The Order of a Path Graph as a Subgraph of a Fan Graph with a Certain Locating Chromatic Number

Des Welyyanti, Fakhri Zikra, Rifda Sasmi Zahra, Lyra Yulianti, and Yanita

***Abstract*—Let $G = (V, E)$ be a connected graph, and let c be a k-coloring of G. A coloring c is called a locating-coloring of G if distinct vertices have distinct color codes. The locating-chromatic number, $\chi_L(G)$, is defined as the minimum number of colors needed to achieve a locating-coloring of G. Research on locating-chromatic numbers has explored various types of graphs. In this paper, we determine the locating-chromatic number of the fan graph F_n by analyzing the range of the order of the path graph P_n, a subgraph of F_n, that corresponds to a specific locating-chromatic number. Specifically, we identify the range of n for which the locating-chromatic number of F_n satisfies the given criteria.**

***Index Terms*—color code, connected graph, fan graph, locating-chromatic number**

I. Introduction

GRAPH theory is a well-known branch of mathematics that has grown significantly over time. One area of study in graph theory is the locating-chromatic number. This concept, introduced by Chartrand et al. in 2022 [3], the locating-chromatic number combines the concepts of graph coloring and partition dimension. The locating-chromatic number of a graph, denoted as $\chi_L(G)$, is the smallest number of colors needed to color the graph so that a k-locating coloring is possible. In simple terms, this means assigning colors to the vertices in a way that allows each vertex to be uniquely identified based on its distance from the colored vertices.

Research on locating-chromatic numbers spans a variety of graph types. These graphs are join graphs in 2014 [2], graphs with dominant vertices in 2015 [11], Halin graphs in 2017 [8], graphs with two homogeneous components in 2017 [12], certain operations on generalized Petersen graphs in 2019 [6], barbell shadow path graphs in 2021 [1], modified paths with cycles having a locating number of four in 2021 [4], book graphs in 2021 [5], certain barbell origami graphs in 2021 [7], and Buckminsterfullerene-type graphs in 2021 [9], among others.

In the study of disconnected graphs, Welyyanti et al. made important progress by finding the locating-chromatic number for certain disconnected graphs in 2014 [10]. Later research examined disconnected graphs made up of paths and cycles in 2019 [13], and those with components such as paths, cycles, stars, or double stars in 2021 [14].

In this paper, we identify the range of n for which the locating-chromatic number of F_n satisfies the given criteria.

II. Fan Graph

The fan graph (F_n) is a graph with $n+1$ vertices. In this graph, one vertex has degree n, two vertices have degree 2, and the remaining vertices have degree 3. The fan graph is constructed by join operation between graph K_1 and P_n for $n \geq 2$. The fan graph has order $n+1$ and size $2n-1$. The vertex and edge sets of fan graph F_n are defined as follows.

$$V(F_n) = \{v_i | 0 \leq i \leq n\}$$
$$E(F_n) = \{v_0 v_j | 1 \leq j \leq n\} \cup \{v_k v_{k+1} | 1 \leq k \leq n-1\}$$

The fan graph F_n for $n \geq 2$ is presented in Figure 1.

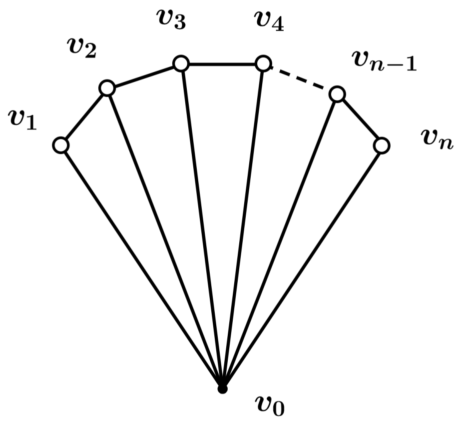

Fig. 1. Fan Graph F_n

III. The Locating-Chromatic Number of a Graph

Let c be a vertex coloring of the graph G such that $c(u) \neq c(v)$ for adjacent vertices u and v in G. Let C_i be the set of vertices colored with color i, known as color classes. Then, let $\Pi = \{C_1, C_2, \ldots, C_k\}$ be the set of color classes from $V(G)$. The color code $c_\Pi(v)$ of a vertex v is a k-ordered tuple $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$, where $d(v, C_i) = \min\{d(v, x) \mid x \in C_i\}$ for $1 \leq i \leq k$. A vertex $v \in V(G)$ is called a dominant vertex if $d(v, C_i) = 1$ for all i where $v \notin C_i$, and $d(v, C_i) = 0$ for the remaining color classes. If

