A Study on the Evaluation of Teaching Effectiveness of Secondary School Mathematics Based on Fuzzy Comprehensive Evaluation

Meng Hu, Mengyao Xu

Abstract--**This paper aims to comprehensively evaluate the effect of secondary school mathematics classroom teaching by establishing a mathematical model. To this end, this paper first through a questionnaire survey, on the basis of statistics and analysis of the data, to construct a new evaluation index system that can truly reflect the effect of secondary school mathematics classroom teaching. On this basis, this paper proposes a two-level fuzzy comprehensive evaluation model for the first time, and explains the evaluation process of the model through a specific example, which realizes the evaluation of the effect of secondary school mathematics classroom teaching from multiple dimensions. Compared with the traditional evaluation method, the evaluation method given in this paper is more scientific, and the evaluation results have more application value.**

Index Term--**fuzzy comprehensive analysis; classroom teaching effectiveness evaluation; secondary school mathematics**

I. Introduction

THE evaluation of teaching effect has become a hot issue in the field of education at home and abroad. Especially in primary and secondary schools, schools are fully aware that the teaching effect of teachers is the core of the survival and development of schools [1]. Evaluation of teaching effect is an extremely important part of the teaching process. The growth of students and the development of schools are closely related to the teaching effect of teachers.

At present, the teaching and research departments at all levels and the teaching and research groups of many schools mostly use the way of lectures to evaluate the effect of classroom teaching, but most of these classroom teaching evaluations stay at the level of experience summary, lack of in-depth understanding of the theoretical level of mathematical science, and lack of understanding of modern educational theory.

Because the effect of secondary school mathematics classroom teaching is affected by many factors, such as teachers' teaching methods, teaching styles, interaction with students, whether to pay attention to teaching students in accordance with their aptitude, and also related to the knowledge level and cognitive ability of evaluators, how to make a comprehensive and objective evaluation of the effect of secondary school mathematics classroom teaching, and in order to promote the construction of secondary school mathematics teachers, to help teachers improve their teaching level and working ability, is an urgent problem to be solved by the school management department.

Based on the construction of two-level evaluation index system, this paper will establish a multi-level fuzzy comprehensive evaluation model, which is different from the traditional scale evaluation method, and make an objective evaluation of teachers' classroom teaching effectiveness from multiple dimensions, aiming to provide a theoretical basis for teachers to optimize classroom teaching design.

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II. Preliminaries

2.1 Fuzzy comprehensive evaluation

Fuzzy comprehensive evaluation is the use of fuzzy mathematical tools to make a judgment about a research object [2]. The fuzzy comprehensive evaluation method typically consists of the following basic steps [3]:

(i) Establishing the evaluation object factors set

$$U=\{u_1,u_2,\cdots,u_n\}.$$

The factors are various attributes or performance indicators that respond to the evaluation object.

(ii) Determining the evaluation level set

$$V=\{v_1,v_2,\cdots,v_m\},$$

i.e., determining the evaluation levels under the indicators $u_1,u_2,\cdots,u_n$. The number of general judgement levels m should not be overly many or few, which are typically classified as very good, good, fair, poor and very poor.

(iii) Determining the fuzzy comprehensive evaluation matrix.

According to the actual problem, the membership degree of the object to be evaluated to the level v_i under the factor u_i is determined, which denotes as r_{ij} $(0\le r_{ij}\le 1, i=1,2,\cdots,n, j=1,2,\cdots,m)$. This determines the fuzzy evaluation matrix $R=(r_{ij})_{n\times m}$.

(iv) Determining the fuzzy weight vector of evaluation indicators.

The vector is determined according to the magnitude of the importance of the evaluation object to each indicator. Set $\tilde{W}=(w_1,w_2,\cdots,w_n)$, and $\sum_{i=1}^{n} w_i=1$.

(v) Determining the comprehensive evaluation model.

The fuzzy weight vector $\tilde{W}$ is combined with the fuzzy evaluation matrix R to form a fuzzy comprehensive evaluation vector $\tilde{B}=(b_1,b_2,\cdots,b_m)$, in which b_j denotes the membership degree of the evaluation object with respect to the level $v_j(j=1,2,\cdots,m)$ as a whole.

2.2 Mathematics classroom teaching evaluation

Teaching evaluation is an activity that judges the value of teaching process and results based on teaching objectives and serves for teaching decision-making. It is a process of judging the real or potential value of teaching activities. Teaching evaluation is the process of studying the value of teachers' teaching and students' learning. Teaching evaluation generally includes the evaluation of teachers, students, teaching content, teaching methods, teaching environment, teaching management and other factors in the teaching process, but mainly the evaluation of students' learning effect and the evaluation of teachers' teaching process. 'Mathematics curriculum standards' [4] pointed out that, mathematics classroom teaching evaluation should do the following aspects:

(i) Enrichment of evaluation methods

According to the contents and characteristics of class-room teaching, different evaluation methods are adopted.

(ii) Multiple evaluation dimensions

In the process of multi-dimensional evaluation of students, we should not only achieve "four bases" and "four abilities", but also pay special attention to the corresponding performance of core literacy, and comprehensively assess and evaluate the formation and development of students' core literacy.

(iii) Diverse evaluation subjects

To conduct a comprehensive investigation of students' learning situation, we should comprehensively use the evaluation subjects such as teachers, students and parents.

(iv) Presentation and application of evaluation results

According to the age characteristics of students, the evaluation results should be presented in a combination of qualitative and quantitative methods, and evaluated in different sections.

