Solution of Discrete Riccati Equation with Multi-period Soliton

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Abstract—**The traditional multi-period soliton method takes too long to solve and the solution results are not satisfactory. To address the above problems, this study proposes a multi-period soliton method for the discrete Riccati equation. In this method, the multi-period soliton solution index of the discrete Riccati equation is firstly selected, and the multi-period soliton solution of the discrete Riccati equation is determined according to the index. The correlation matrix is solved and a positive semi-definite matrix is established. Next, the lower bound and eigenvalues of the matrix are solved using the matrix inequality property inequality and the perturbation parameter. The boundary estimation and eigenvalue properties of the reference matrix and the solution matrix are obtained. Another positive semi-definite matrix is defined. Finally, the soliton solution is constructed to complete the multi-period soliton solution of the discrete Riccati equation. The simulation results showed that the traditional method took more than 5 minutes, while the multi-period soliton solution of the discrete Riccati equation designed in the research only took 2.5 minutes. The solution time of the proposed method was shorter than the traditional method. The optimal solution could be obtained. It showed that the multi-period soliton discrete Riccati equation solution could deepen the understanding for multi-period soliton and nonlinear fluctuation phenomenon, providing important mathematical models and tools for practical application and basic research. These instructions provide guidelines for preparing papers. The multi-periodic soliton method for the discrete Riccati equation proposed in the study can effectively improve the solution time and efficiency, and has certain applications in controller optimization and other aspects.**

Index Terms—**Discrete Riccati equation; Multi-period soliton solution; Matrix equation; Eigenvalue; Linear system; Positive semi-definite matrix**

I. Introduction

In recent years, the research on the stability of linear systems has become very active. There have been research methods about the stability of continuous systems in the left half plane of the complex plane and discrete systems in the unit circle. However, the existing methods have some limitations in practical engineering applications. Firstly, it is often difficult to describe the dynamic characteristics of industrial production processes, production equipment and other controlled objects with accurate mathematical models. Even after obtaining an accurate model, it is hard to obtain timely due to its complexity, analysis and application efficiency [1]. In addition, due to the changing working environment and conditions, the loss of components and the error between the mathematical model and the actual controlled object, there are often many uncertainties in the control system. Meanwhile, in the actual production, time delay phenomenon is also common, such as long pipeline feeding, belt drive, slow chemical reaction process and network control system signal transmission. The uncertainty and time delay make the control system face many special difficulties in theoretical research and practical engineering production. These uncertainties and time delays often lead to system performance degradation and instability. However, the existing methods cannot effectively solve these problems. The solution time is too long and the results are inaccurate in solving multi-period solitons [2-3]. To realize the performance optimization, robustness analysis, filter design and system identification of the control system, and improve the efficiency, stability and accuracy of the engineering process, the multi-period soliton solution of discrete Riccati equation is studied. Discrete Riccati equation plays an important role in many fields of engineering theory, especially in the control system. In addition, the matrix constraint estimation of matrix equation solution also plays an indispensable role in system stability analysis, controller design of time-delay system, maximum cost estimation, numerical algorithm convergence and Riccati differential equation behavior [4-6]. However, as the dimensionality of the matrix equation increases, it is very difficult to solve the solution matrix [7]. Therefore, estimating the solution matrix of matrix equations has important theoretical significance and practical value [8]. In recent years, many scholars have made contributions to solving matrices. Li et al. obtained an exponentially separated variable solution based on the projected Riccati equation. It analyzed the frontal collision and chasing collision between folded solitary waves. The phase shift and the difference during the interaction were obtained. Then the fluctuations were systematically analyzed [9]. Dong et al. analyzed soliton molecules with higher-order corrections for KdV equations using the velocity resonance mechanism and multi-soliton solution. The interaction between solitons and solitons in KdV equations with high order correction was studied based on the analytical and graphical method. The results showed that the KdV equation with higher order correction was a consistent Riccati expansion solvable system [10].

The innovation of the multi-period soliton discrete Riccati equation solution proposed in this study mainly includes the following aspects. The first is to select multi-period soliton solution index. This method introduces the multi-period

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soliton solution as an index to solve the discrete Riccati equation. By selecting the appropriate solution index, the dynamic characteristics and stability requirements of the system can be better reflected. The second is to establish a positive semi-definite matrix. The selected solution index is used to construct a positive semi-definite matrix. The multi-period characteristics of discrete Riccati equation are considered in matrix construction, which makes the solution results more accurate and practical. The third is to adopt matrix inequalities and perturbation parameters. Based on the properties of matrix inequalities and perturbed parameters, the correlation matrix and its eigenvalues are solved. The lower bound estimations are obtained, so that the boundary estimation and eigenvalue properties of the reference matrix and the solution matrix are obtained. The fourth is to construct multi-period soliton solutions.

This research is divided into four parts. The first part provides an introduction of multi-period soliton solutions as well as the discrete Riccati equation. The second part summarizes the research status of solving discrete Riccati equation of multi-period solitons. In the third part, the multi-period soliton solution index of discrete Riccati equation is selected to establish a positive semi-definite matrix. Then, another positive semi-definite matrix is constructed based on the the boundary estimation and the eigenvalue properties of obtained reference matrix and solution matrix. Finally, the soliton solution is constructed to complete the multi-period soliton solution of the discrete Riccati equation. In the fourth part, the multi-period soliton solution of discrete Riccati equation is experimentally verified and the experimental results are analyzed.