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all vertices in G have distinct color codes, then c is called a locating coloring of G. The minimum number of colors required for such a locating coloring is called the locating-chromatic number of G, denoted by $\chi_L(G)$.

The following theorem discusses the locating-chromatic number of a graph, as sourced from Chartrand et al. [3].

Theorem 3.1 ([3]): Let c be a locating-coloring in a connected graph G. If u and v are distinct vertices of G such that $d(u, w) = d(v, w)$ for all $w \in V(G) - \{u, v\}$, then $c(u) \neq c(v)$. In particular, if u and v are nonadjacent vertices of G such that $N(u) \neq N(v)$, then $c(u) \neq c(v)$.

IV. The Order of a Path Graph as a Subgraph of a Fan Graph with a Certain Locating Chromatic Number

In Theorem 4.1, we determine the locating-chromatic number of the fan graph F_n by identifying the range of the order of the path graph P_n, where P_n is a subgraph of the fan graph F_n, with a specific locating-chromatic number. Specifically, we find the range of n for which the locating-chromatic number of F_n meets the given criteria.

Theorem 4.1: Let F_n be a fan graph for $n \geq 2$ and $\chi_L(F_n) = q$ for $q \geq 3$ then

$$\chi_L(F_n) = \begin{cases} 3 & if\ n = 2, \\ 4 & if\ 3 \leq n \leq 7, \\ 5 & if\ 8 \leq n \leq 23, \\ q & if\ (q-2)\sum_{i=3}^{q-1}(i-2) \leq n \leq \\ & \quad \left((q-1)\sum_{i=3}^{q}(i-2)\right) - 1, for\ q \geq 6. \end{cases}$$

Proof: Let F_n be a fan graph for $n \geq 2$, with vertex set $V(F_n) = \{v_0, v_1, v_2, \ldots, v_n\}$, where v_0 is the central vertex of F_n. We will determine the range of n with a specific locating-chromatic number of F_n. Since v_0 is a dominant vertex, it is essential to assign a distinct color to v_0 compared to the other vertices. This ensures that at least two different colors are required for the remaining vertices, excluding the central vertex.

Define a coloring $c : V(F_n) \to \{1, 2, \ldots, q\}$ for $q \geq 3$, where $c(v_0) = q$, and v_0 is the central vertex of F_n colored with q. Since the central vertex v_0 is a dominant vertex, it must be colored differently from the other vertices of F_n. Consequently, there are $q-1$ colors available to be assigned to the remaining vertices $\{v_1, v_2, \ldots, v_n\}$, ensuring that the coloring satisfies the locating-chromatic number of the graph F_n.

Next, we will determine the color combinations for the vertices excluding the central vertex of F_n, considering their neighbors. Each vertex must have a distinct color code that satisfies a q-locating coloring. For a vertex with color i in the graph F_n, which is connected to vertices with color a and color b, is represented as $[a, i, b]$, where $a, b, i \in \{1, 2, \ldots, q-1\}$ with $q \geq 3$, and $a \leq b$ with a and b are not equal to i.

For $q = 3$, we have $c(v_0) = 3$. The color combinations for the vertices other than the central vertex in F_n are presented in Table I.

TABLE I
THE COLOR COMBINATIONS OF ADJACENT VERTICES IN F_n WHEN $\chi_L(F_n) = q$, WITH $q = 3$

$c(v_i)$	1	2
$c(v_{i\pm1})$	$2,2$	$1,1$

Based on Table I, each vertex, except for v_0, can be assigned either color 1 or color 2. Consider the vertices v_1 and v_2. See column two where $c(v_i) = 1$. If $c(v_1) = 1$, then v_1 is adjacent to two vertices with color 2. Conversely, if $c(v_1) = 2$, then v_1 is adjacent to two vertices with color 1.

