The Queueing Inventory System with Working Vacations and Breakdowns

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***Abstract*—This paper considers an M/M/1 queueing inventory system with (s, S) policy. The server may break down during the working vacation and each customer takes one product away after being served. When there is no product in the system, the system starts a working vacation, and the service continues at a lower rate at that time. The server may breakdown only during a working vacation period. When a vacation breakdown occurs, the system immediately stops service and starts repairing. Firstly, we utilize Markov process theory to construct a three-dimensional Markov chain to analyze the stationary process of the system. Using Gaussian iteration and matrix geometry solution method, the steady-state performance measures about queueing and inventory are obtained. Then, through numerical experiments, we analyze the system parameters' influence on performance indexes. Finally, we define a cost function and use the genetic algorithm to obtain the optimal inventory level and the lowest cost at a certain set of parameters.**

***Index Terms*—queueing inventory, working vacation, vacation breakdown, (s, S) policy.**

I. INTRODUCTION

QUeueing inventory system is widely used in production practices, which consists of queueing system and inventory system. In a queueing inventory system, the customer takes away a product when they are served. Therefore, the system should consider not only service operation but also the number of products in stock to ensure the quality of service. For most enterprises, reasonable inventory management is related to the enterprise's profitability. Therefore, the queuing inventory system even its optimal inventory control strategy has been widely concerned by scholars in many fields.

The early literature concerning queuing inventory systems was primarily centered on the realm of manufacturing service. Sigman and Simchi-Levi [1] initially amalgamated queuing service with inventory management theory. They probed into the M/G/1 queue-based inventory system with finite inventory and verified its performance metrics by resorting to approximation techniques. Schwarz et al [2] explored the M/M/1 queueing inventory system for lost sales under various inventory management policies. In the case of the infinite waiting room, they derived that the stationary distribution of the queue length was identical to the classical M/M/1/∞ system. Krishnamoorthy et al [3] deliberated on two models with (s, Q) and (s, S) replenishment policies, and the item provided with probability to a customer. They derived the stationary probability distribution of the joint queue length and inventory level under two control policies and optimized the model to obtain the optimal pairs. Yue et al [4] contemplated two models featuring partially and completely lost sales and acquired the product of the steady-state probability through the quasi-birth and death process. Liu et al [5] investigated perishable items queuing inventory systems with two types of customers. They delineated the service level and established the corresponding optimization model subject to the limitation of the service level. The sensitivity of system parameters and the optimal inventory management policy were explored via numerical experiments. Qin and Yue [6] examined the (s, S) production inventory strategy by taking into account online shopping service time and return-ability. They attained the steady-state joint distribution of queue length and inventory level by means of the product form and employed numerical examinations to scrutinize the parameter influence on the stationary performance measures. Liu [7] studied the M/M/1 production service inventory system with multiple working vacation strategies. They derived a four-dimensional Markov chain based on the (s, S) inventory policy and delineated the cost function. They scrutinized the parameter influence on the performance measures and probed into the optimal inventory policy with different working vacations. Zhang [8] developed several queuing inventory models encompassing working vacations, inventory strategies, and perishable items. He solved the equilibrium equation through the utilization of the quasi-birth and death process and derived the system steady-state probability by means of the recursive method. Subsequently, he demonstrated that the steady-state probability distribution assumes a product form and defined the cost function. Ultimately, he employed a genetic algorithm to acquire the optimal inventory strategy and cost. Levy and Yechiali [9] initially investigated vacation queuing inventory systems and introduced the terms related to vacation and vacation strategies. Selvaraju [10] examined impatient customers in an M/M/1 queue encompassing two distinct working vacations and compared two models to comprehend the influence of parameters on its performance indices. Zhou et al [11] developed the single working vacation and vacation interruption G-queue encompassing setup times. They utilized the quasi-birth-and-death process and matrix geometry to derive the steady-state distribution and stationary queue length. The average waiting time of the working busy period was acquired by solving the distribution function. Yan and Yang [12] conducted an in-depth investigation into two

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distinct customers in the M/M/1 system characterized by two vacation policies. By means of the matrix geometric solution, they deduced the stationary condition and queue length. The model was verified through numerical simulation. Daniel and Ramanarayanan [13] integrated the working vacation into the inventory system. When there is no product, the service initiates a vacation and the customers do not enter the system. They exploited update theory and convolution theory to acquire the steady-state probability. Shan and Yue [14] explored an M/M/1/N queuing inventory system with impatient customers and multiple vacations. They utilized the matrix geometric solution to obtain the probability distribution and performance indicators when there are no products or customers in the system. Fu et al [15], discussed two types of failures with a standby service station and start-up time based on the M/M/1 queuing system. The steady-state equilibrium condition and the steady-state probability vector of the system were derived through the matrix geometry solution method, and the steady-state queuing length of the system was determined. Finally, Matlab was used for numerical analysis of their conclusions. Yang et al [16] employed the quasi-birth-and-death (QBD) process and matrix geometry solution method to investigate an M/M/1 repairable queuing system with two types of server breakdowns and negative customers. They provided stationary conditions, acquired steady-state probability vectors, and calculated some steady-state queueing as well as reliable measures. The server was regarded as stable and reliable in a considerable amount of related literature. Nevertheless, in actual practice, the server might malfunction and require repair. Kalidass and Kasturi [17] investigated a queuing model that could fail at any moment and offer services at a slower pace during a working breakdown. They utilized the probability-generating function to solve the condition of steady-state existence and the probability distribution. They also presented certain performance indicators and numerical examples. Lakshmi et al [18] studied the inventory queuing system with two types of server interruptions, where one interruption occurred during normal working and another interruption occurred during the working breakdown. They employed the matrix geometric solution to obtain the steady-state probability vector in a finite-capacity inventory. In practice, the rationalization of the server's working rate can effectively save costs based on the quantity of products. During working vacations, scenarios such as the departure of the waiter and failure to return promptly or server breakdowns may occur, which constitutes a vacation breakdown. It is particularly essential to optimize the inventory policy by taking into account the cost of repair and customer loss. Hence, this paper considers that the server enters a working vacation when the inventory is depleted. During the working vacation, the server provides services at a lower service rate. Once the working vacation concludes, the server commences operation if the inventory is not depleted. Otherwise, it initiates a new working vacation. It is assumed that server breakdowns only occur during the working vacation. When the server malfunctions, it halts service and enters the repair state immediately. During the vacation breakdown state, newly arriving customers do not enter the system.

II. MODEL DESCRIPTION

We present the M/M/1 queuing system featuring the (s, S) policy, working vacation, and vacation breakdown. The model is delineated as follows:

1) Within the system, there exists merely one server. The arrival pattern of customers adheres to the Poisson process at a rate of λ, and a queue is formed when the number of customers exceeds one within the system. The service principle is first-come-first-served (FCFS), and the service time complies with an exponential distribution characterized by parameter μ_b. After being served, each customer takes one product away, thereby reducing the inventory level of the products by one unit upon the completion of each service.

2)When the inventory becomes depleted, the server initiates a working vacation, and the vacation duration adheres to an exponential distribution with parameter θ. During the working vacation, the service time follows an exponential distribution with parameter μ_v ($\mu_v < \mu_b$). After the working vacation period, if there is at least one product within the system, the server commences a regular busy period. Otherwise, the server initiates another working vacation.

3) Breakdown Procedure: The server is prone to malfunction exclusively during the vacation period. The server lifetime follows an exponential distribution with parameter α during the working vacation time. Once the server malfunctions, it halts the service and initiates the repair promptly. The repair time adheres to an exponential distribution with parameter β. Upon the completion of the repair, the server commences a regular busy period whenever the inventory is not empty. Otherwise, it commences another working vacation.

4) The ordering law for products follows the (s, S) policy. When the stock level reaches the safety stock level s, the system promptly sends an order demand. The inventory level will attain S (where $s < S$) after a replenishment time that follows an exponential distribution with parameter η ($\eta > 0$).

5) The system is a loss system. During a working vacation, provided that the inventory is not empty, arriving customers will access the system and depart after being served. Otherwise, if the system inventory is empty, new arrivals will not access the system. Conversely, in the case of a failure, new arrivals will be precluded from accessing the system, and customers experiencing service interruptions as well as those already in the queue will remain within the system until service is resumed.

6) It is postulated that the customer arrival time, service time, working vacation time, replenishment time, vacation breakdown time, and repair time are mutually independent.

Let $J(t)$ be the state of the server at the moment t:

$$J(t) = \begin{cases} 0, \text{time t in the working vacation period,} \\ 1, \text{time t in the busy period,} \\ 2, \text{time t in the vacation breakdown period,} \end{cases}$$

Figure 2-1 shows how the server state changes with the amount of inventory in this model. Let $N(t)$ be the number of customers in the queueing system at moment t, and $I(t)$ be the inventory level at moment t. Then is a Markov process whose state space is $\Omega = \{(n, 0, 0), (n, 0, 2), (n, 1, 0), \cdots, (n, S, 2); n \geq 0\}$.

