Optimization Plan of EMU Operation Based on Variable Grouping And Unfixed Decoupling Position

Zhiqiang Tian, Boyuan Cai, Junfeng Zhang, Wanpeng Wu, Weigang Yue

Abstract—**The operation plan of EMU (Electric Multiple Unit) trains plays a crucial role in railroad transportation production. Through the reasonable preparation of variable grouping, an EMU operation plan can optimize the trains' operational efficiency, improve transport safety, enhance the level of passenger service, and reduce operating costs. This paper describes the optimization process of variable grouping through the connection network. The objectives are to minimize the costs of the number of EMUs used, connection time, and maintenance. The constraints include connection conditions, maintenance conditions, and connection time requirements. A model is constructed to optimize EMU operation through variable grouping and unfixed decoupling position. To facilitate solving with Gurobi's commercial solver, nonlinear constraints are linearized. Examples are analyzed to validate the model, confirming that operational efficiency improves with unfixed EMU decoupling positions compared to fixed depot decoupling.**

Index Terms—**EMU Operation Plan, Connection Network, Unfixed Decoupling Position, Linearization**

I. Introduction

By the end of 2023, the scale of the railway network had further expanded. The operating mileage of railways nationwide reached 159,000 kilometers, of which high-speed railways reached 45,000 kilometers, accounting for 28.3 percent of the operating mileage of railways nationwide. The nationwide fleet included 78,400 railway coaches, with 35,416 being EMUs, accounting for 45% of the total number of railway coaches owned nationally [1]. The EMU will continue to exhibit its characteristic of speed and safety, playing an increasingly crucial role in China's railway transport. The operation of EMUs is crucial to railway transport systems as a primary transportation strategy. Enhancing EMU operational efficiency through strategic planning aims to minimize inputs and maintenance intervals, thereby reducing costs and conserving resources. The adoption of variable grouping in EMU operations represents a progressive trend and a viable solution to these challenges.

Our scholars have conducted research on EMU operations, routing plans, and operational considerations under maintenance constraints. Lin et al. [2] examined the correlation between EMU routings, EMU quantity, and connection times. They developed an optimization model for EMU routing plans, prioritizing minimal EMU usage as the primary objective and minimizing train connection times as the secondary objective. The model was linearized and solved using a solver. Zhong et al. [3] employed a network approach to depict EMU operation challenges, decomposing them into a primary and a secondary problem. They formulated an optimization model for train service and devised an iterative approximation algorithm using path generation. Finally, they validated the model's soundness and effectiveness using a real-world example. Tong et al. [4] conducted a quantitative analysis of EMU train connection reliability, aiming to minimize total penalty costs. They addressed the unique challenges posed by maintenance or standby task connections of EMUs and constraints such as maintenance capacity. To solve this, they developed an enhanced ant colony algorithm and demonstrated that their optimized EMU operation plan can reduce the occurrence of train connection tasks with low reliability. Li et al. [5] investigated the computer preparation stage of the EMU routing plan, proposing simplified constraints for the computer solution stage based on human-computer interaction methods. They developed an optimization model for the EMU routing plan and designed a particle swarm optimization algorithm. Finally, they validated the effectiveness of the algorithm in solving the EMU routing plan using a practical example. Wang et al. [6] introduced an integer planning model for integrating EMU operation and maintenance plans. They developed a simulated annealing algorithm and validated its effectiveness through case studies that consider simultaneous operational and maintenance constraints. Their optimization goals included minimizing the number of EMUs in use, reducing maintenance costs, and balancing the workload of EMU depot maintenance.

Currently, research on variable grouping EMU problems can be categorized into two main areas, with one focusing on

Manuscript received May 7, 2024; revised September 11, 2024.

This research was supported by the National Natural Science Foundation (No.71761023, No.2161023, No.2361020), the Open Project of Key Laboratory of Intelligent Management and Control of Railway Industry in Plateau Railway Transportation (No. GYYSHZ2302), the Gansu Provincial Science and Technology Department Plan Project (No.22JR5RA379, No.22JR11RA159), and the Gansu Provincial Department of Education Higher Education Research Project (No.2022QB-060).

Zhiqiang Tian is a professor at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: tianzq@mail.lzjtu.cn).

Boyuan Cai is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (Corresponding author, phone: +86 19893170361, e-mail: caiboyuan001@163.com).

Junfeng Zhang is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: 2905537290@qq.com).

Wanpeng Wu is a director at the Beijing Group Company Dispatching Office, Beijing 100000, China. (e-mail: 770893602@qq.com).

Weigang Yue is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: yueweigang2022@163.com).

establishing theoretical foundations for variable grouping EMUs. Kang et al. [7] introduced the concept of variable grouping EMUs, emphasizing the importance of researching and developing these systems while delineating their distinctive operational characteristics. Finally, they analyzed optimal strategies for deploying variable grouping EMUs. Yang et al. [8] addressed the inadequacy of China's fixed grouping EMU capacity regulation. They conducted a detailed analysis of the grouping characteristics and advantages of variable-group EMUs abroad. Furthermore, they examined the application of variable-group EMUs in China's high-speed railway development. Zhai et al. [9] outlined the fundamental characteristics of variable-group EMUs, analyzed the factors influencing their introduction into China's market, suggested potential entry strategies, and concluded that variable-group EMUs have significant development potential on China's current and future high-speed railway lines. Jie et al. [10] introduced three operational modes for coupling and decoupling at EMU stations, analyzed the necessary conditions and requirements for these operations and examined their impact using China Railway Nanchang Bureau Group Company Limited as a case study. The second category involves studying the development of EMU operation plans under variable grouping conditions. Fu et al. [11] meticulously considered EMU-type consistency, coupling and decoupling connection conditions, and maintenance constraints such as mileage limitations for different maintenance levels. Their model aimed to minimize EMU coupling costs and reduce empty travel expenses by integrating EMU operation and maintenance under variable grouping conditions. They employed a non-dominated sequential genetic algorithm to address this complex optimization challenge effectively. Jiang et al. [12] investigated the issue of planning variable-group EMU operations, proposing an efficient approach for managing the coupling and decoupling processes of these EMUs. They developed a comprehensive mixed-integer planning model that considers maintenance constraints, connection conditions, and station capacity limitations of EMUs. Chen et al. [13] redefined the EMU operation challenge as a multi-traveler problem, setting up an EMU connection network. Their optimization objective was to minimize both EMU connection time costs and maintenance time costs. They developed a mixed-integer planning model using variable-group EMUs that adhered to conditions ensuring unique EMU connections, operational line succession, maintenance rules, and capacity constraints. Alfieri et al. [14] introduced a solution method employing an integer multi-commodity flow model to address EMU coupling and decoupling scenarios influenced by varying passenger flows throughout the day. They considered the specific sequencing of EMUs within trains and validated their approach through illustrative examples. Cordeau et al. [15] reformulated the EMU operation challenge into a multi-commodity flow problem. They defined the train sequence as a series of train services operated by trains, accounting for changes in train composition at the start and end of each sequence to reflect EMU coupling and decoupling. They established an EMU allocation and operation model based on these sequences and tackled the problem using methodologies such as branch-and-bound, Benders decomposition, and Dantzig-Wolfe decomposition.

