Robust H-Infinity Control for Input Time-Delay Systems Based on Memory State-Feedback

Xuyang Yuan, Fang Gao*, Qingwen Lv, and Xingyu Chen

Abstract—**This paper presents a method for suppressing disturbances in a category of linear input time-delay systems under H_∞ control. These systems utilize memory state-feedback and possess direct transfer matrices. In the realm of linear systems, the use of Equivalent Input Disturbance (EID) methods has been recognized for their effectiveness in disturbance suppression. Consequently, the primary objective is to achieve superior disturbance suppression capabilities while adhering to the H_∞ control performance criterion. The paper begins by outlining the structure of the linear input time-delay system and its reliance on the EID methodology. Subsequently, an EID estimator is refined to ensure optimal control performance. Stability conditions for the closed-loop system are then formulated as Linear Matrix Inequalities to satisfy the H_∞ control performance requirements. Additionally, a controller featuring memory state-feedback is developed. The efficacy of the presented methodology, along with its advantages over traditional H_∞ control approaches is demonstrated through numerical and practical examples.**

Index Terms—**Disturbance Suppression, H_∞ Control, Input Time-delay Systems, Equivalent Input Disturbance, Linear Matrix Inequalities, Memory State-feedback.**

I. Introduction

DISTURBANCES and time-delays frequently manifest in various real-world systems, including network communication, industrial control, transportation systems, and among others. These time-delays can lead to diminished control efficacy, increased failure rates and decreased system robustness. Consequently, researchers have undertaken significant efforts to address the challenges posed by disturbances and time-delays from these systems [1], [2], [3].

H_∞ control theory is a fundamental aspect of control theories [4], [5] and has emerged as a prominent area of research in recent years. The primary method of this theory is to devise an effective controller that declines the impact of disturbances on the system to ensure the robustness and stability. As a result, H_∞ control plays a critical role in maintaining optimal control capabilities within the system. Furthermore, the integration of H_∞ control methodology with state-feedback has been discussed in previous studies [6], [7]. Studies have shown that memory state-feedback controllers, as opposed to non-memory state-feedback controllers, exhibit reduced conservatism and improved performance, as indicated in [8].

Recently, a variety of methods for suppressing disturbances have been proposed [9], [10]. One of the most promising methods is Active Disturbance Rejection Control (ADRC) technology. ADRC is based on an in-depth analysis of the Proportional-Integral-Derivative (PID) control method [11] and one of its core features is focusing on the observer design and application. The most notable approach is the control strategy based on Disturbance Observer (DOB) as referenced in literature [12]. Common observers include the Extended-State Observer [13], the Sliding-Mode Observer [14], the Proportional-Integral Observer [15], etc. The primary principle of DOB methods is to reshape the disturbance observer and generate a continuous real-time estimation of the disturbance through it, which is subsequently counteracted through system compensation techniques. Nevertheless, it is essential for DOB methods to consider the object's inverse model beforehand, which may pose significant challenges for researchers.

To address the aforementioned issue, we integrate H_∞ control with Equivalent Input Disturbance (EID) technique [16], [17], the EID is additionally considered as one of the technologies utilized by ADRC. It possesses the ability to both reject matched and unmatched disturbances, in addition to estimating disturbances. At the same time, there is no need for a reverse model of the object and previous information about the disturbance.

Therefore, this paper considers the robust H_∞ control in the context of input time-delay systems when exogenous disturbances exist. To achieve preferable control, a novel control law has been developed, incorporating the EID estimation. Besides utilizing memory state-feedback and incorporating input time-delay, the method involves applying the Lyapunov function mode and integrating EID estimation to enhance the stability of an H_∞ controller. The closed-loop stability conditions are shown via Linear Matrix Inequalities (LMIs), it is effective to verify via motor speed control instance simulation and the comparison of the H_∞ control with EID method and without EID method in the presence of both matched and unmatched disturbances.