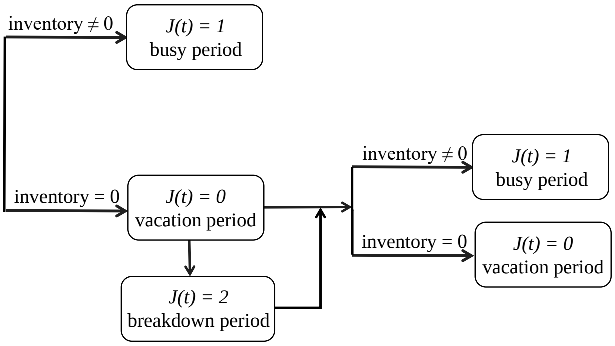

Figure 2-1. Server status change diagram.

Figure 2-2 shows the state transition diagram of the three-dimensional Markov process.

Let the system state be ordered lexicographical. Then the infinitesimal generator Q is the following matrix:

$$Q=\begin{pmatrix} A_0 & C & & & \\ B & A & C & & \\ & B & A & C & \\ & & \ddots & \ddots & \ddots \end{pmatrix},$$

where

$$A_0=\begin{pmatrix} a_0 & & & & & & & h_0 \\ & a_1 & & & & & & h_1 \\ & & a_1 & & & & & h_1 \\ & & & \ddots & & & & \vdots \\ & & & & a_1 & & & h_1 \\ & & & & & a_2 & & \\ & & & & & & \ddots & \\ & & & & & & & a_2 \end{pmatrix},$$

$$A=\begin{pmatrix} a_0 & & & & & & & h_0 \\ & a_3 & & & & & & h_1 \\ & & a_3 & & & & & h_1 \\ & & & \ddots & & & & \vdots \\ & & & & a_3 & & & h_1 \\ & & & & & a_4 & & \\ & & & & & & \ddots & \\ & & & & & & & a_4 \end{pmatrix},$$

$$B=\begin{pmatrix} 0 & & & & & & & & & & & & & & 0\,0\,0\,0 \\ 0 & & & & & & & & & & & & & & 0\,0\,0\,0 \\ \mu_v & & & & & & & & & & & & & & \\ \mu_b & & & & & & & & & & & & & & \\ & 0 & & & & & & & & & & & & & \\ & & \mu_v & & & & & & & & & & & & \\ & & & \mu_b & & & & & & & & & & & \\ & & & & \ddots & & & & & & & & & & \vdots\,\vdots\,\vdots\,\vdots \\ & & & & & \mu_v & & & & & & & & & \\ & & & & & & \mu_b & & & & & & & & \\ & & & & & & & 0 & & & & & & & \\ & & & & & & & & \mu_v & & & & & & \\ & & & & & & & & & \mu_b & & & & & \\ & & & & & & & & & & \ddots & & & & \\ & & & & & & & & & & & \mu_v & & & \\ & & & & & & & & & & & & \mu_b & & 0\,0\,0\,0 \\ & & & & & & & & & & & & & & 0\,0\,0\,0 \end{pmatrix},$$

$$C=\begin{pmatrix} 0 & & & & & & & & & \\ & 0 & & & & & & & & \\ & & \lambda & & & & & & & \\ & & & \lambda & & & & & & \\ & & & & 0 & & & & & \\ & & & & & \lambda & & & & \\ & & & & & & \ddots & & & \\ & & & & & & & \lambda & & \\ & & & & & & & & \lambda & \\ & & & & & & & & & 0 \end{pmatrix},$$

$$a_0=\begin{pmatrix} -(\alpha+\eta) & \alpha \\ \beta & -(\beta+\eta) \end{pmatrix},$$

$$a_1=\begin{pmatrix} -(\alpha+\theta+\eta+\lambda) & \theta & \alpha \\ & -(\eta+\alpha) & \\ & \beta & -(\beta+\eta) \end{pmatrix},$$

$$a_2=\begin{pmatrix} -(\alpha+\theta+\lambda) & \theta & \alpha \\ & -\lambda & \\ & \beta & -\beta \end{pmatrix},$$

$$a_3=\begin{pmatrix} -(\mu_v+\alpha+\theta+\eta+\lambda) & \theta & \alpha \\ & -(\mu_b+\eta+\lambda) & \\ & \beta & -(\beta+\eta) \end{pmatrix},$$

$$a_4=\begin{pmatrix} -(\mu_v+\alpha+\theta+\lambda) & \theta & \alpha \\ & -(\mu_b+\lambda) & \\ & \beta & -\beta \end{pmatrix},$$

$$h_0=\begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \eta \end{pmatrix},\ h_1=\begin{pmatrix} \eta & & \\ & \eta & \\ & & \eta \end{pmatrix}.$$

A_0, A, B, C are all $(3S+2)\times(3S+2)$ order matrices. The matrix Q shows that $\{N(t), I(t), J(t)\}$ is a QBD process.

III. STEADY-STATE CONDITIONS

The system state process $\{N(t), I(t), J(t)\}$ is a quasi birth and death process, which is obtained by making the matrix

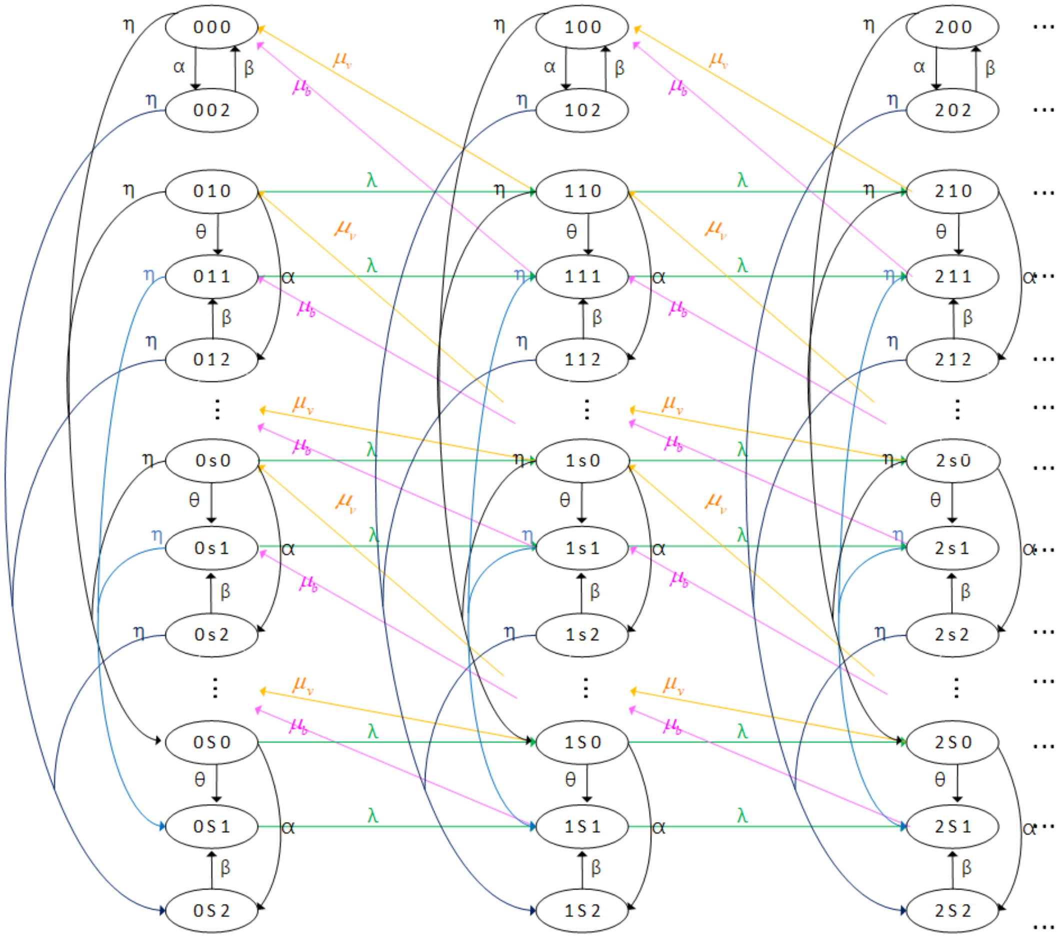

Figure 2-2. State transfer diagram.