Zhu et al. [16] have delved into the realm of flexible train grouping within urban rail transit in China. Their work introduces a concept of flexible train grouping, focusing on analyzing the carbon emission index associated with metro operations. They established a multi-objective planning model under constraints such as the maximum total load rate of trains and the number of trains in operation. To solve this model, they proposed an enhanced quantum genetic algorithm. Through their research, they demonstrated that flexible train grouping, compared to traditional fixed grouping plans, can notably enhance the average total load rate of metro lines without compromising service levels. Importantly, their approach also yields significant energy savings and reductions in emissions for metro operating companies. This was validated through comprehensive example analyses, affirming the effectiveness and practicality of their proposed model. Wan et al. [17] tackled the challenge of optimizing train capacity utilization in urban rail transit systems, particularly addressing issues of insufficient capacity during peak passenger flows and underutilization during off-peak periods. They introduced prospect theory to model passenger path selection behavior, aiming to minimize passenger travel costs, enterprise operating costs, and carbon emissions simultaneously. Their approach involved dividing passenger flows into full-length routes and short-turn routes and constructing a multi-objective optimization model for train operations with reconfigurable train formations. They utilized a quantum genetic algorithm to optimize this model effectively. To validate their approach, they applied it to a specific urban rail transit line, demonstrating the validity and feasibility of their model and algorithm design. Their findings indicated improvements in both operational efficiency and environmental impact mitigation compared to traditional fixed grouping strategies.

Currently, China's railway system has successfully implemented station coupling and decoupling of EMUs under no-load conditions, achieving notable advancements. This paper, therefore, focuses on the operational dynamics of unfixed decoupling positions for EMUs under no-load conditions. It investigates optimization methods for EMU operation plans in this context, aiming to manage station coupling, decoupling, and turnover operations strategically. The goal is to minimize EMU capacity resource wastage, thereby significantly enhancing the adequacy of railway transport supply and improving the efficiency of network resources. This study holds substantial practical significance in advancing supply-side structural reforms within the railway sector.

II. CONSTRUCTION OF EMU CONNECTION NETWORK

A. Problem description

Considering the utilization of China's EMU fleet, which predominantly consists of 8- and 16-car configurations (as depicted in Fig. 1), operational strategies vary based on passenger demand. During peak periods, two 8-car short-group trains can be coupled to form a 16-car long-group train for continuous operation. Conversely, during off-peak times, a 16-car long-group train can be decoupled into two

8-car short-group trains to maintain service continuity.

This study addresses the operational logistics of EMU utilization, particularly the grouping requirements for long-group and short-group trains. It proposes the breakdown of a 16-car long-group train into two 8-car short-group trains. Both configurations share identical departure and arrival station information, times, and travel distances. This necessitates the efficient assignment of EMU trains to fulfill service tasks effectively. Based on the variability in EMU grouping conditions, this paper proposes an operational method where the coupling and decoupling positions of EMUs are not fixed. This approach eliminates the need for EMUs to enter depots for coupling and decoupling procedures, enabling these operations to be conducted directly at stations. The aim is to enhance the efficiency of EMU utilization across the railway network.

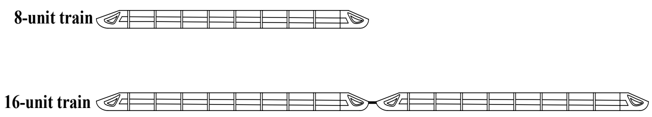

Fig. 1. Types of EMU

B. EMU connection network

The problem of EMU operation is how to arrange the connection relationship between trains reasonably, so we can consider using the connection network to describe the situation. The construction method is as follows:

B.1 Build nodes

(1) Train node

Each train service in the train diagram is represented as a node, categorized into long-group and short-group nodes. Each node includes detailed information such as the train's departure station, destination station, departure time, arrival time, grouping classification, and distance traveled.

(2) EMU depot nodes

The EMU depot is treated as a node in the network, symbolizing the maintenance facility where EMUs start and end their routes. Each depot node includes information regarding the duration that EMUs spend undergoing maintenance at the facility.

B.2 Build task connection arc

(1) Train connection arc

The connection arc denotes the linking relationship between two trains, with the weight on the arc representing the connection time between them. For connections involving short-group or long-group EMUs, this time encompasses the minimum preparation operation duration. When connecting a short-group EMU to a long-group EMU, the connection time incorporates both the minimum preparation operation time and the coupling operation time. Conversely, when connecting a long-group EMU to a short-group EMU, the connection time includes the minimum preparation operation time and the decoupling operation time.

(2) The arc starting from the EMU depot

The arc begins at the EMU depot node and ends at the train service node, signifying that the EMU departing from the depot is assigned to perform the duties of the designated train service.

(3) The arc entering the EMU depot

The arc originates at the train service node. It terminates at the EMU depot node, indicating that the EMU returns to the depot for maintenance and preparation after completing its route task.

B.3 Connection network

The connecting network of the EMU constructed according to the above method is shown in Fig. 2.

