IAENG International Journal of Applied Mathematics

Optimization of Train Operation Plan of Through Operation Between Urban Rail Transit and Suburban Railway

Zhiqiang Tian, Zibo Kong, Huiyuan Han, Wanpeng Wu, Juan Zeng

Abstract—**Based on domestic and international practical experience and relevant research, this study analyzes and summarizes the characteristics of through operation between urban rail transit and suburban railway. The objective is to maximize the total travel time saved by passengers and to increase the revenue of the urban rail transit operating company. A multi-objective mixed-integer nonlinear programming model is developed, then transformed into a mixed-integer linear programming model using the linearization method. It is finally solved using the Gurobi solver. The model's validity is confirmed by designing a case with two rail transit lines. And conducting a sensitivity analysis of the relevant parameters. The study results indicate that the through operation can increase the revenues of urban rail transit and suburban railway by 3,995 yuan and 5,741 yuan, respectively. Additionally, the total time savings for passengers amount to 5,597 minutes.**

Index Terms—**Urban Rail Transit, Suburban Railway, Through Operation, Train Operation Plan.**

I. INTRODUCTION

As the scope of the metropolitan area expands and the distance traveled by commuters increases, the agglomeration effect of the metropolitan area poses a significant challenge to transportation mobility. Many passengers commuting between the suburbs and downtown have to reach their destinations by transfer. Excessive passenger flow at transfer stations significantly affects travel time and operational safety. Given the increasingly diverse passenger travel demands, it is essential to establish a multi-level rail transit operation system. Through operation are effective in reducing the impacts of transfer as a way to address the time demands and direct access needs of passenger trips.

Through operation refers to the integration of facilities, equipment, wiring setup, and dispatching command system infrastructures, enabling urban rail transit trains to cross different lines, share sections with trains from other operators, and achieve unified operational organization [1]. Makoto et al. [2] studied and analyzed the cross-line operation routes of the Tokyo Metro to determine the economic benefits to each operator after the opening of the cross-line operation, and assessed the applicability of the cross-line operation based on these benefits. Yang [3] conducted a study on the operation plan of through trains, focusing on various factors such as line passing capacity, various types of travel demand from passengers, and operation management, and established an optimization model for the through-train operation plan. Li [4] analyzed and studied the through operation mode, established a model of a through operation plan based on passenger flow distribution, and solved it with a genetic algorithm. Li [5] researches on the optimization preparation method of train planning under three typical over-track transportation modes, and uses a genetic algorithm to solve the model. Yang [6] studied the optimization of urban rail transit train planning under through operation conditions, and proposed an optimization model for urban rail transit train planning under through conditions based on the environment of domestic through operation trains and considering the influencing factors of train planning optimization. Duan [7] researches the programming method of urban rail transit train planning under the cross-line operation mode during peak hours, proposes a method for generating the cross-line alternative set based on the cross-line conditions, the conditions of the return station, and passenger flow demand, and constructs an optimization model for the train planning.

Train routing is one of the main contents in the study of train operation plan, and reasonable train routing can improve the operation productivity of the line. Wang et al. [8] studied the urban rail transit network and established a two-layer planning model for train routing plan to derive the train routing plan and train departure frequency. Wang et al. [9] studied the urban rail transit full-length and short-turn routing patterns, constructed a mixed integer nonlinear planning model, and derived the optimal train operation plan under full-length and short-turn routing patterns. Jin [10] conducted a study on the interoperability conditions of urban rail transit

Manuscript received May 21, 2024; revised October 22, 2024.

This research was supported by the National Natural Science Foundation (No.71761023, No.2161023, No.2361020), the Open Project of Key Laboratory of Intelligent Management and Control of Railway Industry in Plateau Railway Transportation (No. GYYSHZ2302), the Gansu Provincial Science and Technology Department Plan Project (No.22JR5RA379, No.22JR11RA159), the Gansu Provincial Department of Education Higher Education Research Project (No.2022QB-060) and the Youth Science Fund of the School of Traffic and Transportation, Lanzhou Jiaotong University(No.YQN202204).

Zhiqiang Tian is a professor at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: tianzq@mail.lzjtu.cn).

Zibo Kong is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (Corresponding author, phone: +86 13180279516, e-mail: kongzb2024@163.com).

Huiyuan Han is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: 2469864985@qq.com).

Wanpeng Wu is a director at the Beijing Group Company Dispatching Office, Beijing 100000, China. (e-mail: 770893602@qq.com).

Juan Zeng is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: zengjuan8880410@126.com).

train routing plan preparation, established an interoperability routing plan preparation model, and solved it with a genetic simulated annealing algorithm. Tan et al. [11] conducted a study on the size of the return mode, which is more complicated, and established a train operation organization optimization model considering the occupation of the train return approach under the size of the return mode, which can achieve the optimization of the train operation organization while clarifying the occupation time of the return train on the internal track resources of the return station. Qian [12] studied the selection method of the operation mode of train interchanges combined with fast and slow trains, and established the optimization model of the traffic scheme under the combination mode of Y-type interchanges and fast and slow trains, and the optimization model of the traffic scheme under the combination operation mode of nested interchanges and fast and slow trains. Zhang [13] conducts research on the optimization of urban rail transit train operating map based on the large and small crossing scheme, and proposes an automatic train operation jumping algorithm given the large and small crossing scheme, which provides a basis for the automatic preparation and adjustment of the operating map of the urban rail transit line.