$H = B + A + C =$

$$H = \begin{pmatrix} u_0 & & & & & & & & h_0 \\ b_0 & u_1 & & & & & & & h_1 \\ & b_1 & u_1 & & & & & & h_1 \\ & & b_1 & \ddots & & & & & \vdots \\ & & & \ddots & u_1 & & & & h_1 \\ & & & & b_1 & u_2 & & & \\ & & & & & b_1 & u_2 & & \\ & & & & & & b_1 & \ddots & \\ & & & & & & & \ddots & u_2 \\ & & & & & & & & b_1 \end{pmatrix},$$

$$u_0 = \begin{pmatrix} -(\alpha+\eta) & \alpha \\ \beta & -(\beta+\eta) \end{pmatrix},$$

$$u_1 = \begin{pmatrix} -(\mu_v+\alpha+\theta+\eta) & \theta & \alpha \\ & -(\mu_b+\eta) & \\ & \beta & -(\beta+\eta) \end{pmatrix},$$

$$u_2 = \begin{pmatrix} -(\mu_v+\alpha+\theta) & \theta & \alpha \\ & -\mu_b & \\ & \beta & -\beta \end{pmatrix},$$

$$b_0 = \begin{pmatrix} \mu_v & 0 \\ \mu_b & 0 \\ 0 & 0 \end{pmatrix}, \; b_1 = \begin{pmatrix} \mu_v & & \\ & \mu_b & \\ & & 0 \end{pmatrix}.$$

Let $\varphi = (\varphi_{0,0}, \varphi_{0,2}, ..., \varphi_{S,2})$ denote the steady-state probability vector of H, substituting φ and H into the normalization condition yields the following results:

$$\begin{cases} \varphi H = 0, \\ \varphi e = 1, \end{cases} \tag{1}$$

where e is a column vector of appropriate dimension with elements all 1.

Substituting φ, e and H into the Eq. (1) obtain:

$$(\alpha+\eta)\,\varphi_{0,0} + \beta\varphi_{0,2} + \mu_v\varphi_{1,0} + \mu_b\varphi_{1,1} = 0, \tag{2}$$

$$\alpha\varphi_{i,0} - (\beta+\eta)\,\varphi_{i,2} = 0, \;\; (0 \le i \le s), \tag{3}$$

$$(\mu_v+\alpha+\theta+\eta)\,\varphi_{i,0} + \mu_v\varphi_{i+1,0} = 0, \;\; (1 \le i \le s), \tag{4}$$

$$\theta\varphi_{i,0} + \beta\varphi_{i,2} - (\mu_b+\eta)\,\varphi_{i,1} + \mu_b\varphi_{i+1,1} = 0, \;\; (1 \le i \le s), \tag{5}$$

$$-(\mu_v+\alpha+\theta)\,\varphi_{i,0} + \mu_v\varphi_{i+1,0} = 0, \;\; (s+1 \le i \le S-1), \tag{6}$$

$$\alpha\varphi_{i,0} - \beta\varphi_{i,2} = 0, \;\; (s+1 \le i \le S-1), \tag{7}$$

$$\theta\varphi_{i,0} + \beta\varphi_{i,2} - \mu_b\varphi_{i,1} + \mu_b\varphi_{i+1,1} = 0, \;\; (s+1 \le i \le S-1), \tag{8}$$

$$-(\mu_v+\alpha+\theta)\,\varphi_{S,0} + \eta\sum_{i=0}^{S}\varphi_{i,0} = 0, \tag{9}$$

$$\alpha\varphi_{S,0}-\beta\varphi_{S,2}+\eta\sum_{i=0}^{S}\varphi_{i,2}=0, \tag{10}$$

$$\theta\varphi_{S,0}+\beta\varphi_{S,2}-\mu_b\varphi_{S,1}+\eta\sum_{i=1}^{S}\varphi_{i,1}=0, \tag{11}$$

$$\varphi_{0,0}+\varphi_{0,2}+\sum_{i=1}^{S}\left(\varphi_{i,0}+\varphi_{i,2}+\varphi_{i,1}\right)=1. \tag{12}$$

From Eq. (2) to Eq. (12), we obtained:

$$\varphi_{0,0}=\frac{\mu_v+\alpha+\theta}{\eta}\varphi_{S,0}-\sum_{i=1}^{S}\varphi_{i,0}, \tag{13}$$

$$\varphi_{i,0}=\left(\frac{\mu_v}{\mu_v+\alpha+\theta+\eta}\right)^{s-i+1}\left(\frac{\mu_v}{\mu_v+\alpha+\theta}\right)^{S-s-1}\varphi_{S,0},\quad(1\le i\le s), \tag{14}$$

$$\varphi_{i,0}=\left(\frac{\mu_v}{\mu_v+\alpha+\theta}\right)^{S-i}\varphi_{S,0},\ \ (s+1\le i\le S-1), \tag{15}$$

$$\varphi_{1,1}=\frac{\alpha+\eta}{\mu_b}\varphi_{0,0}-\frac{\beta}{\mu_b}\varphi_{0,2}-\frac{\mu_v}{\mu_b}\varphi_{1,0}, \tag{16}$$

$$\varphi_{i,1}=-\frac{\theta}{\mu_b}\varphi_{i,0}-\frac{\beta}{\mu_b}\varphi_{i,2}+\frac{\mu_b+\eta}{\mu_b}\varphi_{i-1,1},\ \ (2\le i\le s+1), \tag{17}$$

$$\varphi_{i,1}=-\frac{\theta}{\mu_b}\varphi_{i,0}-\frac{\beta}{\mu_b}\varphi_{i,2}+\varphi_{i-1,1},\ \ (s+2\le i\le S), \tag{18}$$

$$\varphi_{0,2}=\frac{\alpha}{\beta+\eta}\varphi_{0,0}, \tag{19}$$

$$\varphi_{i,2}=\frac{\alpha}{\beta+\eta}\left(\frac{\mu_v}{\mu_v+\alpha+\theta+\eta}\right)^{s-i-1}\left(\frac{\mu_v}{\mu_v+\alpha+\theta}\right)^{S-s-1}\varphi_{S,0},\quad(1\le i\le s), \tag{20}$$

$$\varphi_{i,2}=\frac{\alpha}{\beta}\left(\frac{\mu_v}{\mu_v+\alpha+\theta}\right)^{S-i}\varphi_{S,0},\ \ (s+1\le i\le S-1), \tag{21}$$

$$\varphi_{S,1}=\frac{\theta}{\mu_b}\varphi_{S,0}+\frac{\beta}{\mu_b}\varphi_{S,2}+\frac{\eta}{\mu_b}\sum_{i=1}^{S}\varphi_{i,1}, \tag{22}$$

$$\varphi_{S,2}=\frac{\alpha}{\beta}\varphi_{S,0}+\frac{\eta}{\beta}\varphi_{0,2}+\frac{\eta}{\beta}\sum_{i=1}^{S}\varphi_{i,2}. \tag{23}$$

It can be seen that $\varphi_{0,0},\varphi_{0,2},\varphi_{1,0},\cdots,\varphi_{S,2}$ can all be expressed as $\varphi_{S,0}$ by iterating between equations. From $\varphi e=1$ it is possible to solve for $\varphi_{S,0}$, which in turn gives the value of each probability.

From the matrix geometric solution, the sufficient necessary condition for the normal return of the system state process is $\varphi Ce<\varphi Be$. It is obtained from matrix B and matrix C:

$$\varphi Ce=\lambda\sum_{i=1}^{S}\left(\varphi_{i,0}+\varphi_{i,1}\right),$$

$$\varphi Be=\mu_v\sum_{i=1}^{S}\varphi_{i,0}+\mu_b\sum_{i=1}^{S}\varphi_{i,1}.$$

Therefore, the steady-state equilibrium condition of the system is:

$$\frac{\lambda\sum_{i=1}^{S}\left(\varphi_{i,0}+\varphi_{i,1}\right)}{\mu_v\sum_{i=1}^{S}\varphi_{i,0}+\mu_b\sum_{i=1}^{S}\varphi_{i,1}}<1.$$

Define the steady-state probability vector of the matrix Q as:

$$\pi_{n,i,j}=\lim_{t\to\infty}\pi\left\{N(t)=n,I(t)=i,J(t)=j\right\},(n,i,j)\in\Omega.$$

To accommodate the structure of the transfer rate matrix Q, the steady state probability vector π is chunked as follows:

$$\pi=\left(\pi_0,\pi_1,\pi_2,\cdots\right),$$
$$\pi_n=\left(\pi_{n,0,0},\pi_{n,0,2},\pi_{n,1,0},\cdots,\pi_{n,S,2}\right),n\ge 0.$$

The steady-state probability vector π satisfies the equilibrium equation:

$$\begin{cases}\pi Q=0,\\ \pi e=1,\end{cases}$$

where e is a column vector of appropriate dimension with elements all 1.