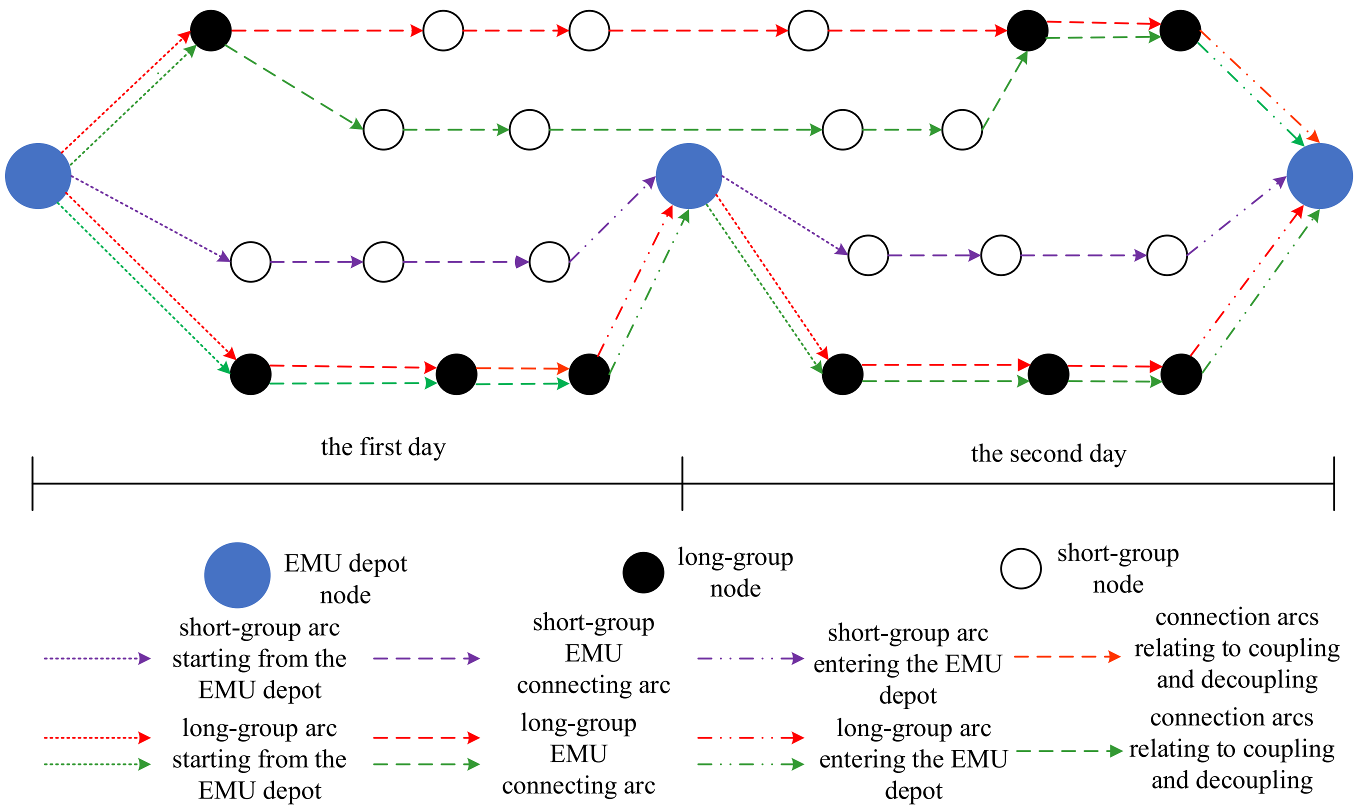

Fig. 2. Train connection network diagram

In this paper, we investigate the operation of EMUs within a 2-day time cycle, exploring two types of routing in the connection network diagram. The first type involves daily interchanges, where the routing contents on the first and second days are identical. That is to say, after the first day of operation of the train set to return to the depot, the next day to continue to carry out the same as the first day of operation of the task. The second type includes two-day routing, where the routing contents differ between the first and second days. That is, the train set in the first day of the end of the operation of the task back to the train station or stop at this station, the next day to carry out the operation of the task is different from the first day (the figure assumes that the station has the parking conditions).

III. Optimization model of EMU operation plan

A. Model assumption

To facilitate the study of the EMU operational plan under variable grouping conditions within one cycle, this paper makes the following assumptions during modeling:

(1) Before preparing the EMU operation plan, the study assumes prior knowledge of arrival and departure times, the grouping size, and the operational mileage for all trains.

(2) In the study area, there is only one type of EMU, and the distinction between trains is solely based on the number of groups. There are no restrictions concerning the kind of EMU when connecting two EMUs.

(3) The EMU depot supplies standard EMUs, which are categorized into required EMUs and standby EMUs. The cumulative mileage of each EMU at the start of its operating cycle is zero.

(4) The study does not account for the positioning movements of EMUs entering and leaving the depot nor for empty runs between operational lines.

(5) Train tasks in operation have their group fixed, there is no decoupling with passengers, and all decoupling occurs before starting or after finishing the execution of the training task.

B. Mathematical model

B.1 Model parameters and variables

The parameters and variables involved in the model are shown in Table I.

TABLE I
PARAMETERS AND VARIABLES INVOLVED IN THE MODLE

S	The station set, consists of station s
K	The EMU depot set, consists of EMU depot k
R	The EMU set， $R=a\bigcup b$ ， consists of station r
a	Necessary EMU set, $a\in R$
b	Standby EMU set, $b\in R$
I	Running line set， $I=P\bigcup O$ ， consists of running line i
P	Long-group EMU set，The train service i in the set is decomposed into two identical short-group trains i',i'' , and a new set P is obtained, $P\in I$,where the actual train servicer of i',i'' is $Q_{i'},Q_{i''}$
O	Short-group EMU set, $O\in I$
t_i^d	The departure time of running line i
t_i^a	The arrival time of running line i
s_i^d	The departure station of running line i
s_i^a	The arrival station of running line i
l_i	The running distance of running line i
t_i	The running time of running line i
L	Maximum operating mileage of primary maintenance
ΔL	Overtime maintenance parameters of primary maintenance mileage
$T_{\min}$	The minimum preparation operation time for EMU connection
T_o	Primary maintenance operation time
T_c	EMU coupling operation time
T_u	EMU decoupling operation time
M	A very large positive integer
$c_{i,j}$	The connection time between running line i and running line j
α	Penalty parameter for the number of EMU used
β	Total connection time penalty parameter
γ	Penalty parameter for maintenance frequency of EMU
$L_{r,i}$	Integer auxiliary variable, EMU r completes the cumulative running mileage of running line i
$\lambda_{i,j}$	0-1 auxiliary variable, if the connection interval between train i and train j has the primary maintenance condition, take 1, otherwise take 0
$x_{k,r}$	0-1 decision variable, if the EMU depot k provides standard EMU r , take 1, otherwise take 0
$u_{k,r}^i$	0-1 decision variable, if the EMU r provided by the EMU depot k undertakes the train service i , take 1, otherwise take 0
$y_{k,r}^{i,j}$	0-1 decision variable, if the EMU r provided by the EMU depot k completes the train task i and then continues the train j , take 1, otherwise 0
$z_{k,r}^{i,j}$	0-1 decision variable, if the EMU r provided by the EMU depot k performs the train task i and then carries out maintenance and then connects the train j , take 1, otherwise 0

B.2 The value of running connection time parameter

$$c_{i,j}=\begin{cases}t_j^d-t_i^a,t_j^d>t_i^a\\ t_j^d-t_i^a+1440,t_j^d<t_i^a\end{cases} \tag{1}$$

Formula (1) expresses that when the departure time of trip i is greater than the arrival time of trip j , the connection time between trip i and trip j is the difference between the two. When the departure time of trip i is less than the arrival time of trip j , the connection time between trip i and trip j is the difference between the two plus one day's time (1440min), that is, at the end of the day, the train returns to the depot or stops at the station with overnight conditions to continue the operation of the next day's mission.