Zhu et al. [14] proposed train plan with a flexible composition of trains and established a multi-objective planning model. They employed an improved quantum genetic algorithm (IQGA) designed to solve the mathematical model. Wan et al. [15] used passenger travel cost, enterprise operation cost, and carbon emission minimization as objective functions in the model. They constructed a multi-objective optimization model for the operation scheme of reconnection marshalling trains. Wang [16] based on analyzing the mutual influence mechanism of fast and slow trains and cross-line operation and its operation organization technology, proposed a calculation model of the opening scheme combining the two. Huang et al. [17] based on the virtual grouping technology on urban rail transit train delay jump-stop adjustment research, to minimize the total passenger travel time as the goal of constructing the delay conditions of the train jump-stop adjustment model, and the use of genetic algorithms for solving the solution, the solution results show that compared to traditional jump-stop adjustment programs, utilizing virtual grouping technology can significantly reduce total passenger travel time.

The development and optimization of a train operation plan are more complex under through operation because multiple operators and a diverse array of passenger types are involved. Compared with the train operation plan under a single operating entity, through operation requires a detailed consideration of the balance of interests among different entities to ensure that the benefits and interests of all parties are not jeopardized. Therefore, when studying through operations, it is necessary to consider the profitability of all operating entities comprehensively and to develop and optimize scenarios without compromising the interests of any party.

Based on domestic and international practical experience and relevant research, this paper analyzes and summarizes the characteristics of urban rail transit over-track suburban railway operation. It takes the maximization of the total travel time saved by passengers and the maximization of the increased revenue of the urban rail transit operating enterprises as the objective function. The paper establishes a dual-objective mixed integer planning model of urban rail transit over-track suburban railway operation and determines the terminal station of the over-track interval as well as the frequency of the over-track train departure. Finally, the accuracy of the model is verified by combining the case, and the research results are fully analyzed.

II. PROBLEM DESCRIPTION

Assuming that the urban rail transit and suburban railway lines are shown in Fig.1. There are m stations in the urban rail transit, denoted as stations $S_i(i=1,2,...,m)$ respectively; There are $n+1$ stations in the suburban railway, denoted as stations $S_i(i=m+1,...,m+n)$ respectively. Before through operation, through passengers transfer at station m to reach their destinations. After through operation, urban rail transit through trains do not turn back at station m. Instead, they're running across the line to the suburban railway. The set of stations through which through trains pass at this point is $S_{though}=\{1,2,...,N\}, m<N\le m+n$.

The problem investigated in this paper is to determine the range of crossing intervals for through trains and the frequency of through train departures, taking into account the maximization of the total travel time saved by the passengers and the maximization of the increased revenue of the urban rail transit operating company.

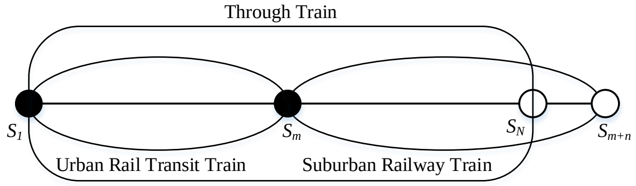

Fig. 1. Schematic diagram of through operation

III. MODEL CONSTRUCTION

A. Model assumption

In order to facilitate the research, this paper assumes the following conditions:

(1) It is assumed that infrastructure conditions support the operation of through trains from urban rail transit to the suburban railway.

(2) Urban rail transit trains, suburban railway trains, and through trains operate in a station-stop mode.

(3) In reality, there are differences in the travel choice behaviors of through passengers. This paper assumes that some through passengers opt for direct through trains, while others choose to transfer to reach their destinations.

(4) Because the impact of through operation on through passenger flow is the largest, and the impact on the passenger flow of other sections is smaller, for this reason, the research object of this paper is limited to through passenger flow, i.e. $O\in\{1,2,...,m\}, D\in\{m,m+1,...,N\}$.

(5) The passenger flow OD between stations is known.

B. Model *parameters* and variables

The parameters and variables involved in the model are presented in Tables I and II.

