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Systematic Literature Review for Optimization System with Advection-Diffusion-Reaction Non-Linear Equation in Water Quality

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Abstract—**Water quality is an essential component of environmental health and sustainability, including various parameters such as chemical, physical, and biological attributes. These parameters determine the suitability of water for consumption, recreation, and ecosystem health. Clean and safe water availability is a crucial foundation for human well-being, ecological balance, and sustainable development, indicating the need to monitor and control water quality to meet human demands. Based on the importance of water quality, this research aimed to provide a comprehensive overview of previous investigations related to the optimization system of the advection-diffusion-reaction (ADR) non-linear equation in water quality. Preferred Reporting Items for Systematic Review and Meta-Analysis (PRISMA) guidelines were conducted to ensure a structured article selection, data extraction, and analysis approach. This was followed by presenting a bibliometric analysis of selected data to support the study of research trends. The results showed that using ADR non-linear equation systems in modeling water quality optimization was a potential area requiring further exploration. Moreover, this research provided valuable information that could be used as a reference for further investigation of water quality optimization. A future research design is also presented.**

Index Terms—**Systematic literature review, Optimization, Water quality, Advection-diffusion-reaction, PRISMA, Sustainable Development Goals (SDGs).**

I. INTRODUCTION

WATER quality is a multidimensional concept including various parameters such as chemical, physical, and biological attributes. These parameters play a crucial role in determining the suitability of water for consumption, recreation, and ecosystem health. Clean and safe water availability is essential for human well-being, ecological balance, and sustainable development. This significance is underscored in the Sustainable Development Goals (SDGs), particularly under Goal 6: Clean Water and Sanitation. However, water quality is intricately related to other SDGs practically [1], indicating the need for monitoring and controlling to meet human needs.

Using mathematical and computational methods has provided opportunities for innovative solutions in understanding, predicting, and managing water quality. These include the Advection-Diffusion-Reaction (ADR) equation, which combines diffusion and advection equations to model the movement, dispersion, and chemical reactions of substances [2], [3]. The general form of the ADR equation for a substance S is given by formulation (1) as follows.

$$\frac{\partial S}{\partial t} = \nabla \cdot (Su) + \nabla \cdot (D\nabla S) + \rho(S), \tag{1}$$

where velocity vector u is the preferred transport direction, D represents the diffusion coefficient, which can potentially vary with spatial coordinates, ρ is a reaction source term, ∇ signifies the gradient operator, and $\nabla\cdot$ indicates the divergence operation [4].

Regarding water quality, the ADR equation is used for modeling pollutants [5], [6], pipeline systems, and water networks [7], [8], [9]. Previous research has explored optimizing, particularly Partial Differential Equation (PDE) based optimization, for problems including the ADR equation [10], [11]. Moreover, Yuan and Liang [12] discuss optimization challenges in identifying sources of groundwater contamination using the equation. Various methods have also been explored focusing on water quality assessment, including probabilistic echo state networks [13], mathematical models in a stream using upwind implicit methods [14], and implicit finite difference simulation of water pollution control [15]. Other reports include an analysis of particulate pollutants in the water-flooded zone [16], a numerical model for water quality assessment with constant absorption [17], numerical simulations of a one-dimensional groundwater pollution measurement model [18] and a groundwater quality assessment model using the cubic spline method in [19].

Based on the above background, this research aimed to comprehensively analyze the methods used to optimize the ADR non-linear equation system for water quality assessment. The analysis followed the Preferred Reporting Items for Systematic Review (PRISMA) to ensure a structured article selection, data extraction, and evaluation method. The PRISMA method is used for a Systematic Literature Review (SLR). The literature review's comprehensiveness is

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enhanced using RStudio software to conduct the bibliometric analysis. Subsequently, the research questions are fundamental for results presentation and discussion.

1) What are the research trends in ADR non-linear equation system optimization for water quality?
2) How do state-of-the-art ADR non-linear equation systems and optimization play a role in determining novel and new research topics?
3) How have optimization models been applied in research related to ADR non-linear equation systems for optimizing water quality?

The organization of this research is as follows: Section II-A. presents the related literature review, while Section II-C discusses the methods. Section III describes the bibliometric analysis and literature review. The discussion is provided in Section IV, while the conclusion is presented in Section V. Table I provides the acronyms list used in this paper to facilitate reading.

TABLE I: The acronyms list used in this paper.

ADR	Advection-Diffusion-Reaction
PRISMA	Preferred Reporting Items for Systematic Review and Meta-Analysis
SDGs	Sustainable Development Goals
SLR	Systematic Literature Review
PDE	Partial Differential Equation
CNKI	Integrated Knowledge Resources System
ACO	Ant Colony Optimization
MOMILP	Multiobjective Mixed-Integer Linear Program
ITSDP	Inexact Downside Risk Control and Two-Stage Stochastic Programming

II. Material and Methods

A. Related Literature Review

The literature review enhances the existing articles on advection-diffusion-reaction (ADR) non-linear equation system optimization in water quality. Six articles are considered references for comparison with the review proposed in this research, as presented in Table II. Based on the results, numerous articles were found conducting reviews on optimization and water quality. These included Wibbenmeyer et al. [20], who examined the economic impact of forest fires on water quality. Dandy et al. [21] explored optimization, including uncertainties in water distribution networks. However, these two articles did not specifically examine the interrelation between optimization and water quality with ADR.

Semasinghe and Rousso [22] conducted a systematic literature review on in-lake mechanisms for controlling manganese, resulting in the selection of 48 articles obtained through Web of Science, ProQuest, and Scopus databases. Meanwhile, Gonzales-Inca et al. [24] reviewed hydrological and hydraulic modeling, including hydrological optimization problems and water quality modeling, using the application of GeoAI and machine learning. The data used were retrieved literature from Web of Science, including Scopus, Springer Link, Wiley Online Library, and MDPI. Subsequently, the data were refined to ensure the information was relevant to GeoAI implementation in hydrological and fluvial research.

TABLE II: Summary of the aspect covered in our article and existing review articles.

Article	SLR	Bibliometric Analysis	Optimization	Water Quality	ADR Equation
[20]	-	-	✓	✓	-
[22]	✓	-	✓	✓	-
[21]	-	-	✓	✓	-
[23]	-	✓	✓	✓	-
[24]	✓	-	✓	✓	-
[25]	-	-	✓	-	✓
Our Article	✓	✓	✓	✓	✓

You et al. [23] discussed bibliometric analysis of low-impact development and its effects on rainstorm management, using data from the Web of Science for international-based articles and Chinas Integrated Knowledge Resources System (CNKI). The research used the CiteSpace software to conduct statistical analyses, including three aspects, namely bibliometrics, keyword hotspot co-occurrence, and clustering, as well as literature co-citation clustering. The results compare the content discussed in both international and articles from China. Based on the analysis, it was concluded that the international-based research tended to focus on parameter optimization, modeling, and improvements. However, the study in China significantly emphasized the challenges faced in sponge city construction.

As shown in Table II, only one article addresses the ADR equation, namely Biesheuvel et al. [25]. This article presented the optimization results of integrating multiple modules for reverse osmosis and electrodialysis, the two membrane technologies for water desalination and treatment. Furthermore, the significance of concentration polarization was assessed by applying a 3D model for cross-current flow within an electrodialysis module.

Based on the reviewed literature, the interconnection between the ADR equation system, optimization, and water quality has yet to be explored. Therefore, this research aimed to provide a comprehensive overview of previous investigations on optimizing the ADR non-linear equation system in water quality.