B.3 Objective function

(1) The purchase cost of EMU is the least

The procurement and maintenance costs associated with the core hardware of high-speed rail systems, particularly the EMUs, are substantial. Variations in the number of EMUs directly impact costs related to crew and maintenance personnel. Thus, minimizing the number of EMUs represents the most effective strategy for reducing economic expenditures. This rationale is encapsulated in the objective function (2).

$$Z_1 = \alpha \sum_{k \in K} \sum_{r \in R} x_{k,r} \tag{2}$$

(2) The cost of EMU connection time is the least

In a feasible connection scheme, optimizing to minimize connection times enhances train connections, reduces inter-cycle operations, and boosts Electric Multiple Unit EMU efficiency by enabling more tasks within a given period. The objective function (3) encapsulates this optimization goal.

$$Z_2 = \beta \sum_{k \in K} \sum_{r \in R} \sum_{i \in I} \sum_{j \in I} c_{i,j} * y_{k,r}^{i,j} \tag{3}$$

(3) The maintenance cost of EMU is the least

When the distance or time of operation for each group of EMUs approaches the first-level maintenance threshold, they are required to return to the nearest maintenance facility for servicing. Minimizing the frequency of these maintenance operations is crucial for reducing EMU maintenance costs. The objective function (4) is formulated to achieve this objective.

$$Z_3 = \gamma \sum_{k \in K} \sum_{r \in R} \sum_{i \in I} \sum_{j \in I} z_{k,r}^{i,j} \tag{4}$$

In summary, the objective function formulated in this study aims to minimize the costs associated with EMU procurement, EMU connection times, and EMU maintenance. The final objective function is expressed as follows (5).

$$Z = Z_1 + Z_2 + Z_3 \tag{5}$$

B.4 Constraint condition

(1) Required EMU constraint

$$\sum_{k \in K} \sum_{r \in R} x_{k,r} = 1 \ , \forall k \in K, r \in a \tag{6}$$

Formula (6) specifies that within the EMU operational plan, EMUs stored in depots are categorized into required EMUs and standby EMUs. In practical implementation, each EMU depot approximates the number of necessary EMUs based on operational task volumes.

(2) Transport task coverage constraint

$$\sum_{k \in K} \sum_{r \in R} u_{k,r}^{i} = 1, \forall i \in I \tag{7}$$

The EMU operation plan is developed, assuming the train diagram is fixed. Equation (7) specifies that each task within the train diagram is assigned to be executed by a single train.

(3) Uniqueness constraint of train connection

$$\sum_{k \in K} \sum_{r \in R} \sum_{i \in I, i \neq j} y_{k,r}^{i,j} = 1, \forall j \in I, i \neq j \tag{8}$$

$$\sum_{k \in K} \sum_{r \in R} \sum_{j \in I, i \neq j} y_{k,r}^{i,j} = 1, \forall i \in I, i \neq j \tag{9}$$

Formulas (8) and (9) demonstrate that each train can have only one designated lead train and one designated rear train. This constraint is logically expressed to enforce uniqueness.

(4) EMU constraints provided by EMU depot

$$x_{k,r} \leq \sum_{i \in I} u_{k,r}^{i}, \quad \forall k \in K, r \in R \tag{10}$$

$$x_{k,r} \geq \sum_{i \in I} u_{k,r}^{i} / M, \quad \forall k \in K, r \in R \tag{11}$$

Formulas (10) and (11) indicate that the EMU depot is not required to provide an EMU if it is not assigned to any tasks in the train diagram. However, if an EMU is assigned to any tasks within the train diagram, the depot must provide that EMU.

(5) EMU connection condition constraint

$$y_{k,r}^{i,j} \leq u_{k,r}^{i} \cdot u_{k,r}^{j}, \forall k \in K, r \in R, i, j \in I, i \neq j \tag{12}$$

Formula (12) indicates that only when the EMU r undertakes the train operation task i and the train operation task j, can the train operation task j be continued after the i operation task is completed.

(6) EMU connection location constraint

$$y_{k,r}^{i,j} \leq 1, \forall k \in K, r \in R, i, j \in I, i \neq j, s_i^a = s_j^d \tag{13}$$

$$y_{k,r}^{i,j} = 0, \forall k \in K, r \in R, i, j \in I, i \neq j, s_i^a \neq s_j^d \tag{14}$$

Formulas (13) and (14) indicate that only when the arrival station of train i is the same as the departure station of train j, train i can connect with train j. If the arrival station of train i is different from the departure station of train j, then train i and train j cannot be connected.

(7) EMU maintenance condition constraints

$$z_{k,r}^{i,j} \leq y_{k,r}^{i,j} \cdot \lambda_{i,j}, \forall k \in K, r \in R, i, j \in I, i \neq j \tag{15}$$

$$\begin{aligned} &\lambda_{i,j} = 1, \forall (t_j^d > t_i^a, t_j^d - t_i^a > To)\text{或} \\ &(t_j^d < t_i^a,\ t_j^d - t_i^a + 1440 > To) \end{aligned} \tag{16}$$

Formulas (15) and (16) indicate that if the EMU r is to carry out primary maintenance between train i and j, then the EMU r must connect with the train j after the completion of the train i and the connection interval has the condition of primary maintenance.

(8) maintenance distance constraint of EMU primary maintenance

$$L_{r,j} = l_j + \sum_{i\in I}(y_{k,r}^{i,j} - z_{k,r}^{i,j}) \cdot L_{r,i}, \quad \forall k \in K, r \in R, i, j \in I, i \neq j \tag{17}$$

$$L_{r,j} \leq (1+\Delta L)\cdot L \tag{18}$$

Formula (17) indicates that if the EMU r performs maintenance after completing the train task i and then continues the train j, then $L_{r,j}$ is the running mileage of the train j. If the EMU r performs the train task i and directly continues the train j, then $L_{r,j}$ is the cumulative running mileage of the EMU after completing the train i plus the running mileage of the train j. Formula (18) indicates that the cumulative operating mileage of the completed train j of the EMU r cannot exceed the maximum operating mileage of the first-level maintenance. The operating mileage of the EMU generally allows extended maintenance, and the extended maintenance ratio ΔL of the first-level maintenance mileage is typically 10 %.

