = \overleftarrow{W}_S(-) \cup \overleftarrow{W}_{DF}(-) \cup \overleftarrow{W}_{DT}(-)$, $\overrightarrow{W}(+) = \overrightarrow{W}_S(+) \cup \overrightarrow{W}_{DF}(+) \cup \overrightarrow{W}_{DT}(+)$.

Let we see the situations such as (Fig. 2.8.).

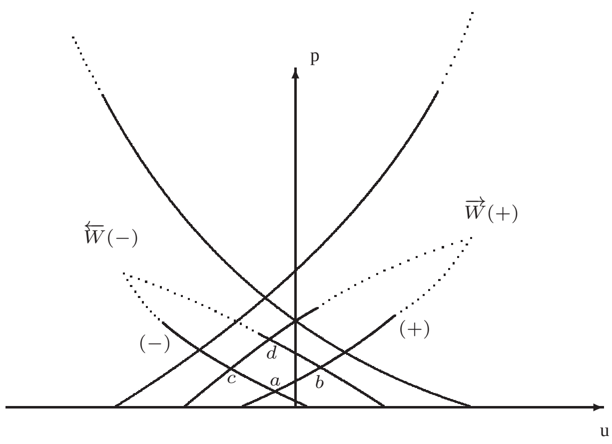


Fig. 2.8. $\overleftarrow{W}_S(-)$ and $\overrightarrow{W}_S(+)$ intersect only once.

If $p_-\tau_-^\gamma = p_+\tau_+^\gamma$. By the MGEC, we select the intersection point d of which we know $T_\delta = 0$ while $T_\delta > 0$ for the other three intersection points a, b, c and it follows that $\overleftarrow{R}$ *or* $\overleftarrow{S} + \overrightarrow{R}$ *or* $\overrightarrow{S}$.

Otherwise, applying the similar means mentioned above, we obtain $\overleftarrow{R}$ *or* $\overleftarrow{S} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$.

When $\overleftarrow{W}(l)$ intersects $\overrightarrow{W}_{DT}(r)$ only, and the solution is $\overleftarrow{DT} + \overrightarrow{DT}$ if $p_-\tau_-^\gamma = p_+\tau_+^\gamma$, or $\overleftarrow{DT} + J + \overrightarrow{DT}$ if $p_-\tau_-^\gamma \neq p_+\tau_+^\gamma$. If not, we see the cases such as (Fig. 2.9.).

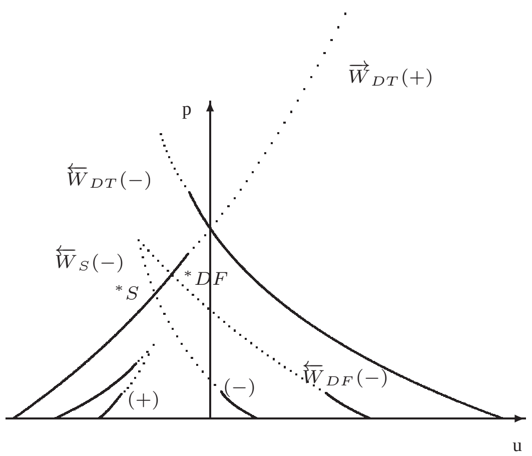
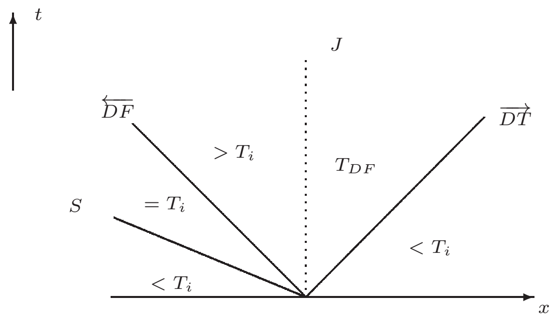
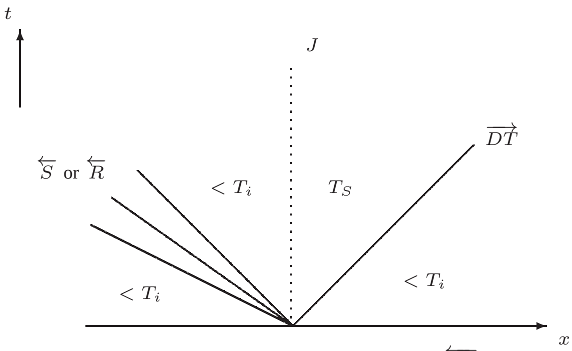


Fig. 2.9. $\overleftarrow{W}(-)$ intersects $\overrightarrow{W}_{DT}(+)$ only.