The system steady-state probability vector has the form of a matrix geometric solution as follows:

$$\pi_n=\pi_0R^n,n>0,$$

where π_0 satisfies the system of equations:

$$\begin{cases}\pi_0\left(A_0+RB\right)=0,\\ \pi_0\left(I-R\right)^{-1}e=1.\end{cases}$$

Obtain the minimum non-negative solution R of equation $R^2B+RA+C=0$. Then, the specific steady-state probability vector is obtained. The steps of the algorithm are as follows:
Step 1: Set $R_0=0$;
Step 2: Execute $R_n=-[{R_{n-1}}^2B+C]A^{-1},n=1,2,...;$
Step 3: If $\|R_n-R_{n-1}\|=\max_{i,j}|a_{ij}(n)-a_{ij}(n-1)|<\varepsilon$, make $R=R_n$, otherwise, loop step 2, where $a_{ij}(n)$ denotes the element of row i and column j of matrix R_n.

IV. SYSTEM PERFORMANCE MEASURES

(1) Expected level of customers in the queue:

$$E_L=\sum_{n=0}^{\infty}\sum_{i=0}^{S}n(\pi_{n,i,0}+\pi_{n,i,1}+\pi_{n,i,2})=\pi_0R(I-R)^{-2}\delta_1,$$

where δ_1 is $(3S+2)$-dimensional unit-column vector.

(2) Expected stock level:

$$E_{inv}=\sum_{n=0}^{\infty}\sum_{i=0}^{S}i(\pi_{n.i.0}+\pi_{n.i.1}+\pi_{n.i.2})=\pi_0(I-R)^{-1}\delta_2,$$

where $\delta_2=(0,0,1,1,1,2,2,2,3,...)^T{}_{1\times(3S+2)}$.

(3) Expected reorder rate:

$$E_{rep}=\eta\sum_{n=0}^{\infty}\sum_{i=0}^{S}\left(\pi_{n,i,0}+\pi_{n,i,1}+\pi_{n,i,2}\right)=\eta\pi_0(I-R)^{-1}\delta_3,$$

where $\delta_3=(1,1,1...,1,0,0,...,0)^T{}_{1\times(3S+2)}$.

(4) Expected reorder quantity:

$$E_P=\sum_{n=0}^{\infty}\sum_{i=0}^{S}(S-i)\left(\pi_{n,i,0}+\pi_{n,i,1}+\pi_{n,i,2}\right)=\pi_0(I-R)^{-1}\delta_4,$$

where $\delta_4=(S,S,S-1,...,S-s,0,..,0)^T{}_{1\times(3S+2)}$.

(5) Expected loss rate of customers:

$$E_{loss} = \lambda \sum_{n=0}^{\infty} \pi_{n,0,0} + \lambda \sum_{n=0}^{\infty} \sum_{i=0}^{S} \pi_{n,i,2}$$
$$= \lambda \pi_0 (I - R)^{-1} \delta_5 + \lambda \pi_0 (I - R)^{-1} \delta_6,$$

where $\delta_5 = (1, 0, 0, ..., 0)^T{}_{1\times(3S+2)}$, and δ_6 is defined as $\delta_6 = (0, 1, 0, 0, 1, ...,)^T{}_{1\times(3S+2)}$.

(6) Service interruption rate:

$$E_i = \sum_{n=0}^{\infty} \sum_{i=0}^{S} \alpha \pi_{n,i,0} = \alpha \pi_0 (I - R)^{-1} \delta_7,$$

where $\delta_7 = (1, 0, 1, 0, 0, ...)^T{}_{1\times(3S+2)}$.

(7) Server repair rate:

$$E_r = \sum_{n=0}^{\infty} \sum_{i=0}^{S} \beta \pi_{n,i,2} = \beta \pi_0 (I - R)^{-1} \delta_6.$$

V. SENSITIVITY ANALYSIS

In this section, we conduct numerical experiments to investigate the sensitivity of the system's performance measures to variations in its parameters.

The figures from 5-1 to 5-4 illustrate the impact of parameter $\alpha, \theta, \lambda, \eta, \mu_v, \beta$ on the average inventory level, E_{inv}.

Figure 5-1 reveals that the average inventory level ascends with the augmentation of α and descends with the escalation of θ. This is attributed to the circumstance that a larger α gives rise to a shorter the average service lifespan, inducing a higher likelihood of service failure and slower inventory depletion, thereby causing a higher inventory level. Conversely, a greater θ brings about shorter average vacation time, longer busy periods within the system, higher service rates during busy periods, faster service velocities, and more rapid inventory consumption, ultimately resulting in a lower average inventory level.

Figure 5-2 depicts that the average inventory level rises along with an augmentation in η and drops with an increase in λ. This is attributed to the fact that a greater η leads to a shorter average replenishment time, giving rise to the timely arrival of replenishment and subsequently elevating the average inventory level. On the contrary, a larger λ is equivalent to a higher customer arrival rate, causing greater inventory consumption and consequently lower average inventory levels.

Figure 5-3 demonstrates that the average inventory level decreases as μ_v increases. This is because a larger value of μ_v leads to a higher service rate, accelerating inventory consumption and thereby reducing the average inventory level.

Figure 5-4 illustrates that the average inventory level decreases as β increases. This is due to the fact that a larger value of β results in shorter repair times, reducing the duration of system faults and leading to decreased inventory levels once normal operations resume.

Figures 5-5 through 5-8 demonstrate the impact of parameters $\alpha, \theta, \lambda, \eta, \mu_v, \beta$ on the interruption rate E_i.

As depicted in Figure 5-5, the service interruption rate rises with increasing parameter α and declines with increasing parameter θ. This can be attributed to the fact that higher values of α lead to shorter average service lifespans and increased failure probabilities, resulting in higher service interruption rates. Conversely, larger values of θ correspond to shorter average vacation times and greater probabilities for system activity during busy periods without service desk failures, thereby reducing service interruption rates.

As depicted in Figure 5-6, the service interruption rate decreases as the parameter η increases and increases with the parameter λ rise. A larger value of η results in a shorter average replenishment time, timely arrival of replenishment, prolonged busy periods at the

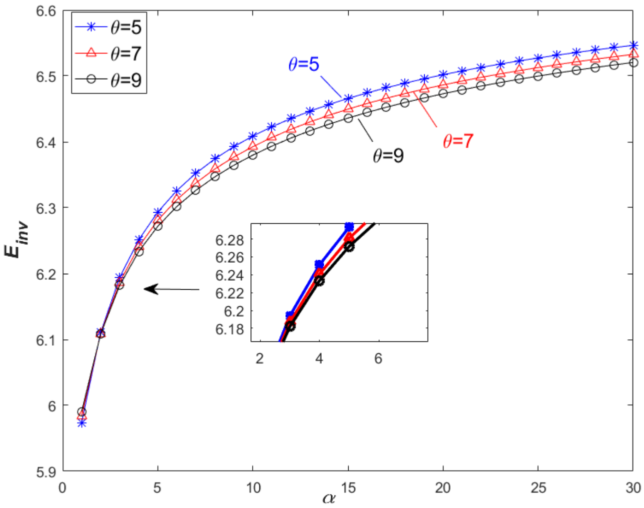

Figure 5-1. Effect of parameters α and θ on the expected stock level. $((s, S, \lambda, \mu_v, \mu_b, \eta, \beta) = (5, 10, 11, 12, 19, 1, 0.8))$.

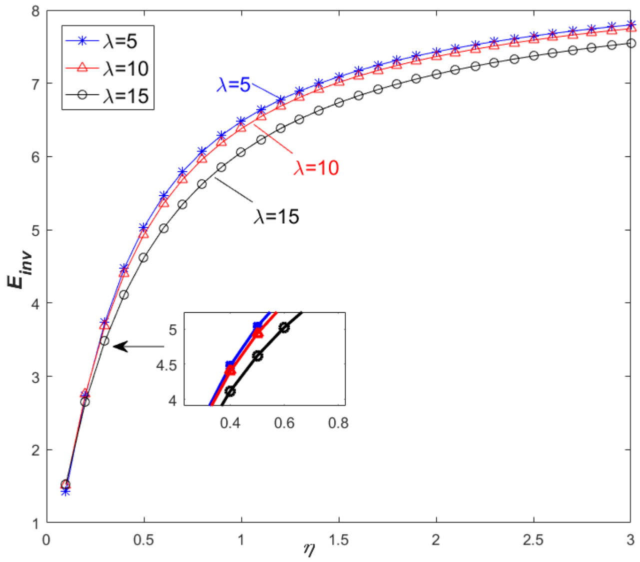

Figure 5-2. Effect of parameters η and λ on the expected stock level. $((s, S, \mu_v, \mu_b, \theta, \alpha, \beta) = (5, 10, 12, 19, 6, 7, 0.8))$.