TABLE I
VARIABLES INVOLVED IN THE MODLE

Symbol	Meaning
C	The increased revenue of the urban rail transit company
c_u	The increased ticket revenue of the urban rail transit company on the urban rail transit line
c_s	The increased ticket revenue of the urban rail transit company on the suburban railway line
c_f	Government subsidy costs obtained by the urban rail transit company
c_x	The line use fee paid by the urban rail transit company
c_g	Increased vehicle fixed costs incurred by the urban rail transit company
c_y	Increased vehicle operating costs borne by the urban rail transit company
S	The increased revenue of the suburban company
s_x	Line use fee obtained by the suburban railway company
s_g	Reduced vehicle fixed costs borne by the suburban railway company
s_y	Reduced vehicle operating costs incurred by the suburban railway company
s_u	The increased ticket revenue of the suburban railway company
s_s	The reduced ticket revenue of the suburban railway company
x_k	0-1 decision variable, if it's the over-track terminal station, take 1, otherwise take 0
q_{ij}	Passenger flow between stations (i) and (j)
Z	Total time saved by passengers
f_g	Frequency of through train departures during the study period
$f_2^{'}$	After the through operation, the frequency of suburban railway trains departure during the study period
ω	Weight coefficient
ξ	0-1 variable

TABLE II
PARAMETERS INVOLVED IN THE MODLE

Symbol	Meaning
f_1	Before the through operation, the departure frequency of urban rail transit trains
f_2	Before the through operation, the departure frequency of suburban railway trains
α	Percentage increase in through passenger flow
β	Percentage of through riders choosing to ride the through train
t_1	Before the through operation, transfer time from urban rail transit transfer to suburban railway at transit station
t_2	Before the through operation, transfer time from suburban railway transfer to urban rail transit at transit station
t_f	Turn-back time for suburban railway trains
δ	The fare rate, and assuming that the subway fare rate is the same as the suburban railway
∂	Rate of government subsidy
l_1	Average station spacing on urban rail transit lines
l_2	Average station spacing on suburban railway lines
ϕ	Unit mileage line use fee for train units
μ_1	Fixed cost per unit of urban rail transit vehicle
μ_2	Fixed cost per unit of suburban railway vehicle
ψ_1	Operating cost per train kilometer for urban rail transit
ψ_2	Operating cost per train kilometer for suburban railway
ϖ_1	Number of urban rail transit train formations
ϖ_2	Number of suburban railway train formations
υ	Time cost per passenger unit
D	Minimum benefit guarantee of urban rail transit company
A	Minimum benefit guarantee of suburban company
T	Time period under study
$f_{\max}$	Passing capacity of suburban railway line
$f_{\min}$	Suburban railway trains meet the minimum departure frequency requirement

C. Mathematical model

The determination of the train operation plan needs to take into account the interests of passengers and each operator. The following is a separate analysis of the cost savings for passengers, and the components of benefits for urban rail transit company and suburban railway company after through operation.

C.1 Total time saved by passengers

The total travel time of passengers mainly includes waiting time, transfer time and on-board time. Assuming that trains arrive at regular intervals and passengers arrive uniformly, the waiting time for passengers is equal to half the departure interval. Assuming that the running speeds of different trains are the same, the change of passenger travel time is only determined by the transfer time and waiting time.

Based on the assumptions of this paper and the above conditions, the total travel time components that can be saved by passengers are as follows:

(1) The starting point is at the urban rail transit station

Before the through operation, passengers would have to take an urban rail transit train to the terminal and then transfer to a city train, a trip that would require a transfer.

$$T_{12}^{+}=\sum_{k=m}^{m+n}x_k(1+\alpha)\sum_{i=1}^{m-1}\sum_{j=m+1}^{k}q_{ij}(\beta(\frac{30}{f_1}+t_1+\frac{30}{f_2}-\frac{30}{f_g})+(1-\beta)(\frac{30}{f_2}-\frac{30}{f_2^{'}+f_g})) \quad (1)$$

（2）The starting point is at the suburban railway station

$$T_{21}^{-}=\sum_{k=m}^{m+n}x_k(1+\alpha)\sum_{i=m+1}^{k}\sum_{j=1}^{m-1}q_{ij}(\beta(\frac{30}{f_2}+t_2+\frac{30}{f_1}-\frac{30}{f_g})+(1-\beta)(\frac{30}{f_2}-\frac{30}{f_2^{'}+f_g})) \quad (2)$$

C.2 Revenues of the urban rail transit company

Changes in the revenues of the urban rail transit company before and after the start of through operation include the following: the increased ticket revenue of the urban rail transit company on the urban rail transit line, the increased ticket revenue of the urban rail transit company on the suburban railway line, government subsidy costs obtained by the urban rail transit company, the line use fee paid by the urban rail transit company, increased vehicle fixed costs by the urban rail transit companies and increased vehicle operating costs by the urban rail transit company.