B. Theoretical Background: General Formulation of ADR

One of the topics of numerical analysis is the non-linear advection-reaction-diffusion equation model. This includes applied mathematics studies, which are widely implemented in chemistry, biology, meteorology, epidemiology, fluid dynamics, computer science, and other applied sciences. Common problems studied using the advection-diffusion-reaction equation model include water pollution, control of meteorological pollution, forest fires, chemical reactions, age structure population dynamics, and the problem of increasing oil recovery. In the issue of water pollution, non-linear advection-diffusion-reaction model studies that have been conducted on groundwater pollution generally use three observation variables.

The study of the advection-diffusion-reaction equation in this research begins with a review of previous studies. The results of this review are presented in Figure 1.

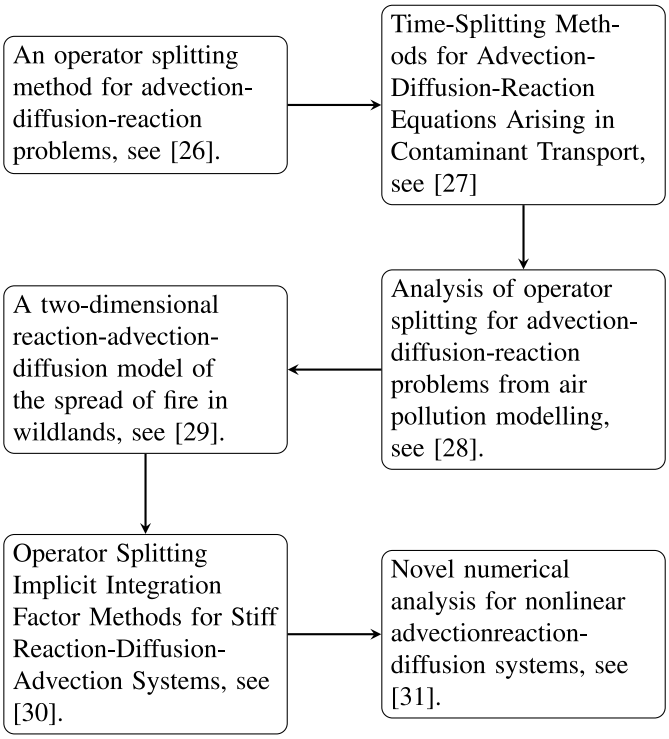

Fig. 1: Flowchart of the Research Process Stages

The general form of the nonlinear advection-diffusion-reaction model [26] is presented in (2).

$$\begin{aligned} \Phi_i \frac{\partial c_i}{\partial t} - \nabla\cdot D(u)\nabla c_i + u\cdot\nabla c_i &= (\tilde{c}_i - c_i)q + \\ &\qquad \Phi_i R_i(c_1, c_2, \cdots, c_N), \\ -\nabla\cdot(k(x)\nabla p) &\equiv \nabla\cdot u \\ &= q, x\in\Omega, t\in J, \qquad (2) \\ c_i(x,0) &= c_{i0}(x), \\ &\qquad i = 1, 2, 3, \cdots, N, \end{aligned}$$

where c_i = free variable (component) that is observed, Ω = domain boundary in R^2, $J = (0, T]$, $q = \max\{q, 0\}$ be the non-zero points observed, R_i = nonlinear reactions, $D(u)$ = tensor and velocity, $\tilde{c}_i$ =the added resources, and $(\widetilde{c_i}) = c_i$ = the produced resources.

The advection-diffusion-reaction system with three free variables is a model for contaminant transport in groundwater. The observed variables (c_i) are hydrocarbon contaminants (H), oxygen (O), and microbiology (M). The discussion is conducted in 2D. Refers to Ricchiuto et al. [32], the numerical solution uses the second-order explicit Runge-Kutta method. The form of the solved model equation is as in (3).

$$\begin{aligned} \frac{\partial H}{\partial t} &- \frac{1}{R_H}\nabla\cdot(D\nabla H - uH) = \frac{q}{R_H}\widetilde{H} + \\ &\qquad \frac{\Phi}{R_H} R_1(O, H, M_t) \qquad (3) \\ \Phi\frac{\partial O}{\partial t} &- \nabla\cdot(D\nabla O - uO) = q\widetilde{O} + \\ &\qquad \Phi R_2(O, H, M_t) \qquad (4) \\ \Phi\frac{\partial M_t}{\partial t} &- \frac{1}{R_m}\nabla\cdot(D\nabla M_t - uM_t) = \frac{q}{R_m}\widetilde{M}_t + \\ &\qquad \Phi R_3(O, H, M_t) \qquad (5) \end{aligned}$$

where R_1, R_2, and R_3 is formulated as follows.

$$\begin{aligned} R_H\frac{dH}{dt} &= M_t.\bar{k}.\left(\frac{H}{K_H + H}\right)\left(\frac{O}{K_O + O}\right) \\ &= R_1(O,\ H, M_t), \qquad (6) \\ \frac{dO}{dt} &= M_t.\bar{k}.F.\left(\frac{H}{K_H + H}\right)\left(\frac{O}{K_O + O}\right) \\ &= R_2(O,\ H\ M_t), \qquad (7) \\ \frac{dM_t}{dt} &= M_t.\bar{k}.Y.\left(\frac{H}{K_H + H}\right)\left(\frac{O}{K_O + O}\right) \\ &\quad + K_c.Y.OC - BM_t \\ &= R_3(O,\ H\ M_t). \qquad (8) \end{aligned}$$

The nonlinear advection-diffusion-reaction equation system is applied to the bioremediation of contaminants in groundwater [27]. The three observed variables are contaminants (S), oxygen (O), and microorganisms (M). The solution is carried out numerically and with analytical analysis. The numerical solution uses a multi-step method with a small time step size.

D. Lanser and J.G. Verwer [28] presented a three-dimensional advection-diffusion-reaction partial differential equation system with time-dependent variables. The analysis of operator splitting aims to provide insight into the splitting errors. Five variables are observed; each does not point to a specific indicator. The general form of the equation model:

$$c_t + uc_x = (kc_z)_z + R(c)$$

with u constant in x, R depending on x and z. The form of the equation model analyzed:

$$\begin{aligned} &\frac{\partial c_1}{\partial t} + u\frac{\partial c_1}{\partial x} = 0, \quad c_1(x,z,0) = c(x,z,0), \\ &\frac{\partial c_2}{\partial t} = \frac{\partial}{\partial z}\left(k\frac{\partial c_2}{\partial z}\right) + b.c.,\ \ c_2(x,z,0) = c_1(x,z,\frac{\pi}{2}), \\ &\frac{\partial c_3}{\partial t} = R(c_3), \quad c_3(x,z,0) = c_2(x,z,\frac{\pi}{2}), \qquad (9) \\ &\frac{\partial c_4}{\partial t} = \frac{\partial}{\partial z}\left(k\frac{\partial c_4}{\partial z}\right) + b.c.,\ \ c_4(x,z,\frac{\pi}{2}) = c_3(x,z,\pi), \\ &\frac{\partial c_5}{\partial t} + u\frac{\partial c_5}{\partial x} = 0, \quad c_5(x,z,\frac{\pi}{2}) = c_4(x,z,\pi), \end{aligned}$$

The movement and transformation of contaminants in groundwater are modeled as a nonlinear parabolic partial differential equation system for advection-diffusion-reaction. The three variables in the system are contaminants, oxygen, and microorganisms [33].