II. Index Selection for Solving Discrete Riccati Equation with Multi-Period Soliton Solution

Soliton (solitary wave) is an energy limited local solution in wave problems. The energy is usually concentrated in a relatively narrow region (or can exist stably in a given region). When two solitons interact with each other, elastic scattering phenomenon will appear (that is, the wave form and wave velocity can quickly recover to the initial state). Solitons can be divided into two categories, topological solitons and non-topological solitons [11]. The necessary condition for the stable existence of topological solitons is the vacuum state, which means that there are different vacuum states or boundary conditions at infinity. When there is a soliton solution, the boundary condition at infinity is different from those without soliton solution. No vacuum state is needed for non-topological solitons. No matter whether there are solitons or not, they all have the same boundary conditions at infinity. Generally speaking, the positive and negative solitons and their sequences of Bell type distribution are non-topological, but the torsional solitons are topological solitons. By simplifying the assumptions of the Riccati equation coefficient, it has a polynomial solution composed of Bell and torsion functions. It contains many solitons solutions with physical meaning. The discrete Riccati equation is shown in equation (1):

$$A(K)*X(K+1)+B(K)*U(K)+C(K)*X(K)=0 \quad (1)$$

In equation (1), $A(K),B(K),C(K)$ represent the constant matrices at K. $X(K),U(K)$ represent the input and output of the system at K. If this simplified assumption is not used, more solutions can be found. Some generalized solutions of ordinary differential equations even can be obtained. An appropriate solution index is very powerful for finding accurate solutions to multi-period soliton solutions [12]. Therefore, the nonlinear transformation of the nonlinear coupled field equation should be determined to simplify the problem. For the discrete control system, the following equation (2) should be defined:

$$sf = \frac{e}{G|df|*f} \quad (2)$$

In equation (2), s represents the initial state of the system. $*$ represents multiplication. e represents the state variable. d represents the control variable. f represents the output variable. G represents the characteristic function of a 2 * 2 matrix [13-14], which is shown in equation (3):

$$G = \begin{cases} Q = |q - xt| \\ \phi q q^{u} + fh \end{cases} \quad (3)$$

In equation (3), Q represents the initial state variable of the system. q represents the perturbation term. h represents a constant. x represents the spatial variable. t represents the time variable. If fh satisfies the symmetry condition, then the coefficient of Q is shown in equation (4):

$$Q = fq + \frac{w}{d} + (b*k) \quad (4)$$

In equation (4), w represents the state variable under that condition. k represents the constant positive. b represents the non-zero parameter. Although the above equation is a simple nonlinear partial differential equation, it is an equation of the same rank. The so-called same rank equation refers to the unknown function appearing in each term of the equation. The sum of the derivatives for each order and the product of times is the same odd number or the same even number. Therefore, a traveling wave transform is required for equation [15], as shown in equation (5):

$$RU = C - \frac{d}{2}u^{t} \cdot Q \quad (5)$$

In equation (5), u^t is the superposition of Riemann wave and long wave. RU represents the traveling wave transformation.

At present, the back scattering method to solve the initial value of the equation is an unsolved problem in soliton theory, because the premise of using the back scattering method is to find the laxness corresponding to the method. Therefore, a nonlinear evolution equation is set up. On the basis of the solution of the nonlinear evolution equation and its corresponding solution for the loose pair, new solutions are obtained using algebraic algorithms and differential operations [16]. At the same time, the integer is determined according to the balance principle, which is shown in equation (6):

$$F = \frac{a}{df} * \frac{s}{f} + df / o \quad (6)$$

In equation (6), F represents positive definite matrix. a represents symmetric positive definite solution. o represents the product of eigenvalues of positive definite solutions.

According to the above equations, the traveling wave solution of nonlinear differential difference equation is obtained, which is shown in equation (7):

$$n(\varsigma e) = a_0 + \frac{\sum_d s_d}{er / o^i} + \frac{\int_s^h vdv}{\int_i^h gdg} + tru \tag{7}$$

When the same power coefficients of the equations are zero, the deterministic nonlinear algebraic equations can be further obtained, as shown in equation (8):

$$x(t) = \int_{hj}^{j} ydy + \frac{p * g}{\sqrt{nf} + s} \tag{8}$$

On this basis, the multi-period soliton solution index of the discrete Riccati equation can be further determined.

III. Determination of the Correlation Matrix for Solving the Discrete Riccati Equation with Multi-Period Soliton Solution

Applying matrix method to analyze the network topology is to use the correlation matrix formed by the correlation between nodes and branches, or the relationship between node pairs to form the adjacency matrix. After operation, a fully connected matrix reflecting the connectivity between any two nodes in all solutions is obtained. Finally, the network topology analysis result is obtained by analyzing the fully connected matrix. A class of Lie algebras and zero curvature equations are used to derive the broad MKdV equation. To obtain another set of foundations of Lie algebra on the complex number set, the incidence matrix forms the node branch incidence matrix and the branch node association according to the node branch association relationship. Boolean matrix multiplication is performed on these two matrices [17] to get the node-to-node association matrix, that is, the adjacency matrix, as shown in equation (9):

$$\frac{s}{\beta|v|} = \begin{cases} \frac{s}{f} + \sqrt{\frac{1}{2}} \\ ds + f \\ \int_o^s fdf + g + h + j_q \end{cases} \tag{9}$$

It satisfies the following relationship [18], which is shown in equation (10):

$$a_0 = \begin{cases} d(ry - fd) + a^0(d + f^2 + y^t) \\ a_2 d + fg_3 sd^t - fh \\ vs\sum_f f + f_s + g^w - \frac{s}{\beta|v|} \end{cases} \tag{10}$$

In equation (10), y is a real constant. a_0 is an integral constant. Therefore, the following development equation can be obtained, which is shown in equation (11):

$$a_0 = \begin{cases} d(ry - fd) + a^0(d + f^2 + y^t) \\ a_2 d + fg_3 sd^t - fh \\ vs\sum_f f + f_s + g^w - \frac{s}{\beta|v|} \end{cases} \tag{11}$$