If $p_-\tau_-^\gamma \neq p_+\tau_+^\gamma$. It follows from $T_S > T_i$, $T_{DF} > T_i$, then $T_\delta(^\star S) = 2$, $T_\delta(^\star DF) = 2$. we obtain $\overleftarrow{DF}$ + $\overrightarrow{DT}$ (Fig. 2.10.).



(i) combustion wave solution containing $\overleftarrow{DF}$



(ii) combustion wave solution containing no $\overleftarrow{DF}$

Fig. 2.10. Solutions corresponding to Subcase 2.3.3.

Otherwise, obtain similarly $\overleftarrow{DF}$ + $\overrightarrow{DT}$ (Fig. 2.10.).

Thus, we get the result as follows.

Theorem 2.1 ([11]) There exists the unique solution of (1), (3) and (4) under the MGEC.

III. Solution of the ignition problem

Now consider the ignition problem (1.1), (3) with (5). We abstract those interesting phenomena and use the same symbol after perturbation in our paper.

Case 3.1. The solution at $(-\varepsilon, 0)$ is $\overleftarrow{SDT} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$, and the solution at $(\varepsilon, 0)$ is $\overleftarrow{S}\, or\, \overleftarrow{R} + J + \overrightarrow{SDT}$. Here we just consider the following subcase $\overleftarrow{SDT} + J + \overrightarrow{S}$ and $\overleftarrow{S} + J + \overrightarrow{SDT}$.

We need only to consider the interactions on the left for we can similarly deal with the interactions on the right. And we denote the state (0) on the left by (l) for simplicity. At first, we recall the following definition for our later discussions.

Definition 3.1 *([14]) The up (down) contact discontinuity* $\acute{J}$ *(or* $\grave{J}$*) connects two states* (l) *and* (r) *which satisfy* $u_l = u_r$, $p_l = p_r$ *and* $\rho_l \lessgtr \rho_r$.

For the above definition, we know the up contact discontinuity $\acute{J}$ means the density jumps increase and the down contact discontinuity $\grave{J}$ means the density jumps decrease.

From [14], we know that $\overrightarrow{S}\overleftarrow{S} \to \overleftarrow{S}J\overrightarrow{S}$, where when $p_l > p_r$, the result is the up contact discontinuity $\acute{J}$ which is $\overrightarrow{S}\overleftarrow{S} \to \overleftarrow{S}\acute{J}\overrightarrow{S}$; when $p_l < p_r$, the result is the down contact discontinuity $\grave{J}$ which is $\overrightarrow{S}\overleftarrow{S} \to \overleftarrow{S}\grave{J}\overrightarrow{S}$.

Furthermore, we have $\grave{J}\overleftarrow{S} \to \overleftarrow{S}\grave{J}\overrightarrow{S}$ and $\acute{J}\overleftarrow{S} \to \overleftarrow{S}\acute{J}\overrightarrow{R}$. Thus, we should investigate the interaction between $\overleftarrow{SDT}$ and $\overleftarrow{S}$ at (x_1, t_1) (Fig. 3.1.).

We can observe that $(m) \in \overleftarrow{W}_{DT}(l)$, $(r) \in \overleftarrow{S}(m)$ and $\overleftarrow{W}_S(l) \cup \overleftarrow{W}_{DF}(l)$ are located below $\overrightarrow{W}_S(m)$ from A. Thus, it follows that $\overleftarrow{W}_S(l) \cup \overleftarrow{W}_{DF}(l)$ are located below $\overrightarrow{R}(m)(u \le u_m)$ and the half straight line $p = p_m(u > u_m)$. Considering the location station of $\overrightarrow{R}(m)$ and $\overrightarrow{R}(r)$, it can be inferred that $\overrightarrow{W}_S(r)$ is located above $\overrightarrow{R}(m)(u \le u_m)$ and the half straight line $p = p_m(u > u_m)$.

Therefore, $\overrightarrow{W}_S(r)$ does not intersect with $\overleftarrow{W}_S(l) \cup \overleftarrow{W}_{DF}(l)$, that is, $\overrightarrow{W}_S(r)$ intersects with $\overleftarrow{W}_{DT}(l)$ only (Fig. 3.2.) and easily know the unique solution is $\overleftarrow{SDT} + \overrightarrow{S}\, or\, \overrightarrow{R}$ as $p_l\tau_l^\gamma = p_r\tau_r^\gamma$; or $\overleftarrow{SDT} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$ as $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$.

