Irreversibility Analysis of Hall Effect on Hydromagnetic Blood Flow in an Inclined Stretching Permeable Vessel

Hammed Fatai Akangbe, Usman Mustapha Adewale and Akanbi Olumuyiwa Olawale

Abstract—**Blood flow rate can be measured by Hall current due to the presence of ions which constitute an electric current in blood. Similarly, magnetic field has the potential application in magnetic drug targeting and adjusting blood flow during surgery. In this work, a theoretical investigation of the irreversibility analysis of unsteady electrically conducting blood flow in an inclined stretching permeable vessel under the influence of Hall current, thermal radiation and magnetic field applied perpendicularly to the flow is undertaken. In order to have a robust analysis of the flow field, temperature, concentration field, entropy generation and Bejan number; formulation of the equations governing the flow is carried out and the resulting coupled partial differential equations are reduced to a set of first order ordinary differential equations with the aid of appropriate similarity variables. A numerical technique, Runge-Kutta with shooting method is applied to solve the dimensionless equations. The impact of flow parameters such as the unsteadiness parameter (A), inclination angle parameter (α), Hall current parameter (m), Hartmann (Ha) and Radiation parameter (Nr), skin friction and Nusselt number are presented using plots and tables. The study reveals that blood flow is accelerated by Hall and radiation parameters while unsteadiness and inclination parameters cause deceleration. Blood temperature is raised by increasing the values of Hartmann number, radiation and inclination angle parameters. However, unsteadiness parameter reduces it. Furthermore, blood entropy generation and Bejan number receive a boost with the enhancement in Hartmann number and radiation parameter.**

Index Terms: Hall current, Hydromagnetic Blood flow, Stretching vessel, Runge-Kutta method, entropy generation

Manuscript received October 20, 2023; revised May 21, 2024.

Hammed, Fatai A. is a lecturer in the Department of Mathematical Sciences, Olabisi Onabanjo University Ago-Iwoye, Nigeria (email: hammed.fatai@oouagoiwoye.edu.ng).

Usman, Mustapha A. is a lecturer in the Department of Mathematical Sciences, Olabisi Onabanjo University Ago-Iwoye, Nigeria (email: usman.mustapha@oouagoiwoye.edu.ng).

Akanbi, Olumuyiwa O. is a Ph.D student in the Department of Mathematical Sciences, Olabisi Onabanjo University Ago-Iwoye, Nigeria (email: pscf20210070@oouagoiwoye.edu.ng).

1 Introduction

The flow of blood plays a crucial role in the transportation of oxygen, nutrients, waste product and supply heat to different body tissues Ribatti [1]. Understanding the flow characteristics of blood and their associated phenomena is of high importance in both clinical and physiological research.

In recent years, the field of hemodynamic has emerged as a powerful tool for investigating the flow behaviour of human blood. Oka and Murata [2] investigated theoretically the flow of blood in a blood vessel which has permeable wall. A study on the blood flow under a post-stenotic dilatation with forced field was carried out by Dhange et. al [3]. A simple mathematical model was presented by Rahman and Hague [4] under the influence of blood pressure gradient and cross-sectional area. McCulloch et. al. [5] studied the mathematical biology of the heat ranging from heat structure to function. Human hypertension blood flow was investigated by Mohammed et. al [6] with the implementation of fractional calculus to understand the structure and functional changes in the large and small arteries affected by hypertension disease. A growing body of literature has examined the magnetohydrodynamics (MHD) on blood flow, highlighting their potential role in modulating hemodynamic parameters and contributing to various physiological processes. Mohamad et. al. [7] studied the effects of heat and mass transfer on MHD blood with a focus on Casson fluid model. The study of blood with gold nano particles in the presence of magnetic dipole was examined in Alam et. al [8]. The MHD blood flow in a stretching permeable vessel under the influence of thermal radiation and chemical reaction was examined by Zigta [9]. Rekha and Usha [10] proposed a mathematical model of blood flow with an externally applied magnetic field in a small blood vessel. Effect of thermal radiation on MHD blood flow and transfer of heat in a stretching permeable capillary was studied by Misra and Sinha [11]. A mathematical study of blood flow characteristics when catheterization is implemented in stenosed artery was studied by [12]. Prakash and Makinde [13] studied the connected actions of axial pressure gradient and externally applied magnetic field on the blood flow through a stenosed artery. The effect of magnetic field on the human blood flow through human arterial system was reported by Sud and Sekhon [14]. Theoretical analysis for magnetohydrodynamic flow with different physical blood parameter was investigated by Misra et. al. [15]-[17].

Reddy et. al. [18] reported a significant impact of MHD flow of blood in an inclined stretching surface under the influence of viscous dissipation, chemical reaction and non-uniform heat source/sink. Majekodunmi et. al [19], conducted a theoretical study of magnetohydrodynamic blood flow in the case of multiple stenosis. Magnetohydrodynamics impact on a generalized power law fluid model of blood flowing through an artery that is with an overlapping shaped stenosis was studied numerically by Zain and Ismail [20]. Mathematical investigation was conducted on thermoregulator influence on blood viscosity in the presence of magnetic field and thermal radiation in human cardiovascular system [21]. Sinha et. al [22] examines the effect of heat transfer on unsteady MHD fluid of human blood under the influence of non-uniform thermal source. Srinivas et al. [23] carried out a study on the effect of chemical reaction and thermal radiation on MHD flow over an inclined surface where the application of the concept of MHD to the dynamics of blood flow was applied. MHD blood flow research was conducted by Tripathi et. al [24] where heat transfer in an inclined stenosed artery under the influence of varying viscosity was considered. A variable viscosity and heat source on MHD blood flow as considered in [25] was investigated. Majeed et. al [26] considered MHD blood flow in a cylindrical tube using fractional model. A case study of unsteady MHD blood flow with single and multiple wall carbon nanotubes and thermal analysis was investigated by Khalid et. al [27]. A study was conducted on Spatio-temporal evolution of MHD blood flow involving heat motions passing through a wavy-walled artery [28]. In a parallel plate channel Latha and Kumar [29] considered unsteady MHD blood flow in a porous medium.

The Hall effect has been extensively studied in various disciplines, such as those encountered in Physics and Electrical engineering. However, its impact on hydromagnetic blood flow in biological system remains relatively unexplored. Effect of Hall current in an oscillatory blood flow, where heat and mass transfer under the impact of Hall effect was examined [30]. Abdullah et. al [31] considered Hall currents along with the magnetic field on the blood velocity and blood concentration with lipoprotein. Umash and Bhupendra [32] examined the human blood flow under the influence of Joule heating, thermal radiation, Hall and ion slip effect. Ascendancy of electromagnetic force and Hall currents on blood flow was investigated by Das et. al. [33]. Asha & Sunitha [34] investigated the effect of thermal radiation and double diffusion with Hall effect on peristaltic blood flow. Influence of magnetic field and Hall effects on human blood flow through an obstructed artery was studied by Mekheimer and El kot [35].

Entropy generation analysis is a powerful tool to quantify the irreversible losses and inefficiencies in a human circulatory system. A number of investigations have been explored in this area. Rashidi et. al. [36] conducted Entropy analysis on magnetic blood flow under the influence of peristaltic wall. Algehyne et. al [37] studied the Entropy minimization and response surface methodology of blood hybrid nanofluid flow in a stenosed artery in the presence of magnetized nanoparticles for effective drug delivery application. A study carried out by [38] analysed the entropy generation on the blood flow with magnetic Zinc-Oxide nanoparticles flowing through Anisotropically Tapered arteries. Shahzad et. al. [39] investigated the entropy analysis and stability of blood flow through infected multiple stenosis artery. An elegant study by Al-Mdallal et. al. [40] investigated the MHD blood flow and Entropy generation for two phase human blood flow in the presence of a curved permeable artery with variable viscosity, heat and mass transfer. Investigation of consequence of thermal radiation in a rotating fluid in the presence of oscillatory vertical plate with variable temperature and mass diffusion was carried out by Antony and Ravikumar [41]. Jaismitha and Sasikumar [46] carried out a study on chemically oscillatory casson hybrid nanofluid flow with heat source/sink in a rotating wavy channel. Effect of various physical parameter such as prandtl number, Hall currents, Soret on MHD flow past in an inclined stretching sheet was investigated by George et al. [47]. Rishu et al. [48] studied the entropy generation of a mediated blood flow of a time-variant multi-stenotic artery. Another recent study by Sakthi et al. [49], carried out a research on the entropy generation of a blood-based hybrid nanofluid in the presence of thermal radiation where channels are converged and diverged.

Motivated through the survey of relevant literature, this study was undertaken to optimize the irreversibility analysis of blood, flowing through an inclined stretching permeable blood vessel. Thus, the objective is to investigate impact of Hall effect on entropy generation of hydromagnetic blood flow in an inclined stretching permeable vessel and examine the behaviour of the flow variables such as velocity, temperature and concentration under the influence of some flow parameters which have not been accounted for in the previous studies by Misra and Sinha [11]. To achieve this, the modified non-linear partial differential equations of the blood flow are formulated, converted to a dimensionless ordinary differential equation and solved numerically by Runge-Kutta via shooting method.

2 Problem Formulation

Consider an unsteady incompressible flow of blood through an inclined stretching permeable narrow artery with angle α to the vertical. It is assumed that the artery is under the influence of a time-dependent magnetic field $B(t)$ acting in a perpendicular direction to the flow. The influence of magnetic field, Hall current, thermal radiation, porosity, Ohmic heating and chemical reaction are incorporated.

The coordinate system, as shown in Figure 1, is chosen such that $x - axis$ is considered along the blood vessel in an inclined upward direction and z-axis normal to plane of the blood vessel. The electron-atom collision frequency is also assumed to be relatively high, so that the Hall effect cannot be ignored. The effect of Hall current gives rise to a force in the z-direction, which induces a cross flow in that direction, and hence the flow becomes three-dimensional, see Figure 1. It is further assumed that the induced magnetic field in comparison to the applied magnetic field is insignificant since the magnetic Reynolds number is much lower than unity, hence it is negligible.

The assumption is justified due to the fact that magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial applications [9]. The velocity slip, thermal slip, solutal slip are incorporated into the flow analysis. At time $t = 0$, the blood artery is assumed to be abruptly stretched along the x-axis with velocity $U_w\,(x,t)$ and concentration $C_w\,(x,t)$ The origin is kept fixed in the fluid medium of ambient temperature T_∞ and ambient concentration of solute C_∞. According to Misra et al. [11] flow in the circulatory system is mainly three-dimensional, however, in many cases blood flow in a narrow vessel such as artery has been analysed as two-dimensional. Following the assumptions made above, the equations that govern the flow of blood for the analysis are considered in the form [9, 11].