$$C=c_u+c_s-c_x-c_g-c_y \tag{3}$$

$$c_u=\sum_{k=m}^{m+n}x_k\alpha\delta l_1(\sum_{i=1}^{m-1}\sum_{j=m+1}^{k}q_{ij}(m-i)+\sum_{i=m+1}^{k}\sum_{j=1}^{m-1}q_{ij}(m-j)) \tag{4}$$

$$c_s=\sum_{k=m}^{m+n}x_k(1+\alpha)\beta\delta l_2(\sum_{i=1}^{m-1}\sum_{j=m+1}^{k}q_{ij}(j-m)+\sum_{i=m+1}^{k}\sum_{j=1}^{m-1}q_{ij}(i-m)) \tag{5}$$

$$c_f=\sum_{k=m}^{m+n}x_k(1+\alpha)\beta\partial l_2(\sum_{i=1}^{m-1}\sum_{j=m+1}^{k}q_{ij}(j-m)+\sum_{i=m+1}^{k}\sum_{j=1}^{m-1}q_{ij}(i-m)) \tag{6}$$

$$c_x=2T\phi\sum_{k=m}^{m+n}x_k l_2(k-m)f_g \tag{7}$$

$$c_g=2\mu_1\varpi_1 f_g\sum_{k=m}^{m+n}x_k\frac{(k-m)l_2}{v} \tag{8}$$

$$c_y=2T\psi_1 f_g l_2\sum_{k=m}^{m+n}x_k l_2(k-m) \tag{9}$$

C.3 Revenues of suburban railway company

The revenues of the suburban railway company experienced several changes following the initiation of through operation. These changes include the line usage fees received from the urban rail transit company, augmented ticket revenue resulting from increased through passenger flow, diminished ticket revenue for the suburban railway after the commencement of through operation, as well as escalated vehicle fixed and operating costs borne by the suburban railway company.

$$S=s_u+s_x+s_g+s_y-s_s \tag{10}$$

$$s_u=\sum_{k=m}^{m+n}x_k\alpha\delta l_2(\sum_{i=1}^{m-1}\sum_{j=m+1}^{k}q_{ij}(j-m)+\sum_{i=m+1}^{k}\sum_{j=1}^{m-1}q_{ij}(i-m)) \tag{11}$$

$$s_s=\sum_{k=m}^{m+n}x_k\beta\delta l_2(\sum_{i=1}^{m-1}\sum_{j=m+1}^{k}q_{ij}(j-m)+\sum_{i=m+1}^{k}\sum_{j=1}^{m-1}q_{ij}(i-m)) \tag{12}$$

$$s_x=c_x \tag{13}$$

$$s_g=2\mu_2\varpi_2(f_2-f_2^{'})(\frac{nl_2}{v}+t_f) \tag{14}$$

$$s_y=2\psi_2 nl_2(f_2-f_2^{'}) \tag{15}$$

C.4 Objective function

The model is characterized as a bi-objective mixed integer nonlinear programming model, which is difficult to solve. Therefore, the introduction of weighting coefficients is considered to transform the original bi-objective problem into a single objective programming problem. The transformed objective function is shown in (16).

$$\max Q=(1-\omega)Z\upsilon+\omega C \tag{16}$$

C.5 Constraint condition

$$c_u+c_s-c_x-c_g-c_y\geq D \tag{17}$$

$$s_u+s_x+s_g+s_y-s_s\geq A \tag{18}$$

$$f_g+f_2^{'}\leq f_{\max} \tag{19}$$

$$f_2^{'}\geq f_{\min} \tag{20}$$

$$\sum_{k=m+1}^{m+n}x_k=1,\forall k\in\{m+1,\ldots,m+n\} \tag{21}$$

$$f_g\in Z^+,\ f_2^{'}\in Z^+ \tag{22}$$

Eq. (17) serves as a constraint aimed at ensuring the operational efficiency of the urban rail transit company; Eq. (18) plays a role in guaranteeing the efficiency of the suburban railway company; Eq. (19) is dedicated to enforcing the interregional passing capacity constraint of the suburban railway system; Furthermore, Eq. (20) imposes a minimum departure frequency constraint on the suburban railway to align with the desired service level for the line; Eq. (21) stipulates the presence of a single terminal station for the through train; Eq. (22) functions to enforce that the train departure frequency remains a positive integer.

D. Linearization of the model

The model proposed in this paper constitutes a mixed-integer nonlinear programming model, presenting challenges in solving nonlinear problems. To address this, the paper employs a linearization method to convert the model into a mixed-integer linear programming model, facilitating precise optimal solutions.

D.1 Variable pool processing

In this paper, the decision variable is the train operation frequency, while the departure interval has an inverse relationship with the frequency of departures, which makes it difficult to directly express it with a simple linear equation when we carry out model construction. Hence, a variable pool selection method is introduced to represent both departure frequency and interval.

The departure frequency is put into the variable pool in the form of alternative values, and a 0-1 variable ξ is set for each alternative value to select. If $\xi=1$ then the value is selected, otherwise $\xi=0$. At the same time, ensure that only one departure frequency is selected for each type of train, and the corresponding departure interval is calculated according to the selected departure frequency.

$$f_g=\sum_{i=0}^{15}i\cdot\xi_i^g \tag{23}$$

$$f_2^{'}=\sum_{i=0}^{15}i\cdot\xi_i^{2'} \tag{24}$$

$$\sum_{i=0}^{15}\xi_i^g=1 \tag{25}$$

$$\sum_{i=0}^{15}\xi_i^{2'}=1 \tag{26}$$

In this procedure, the departure frequency and departure interval are considered parameters, and the decision variables can be converted into several 0-1 variables.