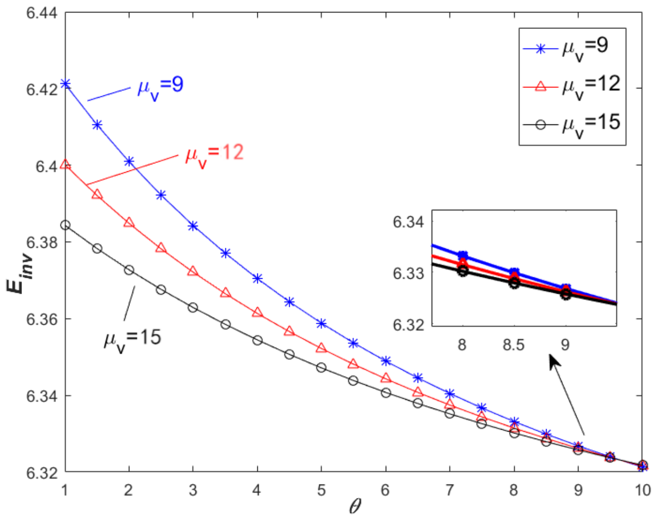

Figure 5-3. Effect of parameters θ and μ_v on the expected stock level. $((s, S, \lambda, \mu_b, \eta, \alpha, \beta) = (5, 10, 11, 19, 1, 7, 0.8))$.

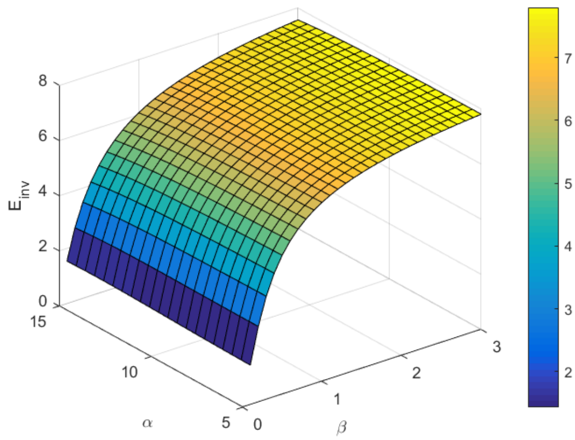

Figure 5-4. Effect of parameters β and α on the expected stock level. $((s, S, \lambda, \mu_v, \mu_b, \eta, \theta) = (5, 10, 11, 12, 19, 1, 6))$.

service, reduced likelihood of entering a work vacation state, lower susceptibility to malfunctioning, and decreased service interruption rate. Conversely, a higher value of λ leads to an increased customer arrival rate, greater inventory depletion by customers, and faster consumption of inventory leading to lower average inventory levels. This makes it easier for the system to enter a work vacation state and increases the probability of breakdowns, resulting in a higher service interruption rate.

Figure 5-7 illustrates that the service interruption rate decreases with an increase in the parameter μ_v. This is due to larger values of μ_v resulting in reduced failure probability and subsequently lowering the service interruption rate.

As shown in Figure 5-8, the service interruption rate increases with parameter β increase. This is because larger values of β lead to shorter repair times and consequently reduce fault period duration. As a result, systems return to normal operation more quickly but are also more likely to fail again; thus increasing the overall service interruption rate.

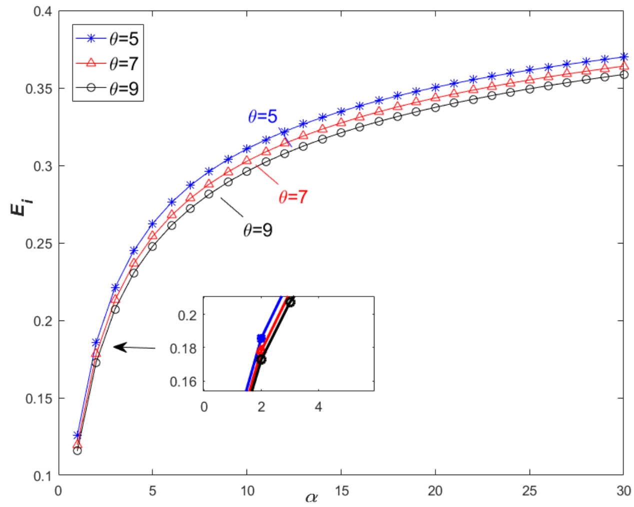

Figure 5-5. Effect of parameters α and θ on the service interruption rate. $((s, S, \lambda, \mu_v, \mu_b, \eta, \beta) = (5, 10, 11, 12, 19, 1, 0.8))$.

The parameters $\alpha, \beta, \theta, \lambda, \eta, \mu_v$ in figures 5-9 to 5-12 correspond to the average replenishment rate E_{rep}.

As depicted in Figure 5-9, the impact of parameter θ on the average replenishment rate is associated with the attendants' average service life. When α is small, an increase in θ decreases the average replenishment rate. This can be attributed to the relatively large safety inventory level ($s = 5$) and maximum inventory

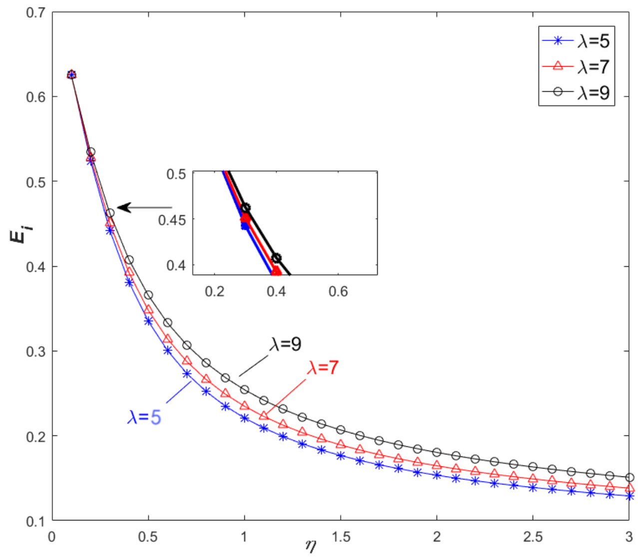

Figure 5-6. Effect of parameters η and λ on the service interruption rate. $((s, S, \mu_v, \mu_b, \theta, \alpha, \beta) = (5, 10, 12, 19, 6, 7, 0.8))$.

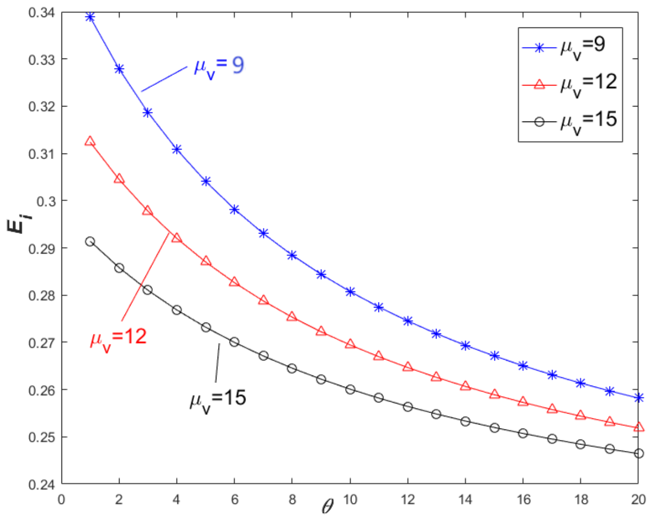

Figure 5-7. Effect of parameters θ and μ_v on the service interruption rate. $((s, S, \lambda, \mu_b, \eta, \alpha, \beta) = (5, 10, 11, 19, 1, 7, 0.8))$.

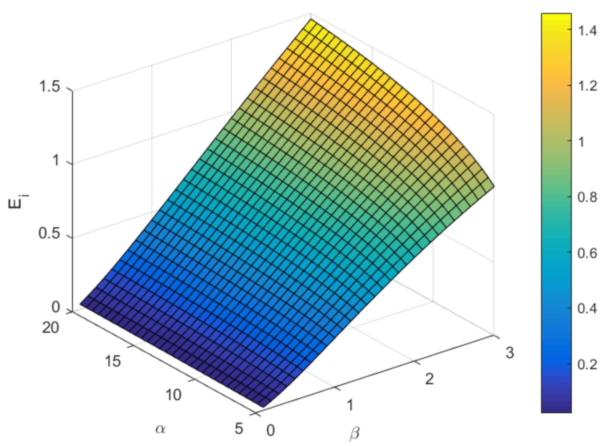

Figure 5-8. Effect of parameters β and α on the service interruption rate. $((s, S, \lambda, \mu_v, \mu_b, \eta, \theta) = (5, 10, 11, 12, 19, 1, 6))$.

level ($S = 10$) selected by the system, resulting in a relatively large average replenishment amount and decreasing the overall replenishment rate. Conversely, when α is larger, an increase in θ results in an increase in the average replenishment rate. A larger θ corresponds to shorter average vacation times for attendants, leading to longer busy periods for the system and faster drops in inventory levels, thereby increasing the replenishment rate.