Based on the above calculations, is spectral pairs are constructed to establish canonical transformations between spectral problems. The search method is one of the most widely used methods in the current topological analysis of equation solutions. This method is to carry out the network topological analysis by searching the adjacent nodes. Topology analysis starts from a certain node, searches the nodes connected with the node through closed switches, and divides them into a group of indicators. Usually, the above conversion depends on the parameters of the search method. Therefore, to write canonical conversion expressions, the study analyzed the problem from the perspective of eigenvalues, as shown in equation (12):

$$U = (-n - sg) + \sum_h o \frac{f_h}{y} \cdot a_0 \tag{12}$$

A standard variation based on equation (12) further yields equation (13):

$$\mu^1 = \begin{cases} d^t + g_i + U(d - k) \\ \int_{i=1}^{e} rdr + f_h d|g| + f^y + j \\ g\phi + h + x\phi + h^i \end{cases} \tag{13}$$

According to the standard change results, the breadth-first search method is used to search nodes. The breadth-first search method is to take the starting node as the center and search outwards in a radial way until all nodes are searched. This method only accesses each node once. Therefore, it has fewer visits to nodes than the depth first search method. The relationship between the breadth-first search method and the nonlinear evolution equation corresponding to the eigenvalues is shown in equation (14):

$$\begin{cases} \varpi_q = (-f + u)\beta \\ \varpi s = B_i + f \\ s(\mu^1) = (f - b) \\ \varpi v = N^t + g^o h \end{cases} \tag{14}$$

In equation (14), ϖ denotes the matrix and the expression equation is shown in equation (15):

$$\varpi = \begin{bmatrix} 0 & -\varpi & -\frac{1}{\varpi} \\ \partial & 0 & -\varpi \\ x & ax & 0 \\ \frac{w}{\wp} & 0 & x_i \\ \frac{1}{\varpi} & 0 & -g \\ f & s & h \end{bmatrix} \tag{15}$$

The study assumes that ϖ is a linear partial differential algorithm with constant coefficients, which further yields equation (16):

$$\begin{cases} \dfrac{x}{zy} + \dfrac{\beta\partial}{\alpha t} = 0 \\ \dfrac{s}{\alpha y} + \dfrac{\partial \wp}{\partial x} = 0 \\ (\alpha y + \partial x) = 0 \end{cases} \tag{16}$$

The relationship between the two equations is established. The original equation solution structure is studied according to the solution and transformation group of another equation. In solving transformation groups, it is required to solve large linear or nonlinear differential system problems [19]. Therefore, some relations of unknown variables are introduced. If these variables can be determined from them, then the equation has this type of solution. Next, based on an example, a more detailed explanation is provided, which is shown in equation (17):

$$\phi(\delta) = \frac{2\sum e + (\wp_f + g_e)}{(d+r)\exp(df+h)} \tag{17}$$

Step 1: The homogeneous balance method is used to balance the highest derivative term and the highest nonlinear term. The value of the balance constant is determined by equation (18):

$$d(u_i, u_o, u\partial) = 0 \tag{18}$$

Step 2: Assuming that the solution of the equation has the following form, as shown in equation (19):

$$u_i = \begin{Vmatrix} n & s_i & g & a \\ n_1 & n_u & f & 0 \\ 0 & -t & 0 & -i \\ g & s & -g & 0 \end{Vmatrix} \tag{19}$$

In equation (19), $\|\cdot\|$ represents the absolute value of the determinant. Step 3: With the help of the symbolic operation software Maple, the algebraic equations obtained above are used to solve the correlation matrix of the equation, which is shown in equation (20):

$$\begin{cases} \partial(\varepsilon) = \dfrac{fg\left(\sqrt{r\wp}\right)}{sd\sqrt{(s)+1}} \\ \partial(\varepsilon)'' = \dfrac{G + g\left(\sqrt{r\wp}\right)}{f\sqrt{(RT)+1}} \end{cases} \tag{20}$$

In equation (20), ε represents a known non-negative matrix. It means that each element is a negative number. According to the above steps, the correlation matrix for solving the multi-period soliton solution of the discrete Riccati equation is obtained.

IV. The Realization of Multi-Period Soliton Solution to Discrete Riccati Equation

According to the multi-period soliton solution index and correlation matrix of the discrete Riccati equation, the continuous Riccati matrix equation is introduced to solve the linear quadratic optimal control problem of the continuous linear time invariant system. It plays a very important role in the the control system. Where the control system expression is as in equation (21):

$$x(t) = \begin{cases} ax + B(t) \\ xt - x(0)_{u_i} \end{cases} \tag{21}$$

In equation (21), $B(t)$ represents the system dimension variable. $x(0)$ represents the system input dimension variable. $x(t)$ represents the system matrix.

Solving node equations requires a large amount of computation. Traditional methods only care about which index value is non-zero after solving, but does not concern about the specific value. Therefore, the index is selected to obtain the solution of the optimal control problem, which is shown in equation (22):

$$ap = \begin{cases} at + gf + f^t \\ asf + h \\ \Delta y + k \end{cases} \tag{22}$$

The above equation must meet one of the following conditions:

Firstly, if norm is bounded, then Δy can be expressed in equation (23):

$$\Delta y = \frac{f}{\sqrt{df} / i} \tag{23}$$

In equation (23), $\sqrt{df}$ represents a constant matrix of known appropriate dimensions. i represents an uncertain parameter.

Secondly, if the non-structure has uncertainty, then Δy can be expressed in equation (24):

$$\Delta y = \frac{u}{\int_c sds} / q \tag{24}$$

In equation (24), u represents the maximum singular value. $\int_c sds$ represents the norm.

Thirdly, strong structure has uncertainty. $\Delta y \le D$. D represents the relationship between the corresponding elements of two matrices.