The first step in solving this involves discretizing the advection-diffusion-reaction equation model using the finite difference method. The numerical solution employs a multi-step integrator, precisely a predictor-corrector method, with variable time step sizes. The Implicit Diagonal Runge-Kutta method, see Kennedy et al. [34], supplies the initial values the predictor-corrector method requires.

The stiffness of the reaction and diffusion necessitates petite time step sizes, while nonlinear advection can lead to sharp gradients in local spatial areas. It is challenging to design a numerical method that can efficiently handle both of these difficulties. The implicit integration factor (IIF) method (see Zhao et al. [30]) and its higher-dimensional analog, compact IIF (CIF) [35], are employed for efficient time-stepping solutions in reaction-diffusion systems with stiff reaction and diffusion. In second-order derivatives, the solution of the equation remains linearly stable without conditions. The equation solution uses an explicit Runge-Kutta method, with the model system solved as follows:

$$\frac{\partial u}{\partial t}+\nabla\cdot(au) = d\Delta u-bu+v, \tag{10}$$

$$\frac{\partial v}{\partial t}+\nabla\cdot(av) = d\Delta v-cv, \tag{11}$$

where

$$u\left(x,y,z,t\right)=(e^{-(b+d)t}+e^{-(c+d)t})\cos\left(x+y+z-at\right), \tag{12}$$

and

$$v\left(x,y,z,t\right)=(b-c)e^{-(c+d)t}\cos\left(x+y+z-at\right). \tag{13}$$

The advection-diffusion-reaction system aims to develop a model for fire spread in wildlands or forests. The model consists of two partial differential equations: one representing the mass formation of each chemical element involved in combustion and the other ensuring enthalpy balance. The elements involved in combustion are represented by pyrolysis gases in the air, namely oxygen, carbon dioxide, water vapor, and nitrogen. The balance factor of enthalpy consists of four elements that ensure combustion occurs: fuel, O_2, CO_2, and H_2O.

The solution employs a fourth-order Runge-Kutta integrator method. The simulation uses various constraints for initial values and boundary conditions. The model equations are structured as follows:

$$\frac{\partial T}{\partial t}=F_T\left(T,X_j\right),\text{dan }\frac{\partial X_j}{\partial t}=Fx_j\left(T,X_j\right), \tag{14}$$

Where T representing the chemical elements involved in combustion, X ensuring combustion occurs, and $j=1,2,3,4$, the equations discussed take the following form:

$$\begin{aligned}\rho c_p\frac{\partial T}{\partial t} = {} & \underbrace{-\rho h_c\frac{M}{M_1}r}_{\text{Combustion}}\underbrace{-\rho c_p u_i^*\frac{\partial T}{\partial x_i}}_{\text{Transport}} \\ & +\underbrace{k\frac{\partial}{\partial x_i}\left(\frac{1}{c_p}\frac{\partial c_pT}{\partial x_i}\right)+k\frac{\partial}{\partial x_i}\left(\frac{1}{c_p}\frac{\partial h_cT}{\partial x_i}\right)}_{\text{Diffusion}} \\ & +\underbrace{C_a(T_{amb}-T)}_{\text{Convection}}+\underbrace{\sigma\varepsilon\left[4dx_i\frac{\partial}{\partial x_i}\left(T^3\frac{\partial T}{\partial x_i}\right)\right]}_{\text{2D-Radiation}} \\ & +\underbrace{\sigma\varepsilon\left[\frac{T_{amb}^4-T^4}{\partial x_3}\right]}_{\text{Pseudo3D-Radiation}}\end{aligned}$$

$$\frac{\partial X_i}{\partial t}=-\frac{\theta_i.M}{\theta_{fuel}.M_{fuel}}r,\quad\text{with } i=1,2,3,4.$$

The model under discussion is the Brusselator Advection-Reaction-Diffusion System (BARD), which refers to two independent variables representing chemical reactants. Its general form is given as follows [31]:

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=d_1\frac{\partial^2 u}{\partial x^2}+f_1\left(u,\ v\right) \tag{15}$$

$$\frac{\partial v}{\partial t}+\frac{\partial v}{\partial x}=d_2\frac{\partial^2 v}{\partial x^2}+f_2(u,\ v) \tag{16}$$

With $u=u(x,t)$ and $v=v(x,t)$ as two different chemical constituents (reactants), where x is the spatial domain of observation and t is the observation time. $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial x}$ denote the advection terms, $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 v}{\partial x^2}$ represent the diffusion factors, d_1 and d_2 are the diffusion coefficients, and $f_1(u,v)$, $f_2(u,v)$ are reaction functions.

The solved model of the equation system takes the form:

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=d_1\frac{\partial^2 u}{\partial x^2}+B-\left(A+1\right)u+u^2v \tag{17}$$

$$\frac{\partial v}{\partial t}+\frac{\partial v}{\partial x}=d_2\frac{\partial^2 v}{\partial x^2}+Au-u^2v \tag{18}$$

with A,B = constants representing concentration during reaction operation, d_1,d_2 = diffusion coefficients. Initial values and boundary values:

$$u(x,0)=f(x),v(x,0)=g(x), \tag{19}$$

$$\frac{\partial u(0,t)}{\partial x}=\frac{\partial u(1,t)}{\partial t}=0,\ \text{and}\ \frac{\partial v(0,t)}{\partial x}=\frac{\partial v(1,t)}{\partial t}=0. \tag{20}$$

The discretization process of the equation system [36] employs finite difference schemes. Numerical schemes are utilized for its solution, ensuring the existence and uniqueness of solutions to the investigated problem.

The following subsection presents a discussion of systematic literature review methods.

C. Systematic Literature Review Methods

The review was conducted following the PRISMA guidelines. Subsequently, a bibliometric analysis was performed on the data obtained from the PRISMA process with the assistance of Rstudio software.

1) Search Strategy and Selection Criteria: This research used four databases for article retrieval: Scopus, Science Direct, Dimensions, and Google Scholar. The keywords used for article search were organized based on the topics listed in Table III. Moreover, some keywords were combined with others; for example, E is a combination of A and B, while F combines B and C.

The search was limited to 2019-2023 and filtered based on several criteria, as shown in Table IV. These criteria included searching through relevant titles, abstracts, and keywords in the databases. Furthermore, data were obtained from articles published in open-access journals, articles were written in English, and the database is research in mathematics.

TABLE III: Number of publications from four databases with five keyword combinations.

Code	Keywords
A	”Advection-Diffusion-Reaction Non-linear Equations”
B	”Water Quality ”
C	”Optimization Model” AND ”Advection-Diffusion-Reaction Non-linear Equations”
D	”Optimization Model” AND ”Water Quality ”
E	Advection-Diffusion-Reaction Non-linear Equations” AND ”Water Quality ”
F	”Water Quality ” AND ”Optimization Model” AND ”Advection-Diffusion-Reaction Non-linear Equations”

TABLE IV: The number of publications from databases with keyword combinations.

Code	Scopus	Science Direct	Dimensions	Google Scholar	Total
A	9	1	500	1	511
B	250	250	500	100	1,100
C	0	0	0	0	0
D	28	3	500	13	544
E	0	0	0	0	0
F	0	0	0	0	0
Total	287	254	1,500	114	2,155

2) PRISMA Method: The articles used were obtained through the PRISMA method [37], according to Abelha et al. [38]. This method provides a standardized and accurate framework for arranging the selection criteria, search strategies, data extraction, and analysis procedures. PRISMA can enhance method precision and results accuracy, serving as a guide for conducting a structured, systematic literature review [39], [40].