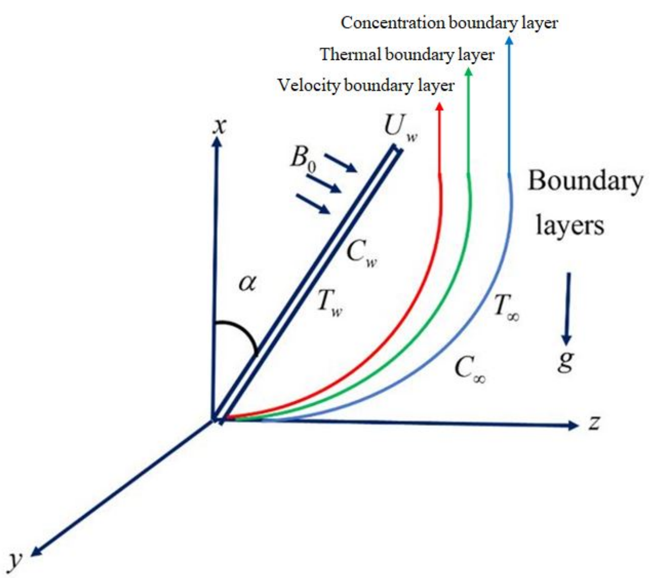

Fig 1: Geometry of Flow

2.1 The Models

Conservation of mass or Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Conservation of momentum or Navier-Stokes equation in x-direction

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{\partial B^2(t)}{\rho(1+m^2)}(u+mw) - \frac{v}{k_1(t)}u + g\Gamma(T-T_\infty)\cos\alpha + g\Gamma^*(C-C_\infty)\cos\alpha \quad (2)$$

Conservation of momentum or Navier-Stokes equation in y-direction

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = v\frac{\partial^2 w}{\partial y^2} + \frac{\sigma B^2(t)}{\rho(1+m^2)}(mu+w) \quad (3)$$

Conservation of energy or Energy equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{\nu}{c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{\sigma B^2(t)(u^2+w^2)}{\rho c_p} \quad (4)$$

Conservation of species concentration (mass diffusion) or Concentration equation

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - R\,(C - C_\infty) \quad (5)$$

Boundary conditions with velocity (N), thermal (K), concentration (P) slips and suction (f_w) parameter

$$\left.\begin{aligned} &u = U_w + N\mu\frac{\partial u}{\partial y},\\ &v = f_w,\\ &w = W_w + N\mu\frac{\partial w}{\partial y},\\ &T = T_w + K\frac{\partial T}{\partial y},\\ &C = C_w + P\frac{\partial C}{\partial y}, y = 0\\ &u \to 0, T \to T_\infty, C \to C_\infty, y \to \infty \end{aligned}\right\} \quad (6)$$

The radiative heat flux in the energy equation is simplified by using Rosseland approximation as follows

$$q_r = -\frac{4\sigma^c}{3k^c}\frac{\partial T^4}{\partial y} = -\frac{16\sigma^c T^3}{3k^c}\frac{\partial T}{\partial y} \quad (7)$$

The term T in Eq. (7) is highly nonlinear hence its solution is very difficult to obtain, therefore an assumption of small temperature differences within the flow is then taken, and this helps to linearize the Rosseland formula about the ambient temperature T. This implies that T_∞. Invoking Eq. (7) in Eq. (4) yields

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^c T^3}{3\rho c_p k^c}\frac{\partial T}{\partial y} + \frac{v}{c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \frac{\sigma B^2(t)(u^2+w^2)}{\rho c_p} \quad (8)$$

Introducing the following transformation variables:

$$\begin{aligned}
\eta &= y\left[\frac{a}{\nu(1-ct)}\right]^{\frac{1}{2}},\\
\psi &= \left[\frac{av}{1-ct}\right]^{\frac{1}{2}} xf(\eta),\\
w &= \left[\frac{ax}{1-ct}\right] g(\eta),\\
\theta(\eta) &= \frac{T-T_\infty}{T_w-T_\infty},\\
\phi(\eta) &= \frac{C-C_\infty}{C_w-C_\infty},\\
u &= \frac{\partial\psi}{\partial y},\\
v &= -\frac{\partial\psi}{\partial x},\\
T &= T_\infty + \frac{cx}{(1-\alpha t)^2}\theta(\eta),\\
C &= C_\infty + \frac{cx}{(1-\alpha t)^2}\phi(\eta)
\end{aligned} \tag{9}$$

Note that ψ is a stream function. Relations (9) is applied to reduce Eqs. (2, 3, 5, 8) to the following system of coupled ordinary differential equations

$$\begin{aligned}
&f^{'''} - A\left[f^{'} + \frac{\eta}{2}f^{''}\right] - f^{'2} + ff^{''} - \frac{Ha^2}{(1+m^2)}(f^{'}+mg)\\
&- Kf^{'} + (Gr\theta + G_c\phi)\cos\alpha = 0
\end{aligned} \tag{10}$$

$$g^{''} - A\left[g + \frac{\eta}{2}g^{'}\right] - f^{'}g + fg^{'} + \frac{Ha^2}{(1+m^2)}(mf^{'}+g) = 0 \tag{11}$$

$$\begin{aligned}
&\frac{1}{Pr}\left[1+\frac{3}{4}Nr\right]\theta^{''} - A\left[\theta + \frac{\eta}{2}\theta^{'}\right] - f^{'}\theta + f\theta^{'} +\\
&EcHa^2(mf^{'}+g) = 0
\end{aligned} \tag{12}$$

$$\frac{1}{Sc}\phi^{''} - A\left[\phi + \frac{\eta}{2}\phi^{'}\right] - f^{'}\phi + f\phi^{'} - \beta\phi = 0 \tag{13}$$

$$\begin{aligned}
&f = 0, f^{'} = 1 + L_f f^{''}(0), g = 0, \theta = 1 + L_t\theta^{'}(0),\\
&\phi = 1 + L_c\phi^{'}(0),\\
&\eta = 0, f^{'} \to 0, g \to 0, \theta \to 0, \phi \to 0, \eta \to \infty
\end{aligned} \tag{14}$$

3 Method of Solution

Application of Runge-Kutta with shooting technique involves the conversion of Eqs. (10-14) to a system of first order ordinary differential equations. If we let $y_1 = f, y_2 = f^{'}, y_3 = f^{''}, y_4 = g, y_5 = g^{'}, y_6 = \theta, y_7 = \theta^{'}, y_8 = \phi, y_9 = \phi^{'}$

Then,

$$\begin{aligned}
&y_1^{'} = y_2, y_1(0) = 0,\\
&y_2^{'} = y_3, y_2(0) = 1 + L_f y_3(0),\\
&y_3^{'} = A\left[y_2 + \frac{\eta}{2}y_3\right] + y_2^2 - y_1y_3 + \frac{Ha^2}{(1+m^2)}(y_2+my_4)\\
&\quad + \; ky_2 - (Gry_6 + Gcy_8)\cos\alpha, y_3(0) = S_1,\\
&y_4^{'} = y_5, y_4(0) = 0,\\
&y_5^{'} = A\left[y_4 + \frac{\eta}{2}y_5\right] + y_2y_4 - y_1y_5\\
&\quad + \; \frac{Ha^2}{(1+m^2)}(my_2+y_4), y_5(0) = S_2,\\
&y_6^{'} = y_7, y_6(0) = 1 + L_t y_7(0),\\
&y_7^{'} = \left[\frac{4\mathrm{Pr}A}{4+3Nr}\right]\left[y_6 + \frac{\eta}{2}y_7\right] + y_2y_6 - y_1y_7\\
&\quad + \; EcHa^2(my_2+y_4), y_7(0) = S_3,\\
&y_8^{'} = y_9, y_8(0) = 1 + L_c y_9(0),\\
&y_9^{'} = ScA\left[y_8 + \frac{\eta}{2}y_9\right] + y_2y_8 - y_1y_9 - \gamma y_8, y_9(0) = S_4
\end{aligned} \tag{15}$$

Runge-Kutta method is employed to obtain the solutions to Equations (15) with a step size 0.001. The results are truncated at a distance where the influence of boundary layers on the blood vessel is less substantial. The numerical method is validated by comparison with previously published works in limited case as displayed in Table 1 Sharidan et. al. [42] and Chamkha et al. [43] and Table 2 Ishak et. al. [44] and Hayat and Qasim [45], the results demonstrate excellent agreement. In this work, the quantities of interest are the skin friction coefficient, rate of heat transfer and mass transfer, these are defined as

$$\left.\begin{aligned}
C_f &= \frac{\tau_w}{\rho U_w^2/2}\\
Nu_x &= \frac{xq_w}{k(T_w-T_\infty)}\\
Sh_x &= \frac{xm_w}{\rho D(C_w-C_\infty)}
\end{aligned}\right\} \tag{16}$$

The terms

$$\begin{aligned}
&\tau_w = \mu\left[\frac{\partial u}{\partial y}\right]_{y=0}, q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} \text{ and}\\
&m_w = -\rho D\left[\frac{\partial c}{\partial y}\right]_{y=0}
\end{aligned} \tag{17}$$

represents the shear stress, surface heat flux and the mass flux respectively. Substituting Eq.(17) in Eq.(16) yields

$$\left.\begin{aligned} C_f &= \frac{1}{2}\mathrm{Re}_x^{-\frac{1}{2}} f''(0) \\ Nu_x &= -\mathrm{Re}_x^{-\frac{1}{2}} \theta'(0) \\ Sh_x &= -\mathrm{Re}_x^{-\frac{1}{2}} \phi'(0) \end{aligned}\right\} \tag{18}$$

4 Entropy Generation

The local volumetric entropy generation rate for blood flow with Hall current, thermal radiation and Ohmic heating is

$$S_G = \frac{k}{T_\infty^2}\left(\left[\frac{\partial T}{\partial y}\right]^2 + \left[\frac{\partial T}{\partial x}\right]^2\right) + \frac{16\sigma T_\infty^3}{3k^c}\left(\left[\frac{\partial T}{\partial y}\right]^2 + \left[\frac{\partial T}{\partial x}\right]^2\right)$$

$$+\frac{\mu}{T_\infty}\left(\left[\frac{\partial u}{\partial y}\right]^2 + \left[\frac{\partial w}{\partial y}\right]^2\right) + \frac{D}{C_\infty}\left(\left[\frac{\partial C}{\partial y}\right]^2 + \left[\frac{\partial C}{\partial x}\right]^2\right) \tag{19}$$

$$+\frac{D}{T_\infty}\left[\left[\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right] + \left[\frac{\partial T}{\partial x}\frac{\partial C}{\partial x}\right]\right] + \frac{\sigma B_0^2}{T_\infty}\left(u^2 + w^2\right)$$

introducing Eq.(9), in Eq.(19) yields

$$Ns = \left[1 + \frac{3}{4}\right]\theta'^2 + \frac{Br}{\Omega}\left(\left(f''^2 + g'^2\right) + Ha^2\left(f'^2 + g^2\right)\right)$$
$$+ \varepsilon\phi'^2 + \lambda\theta'\phi' \tag{20}$$

Now, designating

$$N_1 = \left[1 + \frac{3}{4}\right]\theta'^2 \quad \text{and}$$

$$N_2 = \frac{Br}{\Omega}\left(\left(f''^2 + g'^2\right) + Ha^2\left(f'^2 + g^2\right)\right) + \varepsilon\phi'^2 + \lambda\theta'\phi' \tag{21}$$

The interest here is to obtain the irreversibility ratio to compare the different contributions from the heat energy contributing forces in the blood flow. In this respect, we compare the irreversibility due to heat transfer, N_1 to the irreversibility due to the effective viscous effect, N_2 where,

Bejan number (Be) is given as

$$Be = \frac{N_1}{N_s} = \frac{1}{1+\phi}, \tag{22}$$

And $\phi = \dfrac{N_2}{N_1}$ is the irreversibility distribution ratio, a parameter which measures the rate of the destruction of available energy in the blood flow.

5 Results Validation

Table 1: Comparison of $-f''(0)$ for various values of unsteadiness parameter when $H_a = Gr = Gc = 0$

A	Sharidan et al. [42]	Chamkha et al. [43]	Present
0.8	1.261042	1.261512	1.261042
1.2	1.377722	1.378052	1.377723

Table 2: Comparison of $-\theta'(0)$ for various values of A, f_w and Pr

A	fw	Pr	Ishak et al. [44]	Hayat et al.[45]	Present
0	-1.5	0.72	0.4570	0.45702	0.457028
		1.0	0.5000	0.50000	0.500000
		10.0	0.6452	0.64516	0.645168
	0	0.72	0.8086	0.80863	0.808634
		1.0	1.0000	1.00000	1.000000
		3.0	1.9237	1.92359	1.923655
		10.0	3.7207	3.72159	3.720649
	1.5	0.72	1.4944	1.49436	1.494368
		1.0	2.0000	2.00000	2.000000
		10.0	16.0842	16.09624	16.08421
1.0	-1.5	1.0	0.8095	0.80951	0.809511
	0		1.3205	1.32052	1.320522
	1.5		2.2224	2.22235	2.222355

6 Discussion of Results

To gain deeper insight into the qualitative analysis of this investigation; this section presents the impact of various parameters such as unsteadiness, angle of inclination, Hall current, thermal radiation and Hartmann number on the velocity, temperature and concentration of blood flowing through a stretching vessel. In addition, tables are presented for varying values of skin friction and Nusselt number. For numerical results, we set $\beta = 0.5$, $Nr = 0.5$, $A = 0.5$, $Gr = 2$, $Gc = 2$, $\alpha = \frac{\pi}{4}$, $Sc = 0.6$, Pr= 21,

$Ha = 0.5$, $Ec = 0.5$, $m = 0.5$, $K = 0.5$, $L_f = 1.5$, $L_t = 1$, $L_c = 0.5$. Unless otherwise indicated in the graphs, these values are retained for the purpose of computation.