(9) EMU connection time constraint

1) Short group continuation time constraint

$$t_j^d - t_i^a \geq T_{\min} + (y_{k,r}^{i,j} - 1)\cdot M, \quad \forall k \in K, r \in R, i, j \in O, i \neq j, s_i^a = s_j^d, t_j^d > t_i^a \tag{19}$$

$$t_j^d - t_i^a + 1440 \geq T_{\min} + (y_{k,r}^{i,j} - 1)\cdot M, \quad \forall k \in K, r \in R, i, j \in O, i \neq j, s_i^a = s_j^d, t_j^d < t_i^a \tag{20}$$

2) Long group continuation time constraint

$$\begin{aligned} & t_{j'}^d - t_{i'}^a \geq T_{\min} + (y_{k_1,r_1}^{i',j'} - 1)\cdot M, \\ & t_{j''}^d - t_{i''}^a \geq T_{\min} + (y_{k_2,r_2}^{i'',j''} - 1) * M, \\ & \forall k_1, k_2 \in K, r_1, r_2 \in R, i', i'', j', j'' \in P, \\ & Q_{i'} = Q_{i''}, Q_{j'} = Q_{j''}, s_{i'}^a = s_{j'}^d, t_{j'}^d > t_{i'}^a \end{aligned} \tag{21}$$

$$\begin{aligned} & t_{j'}^d - t_{i'}^a + 1440 \geq T_{\min} + (y_{k_1,r_1}^{i',j'} - 1)\cdot M, \\ & t_{j''}^d - t_{i''}^a + 1440 \geq T_{\min} + (y_{k_2,r_2}^{i'',j''} - 1) * M, \\ & \forall k_1, k_2 \in K, r_1, r_2 \in R, i', i'', j', j'' \in P, \\ & Q_{i'} = Q_{i''}, Q_{j'} = Q_{j''}, s_{i'}^a = s_{j'}^d, t_{j'}^d < t_{i'}^a \end{aligned} \tag{22}$$

3) Coupling connection time constraint

$$t_{j'}^d - t_i^a \geq T_{\min} + T_c + (y_{k,r}^{i,j'} - 1)\cdot M, \quad \forall k \in K, r \in R, i \in O, j' \in P, s_i^a = s_j^d, t_j^d > t_i^a \tag{23}$$

$$t_{j'}^d - t_i^a + 1440 \geq T_{\min} + T_c + (y_{k,r}^{i,j'} - 1)\cdot M, \quad \forall k \in K, r \in R, i \in O, j' \in P, s_i^a = s_j^d, t_j^d < t_i^a \tag{24}$$

4) Decoupling connection time constraint

$$t_j^d - t_{i'}^a \geq T_{\min} + T_u + (y_{k,r}^{i',j} - 1)\cdot M, \quad \forall k \in K, r \in R, i' \in P, j \in O, s_i^a = s_j^d, t_j^d > t_{i'}^a \tag{25}$$

$$t_j^d - t_{i'}^a + 1440 \geq T_{\min} + T_u + (y_{k,r}^{i',j} - 1)\cdot M, \quad \forall k \in K, r \in R, i' \in P, j \in O, s_i^a = s_j^d, t_j^d < t_{i'}^a \tag{26}$$

Formulas (19) to (22) specify that when two short-group EMUs or two long-group EMUs are coupled, their connection time must adhere to the minimum preparation operation time. Formulas (23) to (26) indicate that when connecting two EMUs of different types, coupling and decoupling operations must allocate sufficient time for the process.

B.5 Linearization of the model

Constraint (12) involves a nonlinear term where two variables are multiplied together, which typically complicates solver computations. To simplify, it can be transformed into a linear constraint as follows:

$$\begin{cases} y_{k,r}^{i,j} \geq u_{k,r}^i + u_{k,r}^j - 2, \forall k \in K, r \in R, i, j \in I, i \neq j \\ y_{k,r}^{i,j} \leq (u_{k,r}^i + u_{k,r}^j)/2, \forall k \in K, r \in R, i, j \in I, i \neq j \end{cases} \tag{27}$$

Constraint (15) is transformed into a linear constraint in the same way as constraint (12):

$$\begin{cases} z_{k,r}^{i,j} \geq y_{k,r}^{i,j} + \lambda_{i,j} - 2, \forall k \in K, r \in R, i, j \in I, i \neq j \\ z_{k,r}^{i,j} \leq (y_{k,r}^{i,j} + \lambda_{i,j})/2, \forall k \in K, r \in R, i, j \in I, i \neq j \end{cases} \tag{28}$$

The constraint (16) also has a quadratic term, a linear constraint, which needs to be transformed into a linear constraint by introducing auxiliary integer variables $u_{k,r}^{i,j}$, $v_{k,r}^{i,j}$, let $u_{k,r}^{i,j} = y_{k,r}^{i,j}\cdot L_{r,i}$, $v_{k,r}^{i,j} = z_{k,r}^{i,j}\cdot L_{r,i}$, The linearized constraint is :

$$L_{r,j} = l_j + \sum_{i\in I} u_{k,r}^{i,j} - v_{k,r}^{i,j}, \quad \forall k \in K, r \in R, i, j \in I, i \neq j \tag{29}$$

To restrict the values of the variables $u_{k,r}^{i,j}$ and $v_{k,r}^{i,j}$, constraints (30)-(35) are added.

$$u_{k,r}^{i,j} \leq L_{r,i} \tag{30}$$

$$u_{k,r}^{i,j} \geq L_{r,i} - \lambda\cdot(1 - y_{k,r}^{i,j}) \tag{31}$$

$$0 \leq u_{k,r}^{i,j} \leq \lambda\cdot y_{k,r}^{i,j} \tag{32}$$

$$v_{k,r}^{i,j} \leq L_{r,i} \tag{33}$$

$$v_{k,r}^{i,j} \geq L_{r,i} - \lambda\cdot(1 - z_{k,r}^{i,j}) \tag{34}$$

$$0 \leq v_{k,r}^{i,j} \leq \lambda\cdot z_{k,r}^{i,j} \tag{35}$$

In the formula, λ is the upper bound of the value of $L_{r,i}$, and the condition can be satisfied by taking 1×10^4 in this paper.