As is evident from Figure 5-10, the average replenishment rate escalates with η and λ. This is because the greater the η, the shorter the average replenishment time, the more timely the replenishment arrives, the longer the service is occupied, the higher the customer's demand for products, and thus the average replenishment rate rises; The larger the λ, the higher the arrival rate and the stronger the demand for inventory, leading to an increase in the average replenishment rate.

Figure 5-11 indicates that the average replenishment rate declines as the parameter μ_v increases. This is because the higher the service rate μ_v, the greater the increase in demand for inventory, the faster the inventory depletion. If the replenishment fails to arrive in time, the system enters the state of empty inventory. At this point, new customers in the system do not enter, and the system is in a stagnant state waiting for replenishment, thereby reducing the average replenishment rate.

As is evident from Figure 5-12, the average replenishment rate E_{rep} declines with the augmentation of parameter α and ascends with the escalation of parameter β.This is because the greater the α, the shorter the average lifespan of the service, the higher the probability of failure, the slower the depletion of the inventory level, thereby reducing the average replenishment rate and increasing the average replenishment time. The larger the β, the shorter the repair duration, the shorter the time the system is in the fault period, the quicker the system reverts to the normal working state, and the inventory consumption also rises, thus increasing the replenishment rate.

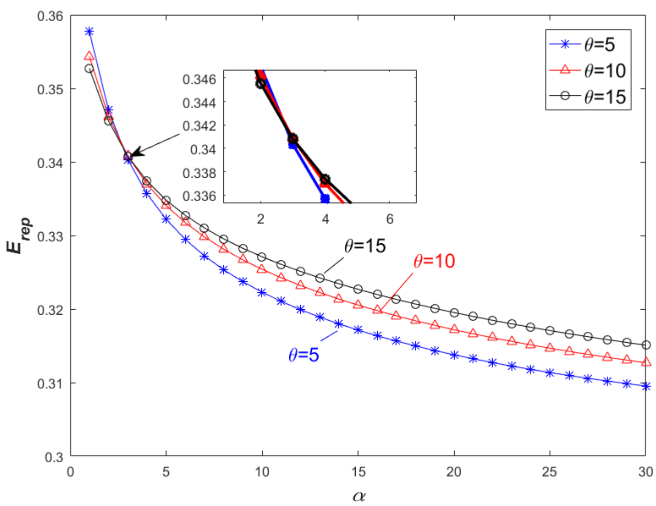

Figure 5-9. Effect of parameters α and θ on the expected reorder rate. $((s, S, \lambda, \mu_v, \mu_b, \eta, \beta) = (5, 10, 11, 12, 19, 1, 0.8))$.

Figure 5-13 to Figure 5-16 demonstrate the influence of the parameters $\alpha, \beta, \theta, \lambda, \eta, \mu_v$ on the average customer loss rate E_{loss}.

It can be observed from Figure 5-13 that the impact of parameter θ on the average customer loss rate is associated with the average service life α. When α is small, the average customer loss rate decreases as θ decreases. This is because when α is smaller, the average service lifespan is longer. When θ is smaller, the average vacation time is longer, and the system is less prone to failure, thereby resulting in a decrease in E_{loss}. When α is larger, the average customer loss rate increases as θ decreases. This is because when α is larger, the average service lifespan is shorter. When θ is smaller, the vacation time is longer, and the possibility of system failure increases at this point, thus causing the average customer loss rate to increase.

Figure 5-14 indicates that the average customer loss rate E_{loss}

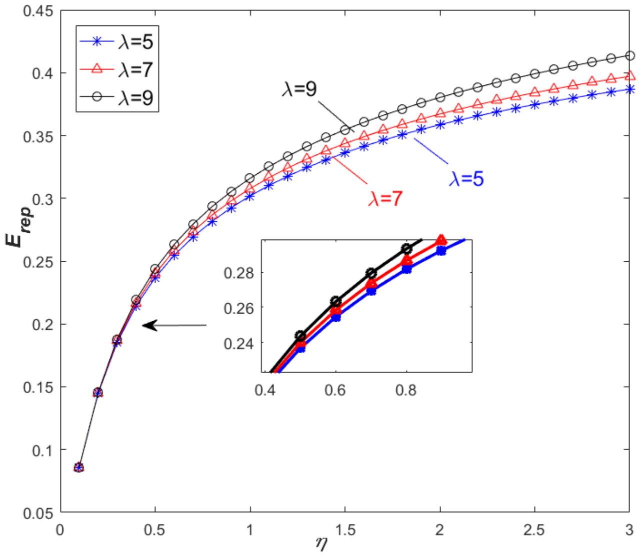

Figure 5-10. Effect of parameters η and λ on the expected reorder rate. $((s, S, \mu_v, \mu_b, \theta, \alpha, \beta) = (5, 10, 12, 19, 6, 7, 0.8))$.

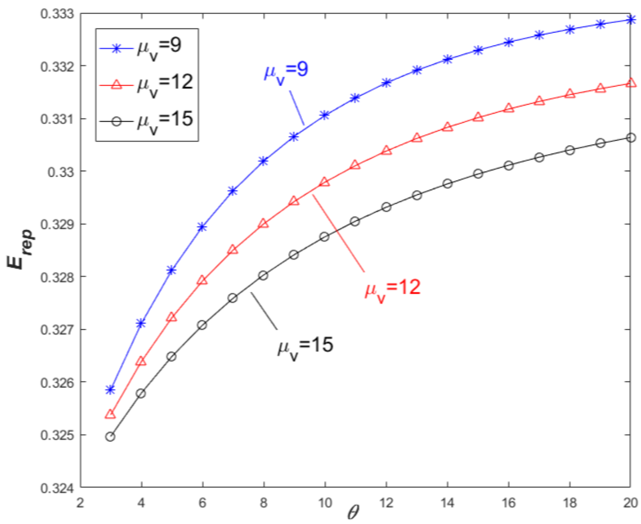

Figure 5-11. Effect of parameters θ and μ_v on the expected reorder rate. $((s, S, \lambda, \mu_b, \eta, \alpha, \beta) = (5, 10, 11, 19, 1, 7, 0.8))$.

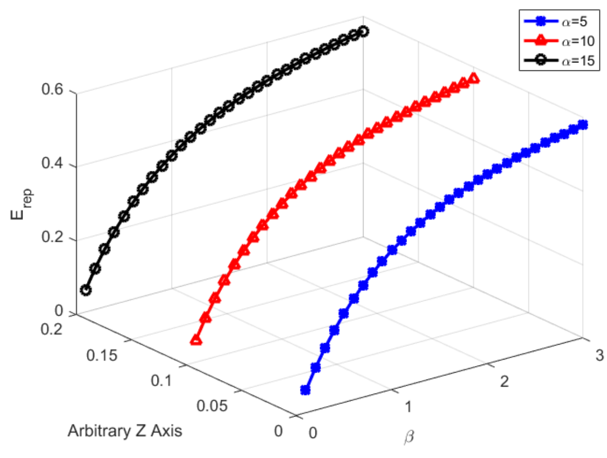

Figure 5-12. Effect of parameters β and α on the expected reorder rate. $((s, S, \lambda, \mu_v, \mu_b, \eta, \theta) = (5, 10, 11, 12, 19, 1, 6))$.

declines as η increases and rises with the escalation of λ. This is due to the fact that the greater the η, the shorter the replenishment time is. With the timely arrival of replenishment, the system is less likely to enter the working vacation stage and less prone to have empty inventory, thereby reducing E_{loss}. The larger the λ, the higher the customer arrival rate becomes. The increased consumption of inventory makes the system more prone to enter the working vacation state, susceptible to failure or empty inventory. At this point, newly arrived customers no longer enter the system, resulting in an increase in the average customer loss rate.

As is evident from Figure 5-15, the average customer loss rate escalates with the growth of parameter μ_v. The greater the μ_v, the higher the service rate, the quicker the inventory consumption, and the system is prone to enter the state of empty inventory, thereby resulting in an increase in the average loss rate.

Figure 5-16 indicates that the average customer loss rate rises along with the growth of parameter α and drops with the increase of parameter β. This is because the greater the α, the shorter the average service lifespan, and the more prone the system is to failure. When a failure occurs, the system suspends its service, thereby causing the average loss rate to increase. The larger the β, the shorter the average repair time, the quicker the system is restored, and the shorter the period during which the system is in a fault state, thus reducing the average customer loss rate.