To save storage space, improve the operation speed, and adapt to large-scale network operations, sparse technology is introduced in the operation process. There are two key points to implementing sparse technology. One is zero storage and zero operation. The other is node number optimization. Zero storage and zero operation can effectively avoid storing and calculating elements that do not affect the calculation results, greatly improving the calculation efficiency of the program. There are different storage methods for sparse matrix, such as scattered format, row and column storage format and triangle retrieval storage format. In this paper, Gauss row elimination method is used. Therefore, row by row storage format is used. The structure body is used to store the non-zero elements in the matrix [20]. The members of the structure body record the position of the first non-zero element in each row of the adjacency matrix, and record the column of the non-zero element in the adjacency matrix. The structure body array is used to store the non-zero elements in the adjacency matrix in row order. The specific calculation matrix is shown in equation (25):

$$F' = \begin{cases} T * f + \dfrac{d}{g} + l \\ \dfrac{s}{o} + k' + h * g \\ \sqrt{a * f} + g_i \end{cases} \tag{25}$$

Aiming at the stability analysis of multi-period soliton solutions, the continuous Lyapunov matrix equation is introduced. It can reduce the difficulty of solving the control system and shorten the solution time. The control matrix is shown in equation (26):

$$|D| = \begin{bmatrix} x_1 & x_2 & K & x_n \\ x_{21} & x_{22} & K & x_{2n} \\ K & K & K & K \\ x_{m1} & x_{m2} & K & x_{mn} \end{bmatrix} \tag{26}$$

The above matrix is a necessary and sufficient condition for asymptotic stability. Given a matrix $|Q|$ arbitrarily, it satisfies [18], which is shown in equation (27):

$$|Q| = \begin{cases} rt * Em \\ \int_i^q w / yn \\ f(t) * h \end{cases} \tag{27}$$

The matrix equation generalization is considered from the real number field to the special region of the complex number field. The special region $|\mu|$ of the complex plane region is defined, which is shown in equation (28):

$$|\mu| = \begin{cases} |Q| * {}^{g}\!/_{f} \\ f * \dfrac{f}{s} \\ dg + gj \end{cases} \tag{28}$$

The above matrix is a positive definite symmetric matrix, then [18], which is shown in equation (29):

$$R|\mu| = \frac{s + gj}{(\Delta d + \Delta r)} \tag{29}$$

Then the solution matrix of the equation satisfies the following inequality, which is shown in equation (30) [18]:

$$R \geq \frac{G^t}{\sum_{i=1} i * r} \tag{30}$$

In equation (30), i, r represent any positive numbers. Assuming that the control matrix obtained from the above unique symmetric positive definite solution matrix needs to satisfy the following equation, as shown in equation (31):

$$A|D| = \begin{cases} Fo(xi) + i \\ \dfrac{d}{cxg} / o \end{cases} \tag{31}$$

By constructing a positive semi-definite matrix, and utilizing the properties of matrix inequality and perturbation parameters, the lower bounds of the solution matrix and its eigenvalues are obtained. However, due to the arbitrariness of matrix selection and some restrictions, the final calculation result may not be optimal. How to select it to achieve the optimal result needs to be further discussed.

Therefore, the properties of the matrix [21], the properties of the matrix inequality, etc. are used to estimate the bounds of the solution matrix and its eigenvalues. Another semi-definite matrix is defined, which is shown in equation (32):

$$|\mu|' = \begin{cases} \dfrac{e}{R} - fh + A|D| \\ uu + W^q \\ G + r_{iu} \end{cases} \tag{32}$$

It needs to meet equation (33):

$$|\mu|' \leq \frac{e^{\frac{1}{2}}}{et(R)} \tag{33}$$

The extended hyperbolic tangent function method is used to solve the soliton solution of the nonlinear evolution equation according to the Riccati equation. The nonlinear evolution equation solution is constructed based on the Riccati equation solution. However, the assumed form of the solution is more extensive in this process. Several new exact solutions to the nonlinear evolution equation are obtained. It has the following form, which is shown in equation (34):

$$su = \sum_{i=1} r + ft + |g| + \sqrt{(oy)} + l|\mu|' + dr|R|^y \tag{34}$$

The elimination process is to perform the elimination operation on the lower triangular nodes in the matrix. When performing the elimination, the sparse matrix technology is used. The zero-row operation is used. In the Gaussian elimination process, each row is generally operated multiple times, and each operation may have an injection. For the generated elements, a length node array is created to store the results of temporary operations. After the elimination operation of each row is finished, the operation result is stored in zero rows and the Gaussian elimination result array. The calculation matrix is shown in equation (35):

$$G|I| = \begin{cases} -\sqrt{bmf(y) + g / ol} \\ re + \dfrac{e}{f} + Gh + su \\ \wp + F_t + hk \\ \sum_{i=1} f_i + ry \end{cases} \tag{35}$$

When Gaussian elimination is used to eliminate the columns of the adjacency matrix, it is equivalent to eliminate nodes in the network. Therefore, the solution nodes need to be re-encoded. When using dynamic stability optimization methods for continuous systems located in the left half plane of the complex plane and discrete systems located in the unit circle, the number of new nodes can only be reduced during the elimination process. But when these nodes are eliminated, new branches may not necessarily appear. In addition, the above two methods count the number of outgoing lines that will be added when the node is eliminated. Then the canceled node that has the least number of outgoing lines for priority numbering is selected, so as to realize the renumbering of dynamic nodes. Obviously, the above numbering method has a large workload, which consumes much time. Therefore, according to the complexity and final numbering effect of different optimal numbering methods, this study combines the topology analysis algorithm and static node optimal numbering method to realize the recoding of dynamic nodes [22]. The specific expression is shown in equation (36):

$$P=\begin{cases}-G\sqrt{b\tan\left(\sqrt{-b_2}\right)}, b<0\\ -G\sqrt{b\coth}\left(\sqrt{-b^u}\right), b<0\\ -\frac{1}{\beta}.b=0, b>0\\ -G\sqrt{b\cot\left(\sqrt{b^i}\right)}, b>0\end{cases} \tag{36}$$