Figure 2 shows the selection process stages with $n_i, i = 1, \ldots, 8$ is the number of references resulting from every PRISMA step.

PRISMA method starts with the initial identification stage, which includes searching and collecting bibliometric data from various databases. Moreover, a more comprehensive explanation of the initial stage is provided in Subsection II-C1.

According to Table IV, 2,155 articles were obtained from four databases using six keyword combinations. These articles were further examined in the screening stage, which consisted of two steps.

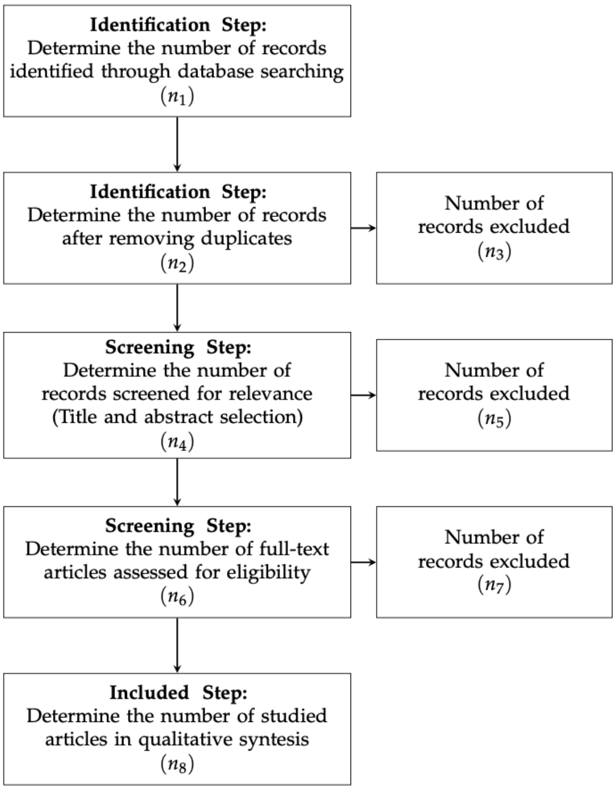

Fig. 2: Selection process based on PRISMA framework.

The first step of the screening stage includes checking for duplicates among the identified articles, which appear in one or more databases with the same title and authors. The duplication check uses the reference manager Jabref (see Simon et al. [41]). After the screening, 1,357 duplicate articles were removed, and the remaining were further analyzed. Subsequently, all articles are assessed for their relevance to the research topic based on title and abstracts. Through this screening stage, twelve articles are identified and referred to as Dataset 1, used in the bibliometric analysis.

The second step is the eligibility stage, where manual filtering is carried out by reading the full text of the selected articles for relevance evaluation. Six articles were identified and referred to as Dataset 2 from this selection process, as shown in Table V.

3) Bibliometric Analysis: Bibliometric Analysis was performed on Dataset 1 using the bibliometrix package in RStudio software to obtain scientific data mapping and comprehensively analyze the available bibliographic information. The diagram depicted in Figure 3 shows the output of bibliometric mapping based on previous research [42] and [43].

The initial step following determining the topic and search platform is bibliometric data mining. This phase is conducted concurrently with the article retrieval process in Section II-C1, resulting in the acquisition of Dataset 1 obtained from four different databases. To facilitate importing into RStudio, the data format was standardized in line with the Scopus data. Subsequently, the adjusted data were input into R-Bibliometrix, a package used for bibliometric analysis [44]. This import process extracted bibliometric parameters for analysis, including authors, affiliations, article citations, scientific production, countries, words, and keywords. The program further generated bibliometric mapping results based on these parameters.

III. RESULTS

A. Results from Bibliometric Analysis

This section discusses the results of the bibliometric analysis of Dataset 1. The bibliometric mapping output steps is shown in Figure 3.

Twelve articles were identified within Dataset 1 based on the main information output. The research was conducted from 2019 to 2023, with an average of 8.67 citations per document. Forty-seven authors were included, with an average of four co-authors per document.

1) Articles' Evolution: An overview of the development of article publication is acquired using R-bibliometrix. From Dataset 1, the number of issues per year is obtained, and average citations per year are presented in Figures 4 and 5.

Based on the results, the highest production was in 2021, with four articles, while the lowest was in 2022, with one article. Meanwhile, the highest average citation count was in 2020, with four articles.

2) Authors' Analysis: Through the Author menu, R-bibliometrix provides information related to authorship from the article data. The data on article production based on the authors countries were obtained. China dominated with 15