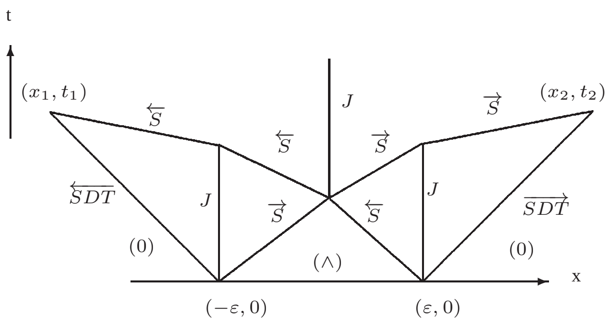


Fig. 3.1. The solution for Case 3.1.

Similarly, it holds that at (x_2, t_2) is $\overleftarrow{S}\, or\, \overleftarrow{R} + J + \overrightarrow{SDT}$ or $\overleftarrow{S}\, or\, \overleftarrow{R} + \overrightarrow{SDT}$.

Theorem 3.1. DT may persist after the perturbation for this case which tells that the combustion wave is stable under this perturbation on the initial data. The contact discontinuities may or may not occur in the process of disturbance.

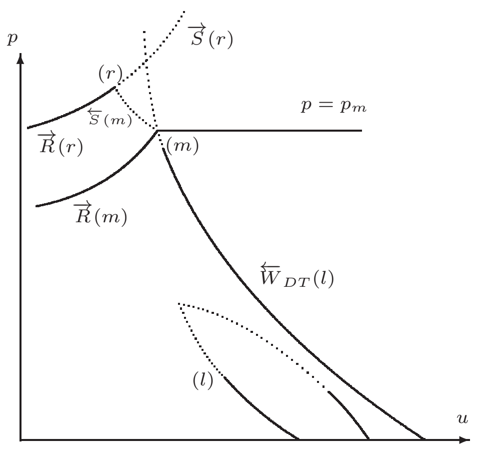


Fig. 3.2. Waves curves of the interaction of $\overleftarrow{SDT}$ and $\overleftarrow{S}$.

Case 3.2. The solution at $(-\varepsilon, 0)$ is $\overleftarrow{WDF} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$, and the solution at $(\varepsilon, 0)$ is $\overleftarrow{S}\, or\, \overleftarrow{R} + J + \overrightarrow{WDF}$. We just study the subcase $\overleftarrow{WDF} + J + \overrightarrow{S}$ and $\overleftarrow{S} + J + \overrightarrow{WDF}$. Here we still need to solve the Riemann problem at (x_1, t_1) which is the first interaction point of $\overleftarrow{WDF}$ and $\overleftarrow{S}$ (Fig. 3.3.). We can deal with the Riemann problem at the point (x_2, t_2) similarly.

If $\overleftarrow{S}$ is weak (Fig. 3.4.), from the argumentations in Case 2.1, notice that $(m) \in \overleftarrow{W}_{DF}(l)$, $(r) \in \overleftarrow{S}(m)$ and (r) is located above $\overleftarrow{W}_{DF}(l)$, it indicates that the temperature is higher than T_i, if $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$, we can obtain the unique solution is $\overleftarrow{WDF} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$ or $\overleftarrow{S}\, or\, \overleftarrow{R} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$. If $\overleftarrow{S}$ is strong, the solution is $\overleftarrow{DT} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$. If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, the solution is $\overleftarrow{WDF} + \overrightarrow{S}\, or\, \overrightarrow{R}$ or $\overleftarrow{S}\, or\, \overleftarrow{R} + \overrightarrow{S}\, or\, \overrightarrow{R}$ or $\overleftarrow{DT} + \overrightarrow{S}\, or\, \overrightarrow{R}$.

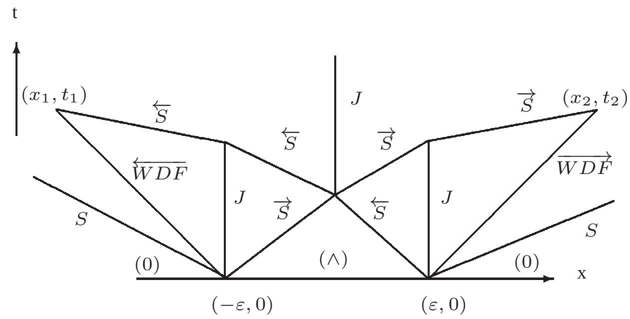


Fig. 3.3. The solution for Case 3.1.

The handling of the right side is similar with the above, and we get $\overleftarrow{R}\, or\, \overleftarrow{S} + J + \overrightarrow{WDF}$ or $\overleftarrow{S}\, or\, \overleftarrow{R} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$ or $\overleftarrow{S}\, or\, \overleftarrow{R} + J + \overrightarrow{DT}$ when $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$; the solution is the same as above just that the contact discontinuity does nor appear when $p_l\tau_l^\gamma = p_r\tau_r^\gamma$.