IV. Case Studies

A. Case Introduction

The map is shown in Table II and contains 28 lines, of which 16 are short-group and 12 are long-group.

TABLE II
TRAIN DIAGRAM INFORMATION

Train Service	Departure Station	Arrival Station	Departure Time	Arrival Time	Running Mileage	Grouping Type
G107	S1	S7	10:03	18:42	2147	16
G108	S7	S4	19:13	23:26	1100	8
G109	S1	S7	8:40	17:19	2147	16
G110	S7	S4	17:50	22:03	1100	8
G111	S4	S7	7:48	11:41	1100	8
G112	S7	S4	19:24	23:27	1100	8
G113	S4	S7	6:25	10:18	1100	8
G114	S7	S4	18:01	22:04	1100	8
G201	S4	S7	7:55	11:35	1100	8
G202	S7	S1	12:17	20:04	2147	16
G203	S4	S7	6:32	10:12	1100	8
G204	S7	S1	10:54	18:41	2147	16
G205	S1	S4	6:00	9:52	1047	8
G207	S1	S4	6:35	10:17	1047	8
G209	S1	S4	14:29	18:46	1047	8
G210	S4	S1	10:11	14:03	1047	8
G211	S1	S4	14:56	19:11	1047	8
G212	S4	S1	10:36	14:28	1047	8
G213	S1	S4	14:42	18:20	1047	16
G214	S4	S1	19:13	23:02	1047	8
G215	S1	S4	13:18	16:56	1047	16
G216	S4	S1	19:48	23:37	1047	8
G218	S4	S1	18:47	22:50	1047	16
G220	S4	S1	17:23	21:26	1047	16
G301	S1	S7	7:00	15:20	2147	16
G302	S7	S1	15:36	22:45	2147	16
G303	S1	S7	8:10	15:30	2147	16
G304	S7	S1	15:46	23:55	2147	16

This paper uses a domestic line diagram as a case study to assess the validity of the model. The conditions of this line are illustrated in Fig. 3. The example features a total of seven stations. Among these, stations S_1、S_4、S_7 are designated as the origin and destination points for operations, respectively. The remaining stations serve as intermediate stops along the route. Station q is noted for its connection to the Electric Multiple Unit (EMU) depot. Additionally, Station S_4 is specified as a location where overnight stays are possible, offering a convenient rest point for train operations.

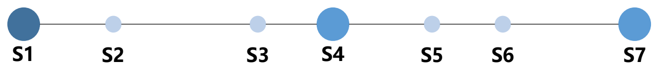

Fig. 3. Schematic diagram of line conditions

B. Calculation and analysis of results

The EMU operation plan is designed with a preparation period of 2 days. For the planned operations, 10 EMUs are required for regular tasks, plus an additional 5 EMUs on standby to handle any unforeseen issues or extra demand. Each train must have a minimum preparation time of 16 minutes between operations to ensure it is ready for the next assignment. The coupling of trains takes 15 minutes, while decoupling requires 10 minutes. Additionally, each EMU is scheduled for first-level maintenance when it accumulates a maximum of 4000 kilometers of mileage. This maintenance process has a duration of 240 minutes. The computational environment for solving this operational model consists of a computer with an AMD Ryzen 55500U processor, which has a base clock speed of 2.10 GHz, and is equipped with 16.0 GB of RAM. The model is addressed using Gurobi version 11.0.0, a commercial optimization solver, and is implemented through Python 3.10 programming language to derive efficient solutions for the EMU operational planning.

After inputting the training data and parameters into the model, we faced a substantial complexity due to the intricate structure of the problem. The model resulted in a total of 285,190 constraints and 76,735 variables, underscoring the extensive and demanding nature of the optimization task. Fig. 4 provides a visual representation of the 'Gap-Time' relationship throughout the solution process. It illustrates how the gap between the current best solution and the optimal solution evolves over time. The figure is instrumental in understanding the dynamics of the optimization process, revealing how the model progresses toward achieving the optimal solution. By tracking the changes in the gap, the figure offers insights into the solver's efficiency and the rate at which it converges to the optimal solution, reflecting the challenges in handling the large-scale problem.

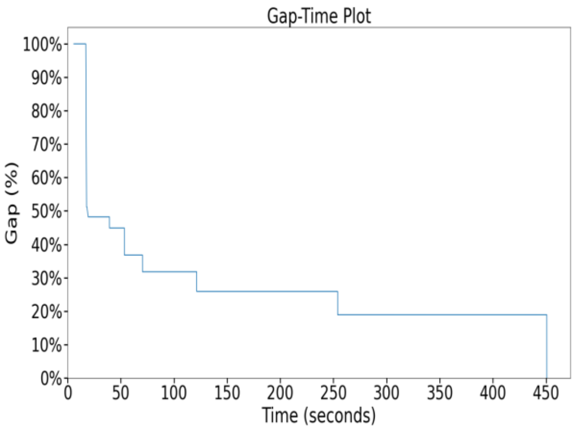

Fig.4.The ' Gap-Time ' diagram in the solution process

According to the figure, the 'GAP-Time' relationship demonstrates that the GAP value reaches 20% at 280 seconds, reflecting a significant deviation between the current solution and the optimal solution at that stage. By 450 seconds, the GAP has reduced to 0%, signifying that the solver has successfully identified the optimal solution. This trend underscores the Gurobi solver's efficiency and effectiveness in refining its solutions over time to achieve optimality for the EMU utilization plan. The detailed results of these calculations, which include specific numerical values, performance metrics, and additional insights, are thoroughly summarized in Table III. This table provides a comprehensive overview of the solution outcomes, illustrating both the accuracy of the final solution and the solver's performance throughout the optimization process. According to the content of the EMU route, the corresponding EMU route diagram is shown in Fig. 5.