Whether students can learn, whether they have progress in their studies, and improving teachers' teaching skills and teaching level are all the contents of mathematics teaching evaluation.

III. Evaluation index system

The current teaching evaluation depends largely on the subjective understanding of the lecture experts. In order to better reflect the teaching quality of teachers and the learning effect of students, many researchers have proposed new index systems for classroom teaching evaluation [5], which improves the objectivity of evaluation by continuously improving evaluation methods.

3.1 Principles for selecting indicators for mathematics classroom teaching evaluation

(i) Principle of advancement

Modern education should not only foster all-round development of human beings, but also focus on the inheritance and development of culture.

(ii) Principle of independence

The evaluation contents of mathematics classroom teaching should be more to avoid repetition and intersection, and it should be as accurate as possible.

(iii) Principle of operability

The principle of operability, namely whether the data related to the indicators are easy to obtain, whether the selected indicators are clear, etc. are all related to the simplicity of the classroom teaching evaluation indicators.

3.2 Evaluation indicators screening for mathematics classroom teaching

3.2.1 Evaluation indicators for mathematics classroom teaching

By reviewing the literature related to the mathematics classroom teaching evaluation and researching secondary school mathematics experts (mainly mathematics teachers engaged in mathematics teaching in No.5 Secondary School and Guanghua Secondary School), the evaluation indicators of the secondary school mathematics classroom teaching are established, with specific contents as shown in Table 1.

3.2.2 Questionnaire design

Design a questionnaire for the importance of teachers' classroom teaching evaluation indicators, and score the importance of each indicator, which provides a scoring specification of 0 to 3 points. It clearly specifies that very important is recorded as 3 points, important is recorded as 2 points, average is recorded as 1 point, and unimportant is recorded as 0 points. Meanwhile, the design of the questionnaire should pay attention to the following issues:

(i) Whether the indicators in the questionnaire are realistic.

(ii) Focus on differentiating the level of importance of each indicator.

(iii) Deletion or modification of indicators in accordance with the survey results.

(iv) Whether the questionnaire is distributed in a reasonable manner.

3.2.3 Indicator screening

The teachers of this survey are composed of 400 teachers including mathematics teachers of No.5, No.6, No.8, No.9, No.21, No.32, No.66, Guanghua secondary schools and some rural secondary schools, and mathematics teachers of tutoring institutions. A total of 400 questionnaires are distributed to the mathematics teachers of Level 7 to Level 9 included in the survey. The statistical results of the questionnaire data are shown in Table 2.

According to the statistical data of Table 2, the total score of each indicator can be calculated, and the specific score is shown in Table 3.

Fig. 1 presents a line graph of the relationship between the indicator and its total score.

3.2.4 Certification of indicators

By analyzing the line graph of Fig. 1, it can be seen that only the indicator "8. Focusing on the improvement of students' mathematical literacy" has a low score, so this indicator is discarded and other indicators are retained. The final retention indicators are shown in Table 4.

From Fig. 1 and Table 4, it can be concluded that in terms of teaching plan, "scientific and rational classroom teaching" is a significant indicator; with regard to teaching attitude, "adequate preparation of lectures" and "strict management of classroom discipline" are significant indicators; and in terms of classroom performance, "degree of memorization of mathematical formulas" is a significant indicator.

3.2.5 Reliability and validity analysis

According to the 16 indicators in Table 4 and the statistical data of the questionnaire survey, SPSS software is used to analyze the reliability and validity of the scale. The

results are shown in Table 5 and Table 6:

TABLE 5 RELIABILITY COEFFICIENTS

Cronbach α	Items
0.858	16

TABLE 6 KMO AND BARTLETT'S TEST

KMO Measure of Sampling		0.870
Bartlett's Test of Sphericity	Approx. Chi-Square	2123.271
	Sig. (p)	0.000

From Table 5 and Table 6, we can see that the Cronbach coefficient α=0.858, close to 1; KMO measure is 0.870 > 0.6; the corresponding probability p of Bartlett's Test of Sphericity is close to 0<0.05. Hence, the scale passes the reliability and validity test, so the research results of this paper are reliable and effective.

IV Comprehensive Fuzzy Evaluation Model

Since there are subjective factors in the process of scoring or selecting indicators by teachers, the indicators themselves are fuzzy and there are multi-level phenomena in the indicators. Therefore, for the sake of making the evaluation results more accurate and reasonable, this paper choose the fuzzy evaluation method to evaluate the secondary school mathematics classroom teaching, which has been described in section II.

Firstly, the weight vectors of the second-level indicators and the fuzzy comprehensive evaluation matrices of the second-level indicators are calculated. The weight vectors of the second-level indicators and the corresponding fuzzy comprehensive evaluation matrices are combined to obtain the fuzzy comprehensive evaluation matrix R of the first-level indicators, the weight vector of the first-level indicators C can also be determined.

Next, the weight vector of the first-level indicators C and the fuzzy comprehensive evaluation matrix R are combined to obtain the fuzzy comprehensive evaluation vector B. The judgment can be made according to the evaluation vector B and the evaluation level set V.

4.1 Description of symbols

The specific meaning of some mathematical symbols used in this paper is shown in Table 7.