Suppose $q = 4$, then $c(v_0) = 4$. The color combinations for the vertices other than the central vertex in F_n are presented in Table II.

TABLE II
THE COLOR COMBINATIONS OF ADJACENT VERTICES IN F_n WHEN $\chi_L(F_n) = q$, WITH $q = 4$

$c(v_i)$	1	2	3
	2,2	1,1	1,1
$c(v_{i\pm1})$	2,3	1,3	1,2
	3,3	3,3	2,2

Based on Table II, each vertex, except for v_0, can be assigned either color 1, color 2, or color 3. Consider the vertex v_1. See column two where $c(v_i) = 1$. If $c(v_1) = 1$, then v_1 is adjacent to two vertices either with color 2, with color 2 and color 3, or with color 3. This is similarly applicable if $c(v_1) = 2$ or $c(v_1) = 3$.

Let $q = k$, then $c(v_0) = k$. The color combinations for the vertices other than the central vertex in F_n are presented in Table III.

TABLE III
THE COLOR COMBINATIONS OF ADJACENT VERTICES IN F_n WHEN $\chi_L(F_n) = q$, WITH $q = k$

$c(v_i)$	1	$\ldots$	$k-2$	$k-1$
	2,2	$\ldots$	1,1	1,1
	$\vdots$	$\ddots$	$\vdots$	$\vdots$
$c(v_{i\pm1})$	2,$k-1$	$\ldots$	1,$k-1$	1,$k-2$
	3,3	$\ldots$	2,2	2,2
	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	3,$k-1$	$\ldots$	2,$k-1$	2,$k-2$
	$\vdots$	$\ddots$	$\vdots$	$\vdots$
	$k-1, k-1$	$\ldots$	$k-1, k-1$	$k-2, k-2$

Based on Table III, each vertex, except for v_0, can be assigned either color 1, color 2, …, or color $k-1$. Consider the vertex v_1. See column two where $c(v_i) = 1$. If $c(v_1) = 1$, then v_1 is adjacent to two vertices that can be colored either color 2, color 3, …, or color $k-1$. For example, if one of these adjacent vertices is assigned color 2, then the other vertex can be assigned one of the colors 2, 3, …, or $k-1$, resulting in $k-1$ possible colorings. Conversely, if one of these vertices is assigned color 3, then the other vertex can be assigned one of the colors 3, 4, …, or $k-1$, resulting in $k-2$ possible colorings. Therefore, for $c(v_1) = 1$, there are $\sum_{i=3}^{k}(i-2)$ possible colorings. This is similarly applicable if $c(v_1) = 2$, $c(v_1) = 3$, ..., $c(v_1) = k - 1$.

The number of possible colorings corresponds to the maximum number of vertices with distinct neighbor combinations. Based on this information, the maximum number of vertices with distinct colorings that can be formed in the graph F_n, while ensuring these vertices have distinct neighbors, is $(k-1)\sum_{i=3}^{k}(i-2)$.

Next, we will determine the values of n that satisfy $\chi_L(F_n) = q$, which will be divided into several cases as follows:

Case 1. For $q = 3$

Define a coloring $c : V(F_n) \to \{1, 2, 3\}$, such that $c(v_0) = 3$. Next, we will determine the values of n that satisfy the 3-locating coloring of the graph F_n. Since $c(v_0) = 3$, then precisely two additional colors will be combined for the vertices other than the central vertex in the graph F_n. However, this requirement of combining two colors for the vertices other than the central vertex is only fulfilled when $n = 2$. The color combinations for the coloring of the graph F_n are as follows:

TABLE IV
THE COLOR COMBINATIONS OF 3-LOCATING COLORING FOR F_n

n	3-locating coloring for F_n $[c(v_0); c(v_1), c(v_2)]$
2	[3;1,2]

Based on Table IV, consider $n = 2$ with $[c(v_0); c(v_1), c(v_2)] = [3; 1, 2]$, This indicates that in the fan graph F_n where $n = 2$, v_0 is assigned by color 3, while v_1 and v_2 are assigned by color 1 and color 2, respectively. The 3-locating coloring of fan graph F_2 is presented in Figure 2.