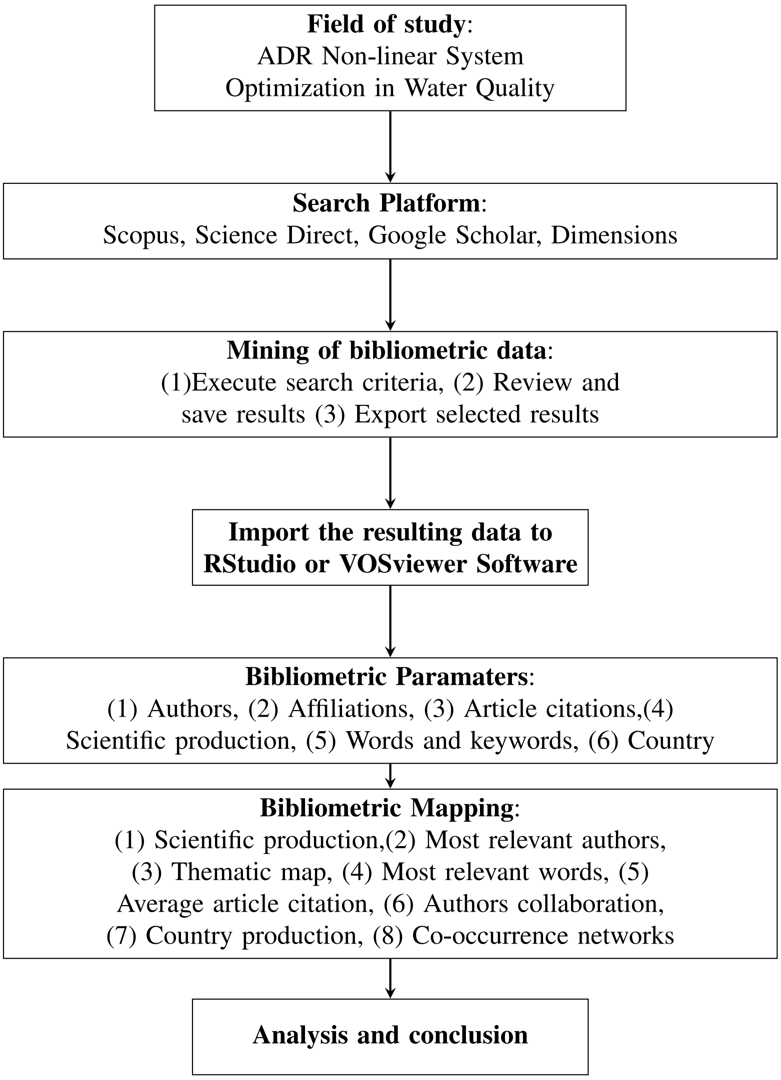

Fig. 3: Bibliometric mapping output steps

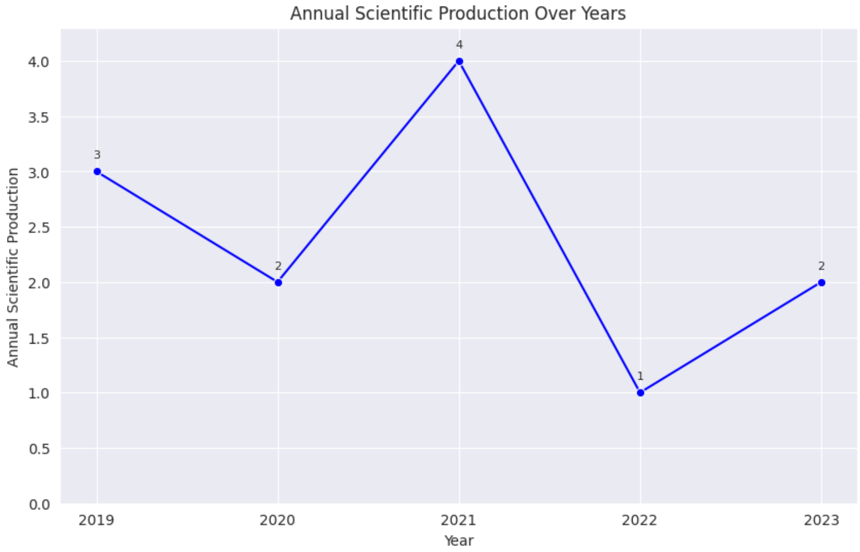

Fig. 4: Annual Scientific Production in Dataset 1 per year

TABLE V: The results of article numbers from the selection based on PRISMA framework.

Code	Total Number	Number of Duplication		Number of Abstract and Title		Number of Full text	
		Included	Excluded	Included	Excluded	Included	Excluded
A	511	198	313	4	194	0	4
B	1,100	56	1,044	4	52	3	1
C	0	0	0	0	0	0	0
D	544	72	472	4	68	3	1
E	0	0	0	0	0	0	0
F	0	0	0	0	0	0	0
Total	2,155	326	1,357	12*	262	6**	6

*Dataset 1 for bibliometric analysis, **Dataset 2 for literature review.

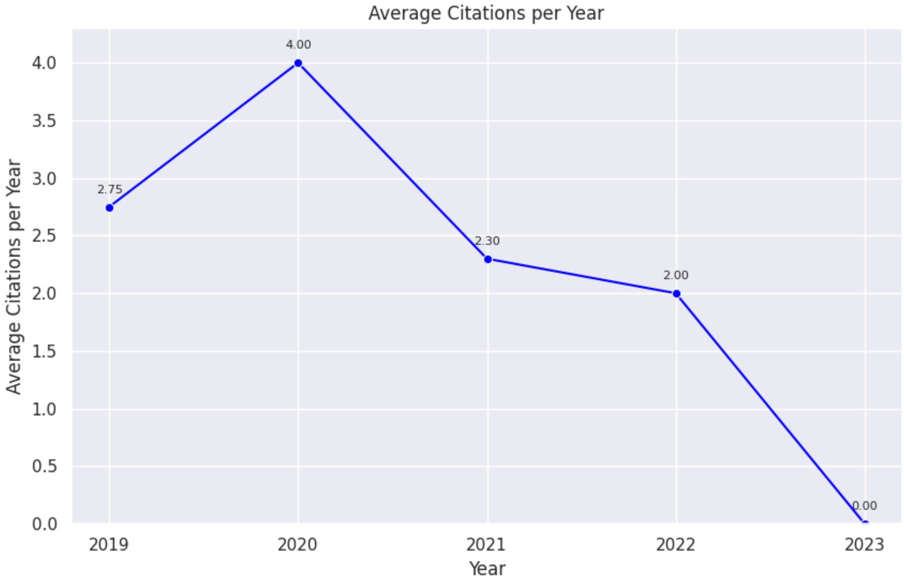

Fig. 5: Average article citations per year.

TABLE VI: Most Cited Country in Dataset 1

Most Cited Country	Number of Documents
USA	31
Denmark	27
China	17
Iran	15

publications, followed by the USA, Iran, and Denmark with 11, 3, and 2 publications, respectively.

Figure 6 and Table VI show production over time and the most cited country. This includes the author with the most relevant production, specifically Liu et al. [45], who has two articles published in 2019, as shown in Table VII.

3) Co-occurrence Network: Based on Figure 3, a co-occurrence network is one of the bibliometric mappings generated by R-bibliometrix or VOSviewer. In this paper, the VOSviewer software, as mentioned in [54], is used to generate the co-occurrence network as can be seen in Figure 7. This shows the co-occurrence network of terms present in the articles' title, abstract, and keywords in Dataset 1. In this paper, the minimum number of occurrences in a keyword is 2. Thus, 34 keywords are obtained. The number of keywords with the total strength of the occurrences links with other keywords is calculated. In this case, the keyword with the greatest total number of occurrences is chosen with the threshold chosen. This network can show clusters of topics formed by terms with similarities. The clustering words in the co-occurrence network can be seen in Table VIII. In this case, optimization and water quality are the largest, indicating their significance in the research topic of Dataset 1.

Furthermore, these two terms are interrelated and situated within the same cluster, suggesting frequent discussion of research topics. In the co-occurrence network, terms related to ADR are not found, particularly in ADR non-linear equation systems. This indicates that research on this topic still needs to be improved regarding its association with optimization and water quality.

4) Words Appearance Analysis: The analysis is related to developing the most frequent words and thematic mapping. The following facts are obtained.

First, Table IX presents the most relevant words of Dataset 1 with minimal occurrences 3 in the keywords within Dataset 1. The word ”Optimization” ranks first with 11 occurrences, followed by Water quality with 10 occurrences.

Second, in Figure 8, this mapping, terms derived from bibliometric data are grouped into clusters of words organized in four quadrants, each representing a frequently discussed theme. The motor themes are represented in Quadrant I, where the cluster shows high centrality and density. This

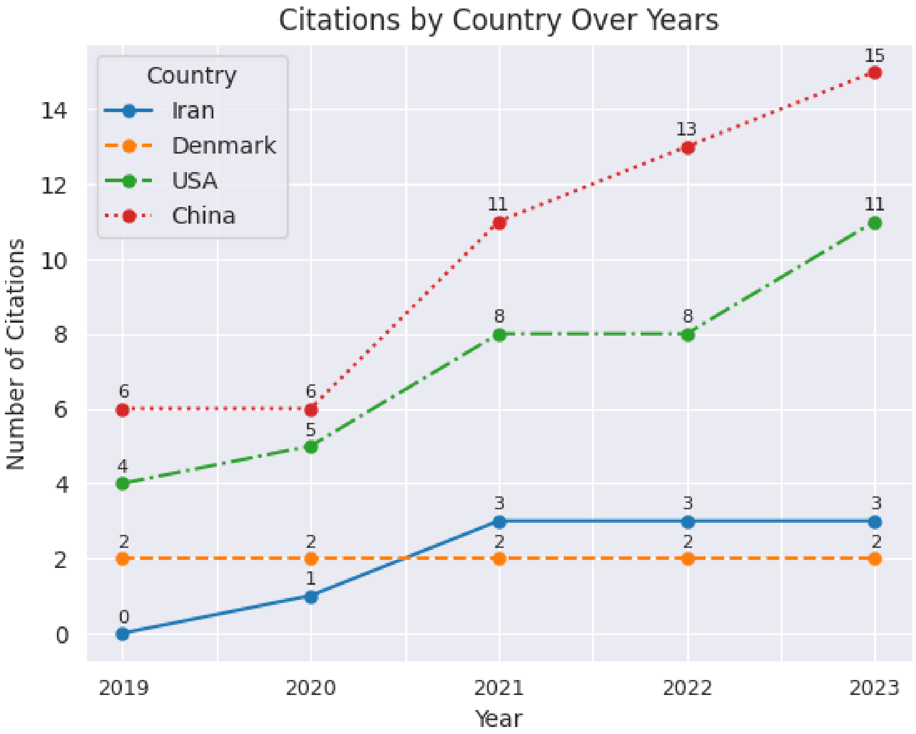

Fig. 6: Country production over time of Dataset 1.