On this basis, the main steps to construct a soliton solution are as follows:

To reduce the amount of injected elements, increase the operation speed, and save storage space, the nodes in the sparse matrix are optimized and numbered. The so-called node optimization numbering is to seek a node numbering method that minimizes the number of injected elements. The optimized coding method is shown in equation (37):

$$H\left|G\right|=\begin{cases}d+sh+p''\\ \sum g+s''g\\ gd+i\end{cases} \tag{37}$$

The solution is determined according to the principle that the highest derivative term of the equation is balanced with the highest power nonlinear term, which is shown in equation (38):

$$d=\frac{a}{sg}+h/\sqrt{sd} \tag{38}$$

Among them, the h matrix is shown in equation (39):

$$H=\begin{Bmatrix}h_1 & h_2 & \mathrm{K} & h_n\\ h_{21} & h_{22} & \mathrm{K} & h_{2n}\\ h_{31} & h_{31} & \mathrm{K} & h_{3n}\\ h_{41} & h_{41} & \mathrm{K} & h_{4n}\end{Bmatrix}. \tag{39}$$

The above calculation describes the interaction between the Riemann wave propagating along the y axis and the long wave propagating along the x axis. A new multi-period soliton solution is obtained based on the Hirota bi-linear method, which satisfies the following matrix conditions, which is shown in equation (40):

$$G\left|i\right|=\begin{cases}\frac{e}{d}/\sum_{i=1} r_i+dgt\\ \left|sd+o\right|-\sqrt{sd+g}\\ d+hh*\frac{s}{\sqrt{vd}}\end{cases} \tag{40}$$

The corresponding solutions of cotangent, cosecant, hyperbolic cotangent, hyperbolic cosecant, and combinations are omitted. The exact solution of the equation can be obtained, which is shown in equation (41):

$$G\left|i\right|'=\begin{cases}\frac{e}{d}/\sum_{i=1} r_i+dgt\\ \left|sd+o\right|-\sqrt{sd+g}\\ d+h*\frac{s}{\sqrt{vd}}\end{cases} \tag{41}$$

According to the above calculations, the exact value for the multi-period soliton solution of the discrete Riccati equation is obtained.

V. Experimental Results and Analysis of the Multi-Period Soliton Solution Method

To verify the validity of the multi-period soliton solution method for the discrete Riccati equation, experiments were carried out. The methods, including the continuous system located in the left half plane of the complex plane and the discrete system located in the unit circle were compared with the proposed method. The solution time and accuracy of different methods were compared.

A. *Experimental Platform Construction*

The simulation experiment was built on the ISO RFF ++ 4.5 platform. The monitoring performance of the software was debugged using a debugger to maximize the accuracy of the experimental results. The ISO RFF ++ 4.5 simulation platform consisted of a display, a controller, a monitoring terminal, a computer and an antenna. The specific parameters of the hardware configuration are shown in Table 1.

In the experiment, the monitoring effect of the computer and the controller were transmitted to the display. The reader was responsible for collecting the experimental data. The sampling rate of the reader was set to 250 $Msample/s$. The monitoring process was realized through wireless network connection. The antenna was selected to connect with 13.56m external coils. During the experiment, the relative position of the reader and display remained unchanged. The experimental environment was temperature controlled to avoid other factors affecting the experimental results. At the same time, the data was collected by computer. The data collection task was to collect the experimental system data in real time, monitor and control its status. The control terminal was used to manage the energy. Energy management utilized the overall information, frequency, time difference, unit power, and interconnection line power of the experimental system to make scheduling decisions. The main goal was to improve the quality of experimental control. The controller analyzed the network. The characteristic of network analysis was to analyze and determine the experimental content based on the experimental system. The aim was to improve the security of the experiment.

In the experiment, professional data analysis software was used to process the experimental data. The analysis software was CIPS_Dview, which can be used to process data from experimental recorder. The experimental data mainly came from two instruments, CIPS/DCVG instrument and Jiaxin stray current detector. The data were stored in excel and txt. The software could intelligently screen the raw data for dynamic detection and remove invalid data from it to reduce data errors during the measurement process. The software presented tedious and abstract static detection data and dynamic detection data to users through graphical comparison analysis, providing them with intuitive and overall data analysis services. The software parameters are shown in Table 2.

TABLE I
EXPERIMENTAL PLATFORM HARDWARE PARAMETERS

Configuration	Specifications and parameters	Quantity
CPU	INTEL Xeon E5620 New official version	2
Motherboard	Supermicro X8DTL-I server motherboard	1
Memory hard drive	8G DDR3 ECC Server memory	2
	SSD solid state hard disk	1
	2.5-inch hard disk (for system backup)	1
	Seagate 2TB Enterprise 64 (Do RAID 1)	2
Power supply	Supermicro 2U standard case	1
Heat sink	2U copper tube radiator	1
Expansion card	Rr2760	/
Case	Super 4U24 bit standard memory	/
RAM	4G DDR3 ECC server memory	/
RAM	8GB	/
Sampling frequency	1 Hz	Keep in experiment

TABLE II
CIPS_DVIEW SOFTWARE PARAMETERS

Serial number	Parameter	Details
1	RAM	32MB
2	Sampling time	1.2 seconds
3	Screen	Display instrument operating status
4	Aisle	Record label, pattern, time, date
5	Operating hours	Powered by lithium battery, power supply time is more than 22 hours
6	Backpack battery	Long-term continuous monitoring
7	Chinese and English operation	Software analysis package

Meanwhile, the software provided protection time ratio, protection distance ratio, dynamic data correction and other functions, providing strong support for users to deeply mining information. In CIPS_Dview, the MVC design pattern was used. A large number of GDI + drawing technologies were used to display the lines. To improve the operability and interaction ability of the data, the C1Chart control in Component One products was selected to present graphic images. The software was used to record and analyze experimental data, and output the experimental results in the form of curves.