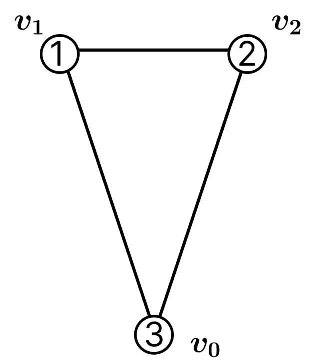

Fig. 2. The coloring of fan graph when $\chi_L(F_n) = 3$

Each vertex has two neighboring vertices, and the colors of these neighbors determine the color code for the vertex. Different color combinations in the neighboring vertices result in distinct color codes. As shown in Figure 2 and Table IV, each vertex has a unique combination of colors in its neighbors, which implies that each vertex has a different color code. Therefore, it is proven that $\chi_L(F_3) = 3$ for $n = 2$.

Case 2. For $q = 4$

Define a coloring $c : V(F_n) \to \{1, 2, 3, 4\}$, such that $c(v_0) = 4$. Next, we determine the values of n that satisfy the 4-locating coloring of the graph F_n. Since $c(v_0) = 4$, then precisely three additional colors will be combined for the vertices other than the central vertex in the graph F_n. However, this requirement of combining three colors for the vertices other than the central vertex is only fulfilled when $3 \le n \le 8$. The color combinations for the coloring of the graph F_n are as follows:

TABLE V
THE COLOR COMBINATIONS OF 4-LOCATING COLORING FOR F_n

n	4-locating coloring for F_n $[c(v_0); c(v_1), c(v_2), \ldots, c(v_n)]$
3	[4;1,2,3]
4	[4;1,2,3,1]
5	[4;1,2,3,1,2]
6	[4;1,2,3,2,3,1]
7	[4;1,2,3,1,3,1,2]
8	[4;1,2,1,3,1,3,2,3]

Based on Table V, consider $n = 8$ with $[c(v_0); c(v_1), c(v_2), \ldots, c(v_n)] = [4; 1, 2, 1, 3, 1, 3, 2, 3]$, This indicates that in the fan graph F_n where $n = 8$, v_0 is assigned by color 4, while v_1 is assigned by color 1, v_2 is assigned by color 2, and so the others. This step also applies to the other vertices. The 4-locating coloring of fan graph F_8 is presented in Figure 3.

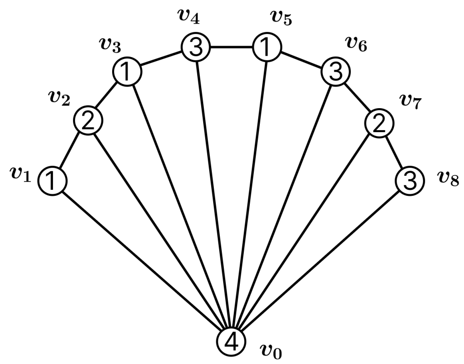

Fig. 3. The coloring of fan graph when $\chi_L(F_8) = 4$

Each vertex has two neighboring vertices, and the colors of these neighbors determine the color code for the vertex. Different color combinations in the neighboring vertices result in distinct color codes. As shown in Figure 3 and Table V, each vertex has a unique combination of colors in its neighbors, which implies that each vertex has a different color code. Therefore, it is proven that $\chi_L(F_n) = 4$ for $3 \le n \le 8$.