TABLE VII: Authors Analysis in Dataset 1

Most Relevant Authors	Number of Documents	Authors Production (2019-2023)	
		Number of Articles	Total Citations per Year
Liu C [45]	2	2	0
Afshar A [46]	1	1	4
Bauer-Gottwein P [47]	1	1	5
Bischel HN [48]	1	1	1
Bozorg-Haddad [49]	1	1	3
Chen X [50]	1	1	5
Chen Y [51]	1	1	4
Cohon JL [52]	1	1	3
Ding X [53]	1	1	2
Duan J [50]	1	1	5

TABLE VIII: Clustering words in the co-occurrence network of Dataset 1

Cluster 1	Cluster 2	Cluster 3	Cluster 4
Article	Cost analysis	Algorithm	Decision making
China	Economic and social effect	performance assessment	learning systems
Economic development	Groundwater resources	quality management	multi-objective optimization models
Environmental policy	Linear programming	reservoirs (water)	multiobjective optimization
Quality control	Optimization	River pollution	neural networks
Resource allocation	Salt water	Rivers	Water pollution
River basin	Water management	Water conservation	Water treatment
Runoff	Water quality		
Water			
Water flow			
Water resource			
Water supply			

indicates strong interconnections among terms and relevance to other clusters. Quadrant II comprises the Niche Themes characterized by high density with low centrality. Clusters in this quadrant have strong internal cohesion but limited interconnections. Meanwhile, Quadrant III is emerging or consists of clusters with low density and centrality, indicating a reduction in the development of new topics. Quadrant IV is the Basic Theme, showing strong interconnections with

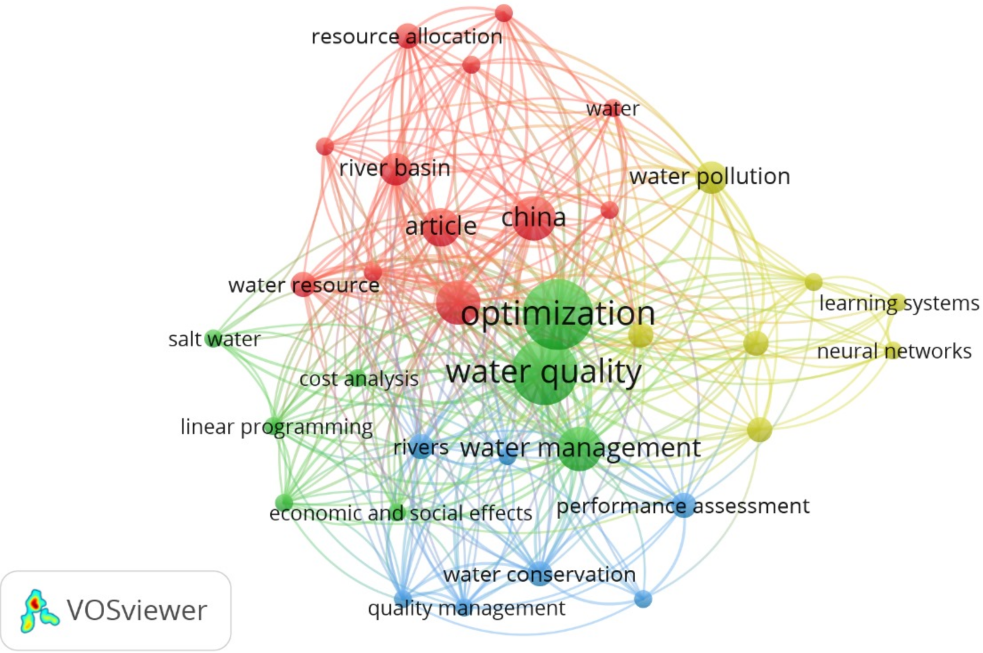

Fig. 7: Co-occurrence network of Dataset 1.

TABLE IX: Most relevant words of Dataset 1 with minimal occurrences 3 in the keywords.

Most relevant words	Occurences	Total link strength
Optimization	11	99
Water quality	10	93
China	6	62
Water supply	6	62
Article	5	57
Water management	6	55
River basin	4	47
Decision making	3	38
Water pollution	4	38
Rivers	3	36
Water resource	3	34
Water conservation	3	36
Resources allocation	3	30
Multi-objective optimization	3	26
Performance assessment	3	23
Water treatment	3	23

other clusters while having weak internal cohesion.

Third, based on Figure 8, the cluster containing the terms optimization and water quality is categorized in the Motor Themes quadrant, indicating high density and centrality. This implies that the cluster has strong interconnections among terms and is highly relevant to others. Moreover, the two other clusters represented by multiobjective optimization and cost analysis are near the Emerging or Declining Themes quadrant. This suggests that the two clusters show relatively low levels of development and relevance.

B. *Result from Systematic Literature Review*

The observations from Dataset 2 are presented as Systematic Literature Review results in this sectiondataset 2 comprised six articles that had passed the complete selection process, published between 2019 and 2023.

A summary of the topics in the six articles and optimization method are provided in Tables X and XI. In contrast, objective functions and constraints of the optimization models are identified in Tables XII and XIII.

IV. Discussion of the Results

A. *State-of-the-art on ADR Equation System Optimization in Water Quality*

This section discusses the answers to the research questions stated in Section II. The state-of-the-art ADR equation system optimization in water quality shows the research trends, significantly determining novel and new topics. This section also discusses applying optimization models in research related to the ADR non-linear equation system for optimizing water quality.

The state-of-the-art ADR and optimization in water quality are presented in Tables X, XI, XII and XIII. These tables discuss the covered topics in each article and explain the objectives and constraints used in optimization models.

TABLE X: Covered water problem topics in Dataset 2.