The experiments used different methods to solve the equations. The solution results are shown in Table 3.

TABLE III
EXPERIMENTAL DATA

Serial number	Correct solution	Serial number	Correct solution
1	52	6	56
2	46	7	146
3	89	8	33
4	32	9	37
5	5	10	59

The stability research methods of continuous systems located in the left half plane of the complex plane and discrete systems located in the unit circle were compared with the method designed in the research. The solution time of different methods was compared. In the experiment, the solution results of different methods were compared with the correct values. If the calculation results were correct, the correct calculation time was recorded. If the calculation was incorrect, the calculation was returned. The error time was accumulated into the experimental time.

In this experiment, the solution time cost was used as a test index. The distributed time algorithm was used to calculate the solution time cost. The function equation is shown in equation (42):

$$H\left|U\right| = \frac{T}{\sum_i A_i \partial} \tag{42}$$

Among them, $H\left|U\right|$ represents the time cost of solving the experiment. T represents the total time spent in solving. $\sum_i A_i$ represents the pulse coefficient, which is only introduced as a parameter. It has no practical calculation significance.

From equation (42), if the $H\left|U\right|$ value is low, the T value is small. Therefore, this solution method has a short solving time, indicating high solving ability. The calculation effect of this method is better.

B. Analysis of the Experimental Comparison Results for Different Solution Methods

To verify the effectiveness of the multi-period soliton solution method of discrete Riccati equation, the stability research method [10] for the continuous system located in the left half plane of the complex plane, and the design method [23] for the discrete system located in the unit circle, were compared with the multi-period soliton solution method of discrete Riccati equation. The solution time comparison results for different methods are shown in Figure 1.

After analyzing the above comparison results, the continuous systems located in the left half plane of the complex plane and the discrete systems located in the unit circle took more than 5 min to solve. The maximum required time was approximately 10 min. The overall calculation time for the multi-period soliton solution of the discrete Riccati equation designed in the research was significantly lower than the other two methods. It only took 2.5 min. From the comparison, the overall solution time for multi-period soliton solutions of the discrete Riccati equation was shorter than that of traditional methods. Therefore, the effectiveness of this method can be proved by the above experiments. It can meet the needs of solving. This is because the method solves the

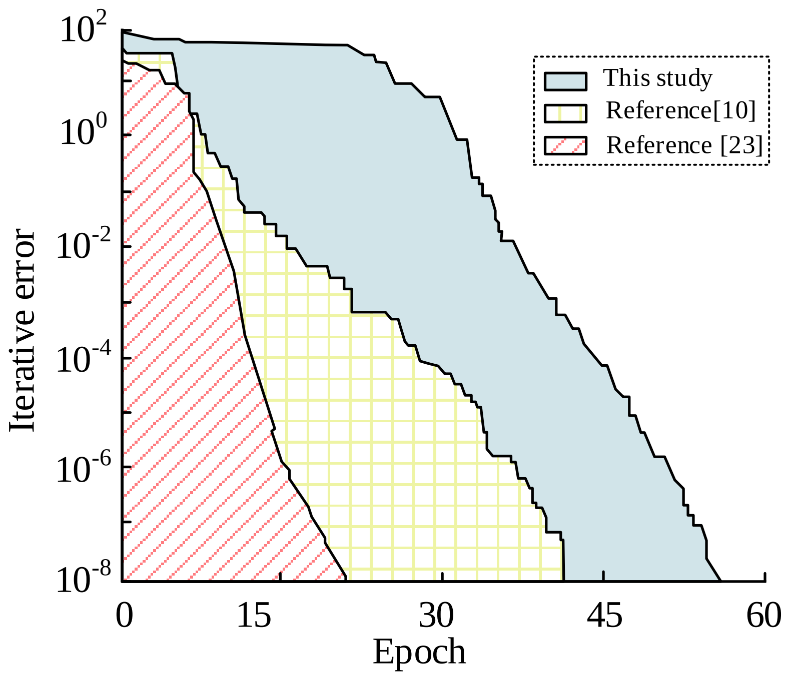

Fig. 2. The comparison of iterations to obtain the corresponding iteration solution.

stability analysis problem for multi-period soliton solutions. A continuous Lyapunov matrix equation was introduced, which can reduce the difficulty of solving the control system, thereby shortening the solution time.

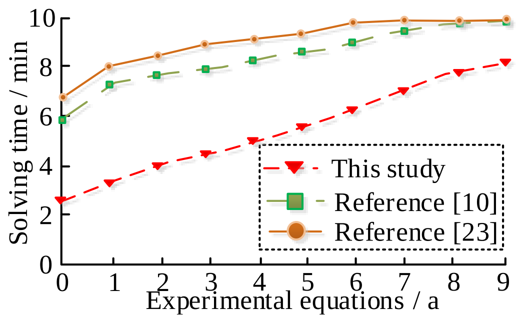

Fig. 1. Comparison of solution time.

TABLE IV
THE UPPER LIMIT OF TIME DELAY OBTAINED UNDER DIFFERENT TIME DELAY VALUES

Method\Time delay value	0.00	0.30	0.70	1.00
This study	2.52	2.90	3.26	3.49
Reference [10]	2.48	2.85	3.19	3.43
Reference [23]	2.40	2.51	3.17	3.42

In control systems, time delay played an important role. The control system used sensors and other devices to perceive and control the state of the physical system. However, there may be time delays between sensors or controllers due to complex factors such as distance, transmission delay, and sampling period, which have a significant impact on the stability and accuracy of the control system. The appropriate upper limit of time delay had a great impact on the stability of controllers, etc. Therefore, the study further compared the upper limit of time delay obtained by three methods under different time delay values, and the comparison results are shown in Table 4.