In order to facilitate a quicker and easier understanding of this article, the following notations are proposed. **Notations** : Positive-definite matrix is denoted as $P>0$. $\begin{bmatrix} N & J \\ \star & A \end{bmatrix}$ demonstrates $\begin{bmatrix} N & J \\ J^{\mathrm{T}} & A \end{bmatrix}$. P^{+} refers to the pseudoinverse of P. $H(s)$ represents the Laplace transform of $h(t)$.

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II. Structure of H_∞ control system based on EID method

Take a plant into account

$$\begin{cases} \frac{dx(t)}{dt} = Ox(t) + Pu(t-\zeta) + P_\omega \omega(t), \\ y(t) = Sx(t) + Tu(t-\zeta), \\ z(t) = Sx(t) + T_\omega \omega(t), \end{cases} \tag{1}$$

where $y(t)$, $x(t)$, $u(t)$, and $z(t)$ are separately the measured output vector, the state vector, the control input vector, and the controlled output vector; $\omega(t)$ refers to the disturbance satisfying $L_2[0,\infty)$; O, P_ω, P, T, S, and T_ω denote real constant matrices of appropriate dimensions, ζ represents a positive time-delay.

According to [16], we can understand the EID principle that a signal ($v_e(t)$), generating on the control input channel, holds an equivalent impact as the external disturbance on the output. Thus, the system (1) turns to the following system (2). As shown in Fig. 1, the control system utilizing the EID technique comprises a plant, a memory state-feedback controller, an EID estimator, and a Luenberger observer respectively.

$$\begin{cases} \frac{dx(t)}{dt} = Ox(t) + Pu(t-\zeta) + Pv_e(t), \\ y(t) = Sx(t) + Tu(t-\zeta), \\ z(t) = Sx(t) + T_\omega \omega(t). \end{cases} \tag{2}$$

In order to estimate the EID, we can make use of an observer, the following equations express its state space.

$$\begin{cases} \frac{d\hat{x}(t)}{dt} = O\hat{x}(t) + Pu_f(t-\zeta) + L(y(t) - \hat{y}(t)), \\ \hat{y}(t) = S\hat{x}(t) + Tu_f(t-\zeta), \end{cases} \tag{3}$$

The utilization of the EID technique results in the following equation

$$\Lambda x(t) = x(t) - \hat{x}(t), \tag{4}$$

the state $\hat{x}(t)$ denotes the remodeling of the state $x(t)$.

$$P^+ := (P^{\mathrm{T}} P)^{-1} P^{\mathrm{T}}. \tag{5}$$

The EID estimate is elucidated in reference [16], therefore

$$\hat{v}_e(t) = P^+ LS\Lambda x(t) + (I - P^+ LT)(u_f(t-\zeta) - u(t-\zeta)), \tag{6}$$

where $\hat{v}_e(t)$ is an estimate of the EID.

The filter selection is characterized by a low-pass configuration [16], with its state-space representation being denoted as

$$\begin{cases} \frac{dx_F(t)}{dt} = A_F x_F(t) + G_F \hat{v}_e(t), \\ \tilde{v}_e(t) = H_F x_F(t). \end{cases} \tag{7}$$

The filter $F(s)$ picks one suitable band of the angle frequency [18].

$$\tilde{V}_e(s) = F(s)\hat{V}_e(s), \tag{8}$$

where $\tilde{v}_e(t)$ is filtered by $\hat{v}_e(t)$.

The control law is depicted as

$$u(t-\zeta) = u_f(t-\zeta) - \tilde{v}_e(t). \tag{9}$$

III. Analysis and design of closed-loop system

Definition 1. *Given a positive scalar γ, an H_∞ controller $u(t)$ is designed such that the system has a given H_∞ control performance, that is:*
(a) The closed-loop system is internally stable when disturbance $\omega(t) = 0$;
(b) Under zero initial condition, $\|z(t)\|_2 < \gamma \|\omega(t)\|_2$, which holds for $\forall\ \omega(t) \in L_2[0,\infty)$, $\omega(t) \neq 0$.