Article	Topics on Water Problem
[45]	Water treatment of the artificial lake in Chengdu, China
[46]	Multiple-pollutant waste load
[47]	Joint water allocation and water quality management in Haihe river basin
[52]	Drinking water system
[55]	Mulched brackish drip irrigation in Xinjiang, China
[56]	Water quality and water utilization in the Fenhe River Basin

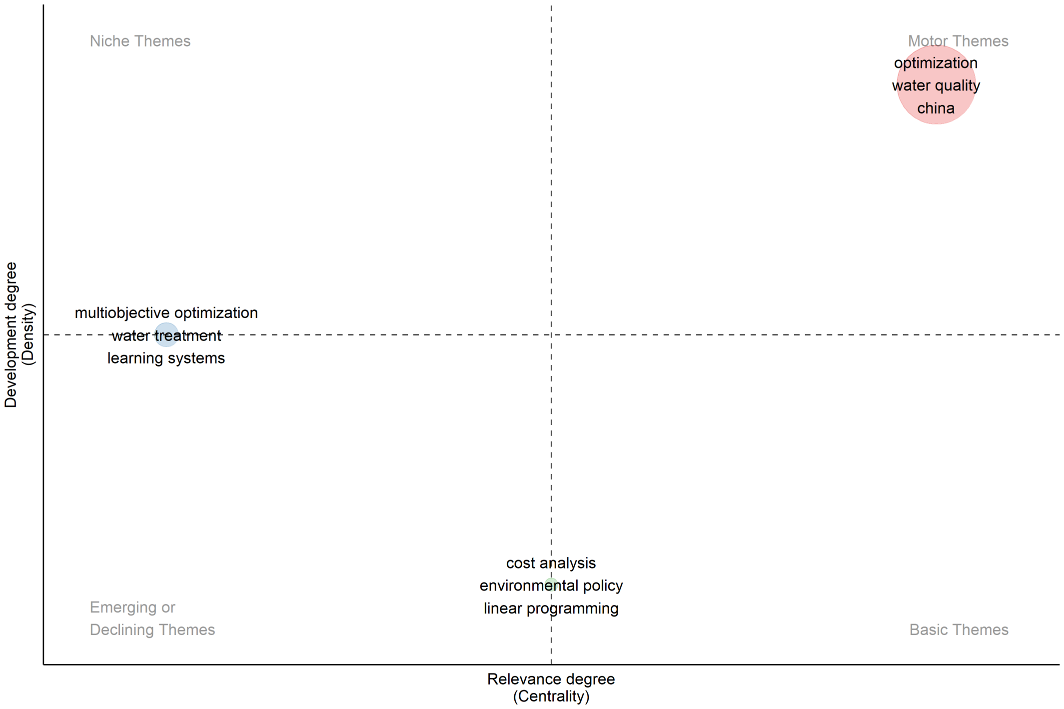

Fig. 8: Thematic mapping of Dataset 1.

TABLE XI: Used optimization methods in Dataset 2.

Article	Method in Optimization Modeling	ADR
[45]	Numerical modeling, surrogate models and multi-objective optimization	-
[46]	Fuzzy Optimization; Simulation-optimization approach; Ant colony optimization (ACO)	-
[47]	Multi-reservoir, multi-temporal, multiobjective linear optimization model	-
[52]	Multiobjective mixed-integer linear program optimization model (MOMILP)	-
[55]	Modelling scenario using HYDRUS	-
[56]	Inexact Downside risk control and two-stage stochastic programming (ITSDP)	-

TABLE XII: Optimization objectives in Dataset 2.

Article	Objective(s)
[45]	Minimizing cost and pollutant concentrations
[46]	Maximizing the value of average NSFWQI in the river water system
[47]	Minimizing total costs
[52]	Minimizing cost, maximizing systemwide chlorine residual and minimizing demand-weighted water age
[55]	Minimizing the impact on the soil environment and water resources
[56]	Maximizing the comprehensive watershed benets, including water benets, water costs, water treatment costs, and downside risks

B. Research Trends and Gaps on ADR Equation System Optimization in Water Quality

Based on the bibliometric analysis, there is continuous development in related publications, as evidenced by a significant increase from 2022 to 2023.

According to Table IX, all keywords show an upward trend, particularly those related to water quality, indicating a significant development. The co-occurrence network in Figure 7 shows the connections between terms that appear in articles within Dataset 1. These connections can be further explored as potential subjects for future research. Additionally, existing and new keywords, such as ADR equation, can be integrated within the network.

C. How Optimization models have been applied in research related to the ADR non-linear equation system for optimizing water quality

Based on the results presented in Tables X and XI, the six articles in Dataset 2 mainly address optimization challenges in water quality problems. The majority of these articles focus on applying their models to water quality issues in China, specifically in regions such as Xinjiang [55], Chengdu [45], Haihe River [47], and Fenhe River [56]. Water quality concerns in China are influenced by factors such as imbalanced water resource distribution, rapid economic growth, and water pollution [57], [58].

Additionally, densely populated areas in Southeast Asian countries are susceptible to various water quality issues due to the deterioration of water resources [59]. This information could be valuable when selecting data for further research applications.

TABLE XIII: Optimization constraints in Dataset 2.

Article	Constraint(s)
[45]	Allowable pollutant concentration at the assessment point, dissolved oxygen concentration, allowable hydraulic loading rate
[46]	Water quality standard intervals
[47]	Demand deficits equality constraint, Surface water resources and water demands
[52]	Satisfying demand, facility capacity, total system capacity, maximum facility capacity, minimum facility capacity, parametric facility cap
[55]	Time-variable flux, time-variable pressure head, atmospheric, no flux boundary condition, solute transport boundary
[56]	Water resource, water sector demand, regional wastewater treatment capacity, regional wastewater reuse capacity, water environment carrying capacity, downside risk

TABLE XIV: The most cited references in this paper.

Author(s)	Citation Freq	Optimization techniques	on water quality?
Wibbenmeyer et al. [20]	2	✓	✓
Semasinghe & Rousso [22]	2	✓	✓
Dandy et al. [21]	2	✓	✓
You et al. [23]	2	✓	✓
Gonzales-Inca et al. [24]	2	✓	✓
Biesheuvel et al. [25]	2	✓	-
Liu et al. [45]	5	✓	✓
Ghorbani et al. [46]	2	✓	✓
Martinsen et al. [47]	4	✓	✓
Schwetschenau et al. [52]	3	✓	✓
Guo et al. [55]	4	✓	✓
Meng et al. [56]	4	✓	✓

The results presented in Dataset 2 showed that the six articles did not use ADR non-linear equations. According to [60], the ADR equation describes phenomena such as mass transport, diffusion, and chemical reactions.

Based on Tables XII and XIII, the six articles did not incorporate all aspects, but the majority of the discussed optimization models primarily focused on minimizing costs, as shown in [45], [47], [52], [56].

Guo et al. [55] addressed a slightly different topic from ADR, precisely the advection-dispersion equation, see [61] and [62] to represent soil salt transportation. This equation was simulated within the model using the HYDRUS software as stated in Guo et al. [55]. As mentioned in [63], The HYDRUS-1D and HYDRUS (2D/3D) are computer software packages that use finite-element models for simulating the one-, two- or three-dimensional movement of water, heat, and multiple solutes in various saturated media.

Despite numerous investigations, ADR has yet to be the leading research focus in optimization modeling for water quality. However, Bigdeli et al. [64] used a generic ADR equation to describe the transport of microplastics in water within a marine microplastics and transport model. Crespo et al. [65] also discuss a control strategy model for treating polluted water resources using a bioreactor.

D. Description to the most cited references in this paper

In this subsection, the most cited references from Sections I to IV are presented in Table XIV. Citations of the articles are calculated by counting their appearance in every section.

Based on the results, 12 references were the most cited, with total citations ranging from two to five. All the references in Table XIV showed that the optimization method was used in solving problems, where only one did not discuss the water quality.

E. Future Research Design

This section presents a discussion on a proposed research design for ADR. The initial stage of this research method begins with formulating the advection-diffusion-reaction equation system for water pollution. Subsequently, this equation model is solved numerically using a system of partial differential equations, the stages of which are presented in Figure 9.