Table 4 shows the upper limit of time delay obtained by three methods under different time delay values. It can be seen that the proposed method in the study took into account more information about time delay and system state, resulting in a higher upper bound on time delay for the control system. This indicated that the proposed method ensures the stability of the controller, further confirming the effectiveness and superiority of the solution of discrete Riccati equation with multi-period solution.

To further validate the number of iterations required to obtain the corresponding iterative solution after applying this method to the system, the simulation experiment was performed. The results are shown in Figure 2.

From Figure 2, for the same iterative error, the continuous systems in the left half plane of the complex plane and the discrete systems in the unit circle had more than 40 iterations to obtain the corresponding iterative solutions. The number of iterations for solving the discrete Riccati equation with multi-period soliton was 21. Therefore, in some cases, the multi-period soliton solution of the discrete Riccati equation designed in this study had fewer iterations. The iteration dropped faster under the same iteration error.

After comparing the number of iterations, to ensure the stability and performance of the computer system, the multi-period soliton solution method was used to verify the discrete Riccati equation and other methods. The CPU usage of the system was tested under high loads. The results are shown in Figure 3.

From Figure 3(a), with the increase of the usage time, the CPU utilization rate of the system tested by the three methods gradually increased. In the early stage, the CPU utilization rate of the continuous system in the left half plane of the complex plane was relatively low. Compared with the continuous system in the left half plane of the complex plane and the discrete system in the unit circle, the CPU utilization rate of the multi-period soliton solution method of the discrete Riccati equation was lower. Combining Figure 2 and Figure 3 as a whole, the multi-period soliton solution of discrete Riccati equation designed in this study not only had fewer iterations to obtain the corresponding iterative solution, but also had lower CPU utilization.

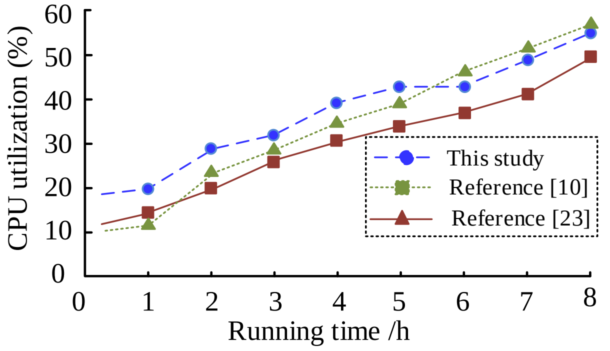

Fig. 3 CPU usage of different methods system

To further verify the validity of the multi-period soliton solution method of the discrete Riccati equation designed in this study, the accuracy of the calculation results was used as a comparison index. The method designed in this study was compared to the the continuous systems in the left half plane of the complex plane and the discrete systems in the unit circle. The results are shown in Figure 4.

From Figure 4, the difference between the solution obtained by the multi-period soliton solution method and the correct solution of the discrete Riccati equation designed in this paper was small. The difference between the solution obtained by the continuous system stability research method located in the left half plane of the complex plane and the

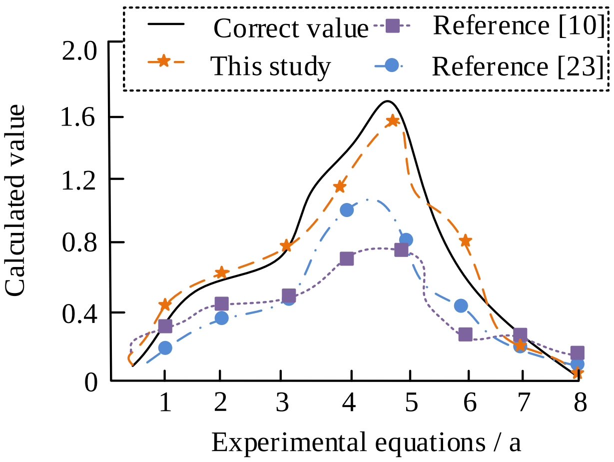

Fig. 4. Comparison of the accuracy of different methods.

discrete system stability research method located in the unit circle and the correct value was large. It showed that the design method could achieve more reliable results, which plays an important role in the stability control of the system. Meanwhile, the study further compared the tracking profiles of the controller system under the action of the three methods.

From Figure 5, the proposed method of the study simultaneously offset the tracking error of the system while ensuring a reduced data sending rate of the system as compared to the other two methods. This indicated that the multi-period soliton method based on the discrete Riccati equation designed by the study was not only able to reduce the tracking error of the controller, but also improved the control accuracy with certain robustness. Finally, the study further compared the curves of the controller in the closed-loop system state under the three methods, as shown in Figure 6.

The closed-loop system curve variations of the three methods under four controller perturbation states are shown in Fig. 6. All the three methods showed curve jitter after the controller perturbations. Whereas, the proposed method of the study was able to become stable quickly after the perturbation. Compared with the literature [10] and [23], the proposed method was less affected by the perturbation. This indicates the superior performance of the controller under this method.