D.2 0-1 variable multiplication

For the case where two 0-1 variables are multiplied ($\xi_1\xi_2$), the auxiliary 0-1 variable can be used instead ($\xi_3 = \xi_1\xi_2$). The corresponding linear constraints are as follows:

$$-\xi_1 + \xi_3 \le 0 \tag{27}$$

$$-\xi_2 + \xi_3 \le 0 \tag{28}$$

$$\xi_1 + \xi_2 - \xi_3 \le 1 \tag{29}$$

In this paper, decision variables of the through terminal station and the selection variables of the departure frequency are 0-1 variables. These decision variables are multiplied in the model. Consequently, the linearization process is accomplished using this approach.

IV. CASE STUDIES

A. Case description

In this case, there are 16 urban rail transit stations and 11 suburban railway stations. The urban rail transit station set is $S_U = \{1, 2, ..., 16\}$ and the suburban railway station set is $S_R = \{17, 18, ..., 27\}$. The urban rail transit realizes through operation through station 16, the Case schematic diagram is shown in Fig.2:

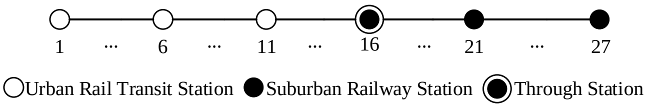

Fig. 2. Through operation lines of a case study

The case study period is during the evening peak hour, and the data in Table III represent the passenger flow during the evening peak hour of a working day.

TABLE III
PASSENGER OD MATRIX

OD	17	18	19	20	21	22	23	24	25	26	27
1	9	48	39	53	7	41	32	14	22	7	56
2	1	14	15	22	7	26	22	12	20	5	25
3	10	74	62	67	27	66	62	23	48	18	66
4	8	30	28	45	12	34	32	18	16	8	28
5	4	31	37	41	10	26	31	9	26	11	34
6	10	74	68	87	27	67	78	33	57	29	54
7	14	55	59	68	24	42	53	42	43	22	75
8	4	22	31	36	6	8	11	11	11	7	36
9	3	40	68	54	39	51	48	12	74	29	88
10	7	63	80	80	25	64	69	44	80	32	99
11	6	86	90	64	26	50	94	47	62	29	76
12	7	69	95	83	34	74	97	60	80	44	126
13	24	145	182	176	76	112	174	83	194	91	200
14	24	231	260	264	81	127	208	139	256	98	344
15	24	225	250	295	90	119	263	187	388	176	491

B. Parameter assignment

TABLE IV
VALUE OF THE PARAMETERS IN THE CASE STUDY

Parameter	Value	Unit
f_1	30	-
f_2	15	-
α	10%	-
β	70%	-
t_1	8	min
t_2	8	min
t_f	2	min
δ	0.3	yuan/person-km
∂	0.1	yuan/person-km
l_1	2	Km
l_2	3	Km
ϕ	11.232	yuan/column-km
μ_1	360	yuan/vehicle
μ_2	360	yuan/vehicle
ψ_1	24	yuan/column-km
ψ_2	48	yuan/column-km
ϖ_1	6	Vehicle
ϖ_2	6	Vehicle
υ	0.1	yuan/min
D	0.3	million yuan
A	0.5	million yuan
T	1	h
$f_{\max}$	24	Vehicle/h
$f_{\min}$	12	Vehicle/h
v	45	Km/h
υ	0.1	yuan/min

C. Solving result

The optimal train operation plan is calculated by substituting the parameters from Table Ⅳ into the model, utilizing the linearization method proposed in this paper and implementing Gurobi programming. Under this plan, the through train terminates at station 21 and operates with a frequency of 10 trips per hour, resulting in a total time savings of 5597 minutes for passengers. Additionally, the revenue of the urban rail transit company increased by 3995 yuan, indicating that the augmented ticket revenue generated by the increased passenger volume surpasses the elevated operational costs incurred by the introduction and operation of the through trains.

D. Sensitivity analysis

D.1 The solution results under different weights

In this paper, a dual-objective planning model is employed and subsequently converted into a single-objective planning model through the introduction of weighting coefficients. The different weight coefficients will significantly affect the solution results of the model, so the sensitivity analysis of the weight coefficients is needed. The outcomes corresponding to different weighting coefficients are presented in Table V and Figure 3.