4.2 Calculation of weights

In the evaluation of secondary school mathematics classroom teaching, assume that there are n first-level indicators, and the sum of the weights of the first-level indicators is 1, that is,

$$C_1 + C_2 + \cdots + C_n = 1. \tag{1}$$

The weight of each first-level indicator is the sum of the corresponding weights of second-level indicators, and

$$C_i = C_{i1} + C_{i2} + \cdots + C_{ik_i}, (i = 1, 2, \cdots, n) \tag{2}$$

that is,

$$\begin{cases} C_1 = C_{11} + C_{12} + \cdots + C_{1k_1}, \\ \cdots \\ C_n = C_{n1} + C_{n2} + \cdots + C_{nk_n}. \end{cases} \tag{3}$$

The process of calculating the weights for the evaluation of secondary school mathematics classroom teaching is organized into the following steps:

(i) The score of the second-level indicators under the i th first-level indicator branch satisfies

$$F_i = F_{i1} + F_{i2} + \cdots + F_{ik_i}, (i = 1, 2, \ldots, n). \tag{4}$$

The total score F of the secondary indicators satisfies

$$\begin{cases} F = F_1 + F_2 + \cdots + F_i, \\ F = F_{11} + F_{12} + \cdots + F_{1k_1} + F_{21} + F_{22} \\ \quad + \cdots + F_{2k_2} + \cdots + F_{n1} + F_{n2} + \cdots + F_{nk_n}. \end{cases} \tag{5}$$

(ii) The weight C_{ij} of the second-level indicator for the secondary school mathematics classroom teaching is calculated as

$$C_{ij} = \frac{F_{ij}}{F}. \tag{6}$$

(iii) According to (2), the weight $C_i (i = 1, 2, \cdots, n)$ of the first-level indicator can be calculated, as shown in Table 8.

From Table 8, the relationship between the weight and the score of each indicator can be obtained, see (7)-(12) :

$$\begin{aligned} F = {} & F_{11} + F_{12} \\ & + F_{21} + F_{22} + F_{23} \\ & + F_{31} + F_{32} + F_{33} + F_{34} \\ & + F_{41} + F_{42} + F_{43}, \end{aligned} \tag{7}$$

$$C_{11} = \frac{F_{11}}{F}, C_{12} = \frac{F_{12}}{F}, \tag{8}$$

$$C_{21} = \frac{F_{21}}{F}, C_{22} = \frac{F_{22}}{F}, C_{23} = \frac{F_{23}}{F}, \tag{9}$$

$$C_{31} = \frac{F_{31}}{F}, C_{32} = \frac{F_{32}}{F}, C_{33} = \frac{F_{33}}{F}, C_{34} = \frac{F_{34}}{F}, \tag{10}$$

$$C_{41} = \frac{F_{41}}{F}, C_{42} = \frac{F_{42}}{F}, C_{43} = \frac{F_{43}}{F}, \tag{11}$$

$$\begin{cases} C_1 = C_{11} + C_{12}, \\ C_2 = C_{21} + C_{22} + C_{23}, \\ C_3 = C_{31} + C_{32} + C_{33} + C_{34}, \\ C_4 = C_{41} + C_{42} + C_{43}. \end{cases} \tag{12}$$

In order to calculate the weights of the first-level indicators of teachers' teaching quality evaluation, we must first calculate the total score of the second-level indicators, and then calculate the weights of the second-level evaluation indicators.

4.3 Calculation of membership matrix

(i) Data statistics

On the basis of the secondary school mathematics classroom teaching effectiveness evaluation system table, combined with the questionnaire data, the score of each evaluation object belonging to each evaluation level under each secondary evaluation indicator can be obtained, as shown in Table 9.

(ii) Calculating the membership degree

The membership degree of the evaluation object

belonging to each evaluation level under the second-level evaluation indicators corresponding to the i th first-level evaluation indicator can be calculated by (13),

$$r_{jz}^{i}=\frac{N_{jz}^{i}}{M}. \tag{13}$$

(iii) Constructing the fuzzy comprehensive evaluation matrix R_i of the second-level evaluation indicators under the i th first-level indicator branch.

According to the calculation results in (ii), the fuzzy comprehensive evaluation matrix R_i can be established,

$$R_i=\begin{bmatrix} r_{11}^{i} & r_{12}^{i} & \cdots & r_{1m}^{i} \\ r_{21}^{i} & r_{22}^{i} & \cdots & r_{2m}^{i} \\ r_{31}^{i} & r_{32}^{i} & \cdots & r_{3p}^{i} \\ \cdots & \cdots & \cdots & \cdots \\ r_{k_i1}^{i} & r_{k_i2}^{i} & \cdots & r_{k_im}^{i} \end{bmatrix}. \tag{14}$$

Based on the weight vector of the second-level evaluation indicators C_i and the fuzzy comprehensive evaluation matrix R_i, the fuzzy composite operation "$\circ$" is performed to obtain the vector B_i, see (15),

$$\begin{aligned} B_i&=\left(c_{i1},c_{i2},\cdots,c_{ik_i}\right)\circ\begin{bmatrix} r_{11}^{i} & r_{12}^{i} & \cdots & r_{1m}^{i} \\ r_{21}^{i} & r_{22}^{i} & \cdots & r_{2m}^{i} \\ r_{31}^{i} & r_{32}^{i} & \cdots & r_{3p}^{i} \\ \cdots & \cdots & \cdots & \cdots \\ r_{k_i1}^{i} & r_{k_i2}^{i} & \cdots & r_{k_im}^{i} \end{bmatrix} \\ &=\left(b_1^{i},b_2^{i},\cdots,b_m^{i}\right). \end{aligned} \tag{15}$$

There are five composite operations of fuzzy relations, i.e., $M(\bullet,+), M(\wedge,\vee), M(\bullet,\vee), M(\wedge,\oplus), M(\bullet,\oplus)$.

(iv) Constructing a fuzzy comprehensive evaluation matrix R under the first-level evaluation indicators.