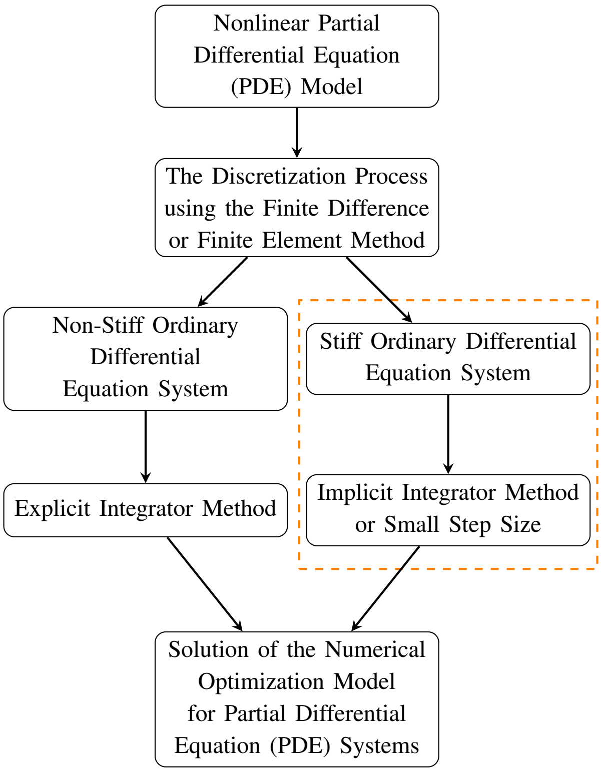

Fig. 9: Stages of numerical solution for nonlinear partial differential equation system

The research methodology used to develop the system follows the waterfall method, consisting of six sequential processes. The general stages of this research are depicted in the research process flow chart in Figure 10.

Figure 10 illustrates the steps undertaken in each research stage, starting from data collection, analysis, model formulation, model testing, and the resulting solutions. Figure 10 describes the research process.

1) Model analysis → determining system requirements, which involves deriving the necessary variables.

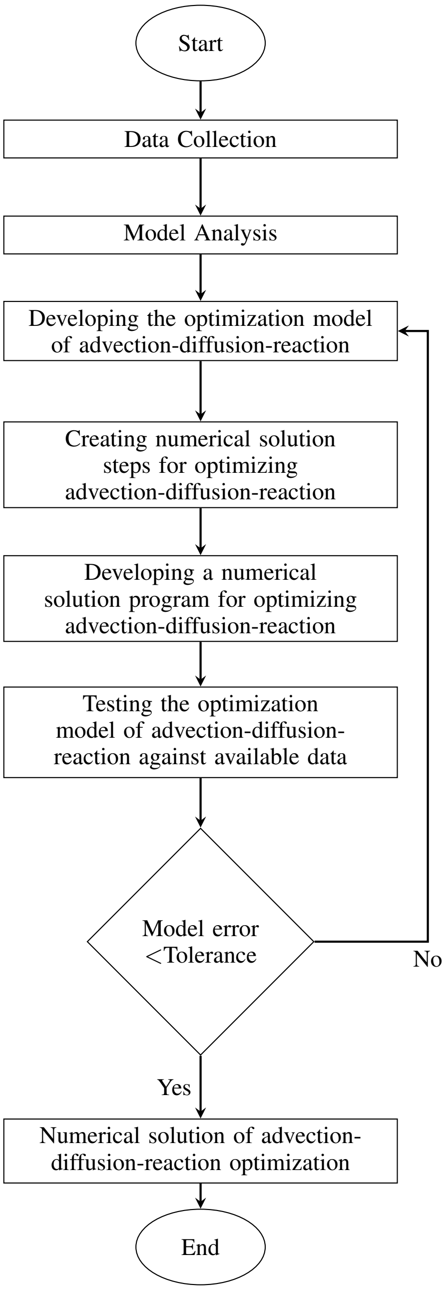

Fig. 10: Flowchart of the Research Process Stages

2) Model design → resulting in an optimized nonlinear advection-diffusion-reaction model.
3) Model implementation → developing the application program.
4) Mode testing → The model is tested using data from the subsequent year's measurements. If the error < tolerance, then the model is considered good/accepted. Application system testing uses black-box testing based on the application menu.

The steps above are embodied in the following software design. System design in software engineering can utilize traditional design approaches, including context diagrams and data flow diagrams (DFD). This design was chosen because the programming language used is not object-oriented. The design is presented in Figures 11 and 12.

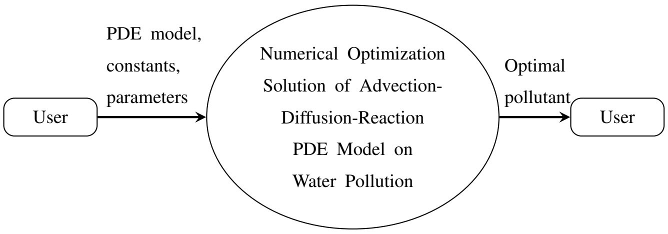

Fig. 11: Context Diagram of Numerical Optimization Solution for Non-linear Advection-Diffusion-Reaction Equation on Water Quality

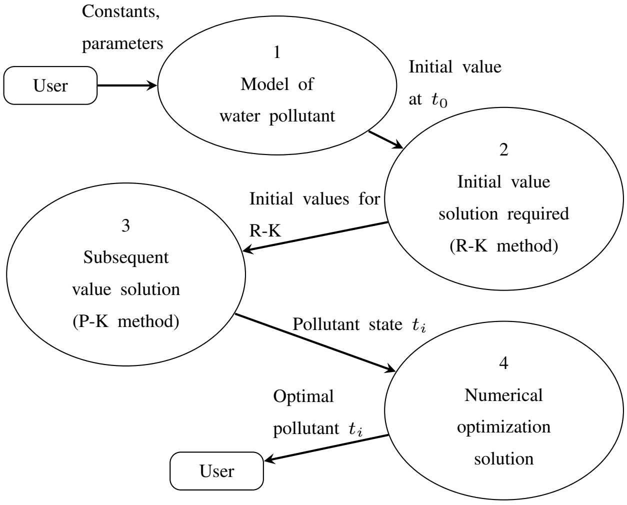

Fig. 12: Level 1 DFD of the Numerical Optimization Solution for Non-linear Advection-Diffusion-Reaction Equation on Water Quality

V. CONCLUSIONS

In conclusion, this research successfully explored the systematic literature review and bibliometric analysis on optimizing the ADR non-linear equation system focusing on water quality. Based on a bibliometric data search, articles were collected from several databases, including Dimensions, Google Scholar, Scopus, and Science Direct. Subsequently, the selection was carried out using the PRISMA framework, resulting in twelve articles for bibliometric analysis in Dataset 1 and six articles used in the state-of-the-art, as presented in Dataset 2. The results showed that research concerning the use of ADR non-linear equations in optimizing water quality models still needed to be explored, indicating potential for further development. This observation was substantiated by bibliometric analysis, showing the absence of keywords related to ADR equations. Several references that resulted in this study, can be considered as resources for future research on the proposed topics. However, the limitations of this research included restricting article search to four databases, namely Scopus, ScienceDirect, Dimensions, and Google Scholar. Additional databases such as Web of Science and Springer could be considered to achieve a more comprehensive scope.

Furthermore, data retrieval was based on specific keywords associated with the topic, indicating the need to explore broader topics for a more extensive and inclusive dataset. A future research design is presented for a recommendation, as seen in Section IV-E, i.e., designing research on finding a numerical optimization solution for a non-linear advection-diffusion-reaction equation on water quality.

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