Case 3. For $q = 5$

Define a coloring $c : V(F_n) \to \{1, 2, 3, 4, 5\}$, such that $c(v_0) = 5$. Next, we will determine the values of n that satisfy the 5-locating coloring of the graph F_n. Since $c(v_0) = 5$, then precisely four additional colors will be combined for the vertices other than the central vertex in the graph F_n. However, this requirement of combining four colors for the vertices other than the central vertex is only fulfilled when $9 \le n \le 23$. The color combinations for the coloring of the graph F_n are as follows:

TABLE VI
THE COLOR COMBINATIONS OF 5-LOCATING COLORING FOR F_n

n	5-locating coloring for F_n $[c(v_0); c(v_1), c(v_2), \ldots, c(v_n)]$
9	[5;1,2,4,1,3,4,2,3,4]
10	[5;1,2,4,1,3,4,3,4,1,4
11	[5;1,2,4,1,3,4,2,3,4,1,4]
12	[5;1,2,3,1,2,4,1,3,4,2,3,4]
13	[5;1,2,4,1,3,4,3,4,2,3,4,1,4]
14	[5;1,2,3,1,2,4,1,3,4,2,3,4,1,4]
15	[5;1,2,4,2,4,1,3,4,3,4,2,3,4,1,4]
16	[5;1,2,3,1,2,4,1,3,4,3,4,2,3,4,1,4]
17	[5;1,2,4,2,4,1,3,4,3,4,2,3,4,3,4,1,4]
18	[5;1,2,3,1,2,4,2,4,1,3,4,3,4,2,3,4,1,4]
19	[5;1,2,4,2,4,1,3,1,3,4,3,4,2,3,2,3,4,1,4]
20	[5;1,2,3,1,2,4,1,3,1,3,4,3,4,2,3,2,3,4,1,4]
21	[5;2,1,2,3,1,2,4,2,4,1,3,4,3,4,2,3,2,3,4,1,4]
22	[5;1,2,3,1,2,4,2,4,1,3,1,3,4,3,4,2,3,2,3,4,1,4]
23	[5;2,1,2,3,1,2,4,2,4,1,3,1,3,4,3,4,2,3,2,3,4,1,4]

Based on Table VI, consider $n = 12$ with $[c(v_0); c(v_1), c(v_2), \ldots, c(v_n)] = [5; 1, 2, 3, 1, 2, 4, 1, 3, 4, 2, 3, 4]$, This indicates that in the fan graph F_n where $n = 12$, v_0 is assigned by color 5, while v_1 is assigned by color 1, v_2 is assigned by color 2, and so the others. This step also applies to the other vertices. The 5-locating coloring of fan graph F_{12} is presented in Figure 4.

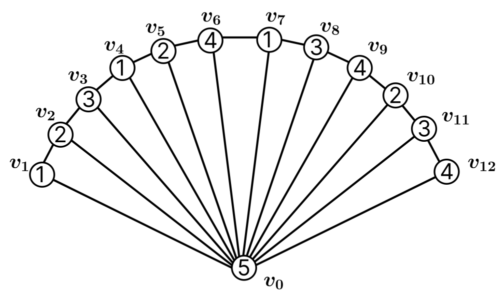

Fig. 4. The coloring of fan graph when $\chi_L(F_{12}) = 5$

Each vertex has two neighboring vertices, and the colors of these neighbors determine the color code for the vertex. Different color combinations in the neighboring vertices result in distinct color codes. As shown in Figure 4 and Table VI, each vertex has a unique combination of colors in its neighbors, which implies that each vertex has a different color code. Therefore, it is proven that $\chi_L(F_n) = 5$ for $9 \leq n \leq 23$.

Case 4. For $q \geq 5$

Define a coloring $c : V(F_n) \to \{1, 2, \ldots, q\}$, such that $c(v_0) = q$. Next, we will determine the values of n that satisfy the q-locating coloring of the graph F_n. Since $c(v_0) = q$, then precisely there are $q - 1$ additional colors will be combined for the vertices other than the central vertex in the graph F_n. However, this requirement is only fulfilled when $n \leq \left((q-1)\sum_{i=3}^{q}(i-2)\right) - 1$.