The fuzzy comprehensive evaluation matrix R of the first-level evaluation indicators can be established by combining the vectors $B_i, i=1,2,\cdots,n$ which have been obtained in (iii), see (16),

$$R=\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}. \tag{16}$$

4.4 Comprehensive evaluation results

(i) After obtaining the weights of the first-level indicators and the fuzzy comprehensive evaluation matrix R of the first-level indicators, the comprehensive evaluation vector B can be obtained as shown in (17),

$$B=C\circ R=C\circ\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}=\left(b_1,b_2,\cdots,b_m\right), \tag{17}$$

where $b_k\left(k=1,2,\cdots,m\right)$ refers to the comprehensive evaluation value of the individual evaluation indicators.

(ii) The normalization of b_k is processed to obtain b_k', and

$$b_k'=\frac{b_k}{\sum_{k=1}^{m}b_k}. \tag{18}$$

(iii) In accordance with the formula (18), the evaluation value can be calculated, namely w-value, which can be used to obtain the evaluation level of secondary school mathematics classroom teaching,

$$w=b_k'\times v^{T}. \tag{19}$$

The w-value of the evaluation scale for the evaluation of the effectiveness of secondary school mathematics classroom teaching lies at the level of the evaluation scale in which the whole comprehensive evaluation is located.

V Case Study Analysis

5.1 Evaluation indicators of teaching quality

The data from the questionnaire can be organized and calculated to obtain the second-level indicator scores, as shown in Table 10.

According to the data in Table 10, combined with the formula (4), the total score of the secondary indicators can be calculated to be 10744.

The weights of the secondary indicators of teaching quality evaluation can be calculated by using the formula (6), as shown in Table 11.

From Table 11, it can be concluded that the difference in the weights of the second-level indicators is small and the sum of the weights is 1.

According to the formula (2), the weight of first-level indicators of teaching quality evaluation can be calculated, as shown in Table 12.

5.2 Fuzzy comprehensive evaluation matrix

Organizing the questionnaire and counting the data of teachers' classroom teaching evaluation. The statistical results are shown in Table 13.

According to the data in Table 13, the fuzzy evaluation value of each indicator is calculated, and the results are shown in Table 14.

According to the data in Table 14, the fuzzy comprehensive evaluation vector under the second-level indicators is calculated, and then the fuzzy comprehensive evaluation matrix under the first-level indicators is constructed. The calculation process is as follows:

(i) Teaching plan indicator evaluation

From (13) and Tables 11 and 14, it can obtain the fuzzy comprehensive evaluation matrix R_1 as shown in (20),

$$R_1=\begin{bmatrix} 0.5 & 0.4 & 0.1 & 0.0 \\ 0.8 & 0.2 & 0.0 & 0.0 \end{bmatrix}. \tag{20}$$

Based on R_1 and C_1, by using the max-min fuzzy composite operation, that is, $M(\wedge,\vee)$, we can get the following results:

$$
\begin{aligned}
B_{11} &= (0.0900 \wedge 0.5) \vee (0.0833 \wedge 0.8) \\
&= 0.0900 \vee 0.0833 \\
&= 0.0900, \\
B_{12} &= (0.0900 \wedge 0.4) \vee (0.0833 \wedge 0.2) \\
&= 0.0900 \vee 0.0833 \\
&= 0.0900, \\
B_{13} &= (0.0900 \wedge 0.1) \vee (0.0833 \wedge 0.0) \\
&= 0.0900 \vee 0.0 \\
&= 0.0900, \\
B_{14} &= (0.0900 \wedge 0.0) \vee (0.0833 \wedge 0.0) \\
&= 0.0 \vee 0.0 \\
&= 0.0,
\end{aligned}
$$

then the evaluation vector B_1 can be derived as shown in (21),

$$
\begin{aligned}
B_1 &= (B_{11}, B_{12}, B_{13}, B_{14}) \\
&= (0.0900, 0.0900, 0.0900, 0.0).
\end{aligned} \tag{21}
$$

(ii) Teaching attitude indicator evaluation

From (13) and Tables 11 and 14, it can obtain the fuzzy comprehensive evaluation matrix R_2 as shown in (22),

$$
R_2 = \begin{bmatrix} 0.7 & 0.2 & 0.1 & 0.0 \\ 0.7 & 0.1 & 0.0 & 0.1 \\ 0.8 & 0.2 & 0.0 & 0.0 \end{bmatrix}. \tag{22}
$$

Based on R_2 and C_2, by using the max-min fuzzy composite operation, that is, $M(\wedge, \vee)$, we can get the following results:

$$
\begin{aligned}
B_{21} &= (0.0784 \wedge 0.7) \vee (0.0803 \wedge 0.7) \\
&\quad \vee (0.0822 \wedge 0.8) \\
&= 0.0784 \vee 0.0803 \vee 0.0822 \\
&= 0.0822, \\
B_{22} &= (0.0784 \wedge 0.2) \vee (0.0803 \wedge 0.1) \\
&\quad \vee (0.0822 \wedge 0.2) \\
&= 0.0784 \vee 0.0803 \vee 0.0822 \\
&= 0.0822, \\
B_{23} &= (0.0784 \wedge 0.1) \vee (0.0803 \wedge 0.0) \\
&\quad \vee (0.0822 \wedge 0.0) \\
&= 0.0784 \vee 0.0 \vee 0.0 \\
&= 0.0784, \\
B_{24} &= (0.0784 \wedge 0.0) \vee (0.0803 \wedge 0.1) \\
&\quad \vee (0.0822 \wedge 0.0) \\
&= 0.0784 \vee 0.0803 \vee 0.0 \\
&= 0.0803,
\end{aligned}
$$

then the evaluation vector B_2 can be derived as shown in (23),

$$
\begin{aligned}
B_2 &= (B_{21}, B_{22}, B_{23}, B_{24}) \\
&= (0.0822, 0.0822, 0.0784, 0.0803).
\end{aligned} \tag{23}
$$