VI. Concluding Remarks

Aiming at many control problems in the control system, this study transformed some control problems into solutions of some matrix equations. Therefore, a method for solving discrete Riccati equations with multi-period solitons was proposed. Through the index of multi-period soliton solution of discrete Riccati equation, the multi-period soliton solution of discrete Riccati equation was obtained. The positive semi-definite matrix was constructed. Based on the properties of matrix inequality and disturbance parameters, the lower bound of the solution matrix and its eigenvalue was calculated. The boundary estimation values and eigenvalue of the reference matrix and the solution matrix were obtained. According to these values, the soliton solution was constructed and the multi-period soliton solution of discrete Riccati equation was completed. The experimental results showed that the solution time of the continuous systems in the left half plane of the complex plane and the discrete systems in the unit circle was more than 5 minutes. However, the overall calculation time for solving the discrete Riccati equation with multi-period solitons was only 2.5 minutes. In addition, the number of iterations for solving the discrete Riccati equation with multi-period solitons in this study was 21 times, while other methods were more than 40 times. Under the same iterative error, the iteration dropped faster. In terms of accuracy, the multi-period soliton solution of discrete Riccati equation designed in this paper had little difference from the correct solution, while the solutions obtained by other methods had great difference from the correct value. This shows that the method can solve the stability analysis problem of multi-period soliton solution, which can reduce the difficulty of solving the control system. It shortens the solving time and obtains more reliable results, which plays an important role in the stability control of the system. The deficiency of this study is that the estimation of the Riccati equation solution in discrete-time algebra is conservative. The solutions of these two equations can be further improved in the future.

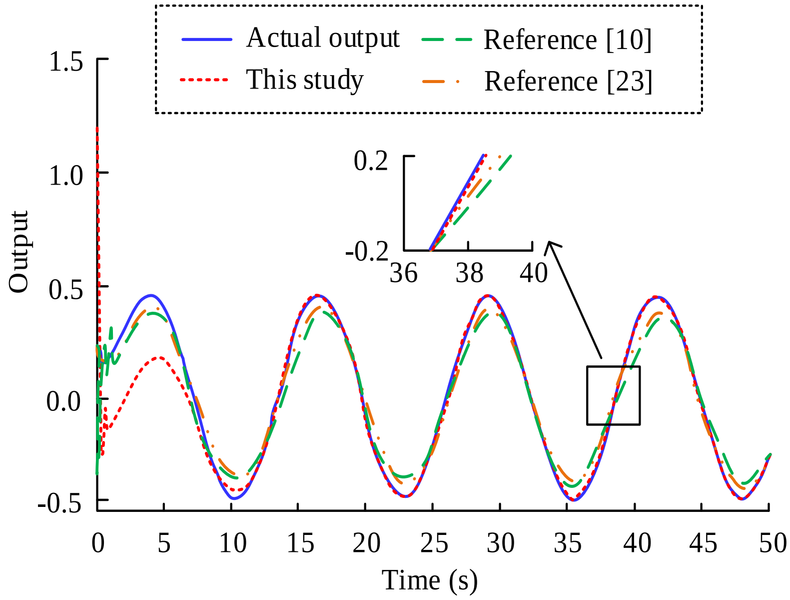

Fig. 5. Comparison of controller system tracking curves under different methods.

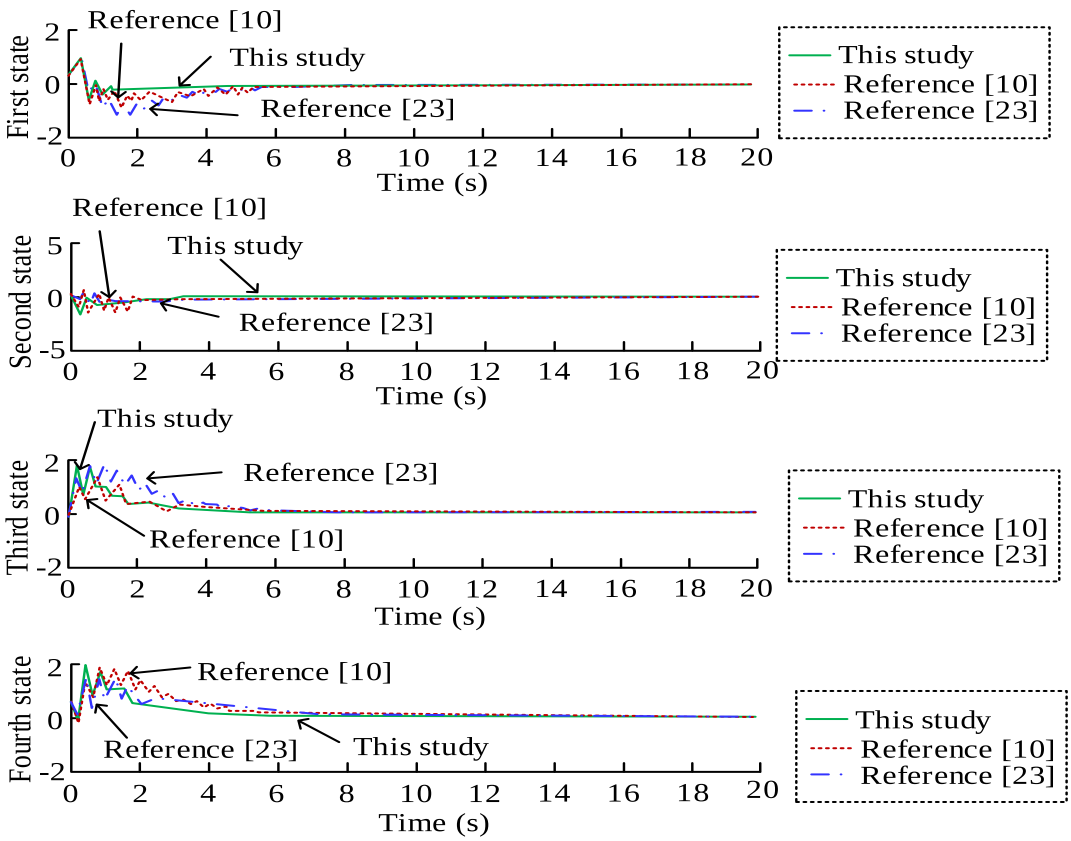

Fig. 6. Comparison of closed-loop system curves under controller interference for different methods

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