Lemma 1 ([19])**.** *Considering a specified symmetric matrix*

$$\varpi = \begin{bmatrix} \varpi_{11} & \varpi_{12} \\ \varpi_{12}^{\mathrm{T}} & \varpi_{22} \end{bmatrix}, \tag{10}$$

equates to the following statements:

(a) $\varpi < 0$*;*
(b) $\varpi_{11} < 0$ *and* $\varpi_{22} - \varpi_{12}^{\mathrm{T}} \varpi_{11}^{-1} \varpi_{12} < 0$*; and*
(c) $\varpi_{22} < 0$ *and* $\varpi_{11} - \varpi_{12} \varpi_{22}^{-1} \varpi_{12}^{\mathrm{T}} < 0$*.*

Then, let

$$\rho(t) = \begin{bmatrix} \hat{x}^{\mathrm{T}}(t) & \Lambda x^{\mathrm{T}}(t) & x_F^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}. \tag{11}$$

Hence, the closed-loop system is well-expressed via the above three states.

We get

$$\begin{aligned} &\dot{\hat{x}}(t) = O\hat{x}(t) + Pu_f(t-\zeta) + LS\Lambda x(t) - LTH_F x_F(t), \\ &\Lambda \dot{x}(t) = (O - LS)\Lambda x(t) + (LT - P)H_F x_F(t) + P_\omega \omega(t), \\ &\dot{x}_F(t) = (A_F + G_F H_F - G_F H_F P^+ LT)x_F(t) \\ &\qquad + G_F P^+ LS\Lambda x(t). \end{aligned}$$

Therefore,

$$\frac{d\rho(t)}{dt} = \bar{O}\rho(t) + \bar{P}u_f(t-\zeta) + \bar{P}_\omega \omega(t), \tag{12}$$

where

$$\bar{O} = \begin{bmatrix} O & LS & \bar{O}_{13} \\ 0 & O - LS & \bar{O}_{23} \\ 0 & G_F P^+ LS & \bar{O}_{33} \end{bmatrix},$$

$$\bar{P} = \begin{bmatrix} P \\ 0 \\ 0 \end{bmatrix}, \bar{P}_\omega = \begin{bmatrix} 0 \\ P_\omega \\ 0 \end{bmatrix}, \bar{S} = \begin{bmatrix} S^{\mathrm{T}} \\ S^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{aligned} &\bar{O}_{13} = -LTH_F, \\ &\bar{O}_{23} = (LT - P)H_F, \\ &\bar{O}_{33} = A_F + G_F H_F - G_F H_F P^+ LT. \end{aligned}$$

The control law in Fig. 1 refers to

$$u_f(t-\zeta) = \bar{K}\rho(t-\zeta), \tag{13}$$

where $\bar{K} = \begin{bmatrix} K_P & 0 & 0 \end{bmatrix}$.

So the following theorem is presented.

Theorem 1. *Given O, P, S, T, P_ω, and T_ω, suppose the existence of symmetric positive-definite matrices Y_1, Y_2, Y_3, X_1, constants κ, $\alpha > 0$, and suitable matrices $\hat{W}$, $\hat{W}_1$, M, system (12) shows gradual stabilization if the condition of the following inequality is satisfied*

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \Theta_{15} \\ \star & -\Theta_{22} & 0 & 0 & 0 \\ \star & \star & -\Theta_{33} & 0 & 0 \\ \star & \star & \star & -\Theta_{44} & 0 \\ \star & \star & \star & \star & -\Theta_{55} \end{bmatrix} < 0, \tag{14}$$