In Figures 2 to 5, blood primary and secondary velocity together with the temperature and concentration are observed to have reduced as a result of the increasing values of unsteadiness parameter. This parameter has an inverse relationship with the blood stretching vessel, hence a rise in the value of (A) will amount to a reduction in blood velocity, temperature and concentration as observed in the plots. In Figure 6, it is obvious that entropy generation is minimised as the unsteadiness parameter grows in values. This can be traced to drop in blood motion, temperature and concentration. Finally, in Figure 7 Bejan number reduces as the unsteadiness parameter rises indicating that fluid friction irreversibility is the major contributor to blood entropy formation. Figures 8 to 11 display the impact of inclination angle on flow velocity, temperature and concentration. Figures 8, 9 and 10 indicate a reduction in blood velocity and temperature. This observation is linked to the fact that inclination angle has the tendency to diminish the impact of buoyancy force on blood momentum. Physically speaking, this is valid since the wall becomes less steep, thus lowering the blood flow rate. However, a reverse phenomenon is observed in Figure 11. Figure 11 shows a significant rise in the concentration profile. Figure 12 reveals a drop in blood entropy formation for a rising values of inclination angle parameter, this is expected as a result of the significant drop in blood motion as indicated in Figures 8 and 9. However, in Figure 13 Bejan number depicts, this implies that heat transfer irreversibility dominates blood entropy production for an increment in the values of inclination angle parameter.