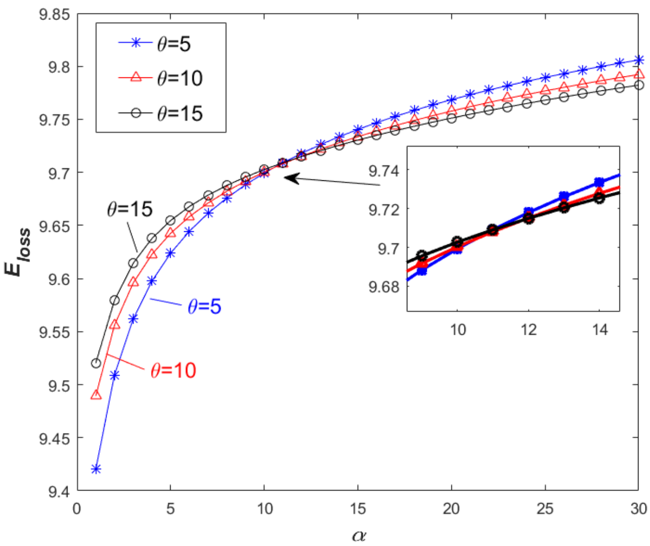

Figure 5-13. Effect of parameters α and θ on the expected loss rate. $((s, S, \lambda, \mu_v, \mu_b, \eta, \beta) = (5, 10, 11, 12, 19, 1, 0.8))$.

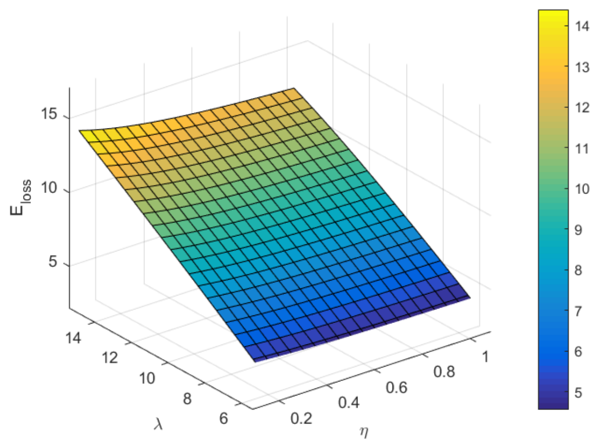

Figure 5-14. Effect of parameters η and λ on the expected loss rate. $((s, S, \mu_v, \mu_b, \theta, \alpha, \beta) = (5, 10, 12, 19, 6, 7, 0.8))$.

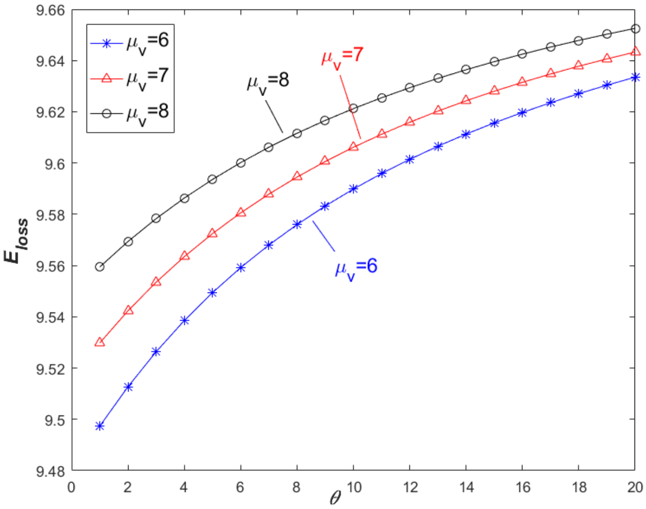

Figure 5-15. Effect of parameters θ and μ_v on the expected loss rate. $((s, S, \lambda, \mu_b, \eta, \alpha, \beta) = (5, 10, 11, 19, 1, 7, 0.8))$.

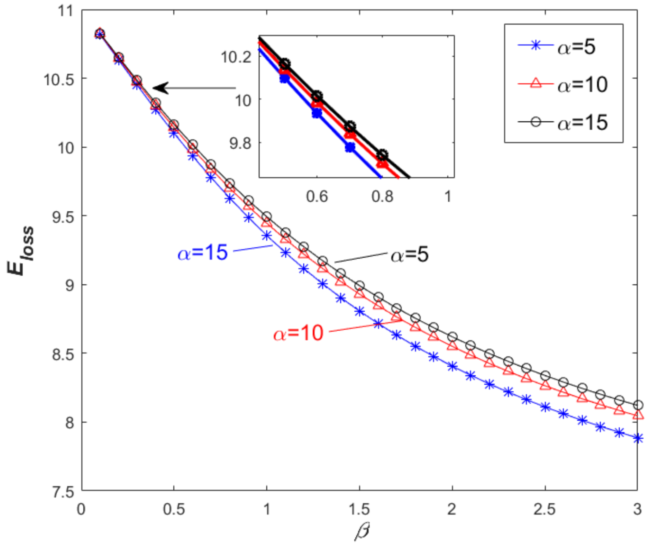

Figure 5-16. Effect of parameters β and α on the expected loss rate. $((s, S, \lambda, \mu_v, \mu_b, \eta, \theta) = (5, 10, 11, 12, 19, 1, 6))$.

VI. COST ANALYSIS AND NUMERICAL RESULTS

Based on the system performance measures, define a cost function $F(s, S)$ of the system and is given by

$$F(s, S) = C_1 E_L + C_2 E_{inv} + C_3 E_{rep} E_p + C_4 E_{rep} + C_5 E_r + C_6 E_{loss} + C_7 E_i$$

where C_1 is the waiting cost per unit of time for customers; C_2 is the inventory holding cost per item per unit of time; C_3 is the reordering cost per unit of product; C_4 is the fixed order cost per replenishment; C_5 is the repair cost per unit of time; C_6 is the cost per unit of time per unit of new arrivals not being able to enter the system; C_7 is the cost per unit of time caused by the service interruption.

Firstly, the value scopes of the safety inventory level s and the maximum inventory level S are delineated, and numerical experiments are executed by means of the genetic algorithm to regulate the values of each parameter within the scopes, thereby ensuring that the optimal inventory strategy corresponds to the minimum cost. Based on this principle, the impact of parameter variations on the system inventory strategy and cost is investigated.

Suppose the parameters of the given system are

$$C_1 = 50,\ C_2 = 0.8,\ C_3 = 4, C_4 = 3.5,$$
$$C_5 = 18,\ C_6 = 12,\ C_7 = 16.$$

Table 6-1 exhibits the effect of the customer arrival parameter λ on the optimal inventory control strategy, while other parameters are set as $\mu_v = 12,\ \mu_b = 19,\ \eta = 1,\ \alpha = 7,\ \beta = 0.8,\ \theta = 6$.As presented in Table 6-1, with the escalation of parameter λ, the demand for inventory escalates, the system safety inventory level s and the system maximum inventory level S gradually ascend, and the cost gradually increases, and the impact on the optimal inventory strategy and cost of the system is notable.

Table 6-1. Impact of parameter λ on optimal inventory strategy and cost.

λ	7	9	11	13	15
(s,S)	(1,8)	(2,12)	(3,19)	(6,28)	(10,35)
$F(s,S)$	104.6218	131.3809	160.2865	193.8386	243.2806

Table 6-2 presents the impact of the service rate parameter μ_v on the optimal inventory control strategy during system work vacations, and other parameters are set as $\lambda = 11,\ \mu_b = 19,\ \eta = 1,\ \alpha = 7,\ \beta = 0.8,\ \theta = 6$. As depicted in Table 6-2, with the escalation of the parameter μ_v, the service rate of attendants during working vacations rises, the system safety inventory level and the system maximum inventory level both decline marginally, and the cost gradually decreases, which has a negligible impact on the optimal inventory strategy and cost of the system.

Table 6-2. Impact of parameter μ_v on optimal inventory strategy and cost.

μ_v	5	7	9	11	13
(s,S)	(4,20)	(4,19)	(4,19)	(3,19)	(3,18)
$F(s,S)$	165.2437	163.4486	161.9897	160.7956	159.8261

Table 6-3 exhibits the influence of the service rate parameter μ_b on the optimal inventory control policy during the regular operation of the system. Other parameters are fixed at $\lambda = 11,\ \mu_v = 12,\ \eta = 1,\ \alpha = 7,\ \beta = 0.8,\ \theta = 6$. As presented in Table 6-3, as the parameter μ_b escalates, the service rate of the attendant in the normal working period ascends, the system safety inventory level s and the system maximum inventory level S both progressively decline, and the cost gradually reduces, exerting a certain effect on the optimal inventory strategy and cost of the system.

Table 6-3. Impact of parameter μ_b on optimal inventory strategy and cost.