TABLE V
OPTIMAL RESULTS WITH DIFFERENT WEIGHTS

Weight coefficient	1	0.8	0.6	0.5	0.4	0.2	0
Q (million yuan)	0.3955	0.4276	0.4597	0.4758	0.4918	0.5239	0.5560
C (million yuan)	0.3955	0.3955	0.3955	0.3955	0.3955	0.132	-0.365
$Z \cdot \upsilon$ (million yuan)	0.5560	0.5560	0.5560	0.5560	0.5560	0.736	0.866
f_g	10	10	10	10	10	7	7
k	21	21	21	21	21	19	19

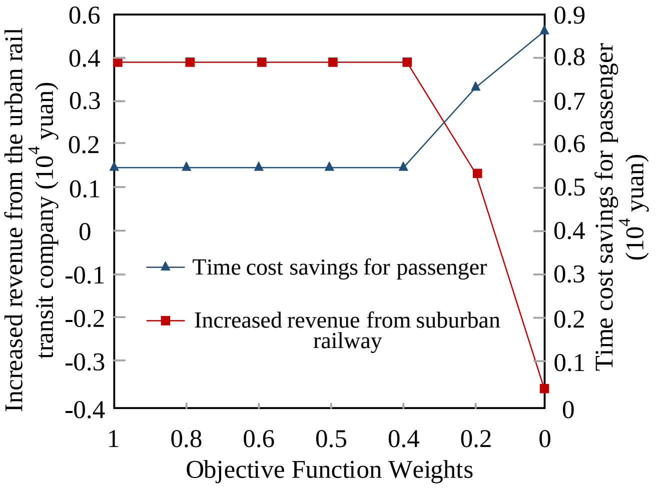

Fig. 3. Impact of different weights on results

As urban rail transit company gradually increases its focus on revenue, the amount of time saved by riders will diminish; conversely, if urban rail transit company focuses more on the time saved by riders, then revenue will decrease. When $0.4 \le \upsilon \le 1$, the increased benefit and time cost savings attain an equilibrium state, where in the increased benefit peaks at 3,955 yuan and the time cost savings amount to 5,560 yuan. When $\upsilon = 0$, the problem is a single-objective planning that only considers maximizing passenger saving time, and the passenger saving time cost is 8660 yuan. Consequently, during this period, the through operation fails to yield economic benefits for the urban rail transit company, resulting in a loss of 3,650 yuan.

D.2 The influence of different through passenger flow on the results

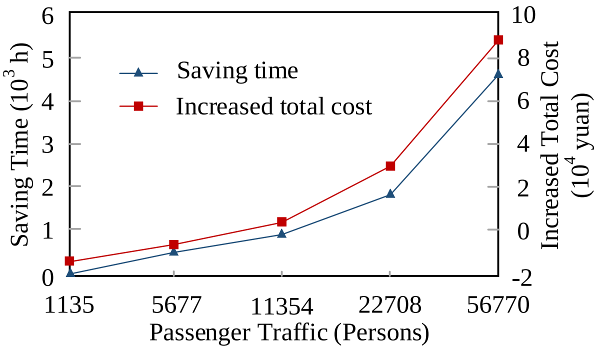

Fig. 4. Optimal results with different through passenger flow

The through passenger flow in Figure 4 takes 0.1 times, 0.5 times, 1 time, 2 times and 5 times of the through passenger flow in the case, respectively. Analysis of the diagram reveals that as the number of through passengers increases, both the total travel time saved and the revenue of the urban rail transit company increase accordingly. The more passenger traffic there is, the more total time passengers save, and the more fare revenue the urban railroad company generates. As a result, different passenger flow conditions affect the final through operating plan; the more through passengers, the more time costs are saved, and the more total costs are added to the urban rail transit operating company. If the total peak-hour through passenger flow exceeds 9,800 passengers, utilizing the through operation organization is deemed appropriate.

D.3 Impact of different interchange walk times on results

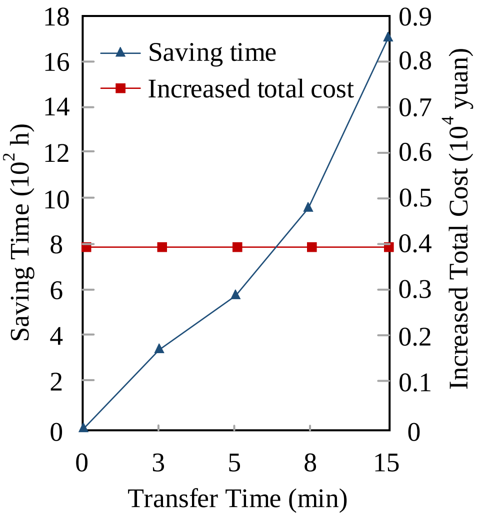

Fig. 5. Optimal results with different transfer time

The through station transfer walk times in Figure 5 are 0, 3, 5, 8, and 15 minutes. From the figure, it is evident that if the transfer time at the through station is short, the total travel time saved for passengers is less. As transfer times at the through station increase, the total travel time saved by passengers increase, but the Metro's total revenue increase remains unchanged. Therefore, it is all the more important to adopt through operation for through stations with long interchange times to save passengers' time costs. By implementing this operational strategy, the Metro can effectively reduce passengers' time costs, enhancing their overall travel experience.