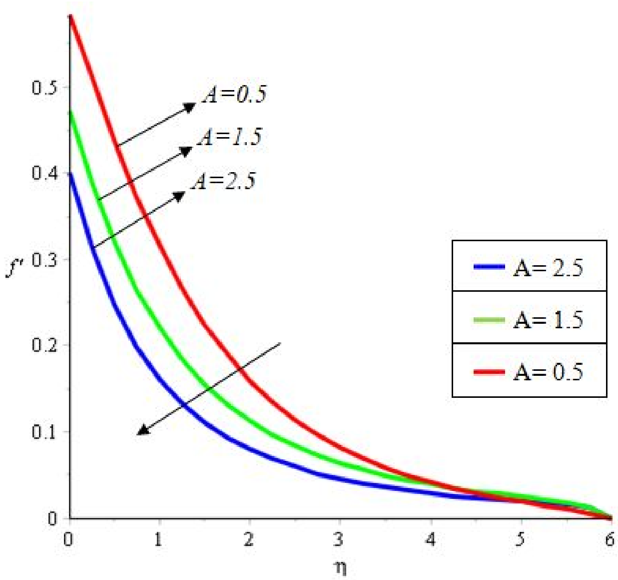

Fig 2: Unsteadiness parameter versus primary velocity

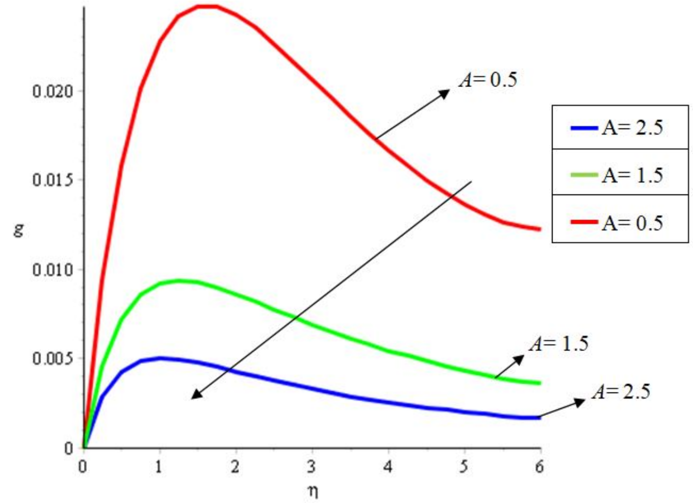

Fig 3: Unsteadiness versus secondary velocity

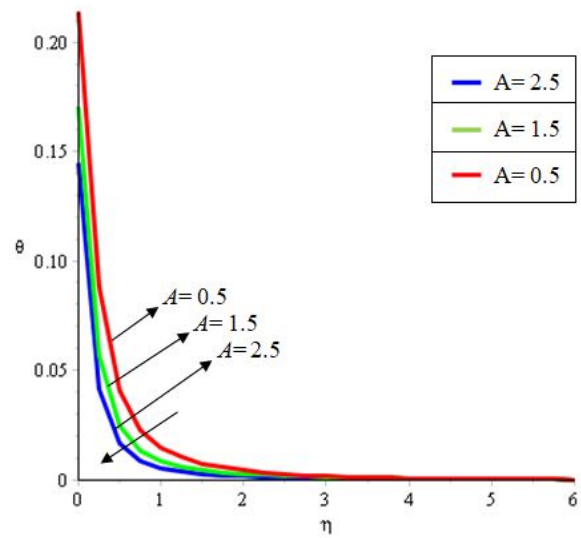

Fig 4: Unsteadiness parameter versus temperature profile

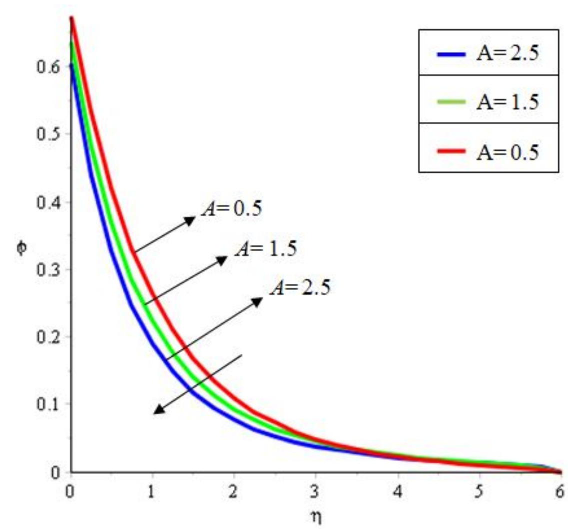

Fig 5: Unsteadiness parameter versus concentration profile

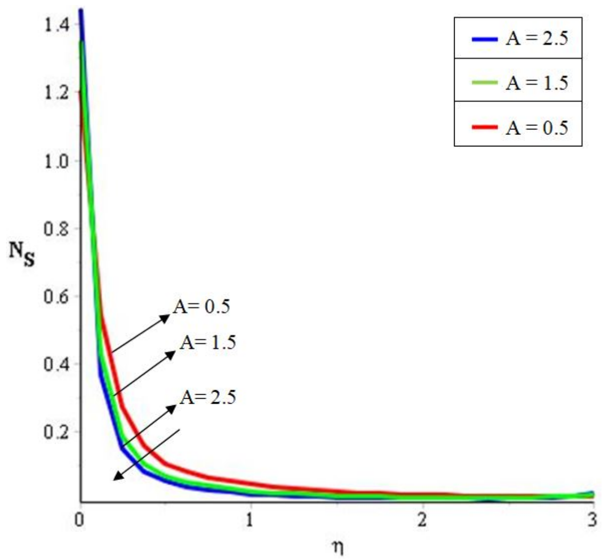

Fig 6: Unsteadiness parameter versus entropy generation

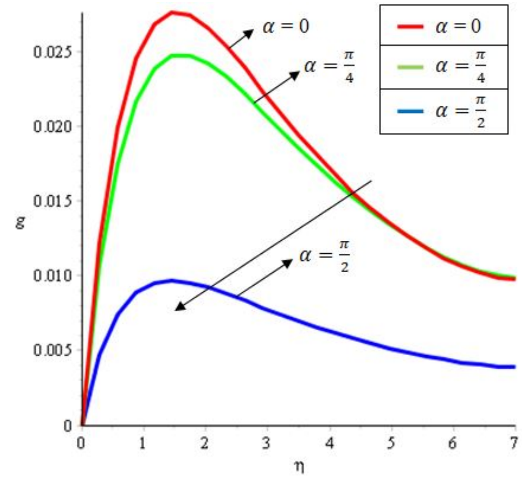

Fig 9: Inclination angle parameter versus secondary velocity

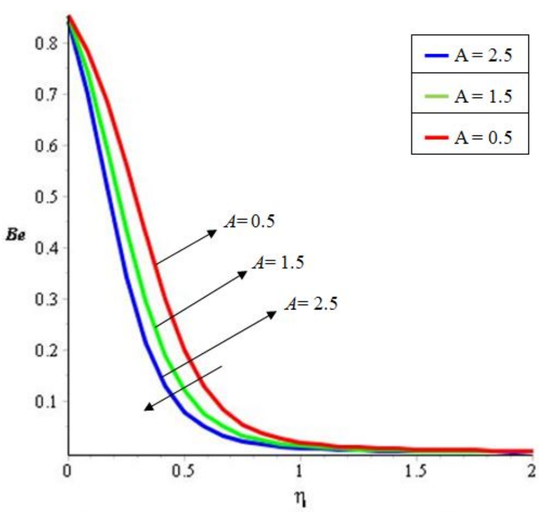

Fig 7: Unsteadiness parameter versus Bejan number

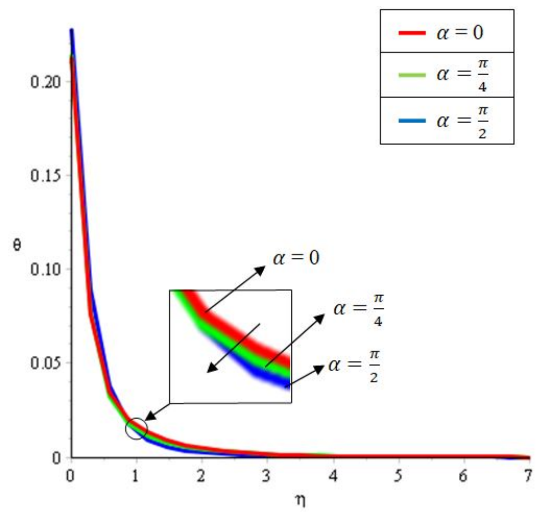

Fig 10: Inclination angle parameter versus temperature profile

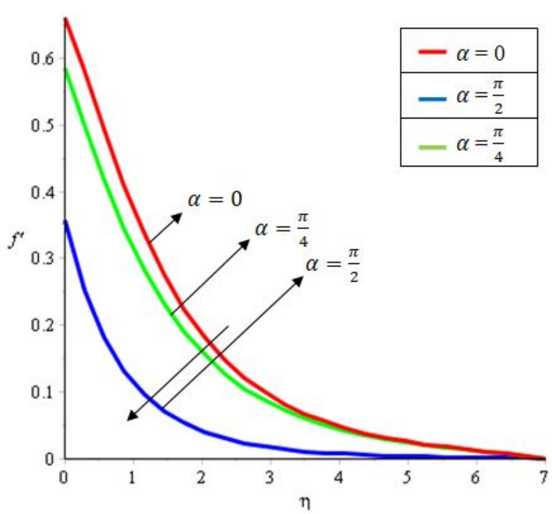

Fig 8: Inclination angle parameter versus primary velocity

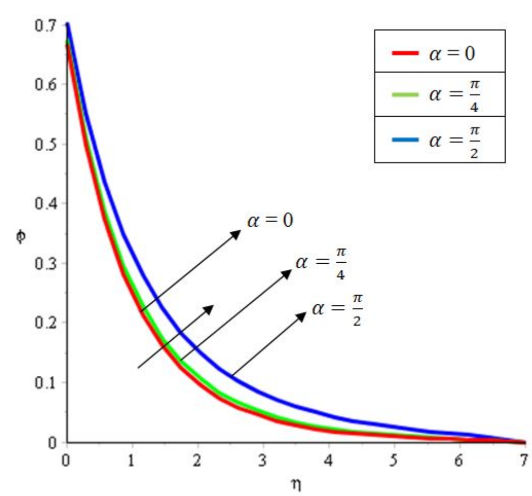

Fig 11: Inclination angle parameter versus concentration profile

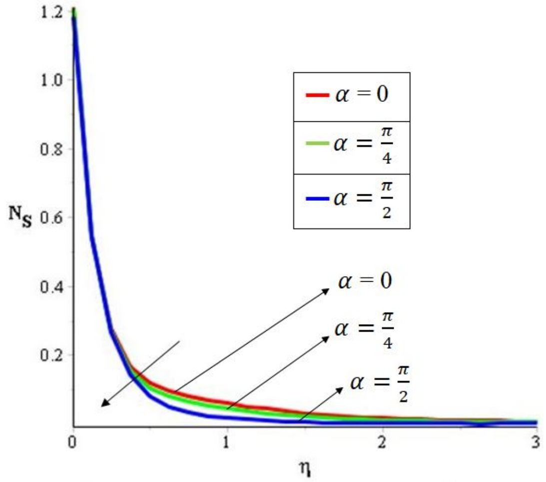

Fig 12: Inclination angle parameter versus entropy generation

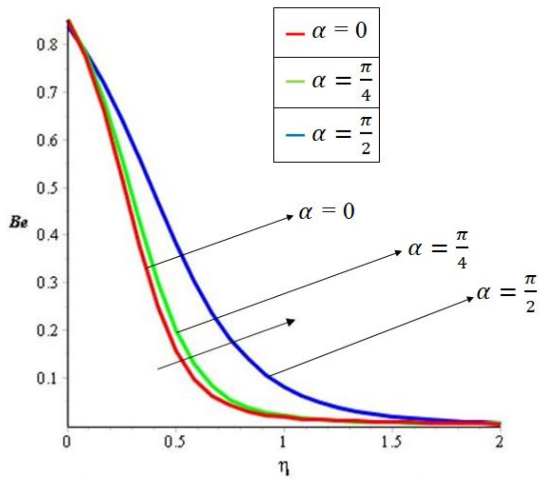

Fig 13: Inclination angle parameter versus Bejan number

In Figure 14 to 15, Hall parameter is plotted against primary and secondary velocity. An increase in blood velocity is registered for various values of Hall parameter. This is physically correct, increasing the value of Hall parameter reduces the damping force of the magnetic field parameter. The implication is that the magnetic resistivity force is weakened, resulting in increased blood flow. Also, Figure 16 illustrates a moderate reduction in Bejan number with increasing Hall parameter values, indicating fluid friction irreversibility dominance over heat transfer irreversibility in entropy production. Figure 17 to 20 illustrate how the blood's motion, temperature and concentration are influenced by variations in the Hartmann number. Figure 17 to 18, it is inferred that the primary velocity is reduced while a reverse scenario is registered for secondary velocity. However, from figure 19 to 20, the temperature and concentration of the flow receive a boost. This decreasing effect in blood velocity is due to the presence of red blood cell (RBC) which contains haemoglobin, and it is paramagnetic in nature. Therefore when an external magnetic field is applied to the blood flow, an interaction between the magnetic field and the blood occurs this produces a strong electromotive field, then a resistive force known as the Lorentz force is generated. Hence the reduction in blood motion. This result validates the fact that blood flow during surgery and treatment of arterial diseases can be adjusted and regulated by the application of an external magnetic field. The existence of Joule heating, which appears in the energy equation, is responsible for the rise in blood temperature and concentration. Furthermore, the presence of a magnetic field induced viscous heating of blood particles, which further raised blood temperature. Blood entropy generation is portrayed in Figure 21 to have reduced as Hartmann number value is enhanced. This is expected since the resistivity force resulting from the presence of the Lorentz force has drastically reduced blood motion. This scenario is corroborated in Figure 22, where Bejan number improved significantly in response to rising Hartmann number values, demonstrating that the formation of entropy is essentially as a consequence of heat transfer irreversibility.