(iii) Teaching methods indicator evaluation

From (13) and Tables 11 and 14, it can obtain the fuzzy comprehensive evaluation matrix R_3 as shown in (24),

$$
R_3 = \begin{bmatrix} 0.6 & 0.3 & 0.1 & 0.0 \\ 0.6 & 0.4 & 0.0 & 0.0 \\ 0.5 & 0.5 & 0.0 & 0.0 \\ 0.4 & 0.6 & 0.0 & 0.0 \end{bmatrix}. \tag{24}
$$

Based on R_3 and C_3, by using the max-min fuzzy composite operation, that is, $M(\wedge, \vee)$, we can get the following results:

$$
\begin{aligned}
B_{31} &= (0.0862 \wedge 0.6) \vee (0.0914 \wedge 0.6) \\
&\quad \vee (0.0823 \wedge 0.5) \vee (0.0836 \wedge 0.4) \\
&= 0.0862 \vee 0.0914 \vee 0.0823 \vee 0.0836 \\
&= 0.0914, \\
B_{32} &= (0.0862 \wedge 0.3) \vee (0.0914 \wedge 0.4) \\
&\quad \vee (0.0823 \wedge 0.5) \vee (0.0836 \wedge 0.6) \\
&= 0.0862 \vee 0.0914 \vee 0.0823 \vee 0.0836 \\
&= 0.0914, \\
B_{33} &= (0.0862 \wedge 0.1) \vee (0.0914 \wedge 0.0) \\
&\quad \vee (0.0823 \wedge 0.0) \vee (0.0836 \wedge 0.0) \\
&= 0.0862 \vee 0.0 \vee 0.0 \vee 0.0 \\
&= 0.0862, \\
B_{34} &= (0.0862 \wedge 0.0) \vee (0.0914 \wedge 0.0) \\
&\quad \vee (0.0823 \wedge 0.0) \vee (0.0836 \wedge 0.0) \\
&= 0.0 \vee 0.0 \vee 0.0 \vee 0.0 \\
&= 0.0,
\end{aligned}
$$

then the evaluation vector B_3 can be derived as shown in (25),

$$
\begin{aligned}
B_3 &= (B_{31}, B_{32}, B_{33}, B_{34}) \\
&= (0.0914, 0.0914, 0.0862, 0.0).
\end{aligned} \tag{25}
$$

(iv) Classroom performance indicator evaluation

From (13) and Tables 11 and 14, it can obtain the fuzzy comprehensive evaluation matrix R_4 as shown in (26),

$$
R_4 = \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.0 \\ 0.6 & 0.3 & 0.1 & 0.0 \\ 0.7 & 0.2 & 0.1 & 0.0 \end{bmatrix}. \tag{26}
$$

Based on R_4 and C_4, by using the max-min fuzzy composite operation, that is, $M(\wedge, \vee)$, we can get the following results:

$$\begin{aligned}B_{41} &= (0.0732 \wedge 0.8) \vee (0.0810 \wedge 0.6) \\ &\quad \vee (0.0881 \wedge 0.7) \\ &= 0.0732 \vee 0.0810 \vee 0.0881 \\ &= 0.0881, \\ B_{42} &= (0.0732 \wedge 0.1) \vee (0.0810 \wedge 0.3) \\ &\quad \vee (0.0881 \wedge 0.2) \\ &= 0.0732 \vee 0.0810 \vee 0.0881 \\ &= 0.0881, \\ B_{43} &= (0.0732 \wedge 0.1) \vee (0.0810 \wedge 0.1) \\ &\quad \vee (0.0881 \wedge 0.1) \\ &= 0.0732 \vee 0.0810 \vee 0.0881 \\ &= 0.0881, \\ B_{44} &= (0.0732 \wedge 0.0) \vee (0.0810 \wedge 0.0) \\ &\quad \vee (0.0881 \wedge 0.0) \\ &= 0.0 \vee 0.0 \vee 0.0 \\ &= 0.0,\end{aligned}$$

then the evaluation vector B_4 can be derived as shown in (27),

$$\begin{aligned}B_4 &= (B_{41}, B_{42}, B_{43}, B_{54}) \\ &= (0.0881, 0.0881, 0.0881, 0.0).\end{aligned} \tag{27}$$

5.3 Comprehensive evaluation

The first-level evaluation indicators contain the teaching plan, teaching attitude, teaching methods and classroom performance situation. By the evaluation vectors B_1, B_2, B_3 and B_4, the first-level fuzzy comprehensive matrix R can be obtained in accordance with (16), as shown in (28),

$$R = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} 0.0900 & 0.0900 & 0.0900 & 0.0 \\ 0.0822 & 0.0822 & 0.0784 & 0.0803 \\ 0.0914 & 0.0914 & 0.0862 & 0.0 \\ 0.0881 & 0.0881 & 0.0881 & 0.0 \end{bmatrix}. \tag{28}$$