μ_b	10	12	14	16	18
(s,S)	(10,30)	(6,27)	(4,21)	(3,16)	(2,14)
$F(s,S)$	197.6323	170.7839	162.4953	158.6780	156.5284

Table 6-4 elucidates the influence of the replenishment parameter η on the optimal inventory control strategy, while the other parameters are set to $\lambda = 11,\ \mu_v = 12,\ \mu_b = 19,\ \alpha = 7,\ \beta = 0.8,\ \theta = 6$. As depicted in Table 6-4, with the escalation of parameter η, the average replenishment time diminishes, the replenishment arrives punctually, the system safety inventory level (s) increases marginally but not conspicuously, the system maximum inventory level (S) drops significantly, and the cost gradually declines, exerting a certain degree of impact on the optimal inventory strategy and cost of the system.

Table 6-4. Impact of parameter η on optimal inventory strategy and cost.

η	0.5	0.7	0.9	1.1	1.3
(s,S)	(2,23)	(3,21)	(3,19)	(3,18)	(4,16)
$F(s,S)$	166.4190	163.2793	161.1324	159.5416	158.2559

Table 6-5 demonstrates the effect of the work leave parameter θ on the optimal inventory control strategy, while other parameters are fixed at $\lambda = 11,\ \mu_v = 12,\ \mu_b = 19,\ \eta = 1, \alpha = 7,\ \beta = 0.8$. As depicted in Table 6-5, as the parameter θ increases, the average working vacation time of attendants declines, the system safety inventory level s slightly reduces, the system maximum inventory level S initially rises and then drops, and the cost gradually declines. However, none of these changes are significant, and the effect on the optimal (s,S) inventory strategy and cost of the system is negligible.

Table 6-5. Impact of parameter θ on optimal inventory strategy and cost.

θ	3	5	7	9	11
(s,S)	(4,18)	(3,19)	(3,19)	(3,19)	(3,18)
$F(s,S)$	162.2217	160.8182	159.8414	159.1421	158.6192

Table 6-6 presents the influence of the service life parameter α on the optimal inventory control strategy. Other parameters are fixed at $\lambda = 11,\ \mu_v = 12,\ \mu_b = 19,\ \eta = 1, \theta = 6,\ \beta = 0.8$. As shown in Table 6-6, as the parameter α increases, the average service life of attendants decreases, the system becomes more prone to failure, the system safety inventory level s and the system maximum inventory level S both increase marginally and then tend to stabilize, and the cost gradually rises, exerting a certain degree of impact on the optimal inventory strategy and cost of the system.

Table 6-6. Impact of parameter α on optimal inventory strategy and cost.

α	0.5	2.0	3.5	5.0	6.5
(s,S)	(2,16)	(3,18)	(3,19)	(3,19)	(3,19)
$F(s,S)$	151.1919	156.1145	158.1745	159.3417	160.0933

Table 6-7 exhibits the effect of the repair time parameter β on the optimal inventory control strategy, while other parameters are fixed at $\lambda = 11,\ \mu_v = 12,\ \mu_b = 19,\ \eta = 1,\ \theta = 6,\ \alpha = 7$. As shown in Table 6-7, with the increase of parameter β, the average maintenance duration of the system is curtailed, the time of the system in the fault period is decreased, the system safety inventory level s is gradually lowered, the system maximum inventory level S is gradually raised, and the cost is gradually augmented, exerting a certain extent of influence on the optimal inventory strategy and cost of the system.

Table 6-7. Impact of parameter β on optimal inventory strategy and cost.

β	0.2	0.4	0.6	0.8	1.0
(s,S)	(6,13)	(5,15)	(4,17)	(3,19)	(3,20)
$F(s,S)$	151.3208	154.4915	157.4570	160.2865	162.9874

It is conspicuously evident from the comprehensive comparison presented in Figure 6-1 that parameters λ, μ_v, η, α, and β exert a significant influence on the optimal cost. The optimal cost function $F(s,S)$ gradually ascends with the increase of parameter lambda and α, β, while it descends with the augmentation of parameter μ_v, η. The influence of work leave time and service rate parameters on the optimal cost function is regarded as insignificant. The most influential parameters λ and β are analyzed as three-dimensional space curves to illustrate their effects on safety stock s, maximum stock S, and cost $F(s,S)$, as shown in Figures 6-2 and 6-3.

VII. CONCLUSION

Based on the M/M/1 queuing inventory system with working vacation, this paper presents the possibility of failure during working vacation, investigates the queuing inventory model of the (s,S) strategy that can fail during working vacation, constructs the state transition matrix, and establishes the three-dimensional Markov chain of the customer number, inventory level, and service status. By using the matrix geometric solution to solve the steady-state equilibrium condition and the steady-state probability vector, the average queue length, average inventory

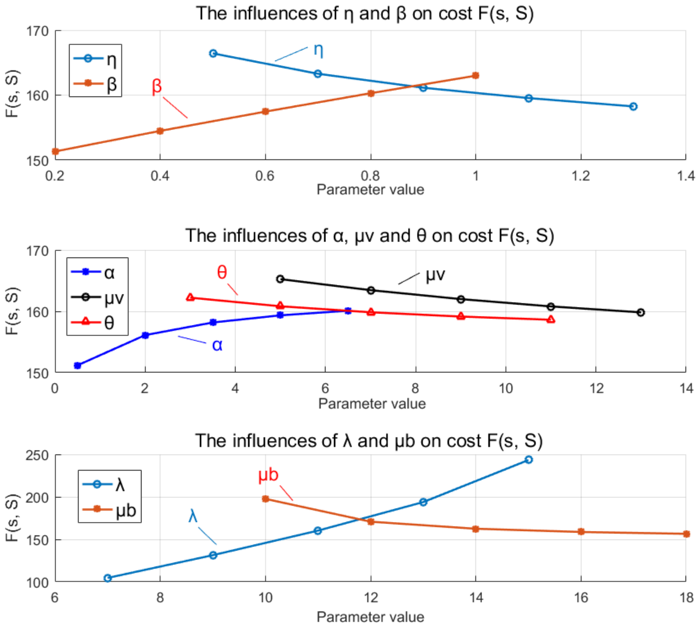

Figure 6-1. The influence of each parameter on the cost $F(s, S)$.

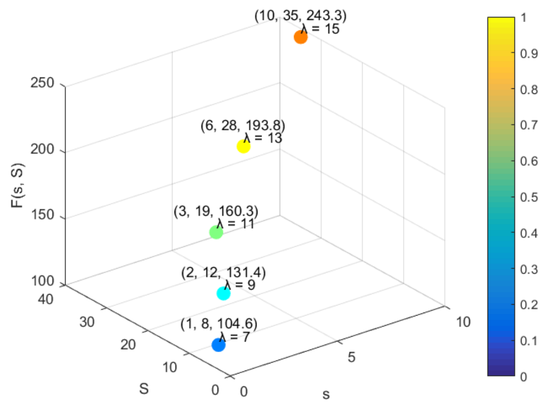

Figure 6-2. Impact of parameter λ on optimal inventory strategy and cost.

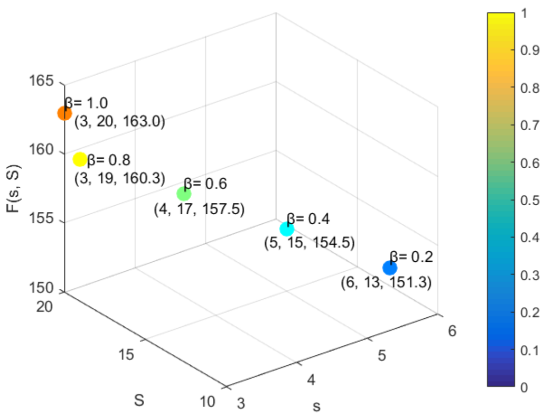

Figure 6-3. Impact of parameter β on optimal inventory strategy and cost.

level, replenishment rate, interruption rate, maintenance rate, and other performance indicators related to queuing inventory were calculated. Furthermore, the impact of the system parameter $\lambda, \mu_v, \eta, \alpha, \beta\theta$ on the system performance index is explored through numerical analysis. On this basis, the cost function is constructed, and the influence of $\lambda, \mu_v, \mu_b, \eta, \alpha, \beta, \theta$ on the optimal cost function $F(s, S)$ is further examined by the genetic algorithm. The results indicate that the customer arrival parameter λ, the service rate parameter μ_b during normal operation, the replenishment rate parameter η, the service life α, and the maintenance rate parameter β have significant influences on the optimal cost. The optimal cost gradually increases with the increase of the parameters η, α, β and decreases with the increase of the parameters μ_b and η. The influences of θ and μ_v on the optimal cost function are not significant.

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