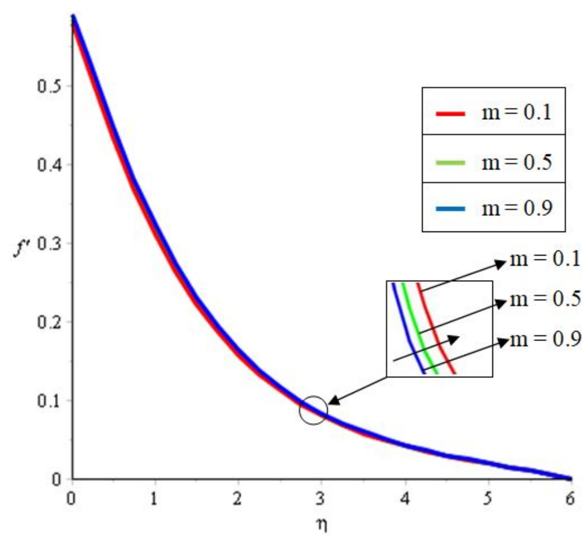

Fig 14: Hall parameter versus primary velocity

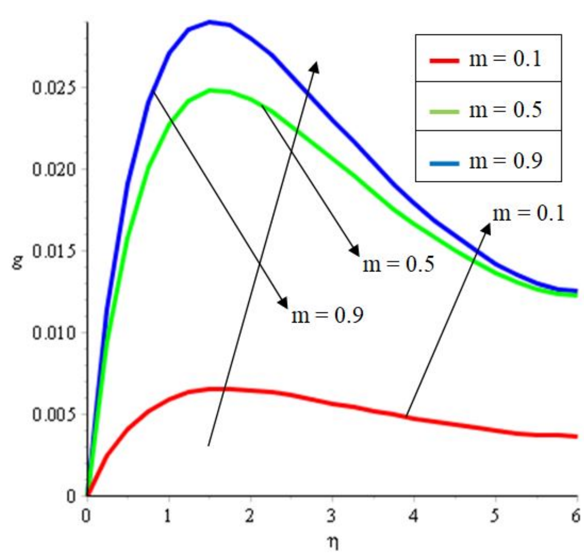

Fig 15: Hall parameter versus secondary velocity

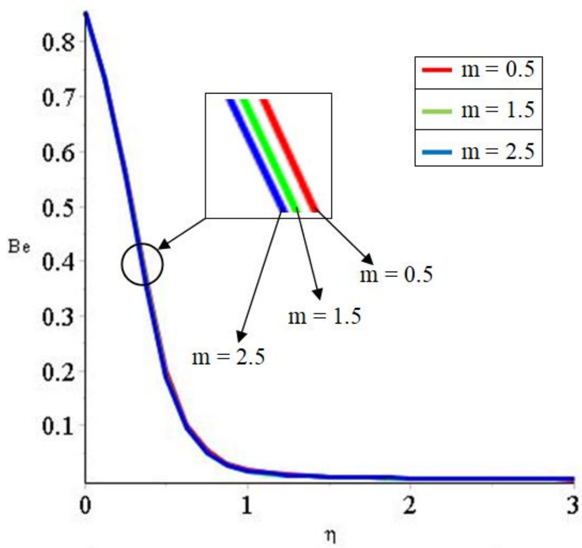

Fig 16: Hall parameter versus Bejan number

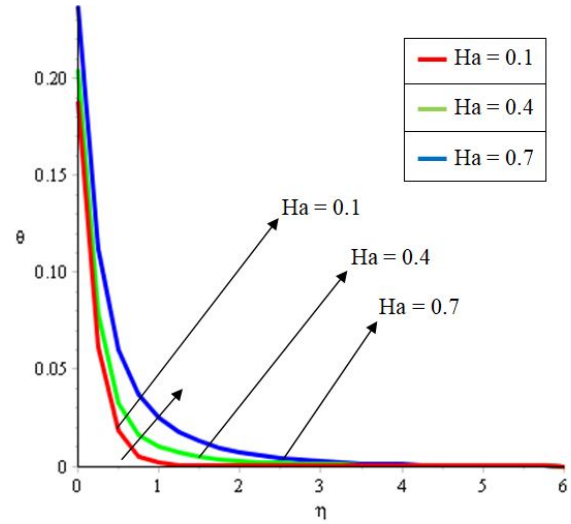

Fig 19: Hartmann number versus temperature profile

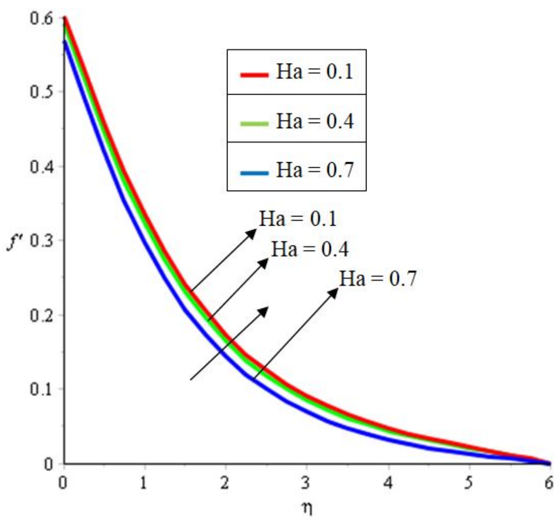

Fig 17: Hartmann number versus primary velocity

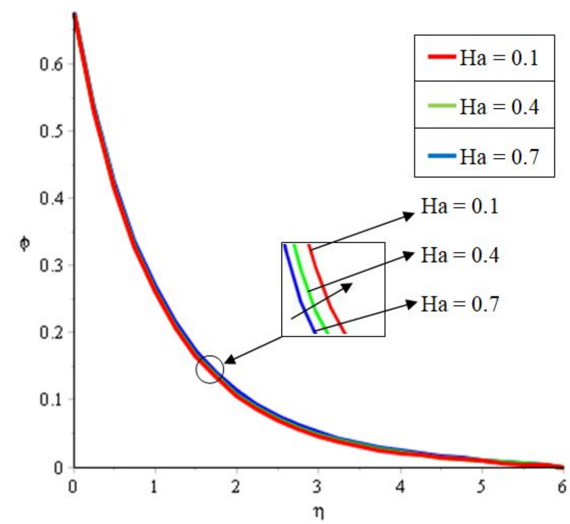

Fig 20: Hartmann number versus concentration profile

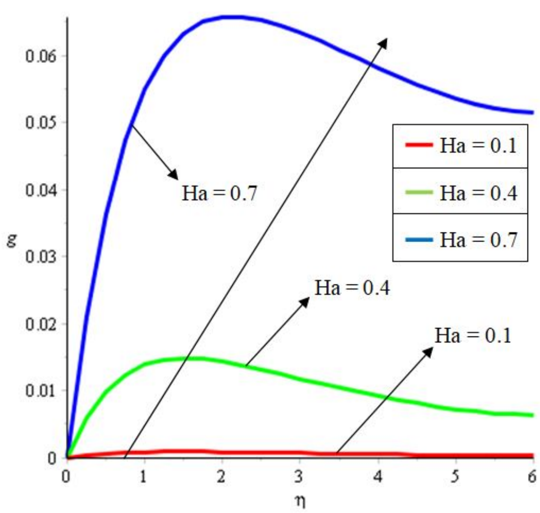

Fig 18: Hartmann number versus secondary velocity

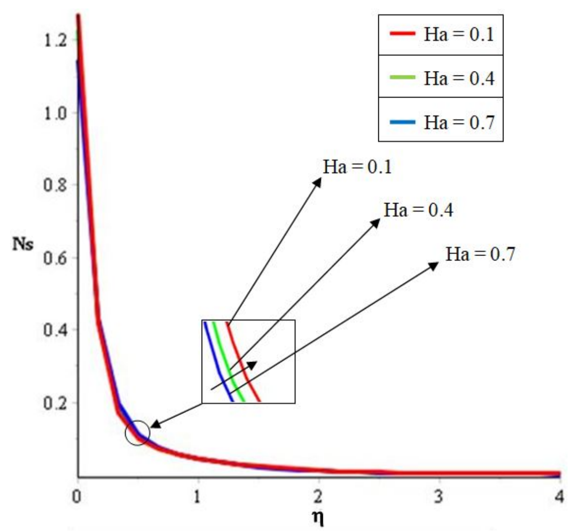

Fig 21: Hartmann number versus entropy generation

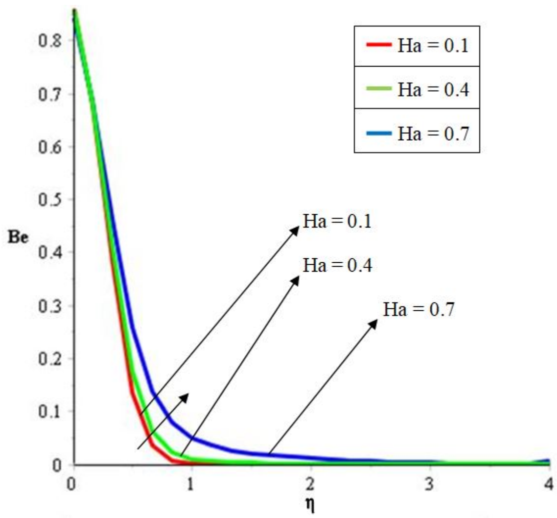

Fig 22: Hartmann number versus Bejan number

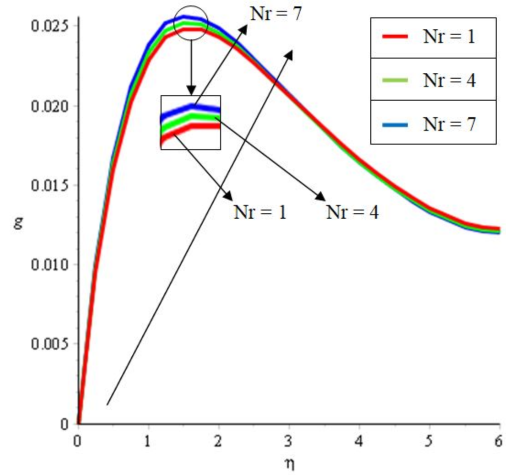

Fig 24: Thermal radiation parameter versus secondary velocity

From Figures 23 to 25 an increment in blood momentum and temperature is registered as thermal radiation parameter varies from 1 to 7. Blood flow momentum and temperature are enhanced due to an increase in the Rosseland mean absorption coefficient term (k^c), which occurs as a denominator in the term that denotes thermal radiation. Pathologists utilize enhanced blood flow to accelerate the healing process when treating and curing injuries. Increases in the thermal radiation parameter will also help patients receiving thermal radiation therapy overcome Lorentz's effect on blood flow by accelerating blood flow. Moreover, high temperatures are applied to human tissue and cancerous tumours as a result of an increase in thermal radiation. Consequently, the tumor-containing cancer cells are killed. An increment in entropy generation and Bejan number is depicted in Figures 26 and 27, the rise in the blood momentum and temperature can be attributed to this observed phenomenon. Furthermore, the enhancement in Bejan number implies the dominance of heat transfer irreversibility in entropy generation.

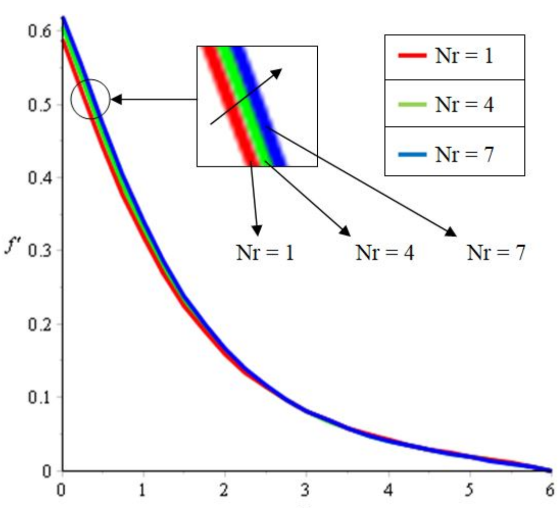

Fig 23: Thermal radiation parameter versus primary velocity

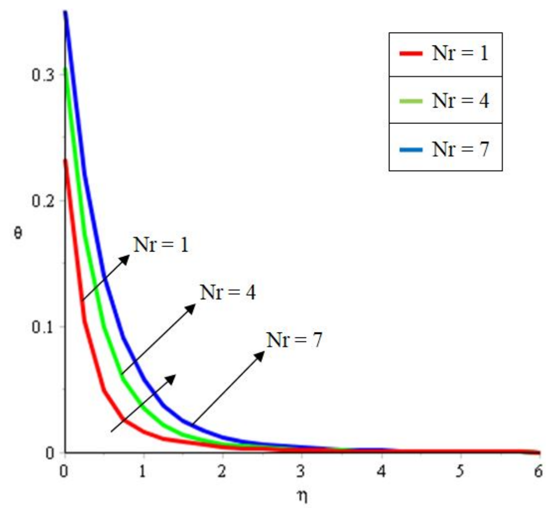

Fig 25: Thermal radiation parameter versus temperature profile

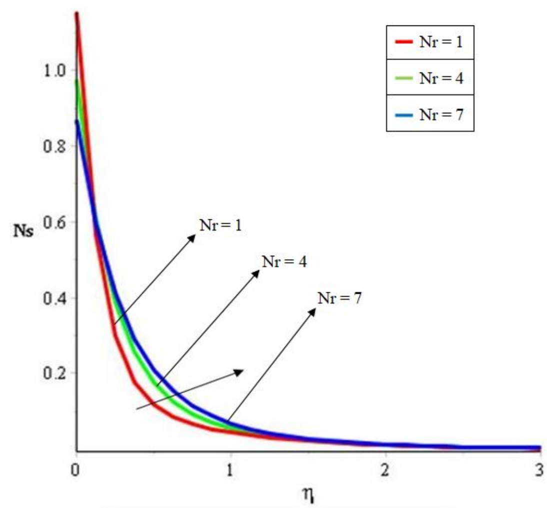

Fig 26: Thermal radiation parameter versus entropy generation

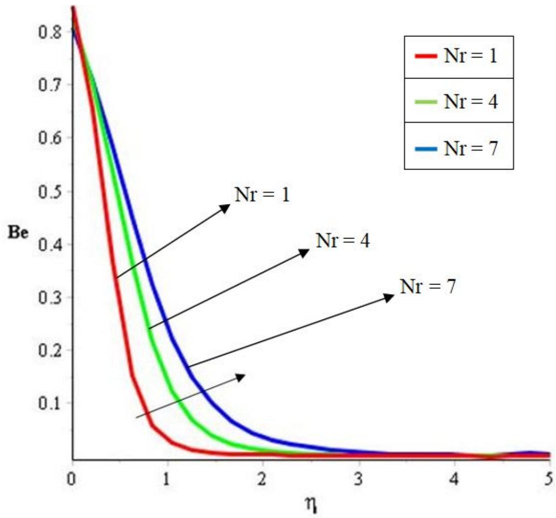

Fig 27: Thermal radiation parameter versus Bejan number

Figure [28] elucidates the correlation between an augmented Prandtl number and the velocity of blood flow. The findings indicate that an escalation in the Prandtl number results in a reduction in blood velocity. This phenomenon is ascribed to the presence of boundary layer effects in blood flow near the walls of blood vessels. An increase in viscosity, corresponding to a higher Prandtl number, may lead to the formation of thicker boundary layers along the vessel walls. Consequently, this could induce a decrease in blood velocity in proximity to the vessel walls.

Figure [29] illustrates the influence of the Prandtl number (Pr) on the temperature distribution within the boundary layer. It is observed that as the Prandtl number increases, there is a decrease in the temperature of the boundary layer. This trend is associated with the diminishment of the thermal boundary thickness with an increase in the Prandtl number. Consequently, the temperature gradient at the surface rises with an escalation in Pr, indicating a concurrent augmentation in the heat transfer rate at the blood vessel wall. This is attributed to the fact that higher Prandtl numbers correspond to lower thermal conductivity in blood, resulting in an increased heat conduction capacity and a reduction in the thermal boundary layer thickness, thereby enhancing the heat transfer rate at the vessel wall.

In Figure [30], it is noticeable that as the Prandtl number rises, there's a reduction in entropy generation (Ns). This reduction occurs because the influence of thermal effects becomes less pronounced compared to viscous effects. Figure [31] demonstrates the dominance of viscosity over thermal effects. Figures [32 & 33] suggest that an increase in the Eckert number leads to higher primary and secondary velocities, indicating a greater proportion of kinetic energy relative to the enthalpy of the blood. This surplus kinetic energy can influence blood flow patterns and velocities, thereby affecting thermodynamic processes.