Based on R and C, by using the max-min fuzzy composite operation, that is, $M(\wedge, \vee)$, we can get the following results:

$$\begin{aligned}b_1 &= (0.1733 \wedge 0.0900) \vee (0.2409 \wedge 0.0822) \\ &\quad \vee (0.3435 \wedge 0.0914) \vee (0.2423 \wedge 0.0881) \\ &= 0.0900 \vee 0.0822 \vee 0.0914 \vee 0.0881 \\ &= 0.0914, \\ b_2 &= (0.1733 \wedge 0.0900) \vee (0.2409 \wedge 0.0822) \\ &\quad \vee (0.3435 \wedge 0.0914) \vee (0.2423 \wedge 0.0881) \\ &= 0.0900 \vee 0.0822 \vee 0.0914 \vee 0.0881 \\ &= 0.0914,\end{aligned}$$

$$\begin{aligned}b_3 &= (0.1733 \wedge 0.0) \vee (0.2409 \wedge 0.0803) \\ &\quad \vee (0.3435 \wedge 0.0862) \vee (0.2423 \wedge 0.0881) \\ &= 0.0 \vee 0.0803 \vee 0.0862 \vee 0.0881 \\ &= 0.0881, \\ b_4 &= (0.1733 \wedge 0.0900) \vee (0.2409 \wedge 0.0822) \\ &\quad \vee (0.3435 \wedge 0.0) \vee (0.2423 \wedge 0.0) \\ &= 0.0900 \vee 0.0822 \vee 0.0 \vee 0.0 \\ &= 0.0900,\end{aligned}$$

then the evaluation vector B can be derived as shown in (29).

$$B = (0.0914, 0.0914, 0.0881, 0.0900). \tag{29}$$

Since

$$0.0914 + 0.0914 + 0.0881 + 0.0900 = 0.3609 \neq 1,$$

normalization is carried out for B as shown in (30),

$$B' = (0.2532, 0.2532, 0.2442, 0.2494). \tag{30}$$

The results are processed after obtaining B'. The evaluation set is set to

V ={very fit, fit, unfit, very unfit}

and the values are 3 points, 2 points, 1 point, and 0 point, respectively. The total score w of the teacher evaluation can be obtained in accordance with (19), as shown in (31),

$$w = (0.2532, 0.2532, 0.2442.2494) \times \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix} = 2.5102.$$

Because the evaluation value of 2.5102 is between 2 and 3, it shows that the teacher's teaching quality evaluation level is between fit and very fit, that is, the students are more satisfied with the teacher's classroom teaching effect.

VI. Conclusion

In view of the study on the evaluation of teaching effectiveness of secondary school mathematics, this paper mainly solves two problems: one is to put forward a new two-level evaluation index system from multiple dimensions, and the other is to combine the fuzzy comprehensive evaluation theory to realize the evaluation of teaching effectiveness of secondary school mathematics based on the indicator system.

This paper studies the evaluation of classroom teaching effectiveness of secondary school mathematics from the perspective of the teacher. The effectiveness of teaching can also be evaluated in a multidimensional manner from the perspectives of the students and the teachers listening to the lectures. Besides, one may combine fuzzy comprehensive evaluation theory with differential equation theory (see [6-10]) to make a dynamic comprehensive evaluation of teaching effectiveness of secondary school mathematics, which we leave for future work.

TABLE 1 THE PRESET SYSTEM OF TEACHERS' TEACHING EVALUATION INDICATORS

Overall target	First-level indicators	Second-level indicators
U : Secondary school mathematics classroom teaching effect evaluation	U_1 : Teaching plan	U_{11} : Scientific and rational classroom teaching
		U_{12} : Completion of classroom assignments on a regular basis
	U_2 : Teaching methods	U_{21} : Multimedia teaching
		U_{22} : Expand students' vision of mathematics
		U_{23} : Meticulous teaching, in-depth, with lots of examples
		U_{24} : Focusing on the enhancement of students' mathematical literacy
	U_3 : Teaching attitude	U_{31} : Adequate preparation of lectures
		U_{32} : Strict management of classroom discipline
		U_{33} : Timely feedback on issues raised by students
		U_{34} : The class time is arranged properly
	U_4 : Classroom performance	U_{41} : Percentage of students answering questions correctly
		U_{42} : The extent to which students are motivated to answer questions
		U_{43} : Degree of formula familiarity

TABLE 2 SURVEY STATISTICS ON THE IMPORTANT OF EVALUATION INDICATORS

Indicators	Very important	Important	Average	Unimportant
1. Teaching plan	200	144	56	0
2.Classroom teaching is scientific and reasonable	204	160	36	0
3. Timely completion of classroom tasks	200	96	104	0
4. Teaching methods	176	108	116	0
5. Multimedia teaching	172	116	96	16
6. Expand students' vision of mathematics	172	120	108	0
7. Meticulous teaching, in-depth, with lots of examples	172	144	80	4
8. Focusing on the enhancement of students' mathematical literacy	80	84	236	0
9. Teaching attitude	216	108	76	0
10. Adequate preparation of lectures	184	152	64	0
11. Strict management of classroom discipline	224	136	40	0
12. Timely feedback on issues raised by students	176	132	92	0
13. The class time is arranged properly	184	124	92	0
14. Classroom performance	180	152	68	0
15. Percentage of students answering questions correctly	156	80	160	4
16. The extent to which students are motivated to answer questions	176	120	104	0
17. Degree of familiarity with mathematical formulas	192	164	44	0