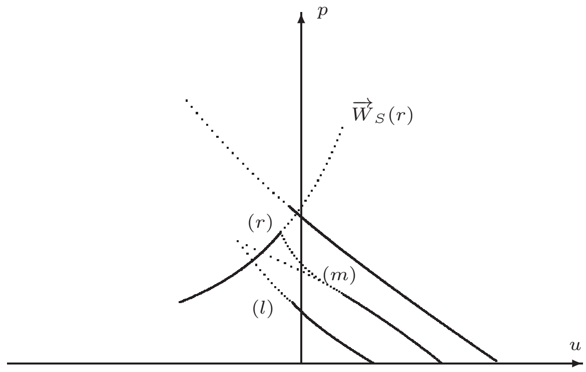


Fig. 3.4. Waves curves in the (u, p).

Theorem 3.2. DF may be extinguished or transformed into DT after the such small perturbation. In this case, we observe that the important combustion phenomenon of the transition of DT and DF which plays the important role in the study of the combustion problems.

Case 3.3. The solution at $(-\varepsilon, 0)$ is $\overleftarrow{S}\, or\, \overleftarrow{R} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$, and the solution at $(\varepsilon, 0)$ is $\overleftarrow{S}\, or\, \overleftarrow{R} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$. We just investigate the subcase $\overleftarrow{S} + J + \overrightarrow{S}$ and $\overleftarrow{S} + J + \overrightarrow{S}$. Again we want to resolve the interaction between $\overrightarrow{S}$ and $\overleftarrow{S}$ at (x_1, t_1) (Fig. 3.5). The Riemann problem at (x_2, t_2) can be treated similarly.

If $\overleftarrow{S}$ is strong, we can see that $\overrightarrow{W}_S(r)$ intersects with $\overleftarrow{W}_{DT}(l)$ uniquely and the unique solution is $\overleftarrow{DT} + J + \overrightarrow{S}\, or\, \overrightarrow{R}$. If $\overleftarrow{S}$ is weak, we can see the case such as (Fig. 3.6.). From the statements in Case 2.1, it reveals that

$\overleftarrow{WDF} + J + \overrightarrow{S} or \overrightarrow{R}$ or $\overleftarrow{S} or \overleftarrow{R} + J + \overrightarrow{S} or \overrightarrow{R}$.

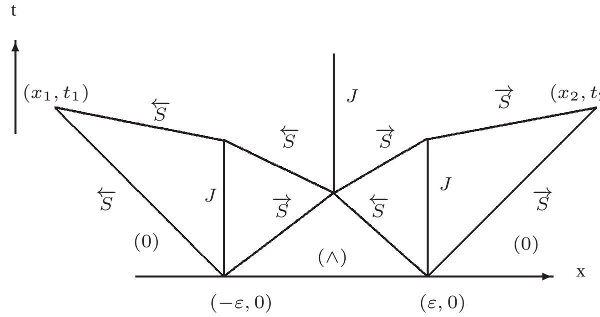


Fig. 3.5. The solution for Case 3.3.

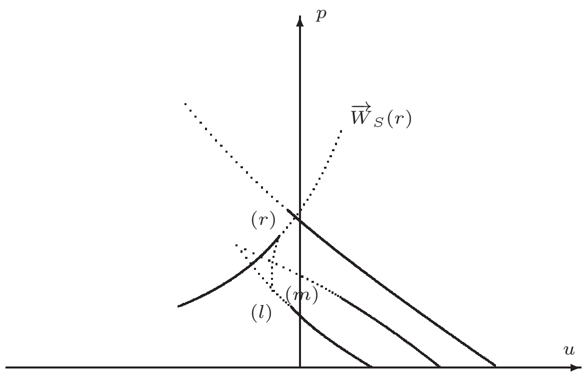


Fig. 3.6. Waves curves in the (u, p).

Theorem 3.3. The burning phenomenon occurs after the perturbation. And we capture the instability of the unburnt gas which is important for our study.

IV. Conclusion

The above discussion is particularly crucial to our research. And the present study draws on the experience of the future research on the high-dimensional problems for the Magnetogasdynamics system. Next, the further research is planned on the ultimate behavior of the solution of SZND magnetogasdynamic. We plan to compare its solution of the SZND model with the solution of the system (1) in order to make the inquiry of the intrinsic mechanism of combustion.

Now we get the main conclusion.

Theorem 4.1 It holds the unique solution of the perturbed problem (1), (3) and (5) under the MGEC.

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