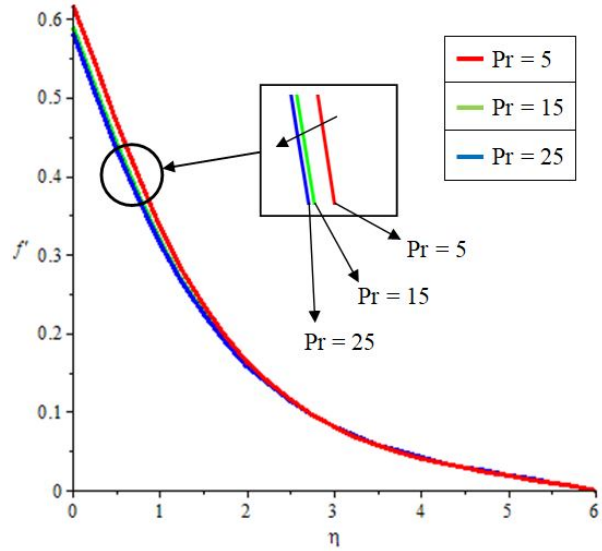

Fig 28: Velocity versus Prandtl number

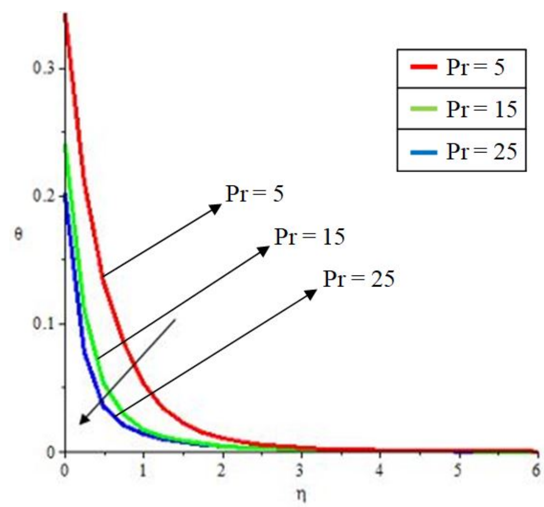

Fig 29: Temperature versus Prandtl number

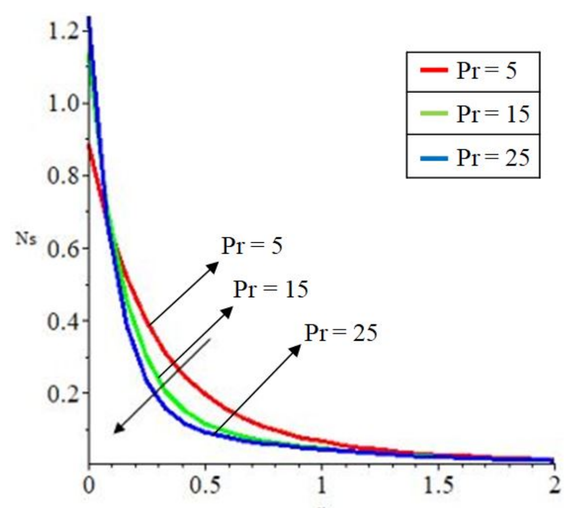

Fig 30: Entropy generation versus Prandtl number

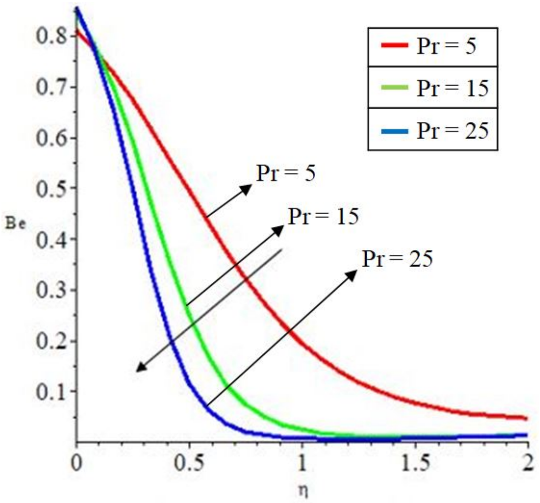

Fig 31: Bejan number versus Prandtl number

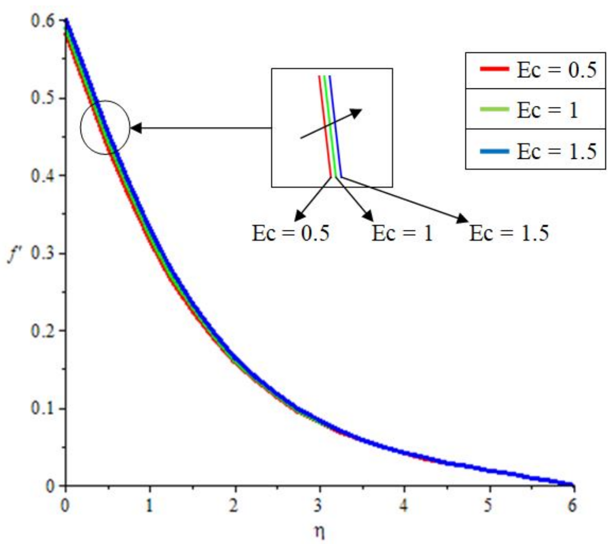

Fig 32: Primary Velocity versus Eckert number

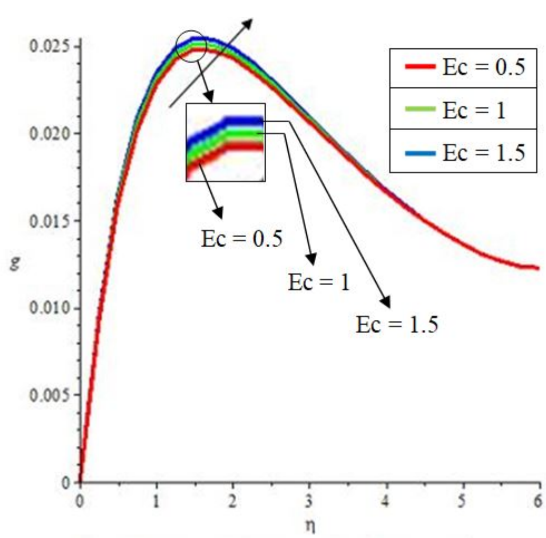

Fig 6.33: Secondary Velocity versus Eckert number

Figure [34] illustrates a sharp rise in temperature with increasing Eckert number, attributed to factors such as frictional heating, improved mixing, and energy dissipation. Figure [35] reveals that entropy generation escalates with an increase in the Eckert number due to alterations in heat transfer, shear stress, and viscous dissipation resulting from the higher Eckert number. Figure [36] indicates that blood flow experiences thermal dominance as the Eckert number increases.

Figure [37] demonstrates the correlation between the velocity of blood and the Schmidt number, revealing a decrease in blood velocity as the Schmidt number increases. This suggests that momentum diffusion becomes relatively more significant compared to mass diffusion. A higher Schmidt number, associated with increased viscosity, results in greater resistance to flow, thus causing a deceleration in blood velocity.

In Figure [38], the impact of the Schmidt number on blood concentration is depicted. It is observed that as the Schmidt number increases, there is a decrease in blood concentration. This phenomenon can be attributed to the indirect influence of the Schmidt number on biochemical reactions or processes affecting solute concentrations. Changes in viscosity, influenced by the Schmidt number, may affect enzyme activity or the binding affinity of solutes to proteins, thereby leading to variations in solute concentration. Physiologically, this suggests a slower diffusion of blood within the vessel. Figure [39] indicates a decrease in entropy generation with an increase in the Schmidt number. This is because a higher Schmidt number promotes smoother blood flow conditions, reducing shear stress along vessel walls and consequently decreasing entropy generation within the cardiovascular system.

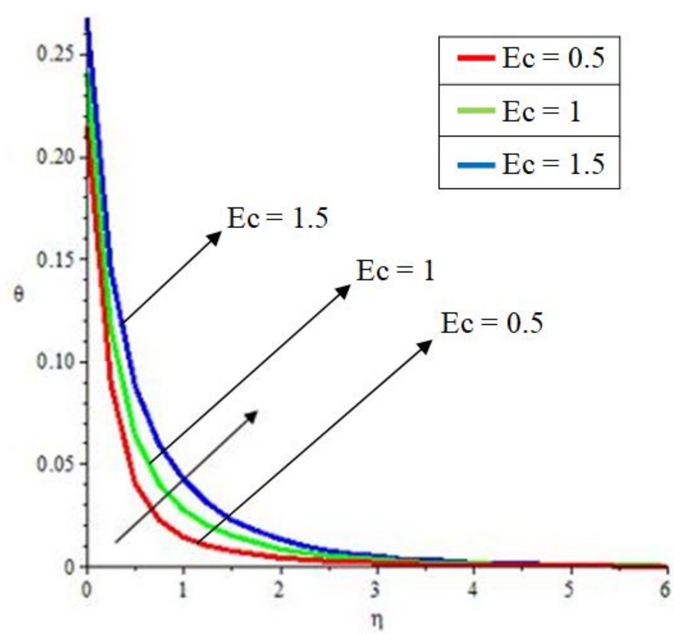

Fig 34: Temperature versus Eckert number

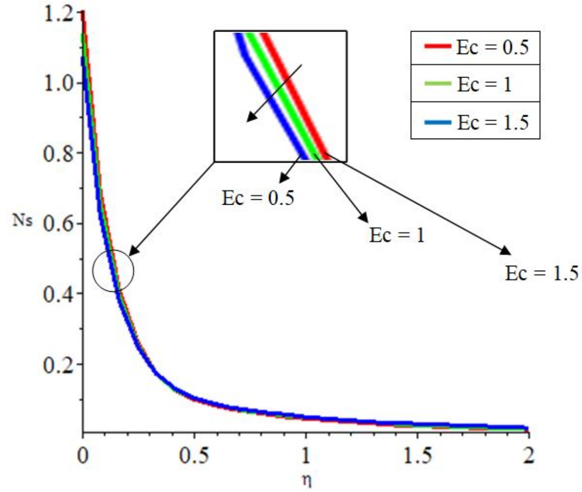

Fig 35: Entropy generation versus Eckert number

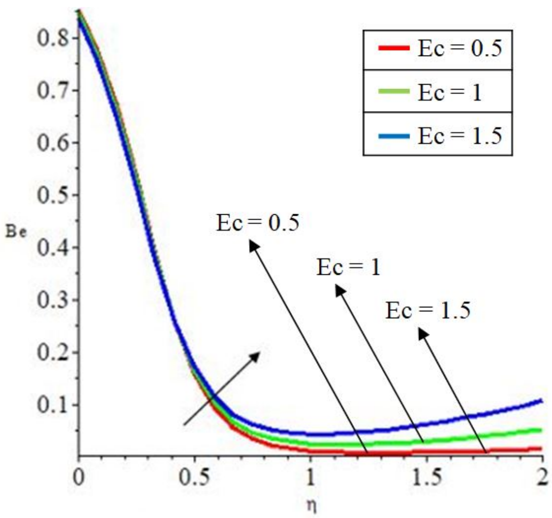

Fig 36: Bejan number versus Eckert number

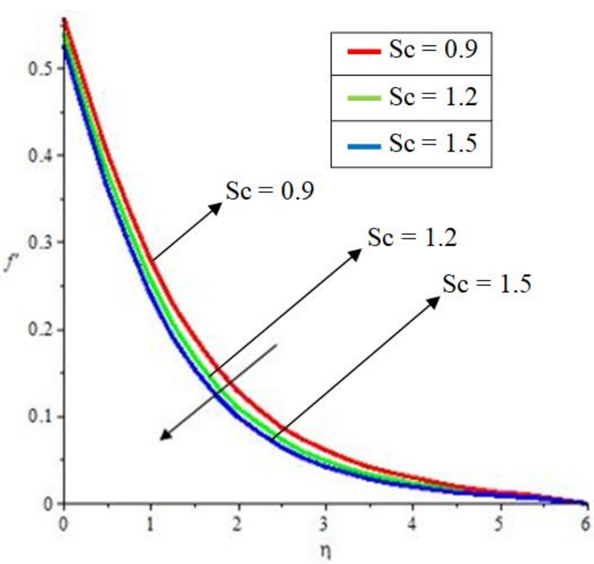

Fig 37: Velocity versus Schmidt number

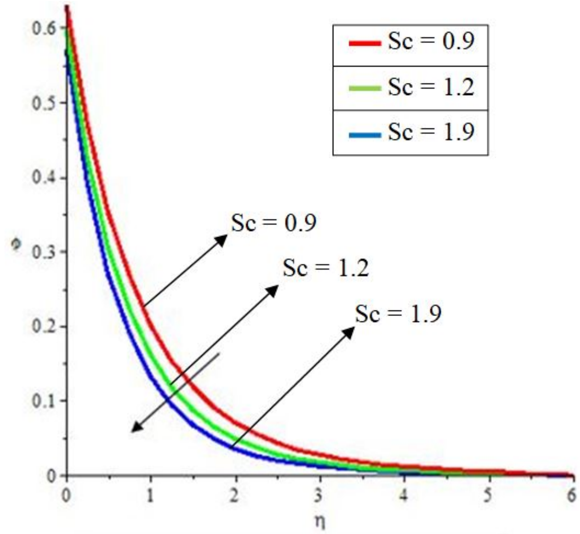

Fig 38: Concentration versus Schmidt number

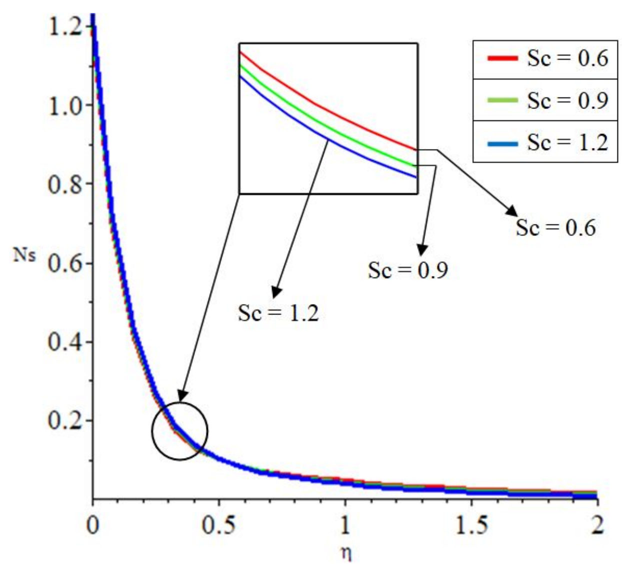

Fig 39: Entropy generation versus Schmidt number

In Figure [40], the prevalence of heat transfer irreversibility in entropy generation is suggested. Figure [41] illustrates a reduction in blood velocity with an increase in the chemical reaction parameter. This decrease is attributed to chemical reactions occurring within the blood, such as those involved in oxygen transport or pH regulation, which alter blood properties and interactions with vessel walls, consequently impacting flow dynamics and lowering velocity.