TABLE III

OPTIMIZED EMU ROUTING PLAN

Serial Number	Route Content	Coupling and Decoupling	Number of EMUs
1	G107-G108 -G111-G202	Coupling、Decoupling	1
2	G107-G112 -G201-G202	Coupling、Decoupling	1
3	G205-G210-G209-G214	not	1
4	G207-G212-G211-G216	not	1
5	G213-G218	not	2
6	G215-G220	not	2
7	G109-G114 -G203-G204	Coupling、Decoupling	1
8	G109-G110 -G113-G204	Coupling、Decoupling	1
9	G301-G302	not	2
10	G303-G304	not	2

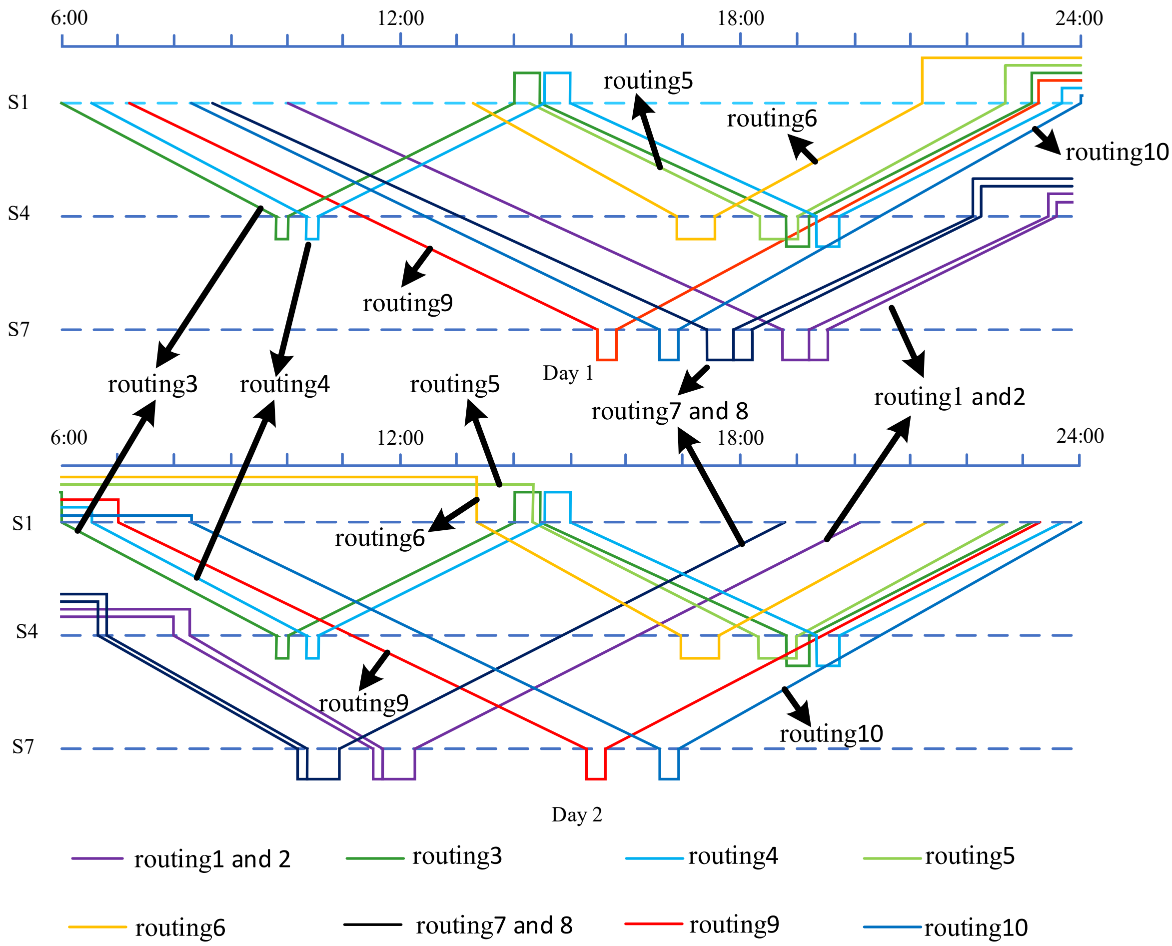

Fig.5. EMU route diagram

The results show a total of 10 routes, including four routes spanning two days and six routes lasting one day each. To complete all train tasks, 14 standard EMUs are required, with 4 station coupling and decoupling operations, along with 12 primary maintenance operations.

According to the content of the EMU route, the running mileage of each EMU and the mileage of the first level of loss can be found, as shown in Fig. 6.

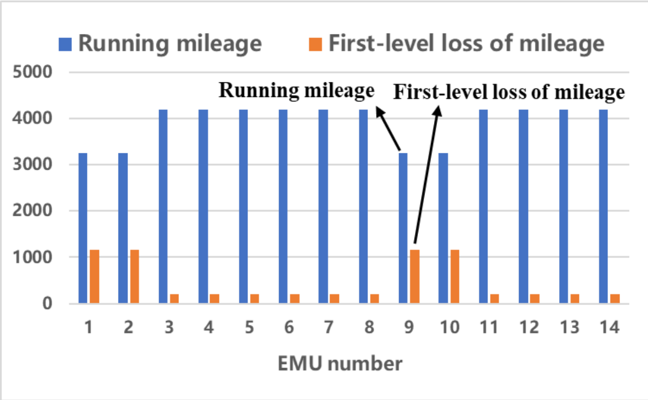

Fig.6. EMU operation and first-level loss mileage

The transportation tasks of all EMUs are well-balanced, with each vehicle averaging approximately 4000 kilometers in transportation mileage. The EMUs demonstrate high utilization efficiency, and the loss of available mileage in the first-level pickup is minimal.

To further evaluate the impact of the EMU operation plan based on the proposed unfixed decoupling positions in this paper, a comparison is made with the EMU operation plan utilizing fixed decoupling positions. Table IV presents the findings.

TABLE IV
COMPARISON RESULTS OF TWO SCHEMES

Compilation Method	Number of EMUs Required	Number of First-level Maintenance	First-level loss of mileage	The Average Number of Train Kilometers
Unfixed decoupling position	14	14	3012km	3214km/train
decoupling of EMU depot	16	16	3263km	2948km/train

The table indicates a reduction of 2 EMUs required, a decrease of 2 instances of first-level maintenance, a reduction of 251 km in first-level loss mileage, and an average increase of 194 km per train in EMU travel distance. Furthermore, the operational approach of EMUs with non-fixed decoupling positions proposed in this paper enhances the flexibility of EMU connections and reduces both empty mileage and idle time at EMU stations. This is achieved by eliminating the need for EMUs to return to the station for coupling and uncoupling.

In summary, the proposed optimization strategy enhances the flexibility of EMU operations, optimizes the operation plan, and further improves EMU efficiency.

C. Calculation and analysis of results

As EMU body equipment and decommissioning technology continue to advance, the decoupling time of EMUs will gradually decrease, directly impacting the preparation of EMU operation plans. The following section will focus on analyzing the impact of the time-consuming coupling and decoupling operations on the results.

The time taken for coupling and decoupling operations of the EMU at the station directly influences the connectivity of trains and the efficiency of these operations. Therefore, assuming other parameter values remain unchanged, the method of comparing control variables is employed to analyze the sensitivity of the time-consuming parameters related to EMU coupling and decoupling operations within the time interval of [5, 50]. The analysis results are depicted in Fig. 7.