where

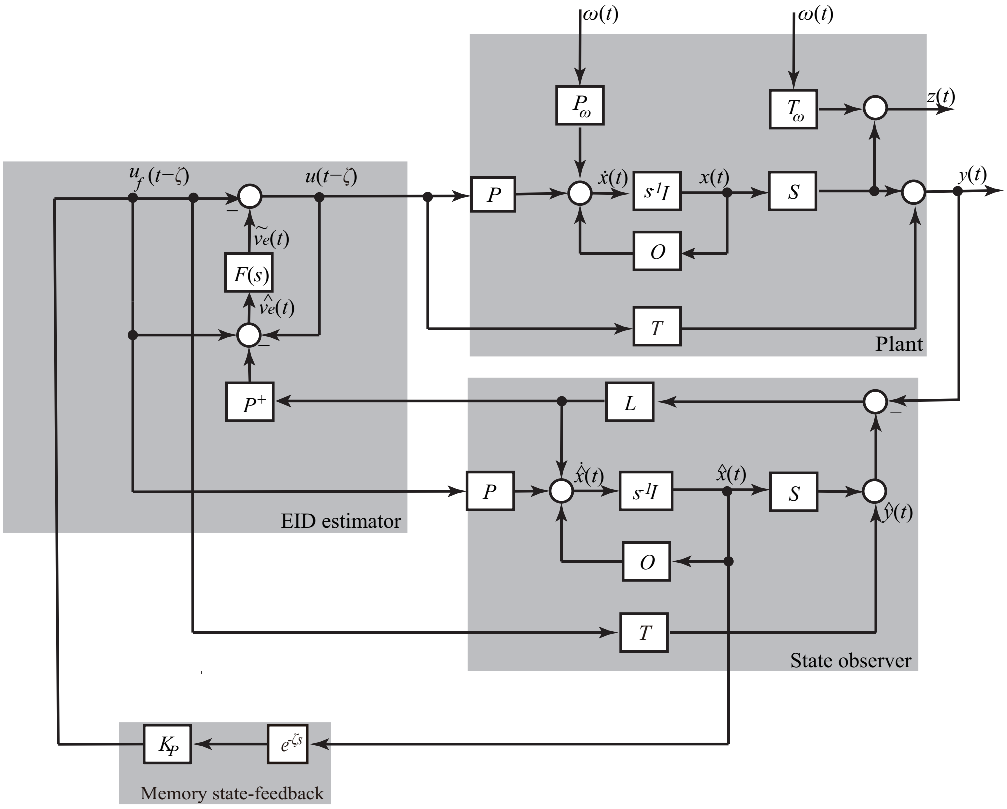

Fig. 1: H_∞ memory state-feedback built on EID control system structure.

$$\Theta_{11} = \begin{bmatrix} \theta_{11} & MS & \theta_{13} \\ \star & \theta_{22} & \theta_{23} \\ \star & \star & \theta_{33} \end{bmatrix},$$
$$\theta_{11} = OX_1 + X_1O^{\mathrm{T}},$$
$$\theta_{13} = -MTH_F,$$
$$\theta_{22} = O\alpha - MS + (O\alpha - MS)^{\mathrm{T}},$$
$$\theta_{23} = MTH_F - PH_F\alpha + (G_FP^+MS)^{\mathrm{T}},$$
$$\theta_{33} = A_F\alpha + G_FH_F\alpha - G_FH_FP^+MT + (A_F\alpha + G_FH_F\alpha - G_FH_FP^+MT)^{\mathrm{T}},$$
$$\Theta_{12} = \begin{bmatrix} \kappa PW_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\Theta_{13} = \mathrm{diag}\{X_1, \alpha I, \alpha\},$$
$$\Theta_{14} = \begin{bmatrix} X_1S^{\mathrm{T}} \\ \alpha S^{\mathrm{T}} \\ 0 \end{bmatrix},$$
$$\Theta_{15} = \begin{bmatrix} X_1S^{\mathrm{T}}T_\omega \\ \alpha S^{\mathrm{T}}T_\omega + P_\omega \\ 0 \end{bmatrix},$$
$$\Theta_{22} = \Theta_{33} = \mathrm{diag}\{\kappa Y_1, \kappa Y_2, \kappa Y_3\},$$
$$\Theta_{44} = I,$$
$$\Theta_{55} = T_\omega^{\mathrm{T}}T_\omega - \gamma^2 I.$$

Using the LMI toolbox calculations, we deduce that the controller of memory state-feedback gain and observer gain can be obtained by

$$K_P = \hat{W}_1Y_1^{-1}, \quad L = M\alpha^{-1}. \tag{15}$$

Proof: In accordance with (13), the system is characterized by the following description

$$\frac{d\rho(t)}{dt} = \bar{O}\rho(t) + \bar{P}\bar{K}\rho(t-\zeta) + \bar{P}_\omega\omega(t). \tag{16}$$