D.4 The impact of the amount of line usage fees on the interests of operators

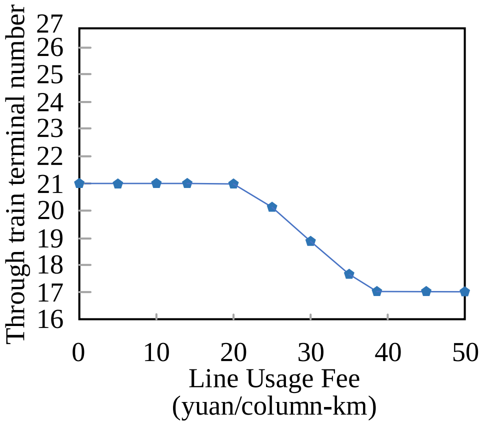

Fig. 6. Optimal results with different line usage fee

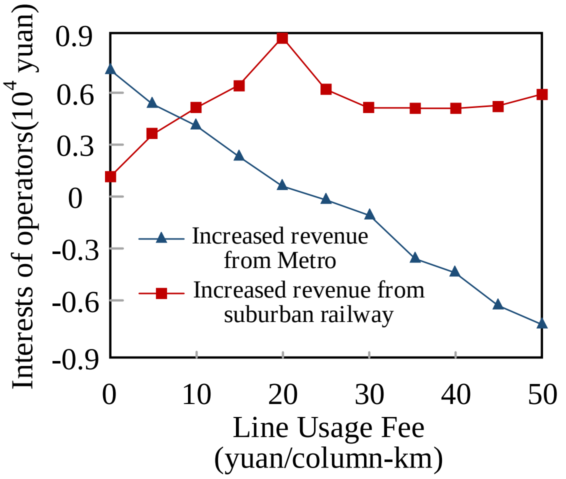

Fig. 7. Optimal results with different line usage fees

It is evident in Figure 6 through interval with the increase of line usage fee is a segmented rule of change, when the line usage fee is less than 20 yuan/column-km, the through interval does not change with the change of line usage fee; when the line usage fee is 20-39 yuan/column-km, the length of the through interval decreases with the increase of line usage fee; when the line usage fee is more than 39 yuan/column-km, the length of the through interval does not change with the change of line usage fee, and the length of the through interval is smaller.

It is evident in Figure 7 that the interest of the suburban railway company increases and then decreases with the rise of the line usage fee, and it reaches its maximum when the line usage fee is around 20 yuan/column-km. This phenomenon is mainly due to the following reasons: when the lease fee is smaller, the length of the through interval remains unchanged, and the other related revenue expenses of the suburban railway company remain unchanged, but the received lease fee increases; however, as the lease fee increases further, the length of the through interval decreases, and consequently, the number of trains operated by the suburban railway company on its own line increases, thus reducing the overall benefits.

The benefit to the urban rail transit company continues to decrease as the line usage charge increases, and the increased benefit to the urban rail transit company becomes negative when the line usage charge reaches 25 yuan/column-km. The primary reason is that the length of the through interval remains the same, while the line usage fee paid is increasing, and when the line usage fee reaches 25 yuan/column-km, the urban rail transit company incurs losses, meaning the increase in fare revenue is less than the increase in cost.

In summary, the magnitude of the line usage fee significantly impacts the urban rail transit company's interests, influencing the through interval length. Once it surpasses 20 yuan/column-km, the operating company's interests decrease, while those of the suburban railway company post through operation increase substantially compared to the urban rail transit company. Therefore, to optimize the overall effectiveness of through operations, the suburban railway company may consider reducing the line usage fee for urban rail transit post through operation.

D.5 The impact of different government subsidy rates on the objective function

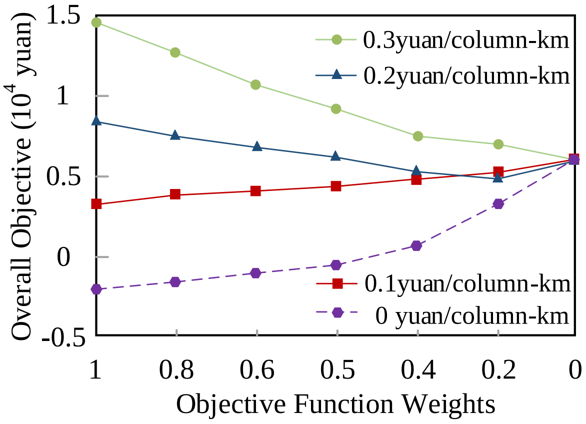

Fig. 8. The impact of different government subsidy rates on the objective function

The cost of government subsidies impacts the final train operation plan, which in turn impacts Metro's and the time savings for passengers. Consequently, this paper examines the effect of varying government subsidy rates on the objective function, as illustrated in Figure 8.

When the weight coefficient of the objective function is larger, the economic benefits to the subway company become more significant. Consequently, different government subsidy rates will substantially affect the results; a higher government subsidy rate leads to greater economic benefits for the subway company. Conversely, when the weight coefficient of the objective function is smaller, the subway company prioritizes passenger interests, especially when $\upsilon = 0$, the subway does not take into account their own economic gains, only maximize the interests of passengers. In such cases, different subsidy rates do not influence the final outcome. If the government does not provide any financial subsidies, the subway company will incur a loss in economic returns following the operation. Conversely, excessively high subsidy rates will significantly boost subway revenue but also impose greater financial pressure on government departments. Therefore, the government should develop a reasonable financial subsidy program based on its financial situation and the demand for passenger flow to ensure the rationality of the through operation plan.