TABLE 3 SCORES OF THE EVALUATION INDICATORS

Indicators	Total scores
1. Teaching plan	944
2. Classroom teaching is scientific and reasonable	968
3. Timely completion of classroom tasks	896
4. Teaching methods	860
5. Multimedia teaching	844
6. Expanding students' vision of mathematics	864
7. Meticulous teaching, in-depth, with lots of examples	884
8. Focusing on the enhancement of students' mathematical literacy	644
9. Teaching attitude	940
10. Adequate preparation of lectures	920
11. Strict management of classroom discipline	984
12. Timely feedback on issues raised by students	884
13. The class time is arranged properly	892
14. Classroom performance	912
15. Percentage of students answering questions correctly	788
16. The extent to which students are motivated to answer questions	872
17. Degree of familiarity with mathematical formulas	948

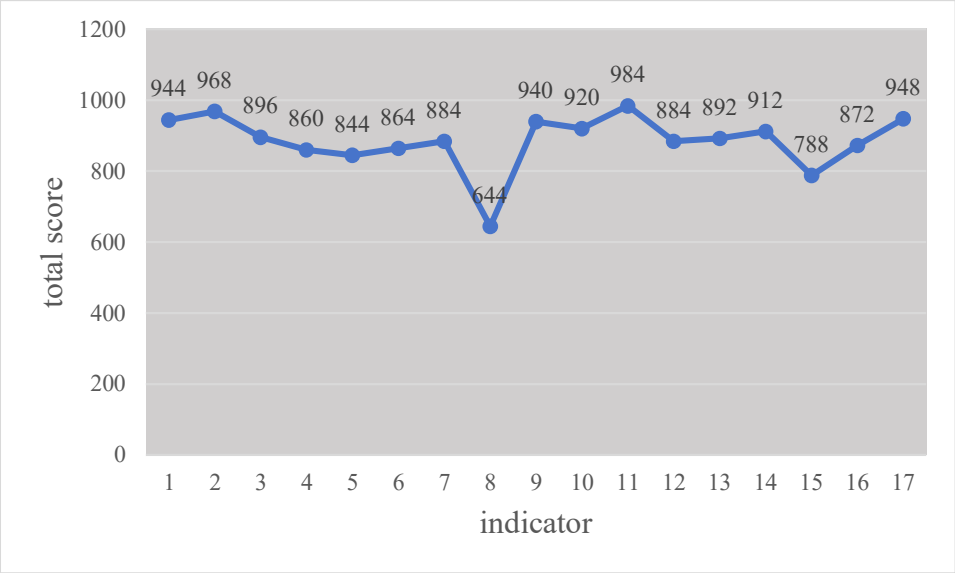

Fig. 1 Line graph of the evaluation indicators' score

TABLE 4 THE EVALUATION INDEX SYSTEM

Overall target	First-level indicators	Second-level indicators
U : Secondary school mathematics classroom teaching effect evaluation	U_1 : Teaching plan	Scientific and rational classroom teaching
		Timed completion of classroom tasks
	U_2 : Teaching methods	multimedia teaching
		Expand students' vision of mathematics
		Meticulous teaching, in-depth, with lots of examples
	U_3 : Teaching attitude	Adequate preparation of lectures
		Strict management of classroom discipline
		Timely feedback on issues raised by students
		The class time is arranged properly
	U_4 : Classroom performance	Percentage of students answering questions correctly
		Degree to which students are motivated to answer questions
		Mathematical formulae familiarity

TABLE 7 DESCRIPTION OF SYMBOLS

Symbols	Description
U_i	The i th first-level indicator
C_i	The weight corresponding to the i th first-level indicator
K_i	The second-level indicators under the i th first-level indicator
C_{ij}	The weights corresponding to the j th second-level indicator under the i th first-level indicator branch
F	Total score of the questionnaire
F_i	Total score of the second-level indicators under the i th first-level indicator branch
F_{ij}	Score for the j th second-level indicator under the i th first-level indicator branch
k_i	The number of secondary indicators under the i th first-level indicator branch
v_i	The i th evaluation level
N	Data frequency under each evaluation level
R	Fuzzy comprehensive evaluation matrix
r	Membership degree
M	Sample size

TABLE 8 TEACHING EVALUATION INDEX SYSTEM

Overall target	First-level indicators	Weights of the first-level indicators	Second-level indicators	Weights of the second-level indicators	Scores for the second-level indicators
U: Secondary school mathematics classroom teaching effect evaluation	U_1: Teaching plan	C_1	U_{11}: Scientific and reasonable classroom teaching	C_{11}	F_{11}
			U_{12}: Complete the task of classroom teaching regularly	C_{12}	F_{12}
	U_2: Teaching attitude	C_2	U_{21}: Multimedia teaching	C_{21}	F_{21}
			U_{22}: Expand students' vision of mathematics	C_{22}	F_{22}
			U_{23}: Teaching carefully, explain profound theories in simple terms, with lots of examples	C_{23}	F_{23}
	U_3: Teaching methods	C_3	U_{31}: Fully prepare the teaching contents	C_{31}	F_{31}
			U_{32}: Strict management of classroom discipline	C_{32}	F_{32}
			U_{33}: Timely feedback on students' questions	C_{33}	F_{33}
			U_{34}: The class time is arranged properly	C_{34}	F_{34}

	U_4: Classroom performance	C_4	U_{41}: Students' correct rate of answering questions	C_{41}	F_{41}
			U_{42}: Students' activeness in answering questions	C_{42}	F_{42}
			U_{43}: The degree of familiarity with mathematical formulas	C_{43}	F_{43}