In Figure [42], the influence of the chemical reaction parameter on the concentration profiles of blood is depicted. It is observed that as the chemical reaction parameter increases, blood concentration decreases. This suggests that higher values of the chemical reaction parameter led to a decrease in molecular diffusivity, indirectly causing a reduction in blood concentration. Figure [43] demonstrates the effect of the chemical reaction parameter on temperature profiles. As the chemical reaction parameter increases, the temperature decreases. This is due to changes in biochemical processes such as metabolic activity and hormonal regulation.

Figure [44] depicts the relationship between the chemical reaction parameter and entropy generation. It is noted that an increase in the chemical reaction parameter promotes entropy generation. This is attributed to heightened molecular motion, energy dissipation, production of waste products, increased system complexity, and deviation from thermodynamic equilibrium. In Figure [45], an increase in the Bejan number is observed, indicating thermal dominance in entropy generation. This is primarily attributed to metabolic heat generation and heat transfer processes associated with chemical reactions occurring as part of metabolic processes such as cellular respiration, hormone regulation, and digestion. These reactions release energy, much of which is in the form of heat, thus an increase in chemical reactions leads to a higher rate of metabolic heat generation.

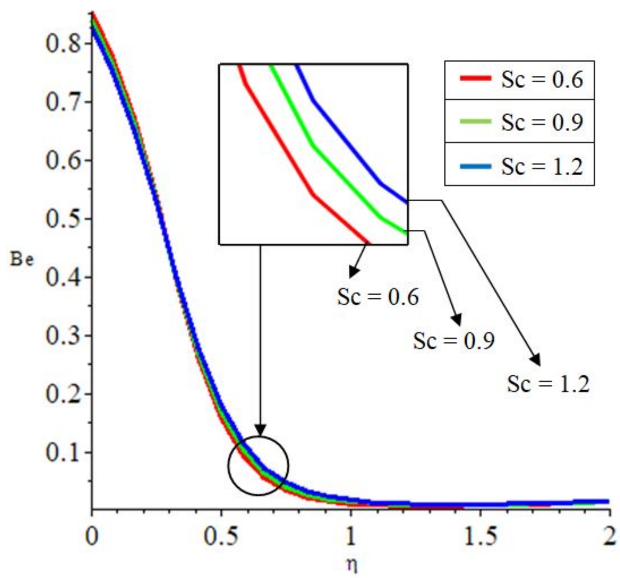

Fig 40: Bejan number versus Schmidt number

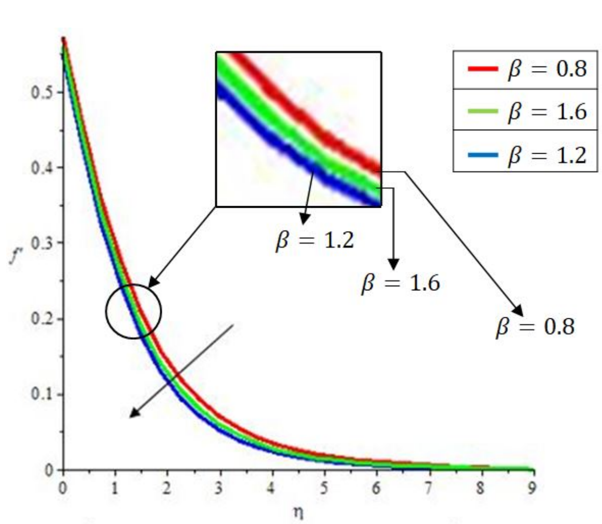

Fig 41: Velocity versus Chemical reaction parameter

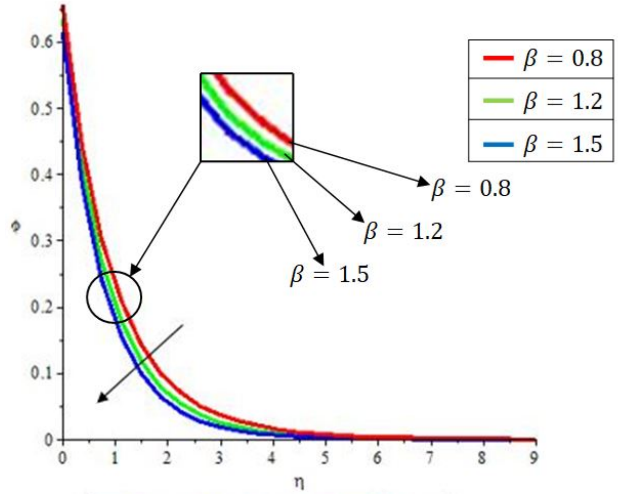

Fig 42: Concentration versus Chemical reaction parameter

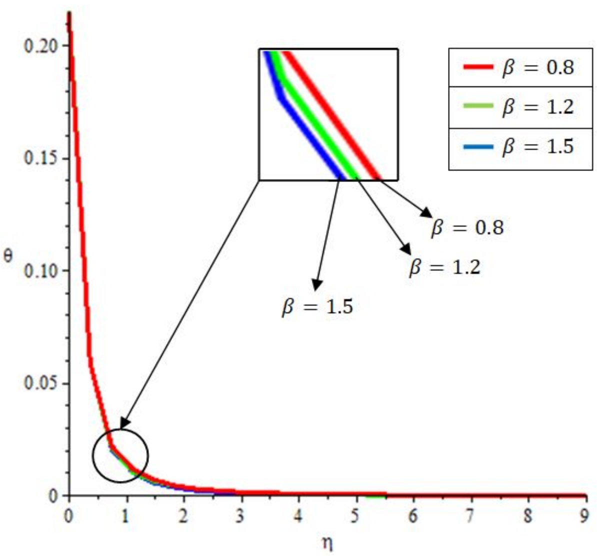

Fig 43: Temperature versus Chemical reaction parameter

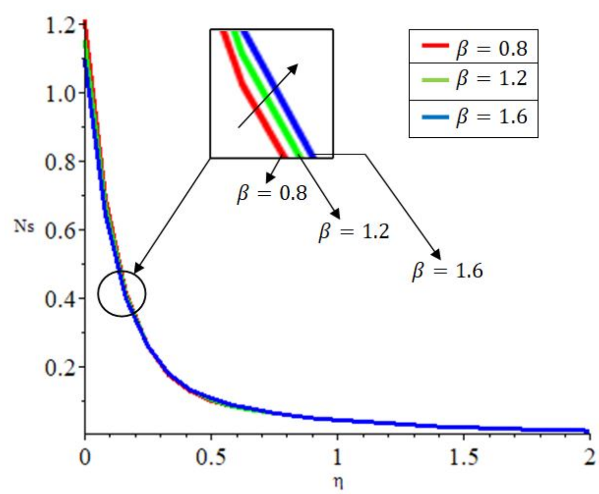

Fig 44: Entropy generation versus Chemical reaction parameter

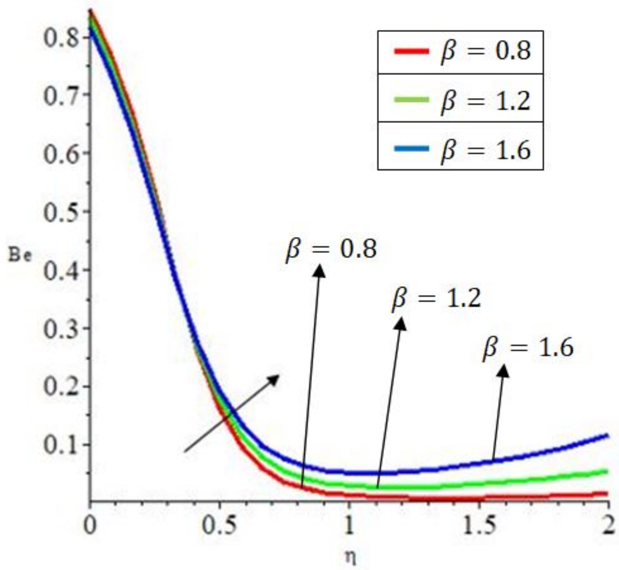

Fig 45: Bejan number versus Chemical reaction parameter

7 Conclusion

The unsteady two-dimensional hydromagnetic radiative blood flow in an inclined stretching vessel is theoretically analysed. The equations for the blood flow are obtained and converted to system of ODEs with appropriate similarity variables and the resulting dimensionless equations are numerically solved. Tables and Plots are presented to illustrate the impact of various physical parameters on the flow. The theoretical investigation of blood flow in a stretching permeable vessel will be of immense benefits for researchers, clinical engineers and medical practitioners to estimate blood flow velocity, temperature and concentration. In addition, it is applicable in areas such as drug targeting, cancer tumour treatment, magnetic devices for cell separation, adjusting blood flow during surgery and treatment of arterial diseases. The main highlights of the results are:

- Blood flow is accelerated by Hall and radiation parameters while unsteadiness and inclination parameters cause deceleration,
- Blood temperature is raised by increasing the values of Hartmann number, radiation and inclination angle parameters. However, unsteadiness parameter reduces it,
- Blood concentration is enhanced by inclination angle parameter and Hartmann number,
- Blood entropy generation and Bejan number receive a boost with the enhancement in Hartmann number and radiation parameter.

8 Nomenclature

A = Unsteadiness parameter

α = Inclination angle parameter

m = Hall current parameter

Ha = Hartmann Number

Nr = Radiation parameter

Cf = Skin friction parameter/coefficient

Nu = Nusselt number

u = Velocity component in $x-$direction

ν = Velocity component in $y-$direction

w = Component of dimensional velocity

x = $x-$axis is taken along the flow direction

y = $y-$axis is normal along the flow direction

t = Time

v = Kinematic viscosity of blood

σ = Electrical conductivity of blood

B = Time dependent magnetic field intensity

ρ = Density of blood

k_1 = Time – dependent permeability parameter of blood

g = Acceleration due to gravity

Γ = Coefficient of thermal expansion

Γ^* = Coefficient of mass expansion

T = Temperature of blood at any point in the vessel

T_∞ = Ambient temperature of the blood

C = Concentration of the blood

C_∞ = Ambient concentration of the blood

k = Thermal conductivity of the blood

C_p = Specific heat at constant pressure

q_r = Radiative heat flux of the blood

D = Molecular diffusivity

R = Chemical reaction rate

u_w = Stretching velocity

k^c = Rosseland mean absorption coefficient term

N = Velocity – slip factor

K = Thermal – slip factor

P = Concentration – slip factor

μ = Dynamic viscosity of the blood

f_w = Injection/Suction velocity

T_w = Wall temperature of the blood vessel

η = Similarity variable

ψ = Stream function

k_c = Porosiry parameter

Ω = Dimensionless temperature difference

λ_1 = Diffusive parameter

ε = Dimensionless constant parameter

Br = Brinkman number

Ns = Total entropy generation

N_1 = Irreversibility due to heat transfer

N_2 = Irreversibility to viscious effect

ϕ = Irreversibility distribution ratio

C_w = Concentration of the blood at vessel wall

Ec = Eckert number

Sc = Schimdt number

Pr = Prandtl number

β = Chemical reaction parameter

Gr = Thermal Grashof number

Gc = Solutal Grashof number

REFERENCES

[1] D. Ribati, "William Harvey and the discovery of the circulation of the blood". *Journal of Angiogenesis Research*, Vol 1, pp 1-2, September, 2009.

[2] S. Oka and T. Murata. "A theoretical study of the flow of blood in a capillary with permeable wall". *Int. J. Appl Phys*, Vol 9, no 4. pp 345-352, October, 1970.

[3] M. Dhange, G. Sankad, R. Safdar, W. Jamshed, M.R. Eid, U. Bhujakkanavar, S. Gouadria. and R. Cholukh. "A mathematical model of blood flow in a stenosed artery with post-stenotic dilatation and a forced field". *PLOS ONE*, Vol 17, no. 7, 2022 e0266727. https.//doi.org/1013711. Journal pone 0266727.

[4] M. S. Rahman, and M. A. Haque. "Mathematical modelling of Blood flow. International conference on informatics, Electronics & vision" *(IEEE/OSA.IAPR)*, pp 672-676, 2012.