The selection of a suitable Lyapunov function is

$$V(\rho_t) = \rho^{\mathrm{T}}(t)\hat{N}\rho(t) + \int_{t-\zeta}^{t}\rho^{\mathrm{T}}(s)\hat{R}\rho(s)\mathrm{ds}, \tag{17}$$

where matrices $\hat{N}$, $\hat{R} > 0$, $\hat{N} = \mathrm{diag}\{\hat{N}_1, \hat{N}_2, \hat{N}_3\}$, $\hat{R} = \mathrm{diag}\{\hat{R}_1, \hat{R}_2, \hat{R}_3\}$ are decided.

Consider H_∞ performance index

$$J_{z\omega} = \int_0^\infty [z^{\mathrm{T}}(t)z(t) - \gamma^2\omega^{\mathrm{T}}(t)\omega(t)]\mathrm{dt}. \tag{18}$$

By the third equation of (2), we can rewrite the equation as follows

$$z(t) = \bar{S}\rho(t) + T_\omega\omega(t). \tag{19}$$

It is simple to obtain $V(0) = 0$ and $V(\infty) \geq 0$. Therefore, we have

$$J_{z\omega} \leq \int_0^\infty [\dot{V}(\rho_t) + z^{\mathrm{T}}(t)z(t) - \gamma^2\omega^{\mathrm{T}}(t)\omega(t)]\mathrm{dt}. \tag{20}$$

Computing the derivative about $V(\rho_t)$ in (17) gives

$$\dot{V}(\rho_t) = 2\rho^{\mathrm{T}}(t)\hat{N}\dot{\rho}(t) + \rho^{\mathrm{T}}(t)\hat{R}\rho(t) - \rho^{\mathrm{T}}(t-\zeta)\hat{R}\rho(t-\zeta). \tag{21}$$

Substituting (19) (21) to (20), we obtain

$$J_{z\omega} \leq \int_0^\infty \eta_1^{\mathrm{T}}(t)\Psi\eta_1(t), \tag{22}$$

where

$$\eta_1(t) = \begin{bmatrix} \rho^{\mathrm{T}}(t) & \rho^{\mathrm{T}}(t-\zeta) & \omega^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}},$$

$$\Psi = \begin{bmatrix} \Sigma & \hat{N}\bar{P}\bar{K} & \hat{N}\bar{P}_\omega + \bar{S}^{\mathrm{T}}T_\omega \\ \star & -\hat{R} & 0 \\ \star & \star & T_\omega^{\mathrm{T}}T_\omega - \gamma^2 I \end{bmatrix}, \tag{23}$$

where

$\Sigma = \hat{N}\bar{O} + \bar{O}^{\mathrm{T}}\hat{N} + \hat{R} + \bar{S}^{\mathrm{T}}\bar{S}$.

According to Lemma 1, (23) can be equivalent to

$$\begin{bmatrix} \Sigma_{11} & \hat{N}\bar{P}\bar{K} & I & \bar{S}^{\mathrm{T}} & \Sigma_{15} \\ \star & -\hat{R} & 0 & 0 & 0 \\ \star & \star & -\hat{R}^{-1} & 0 & 0 \\ \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & T_\omega^{\mathrm{T}}T_\omega - \gamma^2 I \end{bmatrix} < 0, \tag{24}$$

where

$\Sigma_{11} = \hat{N}\bar{O} + \bar{O}^{\mathrm{T}}\hat{N}$,

$\Sigma_{15} = \hat{N}\bar{P}_\omega + \bar{S}^{\mathrm{T}}T_\omega$.