Suppose $\chi_L(F_n) = q - 1$, then it will be satisfied by $n \leq \left((q-2)\sum_{i=3}^{q-1}(i-2)\right) - 1$. This implies that $\chi_L(F_n) = q$ is only satisfied when $n \geq \left((q-2)\sum_{i=3}^{q-1}(i-2)\right)$. Thus, $\chi_L(F_n) = q$ can be satisfied by $(q-2)\sum_{i=3}^{q-1}(i-2) \leq n \leq \left((q-1)\sum_{i=3}^{q}(i-2)\right) - 1$.

The coloring of vertices other than the central vertex in the graph F_n will be formed into several color combinations, where each combination contains two or three colors written in the form $[a, b]$ or $[a, b, c]$, with $a, b, c \in \{1, 2, .., q - 1\}$ and $a < b < c$. These color combinations will be written in such a way that they are sorted from left to right. The color combinations for the q-locating coloring of vertices other than the central vertex in the graph F_n, where $c(v_0) = q$ and for maximum n, are as follows.

$[2], [1, 2, 3], [1, 2, 4], \ldots, [1, 2, q-2], [1, 2, q-1], [2, q-1]$
$[1, 3], [1, 3, 4], [1, 3, 5], \ldots, [1, 3, q-2], [1, 3, q-1],$
$[3, q-1]$
$\vdots$
$[1, q-3], [1, q-3, q-2], [1, q-3, q-1], [q-3, q-1]$
$[1, q-2], [1, q-2, q-1], [q-2, q-1]$
$[2, 3], [2, 3, 4], [2, 3, 5], \ldots, [2, 3, q-2], [2, 3, q-1]$
$[2, 4], [2, 4, 5], [2, 4, 6], \ldots, [2, 4, q-2], [2, 4, q-1]$
$\vdots$
$[2, q-3], [2, q-3, q-2], [2, q-3, q-1][2, q-2],$
$[2, q-2, q-1]$
$[3, 4], [3, 4, 5], [3, 4, 6], \ldots, [3, 4, q-2], [3, 4, q-1]$
$[3, 5], [3, 5, 6], [3, 5, 7], \ldots, [3, 5, q-2], [3, 5, q-1]$
$\vdots$
$[3, q-3], [3, q-3, q-2], [3, q-3, q-1][3, q-2],$
$[3, q-2, q-1]$
$\vdots$
$[q-5, q-4], [q-5, q-4, q-3], [q-5, q-4, q-2],$
$[q-5, q-4, q-1]$
$[q-5, q-3], [q-5, q-3, q-2], [q-5, q-3, q-1]$
$[q-5, q-2], [q-5, q-2, q-1]$
$[q-4, q-3], [q-4, q-3, q-2], [q-4, q-3, q-1]$
$[q-4, q-2], [q-4, q-2, q-1]$
$[q-3, q-2], [q-3, q-2, q-1], [1, q-1]$

Based on the color combinations above, the q-locating coloring of vertices other than the central vertex in the graph F_n for $q \geq 6$, can be combined from the results of q-locating coloring combinations of vertices other than the central vertex in the graph F_n for maximum n, by removing certain color combinations based on the following criterias:

1) For any color combination that is containing two vertices, it can be removed.
2) For any color combination that is containing three vertices, $[a, b, c]$ where $a, b, c \in [1, q-1], a < b < c$, and $c \neq q - 1$, it can be removed.

From the list above, we can conclude that $\chi_L(F_n) = q$, for $q \geq 6$ can be fulfilled when $(q-2)\sum_{i=3}^{q-1}(i-2) \leq n \leq \left((q-1)\sum_{i=3}^{q}(i-2)\right) - 1$. ∎

V. Conclusion

Let F_n be a fan graph for $n \geq 2$ and $\chi_L(F_n) = q$ for $q \geq 3$ then

$$\chi_L(F_n) = \begin{cases} 3 & if\ n = 2, \\ 4 & if\ 3 \leq n \leq 7, \\ 5 & if\ 8 \leq n \leq 23, \\ q & if\ (q-2)\sum_{i=3}^{q-1}(i-2) \leq n \leq \\ & \quad \big((q-1)\sum_{i=3}^{q}(i-2)\big) - 1, for\ q \geq 6. \end{cases}$$

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