[5] A. McCulloch, J. Bassingthwaighte, P. Hunter, and D. Noble. "Computation biology of the heat from structure to function". *Prog. Biophy. Mol. Biol*, pp 151-559, 1998.

[6] A. B. Mohamed, A. Yasser. and L. Taous-Meriem. "Human Hypertension Blood flow Model using Fractional Calculus". *Frontiers in physiology*, Vol 13, 838593 doi:105389/phys 2022838593. 2022.

[7] M. R. Mohammad, Y. Zhigang, M. B. Muhammad and A. A. Munawwar. "Heat and Mass Transfer Analysis on MHD Blood Flow of Casson Fluid Model due to Peristaltic Wave". *Journal of Thermal Science*, Vol 22, pp 1-3. 2016.

[8] M. J. Alam, M. G. Talukder, E. Tzintzilakis. and M. Ferdows. "Fluid Flow and Heat Transfer, the study of Blood with Gold nanoparticles over a stretching shear in the presence of magnetic dipole". *Fluids* Vol 6, pp 113, 2021.

[9] B. Zigta. "Effect of thermal radiation and chemical reaction on MHD flow of blood in a stretching permeable vessel". *Int. J. of Applied mechanics and Engineering.* Vol 25, no. 3, pp 198-211, 2020.

[10] B. Rekha, and A. Usha. "Mathematical Model of Blood flow in small blood vessel in the presence of magnetic field". *Applied Mathematics*, Vol 2, pp 264-269. 2011

[11] J. C. Misra, and A. Sinha. "Effect of thermal radiation on MHD flow of Blood and heat transfer in a permeable capillary in stretching motion". *Mass Transfer*, Vol 49, pp 617-628, 2013.

[12] R. P. Ganga. "Mathematical study of Blood flow characteristics during catherization in Stenosed Artery". *Dissertation of master of philosophy kathmandu university Nepal*, 2013.

[13] J. Prakash, and O. D. Makinde. "Radiative heat transfer to blood flow through a stenotic artery in the presence of magnetic field". *Latin American Applied Research*, Vol 41, pp 273-277, 2011.

[14] V. K. Sud, and G. S. Sekhon. "Blood flow through the human arterial system in the presence of a steady magnetic field". *Physiol Med Biol*, Vol 34, pp 795-805, 1989.

[15] J. C. Misra, A. Sinha. and G. C. Shit. "A numerical model for the magnetohydrodynamic flow of blood in a porous channel". *J. Mech Med Biol*, Vol 11, pp 547-562, 2011.

[16] J. C. Misra, G. C. Shit. and H. J. Rath. "Flow and heat transfer of MHD viscoelastic fluid in a channel with stretching walls. Some application to hemodynamic". *Compt. Fluids*, Vol 37, pp 1-11, 2008.

[17] J. C. Misra, A. Sinha, and G. C. Shit. ”Flow of a bio magnetic viscoelastic fluid: application to estimation of blood flow in arteries during electromagnetic hyperthermia, a therapeutic procedure for cancer treatment”. *Appl. Math Mech Eng. Edu.* Vol 31, pp 1405-1420, 2010.

[18] S. R. Reddy, R. P. Balaanki, and S. Suneetha. ”Magnetohydrodynamic flow of blood in a permeable inclined stretching surface with viscous dissipation, non-uniform heat source/sink and chemical reaction”. *Frontier in heat and mass transfer.* Vol 10, no. 22, pp 1-10, 2018.

[19] J. T. Majekodunmi, K. A. M. Khazali, and N. B. Abdullah. ”Numerical study of MHD blood flow through an Artery with multiple stenosis”. *10P Conf.ser.mater.sci. eng, 864012199*, pp 1-19, 2020.

[20] N. M. Zain, and Z. Ismail. ”Numerical solution of MHD effects on a generalized power law fluid model of blood flow through a bifurcated artery with an overlapping shaped stenosis”. *PLOS ONE*, Vol 18, no. 2, eco276576. https://doi.org/co137/journal pone 0216576, 2023.

[21] M. Priyadharsini, and M. G. David. ”Mathematical modelling and Analysis of thermoregulation effects on blood viscosity under magnetic effects and thermal radiation in a permeable stretching capillary”. *Journal of Thermal Biology*, pp 111-122, (103398), 2023

[22] A. Sinha, J. C. Misra, and G. C. Shit. ”Effect of heat transfer on unsteady MHD flow of blood in a permeable vessel in the presence of non-uniform heat source”. *Alexandria Engineering Journal*, 2016.

[23] S. Srinivas, P. B. Reddy, and S. R. V Bhuvanagiri S. R. V. ”Effect of chemical reaction and thermal radiation on MHD flow over an inclined permeable stretching surface with non-uniform heat source/sink: An application to the dynamics of blood flow”. *Journal of Mechanics in Medicine & Biology*, Vol 14, no. 5, 2016.

[24] B. Tripathi, and B. K. Sharma. ”MHD blood flood and heat transfer through an inclined porous stenosed artery with variable viscosity”. *Physics.flu.dynarxiv.161003470*, 2016.

[25] T. Bhavya, and K. Bhupendra. ”Effect of Heat transfer on MHD blood flow through an inclined stenosed porous artery with variable viscosity and heat source”. https://9PI.sematicscholar.org/carpusID: 214688547, 2018.

[26] S. Majeed, F. Ali, A. Imtiaz, I. Khan and M. Andualem. ”Fractional model of MHD blood flow in a cylinder containing magnetic particles”. *Sci.Rep 12418.* http”//doi.org/10.1038/341598-021-04088-9, 2022.

[27] A. Khalid, I. Khan, A. Khan, S. Shafie and I. Tilili. ”Case study of MHD blood flow in a porous medium with CNTS and thermal analysis”. *Case studies in Thermal Engineering*, Vol 12, pp 314-380, 2018.

[28] S. Majee, S. Maiti, G. C. Shit and D. K. Maiti. ”Spatio-temporal evolution of MHD blood flow and heat dynamics through a porous medium in a wavy-walled artery computers in biology and medicine”, 135104595. ISSN10010-4825, 2021.

[29] R. Latha, and B. Kumar. ”Unsteady MHD blood flow through porous medium in a parallel plate channel”. *10P Conf.services: Materials science and Engineering 263062020*, 2017.

[30] S. Priyadharsini, G. J. Raj, and S. Ramesh. ”Effect of Hall Current in an oscillatory blood flow”. Journal of Applied science and computations, Vol 5, no. 3, pp 1269-1279, 2019.

[31] I. Abdullah, N. Naser, A. Talib, and S. Mahali. ”Effects of magnetic field and hall current to the blood velocity and LDL transfer”. *Journal of Physics: conference services* 63302133. Doi:10.1088/1742-6596/633/012133, 2015.

[32] K. Umash, and K. S. Bhupendra. ”Hall and ion ship effects on hybrid nano particles (Au-GO/Blood) flow through a catharized stenosed artery with thrombosis. Proceedings of the institution of mechanical engineers”, *Journal of mechanical engineering science*, Vol 237, no. 10, pp 2256-2278, 2022. Doi: 10.1177/09544062221136710.

[33] S. Das, T. K. Pal, R. N. Jana, and B. Girl. ”Ascendancy of electromagnetic force and hall currents on blood flow carrying Cu-AU NPs in a non-uniform endoscopic annulus”. *Microvascular Research*, pp 138, 2021.

[34] S. K. Asha, and G. Sunitha. ”Thermal radiation hall effects on peristaltic blood flow with double diffusion in the presence of nanoparticles”. *Case Studies in Thermal Engineering* Vol 17, 2020.

[35] K. S. Mekheimer, and M. A. El kot. ”Influence of magnetic field and hall currents on blood flow through a stenotic artery”. *Applied mathematics and mechanics*, Vol 29, pp 1093-1104. https://doi.org/10.1007/S10483-008-0813-x, 2008.

[36] M. R. Rashidi, M. M. Bhatti, M. A. Abbas. and M. E. Ali. ”Entropy generation on MHD Blood flow of nano fluid due to peristalic waves. Entropy, Vol 18, no. 4, pp 17. https://doi.org/10.3390/ei8040117, 2016.

[37] E. A. Algehyne, N. A. Ahammad, M. E. Elnair, M. Zidan, Y. Y. Alhusayni, B. O. El-Bashir, A. Saeed, A. S. Alshomrani, and F. Alzahrani. ”Entropy optimization and response surface methodology of blood hybrid nano fluid flow through composite stenosis artery with magnetized nano particles (Au-Ta) for drug delivery application”. *Sci Rep*, Vol 13, 9856. https://doi.org/10.1038/841598-023-36931-6, 2023.

[38] L. Zhang, M. M. Bhatti, M. Marin, and K. S. Mekheimer. "Entropy Analysis on the Blood flow through Anisotropically Tapered Arteries Filled with magnetic Zinc-oxide (ZnO) nano particles". *Entropy Basel*, Vol 22, no. 10, pp 1070, doi 10. 33901e22101070, 2020.

[39] M. H. Shahzad, A. U. Awan, S. Akhtar, and S. Nadeem. "Entropy and Stability analysis on blood flow with nano particles through a stenosed artery having permeable walls". *Scipros*, Vol 105, no. 2, doi: 10. 1177/00368504221096000. PMID: 35491727, 2022.

[40] Q. Al-Mdallal, G. M. Rahimi, B. Sharma, and C. Kumawat. "Entropy generation for MHD two phase blood flow through a curved permeable artery having variable viscosity with heat and mass transfer". *International communications in heat and mass transfer*. Vol 133, pp 105-135, 2022.

[41] G. Antony, A. Aravind, and J. Ravikumar. "Effects of Thermal radiation in a rotating fluid on an oscillatory vertical plate with variable temperature and mass diffusion", *IAENG International Journal of Applied Mathematics*, Vol 54, no. 1, pp 128-139. 2024.

[42] S. Sharidan, T. Mahmood, and I. Pop. "Similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet". *International Journal of Applied Mechanics*, Vol 11, pp 647–654, 2006.

[43] A. J. Chamkha, A. M. Aly, and M. A. Mansour. (2010). "Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects". *Chemical Engineering Communications*, Vol 197, no. 6, pp 846-858. https://doi.org/10.1080/00986440903359087. 2010.

[44] A. Ishak. "Unsteady MHD and Heat Transfer over a stretching Plate". *Journal of Applied Sciences*, Vol 10, no. 18, pp 2127-2131, 2010. https://scialert.net/abstract/?doi=jas.2010.2127.2131

[45] T. Hayat, and M. Qasim. "Influence of thermal radiation and Joule heating on MHD flow of a Maxwell fluid in the presence of thermophoresis". *International Journal of Heat and Mass Transfer*, Vol 53, pp 4780-4788, https://doi.org/10.1016/j.ijheatmasstransfer. June, 2010.

[46] B. Jaismitha, and J. Sasikumar. "Chemically reactive oscillatory casson hybrid nanofluid flow with heat generation/absorption phenomenon through radiating wavy channel". *IAENG International Journal of Applied Mathematics*, Vol 53, no. 4, pp 1148-1161, 2023.

[47] B. George, T. K. Martin, and M. M. William. "Effective prandtl number, Hall currents, soret and Dufour effect on MHD flow past an inclined stretching sheet with applied magnetic field and heat generation", *IAENG International Journal of Applied Mathematics*, Vol 53, no. 4, pp 1201-1210, 2023.

[48] G. Rishu, B. K. Sharma, M. Qasem. and H. V. B. Mittal. "Entropy generation and shape effects analysis of hybrid nanoparticles (Cu-Al_20_3/blood) mediated blood flow through a time-variant multi-stenotic artery". *International Journal of Thermofluids*, Vol 18, 2023.

[49] I, Savthi, D. Raja, A. Bala, and P. Reddy. "Entropy generation for the MHD flow of a blood-based hybrid nanofluid by thermal radiation over converging and diverging channels". *Numerical Heat transfer, Part B: Fundamentals*, Vol 85, no. 3, pp 258-285, Doi:10.1080/10407790.20232232540. 2023.