TABLE 9 FREQUENCY OF EACH EVALUATION LEVEL

Frequencies		Evaluation levels				
		v_1	v_2	v_3	...	v_m
Indicators	U_{i1}	N_{11}^i	N_{12}^i	N_{13}^i	...	N_{1m}^i
	U_{i2}	N_{21}^i	N_{22}^i	N_{23}^i	...	N_{2m}^i
	...	...	...	...	...	...
	U_{ik_i}	$N_{k_i1}^i$	$N_{k_i2}^i$	$N_{k_i3}^i$	...	$N_{k_im}^i$

TABLE 10 SCORES FOR TEACHING EVALUATION INDICATORS

Second-level indicators	Scores for the second-level indicators
U_{11}: Scientific and reasonable classroom teaching	F_{11}: 968
U_{12}: Complete the task of classroom teaching regularly	F_{12}: 896
U_{21}: Multimedia teaching	F_{21}: 844
U_{22}: Expand students' vision of mathematics	F_{22}: 864
U_{23}: Teaching carefully, explain profound theories in simple terms, with lots of examples	F_{23}: 884
U_{31}: Fully prepare the teaching contents	F_{31}: 920
U_{32}: Strict management of classroom discipline	F_{32}: 984
U_{33}: Timely feedback on students' questions	F_{33}: 884
U_{34}: The class time is arranged properly	F_{34}: 892
U_{41}: Students' correct rate of answering questions	F_{41}: 788
U_{42}: Students' activeness in answering questions	F_{42}: 872
U_{43}: The degree of familiarity with mathematical formulas	F_{43}: 948

TABLE 11 WEIGHTS OF THE SECOND-LEVEL INDICATORS

Overall Target	Second-level indicators	Weights of the second-level indicators
U: Secondary school mathematics classroom teaching effect evaluation	U_{11}: Scientific and reasonable classroom teaching	0.0901
	U_{12}: Complete the task of classroom teaching regularly	0.0834
	U_{21}: Multimedia teaching	0.0786
	U_{22}: Expand students' vision of mathematics	0.0804
	U_{23}: Teaching carefully, explain profound theories in simple terms, with lots of examples	0.0823
	U_{31}: Fully prepare the teaching contents	0.0856
	U_{32}: Strict management of classroom discipline	0.0916

	U_{33} : Timely feedback on students' questions	0.0823
	U_{34} : The class time is arranged properly	0.0830
	U_{41} : Students' correct rate of answering questions	0.0733
	U_{42} : Students' activeness in answering questions	0.0812
	U_{43} : The degree of familiarity with mathematical formulas	0.0882

TABLE 12 WEIGHTS OF THE FIRST-LEVEL INDICATORS FOR THE EVALUATION

Overall target	First-level indicators	Weights of the first-level indicators
U : Secondary school mathematics classroom teaching effect evaluation	U_1 : Teaching plan	0.1733
	U_2 : Teaching attitude	0.2409
	U_3 : Teaching methods	0.3435
	U_4 : Classroom performance	0.2423

TABLE 13 STATISTICS OF TEACHERS' TEACHING EVALUATION

No.	Survey contents	Evaluation levels			
		Perfect fit	Fit	Unfit	Very unfit
1	Properly organize classroom instruction based on teacher's knowledge of classroom contents	200	160	40	0
2	According to the teaching plan, enhance the enthusiasm of the students to complete the classroom tasks regularly	320	80	0	0
3	Combine with modern multimedia to make teaching more convenient	280	80	40	0
4	Include the history of mathematics in the teaching process to broaden students' horizons of mathematics teaching	280	40	0	40
5	Provide detailed teaching, in-depth analysis of problems and examples in the teaching process	320	80	0	0
6	Teachers fully prepare the teaching contents according to the course contents	240	120	40	0
7	Take students as the main focus in the teaching process, pay attention to students' behavior, and strictly manage classroom discipline	240	160	0	0
8	Answer students' questions before, during and after class in a timely manner	200	200	0	0
9	Prepare lessons before class to fully arrange the classroom teaching time, so as to rationally utilize the teaching time	160	240	0	0
10	The correctness of students' answers to questions raised by the teacher in class	320	40	40	0
11	Whether students are active in answering questions raised by the teacher in class	240	120	40	0
12	The degree of memorization of mathematical formulas	280	80	40	0

TABLE 14 THE FUZZY EVALUATION VALUE OF INDICATORS

First-level indicators	Second-level indicators	Evaluation indicators			
		Perfect fit	Fit	Unfit	Very unfit
U_1 : Teaching plan	U_{11} : Scientific and reasonable classroom teaching	0.5	0.4	0.1	0.0
	U_{12} : Complete the task of classroom teaching regularly	0.8	0.2	0.0	0.0
U_2 : Teaching attitude	U_{21} : Multimedia teaching	0.7	0.2	0.1	0.0
	U_{22} : Expand students' vision of mathematics	0.7	0.1	0.0	0.1
	U_{23} : Teaching carefully, explain profound theories in simple terms, with lots of examples	0.8	0.2	0.0	0.0
U_3 : Teaching methods	U_{31} : Fully prepare the teaching contents	0.6	0.3	0.1	0.0
	U_{32} : Strict management of classroom discipline	0.6	0.4	0.0	0.0
	U_{33} : Timely feedback on students' questions	0.5	0.5	0.0	0.0
	U_{34} : The class time is arranged properly	0.4	0.6	0.0	0.0
U_4 : Classroom performance	U_{41} : Students' correct rate of answering questions	0.8	0.1	0.1	0.0
	U_{42} : Students' activeness in answering questions	0.6	0.3	0.1	0.0
	U_{43} : The degree of familiarity with mathematical formulas	0.7	0.2	0.1	0.0

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