V. CONCLUSION

The through operation mode of urban rail transit and suburban railway effectively reduces the need for passenger transfers, shortens passenger travel times, and enhances passenger service levels. This paper establishes a dual-objective integer planning model for urban rail transit over-track suburban railway operation plan, adopts the linearization method to process it, and finally solves it by using the Gurobi solver, and the case verifies the validity of the model. The primary research conclusions obtained in this paper are as follows:

1) Under the through operation, the total travel time for through passengers is reduced; the pressure of passenger flow at the interchange station is alleviated. Moreover, under a reasonable operational organization scheme, the increase in ticket revenue for the operator will exceed the rise in operating costs associated with the through operation, thereby enhancing the overall revenue for the operator.

2) The operation of through trains can efficiently reduce the inconvenience caused by passenger transfers, diminish the travel time of passengers, improve the service level of urban rail transit for passengers, and heighten its attractiveness.
3) The design of the through operation plan should take into full consideration the interests of the urban rail transit companies and the suburban railway companies to ensure that both sides will conduct through operation without jeopardizing their interests. Additionally, the government may need to provide certain subsidies to the urban rail transit companies to ensure that the urban rail transit companies and the suburban railway companies will increase their revenues by running through trains.
4) The rail transportation planning and design departments should consider through operation in advance during planning and design, and prepare corresponding conditions to support the operation of subsequent through trains. Relevant departments should consider the influence of various factors comprehensively and reasonably establish the relevant parameters before formulating the through operation plan.

The model in this paper solely focuses on through passenger flow. Additionally, the model assumes a fixed value for the proportion of through passenger travel choice behavior. This assumption presents a disparity with the real-world scenario. Further investigation will be conducted on these aspects in subsequent research.

REFERENCES

[1] Mao B H, Liu M J, Huang R, et al. Operational theories and key technologies of rail transit networks[M]. Beijing: Science Press, 2011.

[2] Makoto A. Railway operators in Japan 4: Central Tokyo[J]. Railway Operator, 2002, 30(3): 42-53.

[3] Yang A A. Research on urban rail transit operation plan under multi-operation mode[D]. Beijing University of Technology, 2019.

[4] Li Y W. Analysis on the operation mode of suburban railway and urban rail transit[D]. South Jiaotong University, 2020.

[5] Li M G. Research on optimization method of train running scheme under subway and suburban railroad over-track operation [D]. Beijing Jiaotong University, 2016.

[6] Yang Y F. Research on optimization of train running scheme of urban rail transit under through running conditions [D]. Beijing Jiaotong University, 2018.

[7] Duan X Y. Research on train running scheme of urban rail transit under cross-line operation mode [D]. Beijing Jiaotong University, 2022.

[8] Wang Y L, Zhang X C, et al. Optimization method of network train operation plan for urban rail transit[J]. China Railway Science, 2012, 33(5): 120-126.

[9] Wang Y Y, Ni S Q. Optimization of train schedules of full-length & short-turn operation modes in urban rail transit[J]. Journal of The China Railway Society, 2013, 35(7): 1-8.

[10] Jin T F. Optimization of train routing plan for urban rail transit under the condition of interconnection[D]. Beijing Jiaotong University, 2018.

[11] Tan B B. Research on optimization of train operation chart of urban rail transit considering the occupation of folding approach [D]. Beijing Jiaotong University, 2022.

[12] Qian Z W. Research on train operation organization of urban rail transit based on the combination of interchanges and fast and slow trains [D]. Nanjing University of Science and Technology, 2021.

[13] Zhang M. Research on the optimization of urban rail train operating map based on large and small crossing scheme[D]. Beijing Jiaotong University, 2019.

[14] Zhu C, Yang X, Wang Z, et al. “Optimization for the Train Plan with Flexible Train Composition Considering Carbon Emission,” Engineering Letters, vol. 31, no.2, pp562-573, 2023.

[15] Wan S, Zhu C, Fang J, et al. “Operation Optimization of Reconnection Marshalling Trains Considering Carbon Emission,” IAENG International Journal of Applied Mathematics, vol. 53, no.2, pp601-612, 2023.

[16] Wang L. Research on the optimization of the combined operation plan of express-local train and cross-line operation of urban rail transit[D]. Beijing Jiaotong University, 2019.

[17] Huang J Y, Zhang Q. Research on the adjustment of urban rail transit train delay jump-stop based on virtual grouping technology[J]. Railway Transportation and Economy, 2024, 46(02): 151-158.

Zhiqiang Tian was born in Gansu, China, in 1981. In October 2011, he obtained a doctor's degree in the Planning and Management of Traffic and Transportation in Southwest Jiaotong University. He is now a professor of the School of Traffic and Transportation of Lanzhou Jiaotong University. He has rich scientific research achievements, and has published more than a few papers in domestic academic journals and international conferences.