Let $\hat{N}_i^{-1} = X_i \ (i = 1, \cdots, 3)$, $\hat{R}_i^{-1} = \kappa Y_i \ (i = 1, \cdots, 3)$, $X = \mathrm{diag}\{X_1, \alpha I, \alpha\}$, $Y = \mathrm{diag}\{Y_1, Y_2, Y_3\}$. Pre-multiplication and post-multiplication on the matrice of inequality (24) by $\mathrm{diag}\{\hat{N}^{-1}, \hat{R}^{-1}, I, I, I\} = \mathrm{diag}\{X, \kappa Y, I, I, I\}$, making $\hat{W} = \bar{K}Y$ get

$$\begin{bmatrix} \Delta_{11} & \kappa\bar{P}\hat{W} & X & X\bar{S}^{\mathrm{T}} & \Delta_{15} \\ \star & -\kappa Y & 0 & 0 & 0 \\ \star & \star & -\kappa Y & 0 & 0 \\ \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & T_\omega^{\mathrm{T}}T_\omega - \gamma^2 I \end{bmatrix} < 0, \tag{25}$$

where

$\Delta_{11} = \bar{O}X + X\bar{O}^{\mathrm{T}}$,

$\Delta_{15} = \bar{P}_\omega + X\bar{S}^{\mathrm{T}}T_\omega$.

Then, substituting (12), (13) and (15) into (25) yields (14).

Therefore, if $(23) < 0$ implies that $(20) < 0$, according to the previous definition, the system attains the H_∞ performance index.

We could know $\frac{d\rho(t)}{dt} < 0$ when $(14) < 0$ stands. Thus, in situations where the disturbance $\omega(t)$ is 0, the system (16) is progressively stable if $\frac{d\rho(t)}{dt} < 0$.

Therefore, the system (16) is gradually stable when $(14) < 0$ stands.

This completes the proof. □

IV. Simulation

Instance 1: Select a set of parameters for plant (1) as follows:

$$O = \begin{bmatrix} -8 & 0 \\ 0 & -6 \end{bmatrix}, \ P_\omega = \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$

$$P = \begin{bmatrix} 12 \\ 4 \end{bmatrix}, \ S = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^{\mathrm{T}},$$

$$T_\omega = 0.4, \gamma = 0.9, T = 0.1,$$

$$\zeta = 1, \alpha = 1.07, \kappa = 0.2.$$

Apply the disturbance

$$\omega(t) = 1.25\sin 1.5\pi t + 1.25\cos 1.5\pi t + 1.25\sin 2.5\pi t. \tag{26}$$

We choose the parameters of $F(s)$, $A_F = -101$, $G_F = 100$, $H_F = 1$ [16].

Through the utilization of MATLAB software, it is possible to compute the LMI outcome corresponding to the Theorem 1, the H_∞ memory state-feedback controller parameter and observer parameter were derived as follows:

$$K_P = \begin{bmatrix} -0.1928 & -0.2722 \end{bmatrix},$$

and

$$L = \begin{bmatrix} 0.7904 & 2.5254 \end{bmatrix}^{\mathrm{T}}.$$

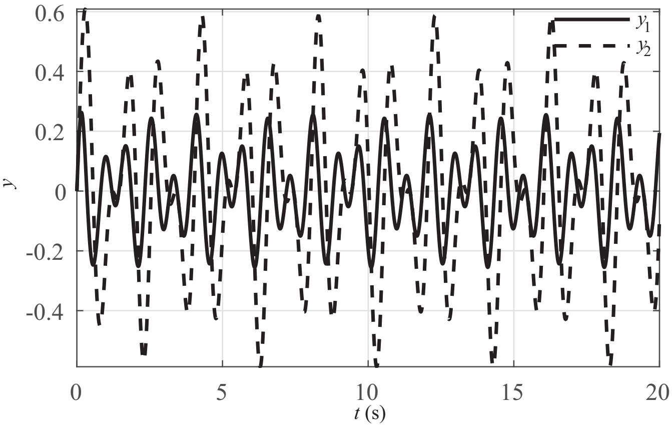

Fig. 2: Output response of our method (y_1) and H_∞ memory state-feedback method (y_2) for $P_\omega \neq P$.