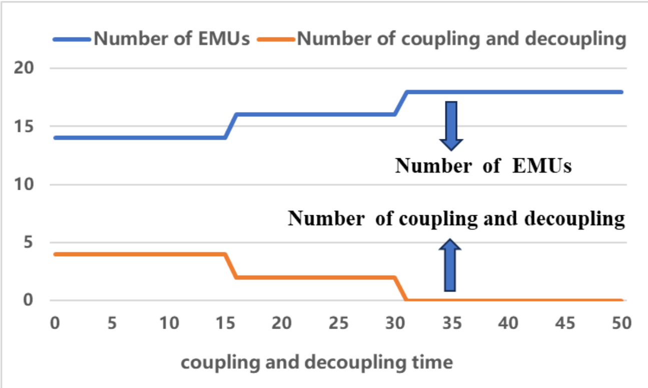

Fig.7. The change of EMU operation index under different coupling and decoupling operation time

As observed from the figure, as the time-consuming operations for coupling and decoupling increase, the frequency of EMU coupling and decoupling gradually decreases while the number of EMUs in use and the associated costs gradually rise. When the coupling and decoupling time is less than 15 minutes, the frequency of coupling and decoupling operations stabilizes, resulting in the minimum number of EMUs being utilized. Exactly. Reducing the decoupling time ensures that trains can maintain their connection efficiently at stations. This means most trains can quickly decouple and couple, maximizing the utilization of EMUs.

Based on the sensitivity analysis results, improving the coupling and decoupling technology of EMUs to achieve rapid operations at stations holds significant importance. This enhancement can notably enhance the operational efficiency of EMUs and reduce costs associated with high-speed railway operations.

V. CONCLUSION

1) The proposed operation mode involves implementing an unfixed decoupling position for EMUs. To simplify this model, the approach suggests dividing long-group EMUs into two identical short-group trains. This method aims to streamline operations and enhance flexibility in managing EMU configurations.

2) An optimization model is developed to describe the operational process of EMUs within the railway network, mainly focusing on EMUs with unfixed decoupling positions.

3) The solver is employed to resolve the case, confirming the validity of the model. The operation mode utilizing

unfixed decoupling positions effectively reduces the quantity of EMUs and enhances their operational efficiency.

4) Based on sensitivity analysis of coupling and decoupling times, it is evident that these operations significantly impact EMU operational efficiency. Given the high-speed railway network context, innovation in achieving rapid EMU coupling and decoupling is essential. This innovation aims to offer diverse solutions for reducing railway costs and enhancing operational efficiency.

REFERENCES

[1] National Railway Administration of People's Republic of China, 2023 *Railway Statistical Bulletin*, 2024.

[2] B.L. Lin, Y.M. Shen, and W.J. Zhong, "Optimization of the electric multiple units circulation plan with the minimum number of train-set," *China Railway Science*, vol. 44, no. 5, pp. 210-221, 2023.

[3] Q.W. Zhong, Y.X. Zhang, and D. Wang, "Optimization model and algorithm for train-set scheduling based on trip sequence," *Journal of Southwest Jiaotong University*, vol. 56, no. 2, pp. 385-394, 2021.

[4] S. Tong, S.K. Chen, and G.H. Liu, "Optimum circulation model for electrical multiple units considering reliability of their operational connections," *Journal of the China Railway Society*, vol. 42, no. 4, pp. 27-34, 2020.

[5] H. Li, B.M. Han, and Q. Zhang, ''Research on optimization model and algorithm of EMU circulation plan," *Journal of the China Railway Society*, vol. 35, no. 3, pp. 1-8, 2013.

[6] Z.K. Wang, T.Y. Shi, and W.J. Zhang, "Model and algorithm for the integrative scheduling of EMU utilisation plan and maintenance plan," *China Railway Science*, vol. 33, no. 3, pp. 102-108, 2012.

[7] H.J. Kang, P. Zhao, and S. Yang, "Rational application mode and application scope analysis of variable configuration EMU train," *China Transportation Review*, vol. 39, no. 9, pp. 64-67, 2017.

[8] S. Yang, P. Zhao, and H.J. Kang, "Applicability analysis of variable configuration EMU train for high-speed railway," *Railway Transport and Economy*, vol.39, no. 11, pp. 67-71, 2017.

[9] R.X. Zhai, P. Zhao, and W.B. Song, "A study on market entry strategies of variable configuration EMU trains," *Railway Transport and Economy*, vol.39, no. 12, pp. 22-26, 2017.

[10] H. Jie, "Research on uncoupling and re-coupling operation scheme of EMU trains at stations," *China Railway*, no. 10, pp. 40-45, 2021.

[11] J.J. Fu, X.C. Zhang, and Z.M. Wang, "Integrated model method for multiple-unit EMU operation and maintenance based on pareto frontier analysis," *Journal of Transportation Systems Engineering and Information Technology*, vol. 19, no. 6, pp. 1009-6744, 2019.

[12] A.P. Jian, "Research on the method of electrical multiple units scheduling problem based on variable grouping," *Beijing Jiaotong University*, 2019.

[13] X.Y Chen, "Research on EMU operation plan based on variable formation mode," *Beijing Jiaotong University*, 2020.

[14] Alfieri A, Groot R, and Kroon L, "Efficient circulation of railway rolling stock," *Transportation Science*, vol. 40, no. 3, pp. 378-391, 2006.

[15] Cordeau J F, Soumis F, and Desrosiers J, "Simultaneous assignment of locomotives and cars to passenger trains," *Operations research*, vol. 49, no. 4, pp. 531-548, 2001.

[16] Z.F. Zhu, X.E. Yang, and Z.F. Wang, "Optimization for the train plan with flexible train composition considering carbon emission," *Engineering Letters*, vol. 31, no. 2, pp. 562-573, 2023.

[17] S.X. Wan, C.F. Zhu, and J.H. Fang, "Operation optimization of reconnection marshalling trains considering carbon emission," *IAENG International Journal of Applied Mathematics*, vol. 53, no. 2, pp. 601-612, 2023.

Zhiqiang Tian was born in Gansu, China, in 1981. In October 2011, he obtained a doctor's degree in the Planning and Management of Traffic and Transportation in Southwest Jiaotong University. He is now a professor of the School of Traffic and Transportation of Lanzhou Jiaotong University. He has rich scientific research achievements, and has published more than a few papers in domestic academic journals and international conferences.