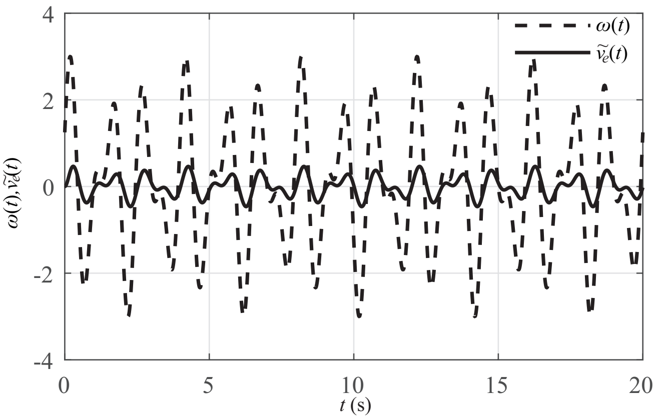

Fig. 3: Simulation result for $P_\omega \neq P$ between disturbance $\omega(t)$ and disturbance estimate $\tilde{v}_e(t)$.

To showcase the effectiveness of our method in suppressing disturbances, we analyze and contrast the outcomes obtained through simulations. Fig. 2 and Fig. 4 denote output response of our method (y_1) and H_∞ memory state-feedback method (y_2) for $P_\omega \neq P$ and $P_\omega = P$. Fig. 3 and Fig. 5 denote simulation results between disturbance $\omega(t)$ and disturbance estimate $\tilde{v}_e(t)$ for $P_\omega \neq P$ and $P_\omega = P$.

In Fig. 2, the result of our method represented the system was stable. The peak-to-peak value (PPV) of output error in the steady-state approached nearly 0.26 while the PPV of H_∞ memory state-feedback control method was 0.6. Comparing simulation results between Fig. 2 and Fig. 4, our method can suppress matched disturbances and unmatched disturbances more effectively.

In Fig. 3, the PPV of disturbance estimate $\tilde{v}_e(t)$ was nearly 0.4593 while the PPV of disturbance $\omega(t)$ was 2.988. These results suggest that EID technology can effectively mitigate the impact of disturbances on the system output performance. The discrepancy in PPV between $\tilde{v}_e(t)$ and $\omega(t)$ in Fig. 5 was denoted as 0.091, which indicates the EID method's ability to accurately estimate unknown and external disturbances. Consequently, these findings demonstrate the efficacy of the proposed methodology.

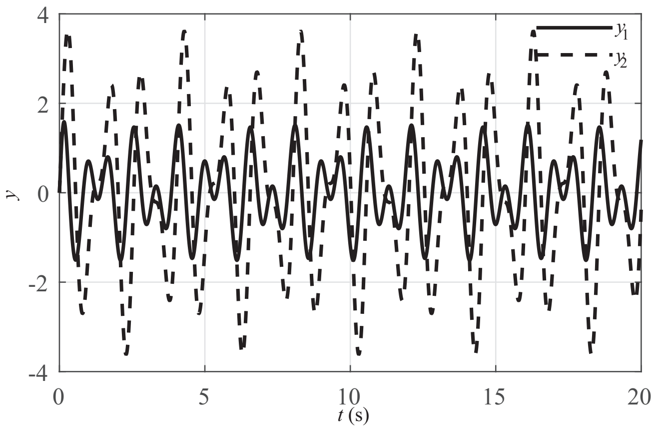

Fig. 4: Output response of our method (y_1) and H_∞ memory state-feedback method (y_2) for $P_\omega = P$.

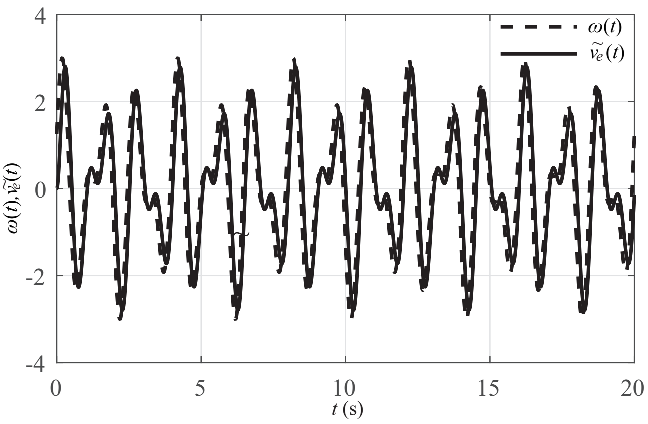

Fig. 5: Simulation result for $P_\omega = P$ between disturbance $\omega(t)$ and disturbance estimate $\tilde{v}_e(t)$.

Instance 2: The instance applies our method in a classical system named the rotary-speed control in [20]. The control system includes a driver, a direct current motor, a high-performance desk computer(equivalent to a controller), and an optical encoder.

Suppose that $\bar{x}(t) = \begin{bmatrix} \bar{y}(t) & \bar{i}(t) \end{bmatrix}^{\mathrm{T}}$, where $\bar{i}(t)$, $\bar{y}(t)$, $\bar{u}(t)$ are armature current, rotary speed and access voltage severally. Choose 0.002s as the sampling time. According to a least-squares approach, we found a series of the following parametric values of the rotary speed control instance in (1), finally considered in the case of $T \neq 0$. We obtained

$$O = \begin{bmatrix} -331.2 & -684.8 \\ 355.2 & -696,1 \end{bmatrix},\ P = \begin{bmatrix} 19.52 \\ -12.44 \end{bmatrix},$$
$$P_\omega = P,\ S = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$T = 0.1, T_\omega = 0.4, \gamma = 0.9,$$
$$\zeta = 1, \alpha = 5.34, \kappa = 0.2.$$

We found a feasible solution for LMI (14), the result of controller gain was

$$K_P = \begin{bmatrix} -0.1321 & -0.1865 \end{bmatrix},$$

and observer gain was

$$L = \begin{bmatrix} -52.27 & -114.14 \end{bmatrix}^{\mathrm{T}}.$$

To reflect the superiority of our method, we compared the output response $y(t)$ of our method and H_∞ memory state-feedback method in the rotary-speed control system in Fig. 6.

From Fig. 6, the PPV of our proposed method amounts to 53.4% of the PPV associated with H_∞ memory state-feedback control. Therefore, our method has significant disturbance suppression performance. This also proves the efficacy of our method.

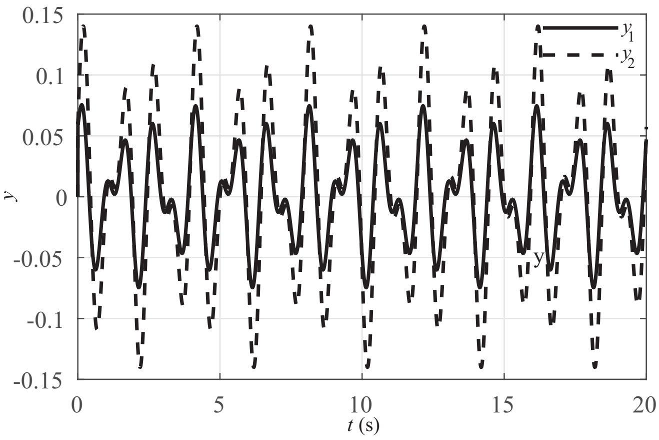

Fig. 6: Output response of our method (y_1) and H_∞ memory state-feedback method (y_2) in the rotary-speed control system.

V. Conclusion

This paper aims to demonstrate a method for effectively suppressing disturbances in linear input time-delay systems through H_∞ control, and the method involves incorporating memory state-feedback and utilizing the direct transfer matrix within the system. A robust control method appears by building on the EID technique. It can drop the impact of disturbance effectively on the system output and reject matched and unmatched disturbances to achieve satisfactory control performance. Using Lyapunov stability theories, the stability conditions of the closed-loop system are given as LMIs. Utilizing the EID estimation technique and incorporating H_∞ control principles, design a memory state-feedback controller with input time-delay. Numerical instances, a practical instance and the simulation outcomes illustrate the effectiveness and satisfactory control capabilities of our proposed method.

This approach holds significant theoretical importance for enhancing the disturbance rejection capabilities of uncertain input time-delay systems and nonlinear input time-delay systems, and will be implemented in the future.

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