# **Abstract**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithm**<br>**Algorithmen**<br>**Algorithmen**<br>**Algorithmen**<br>**Algorithmen kobot based on the Beetle Ante**<br>**kind of two-degree-of-freedom (2-DOF) upper limb exoskeleton**<br>**kind of two-degree-of-freedom (2-DOF) upper limb exoskeleton**<br>**kind of two-degree-of-freedom (2-DOF) upper limb exoskeleton**<br> Figure Limb Exoskeleton<br>
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human limb. M</sub> **procedure Antennae Search<br>etle Antennae Search**<br>ithm<br>yun He, Lie Yu and Lei Ding<br>engaged in heavy lifting tasks. The purpose of such robots is<br>usually to enable the robotic limb to move in unison with the<br>human limb. More IAENG International Journal of Applied Mathematics<br>
Robust PID Control for Upper Limb Exoskeleton<br>
Robot based on the Beetle Antennae Search<br>
Algorithm<br>
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<b>PIDMONG A CON** *Abstract***—This paper presents a computational model for one usually to enable the robotic linkind of two-degree-of-freedom (2-DOF) upper limb exoskeleton luman limb. Moreover, the hum<br>robot. This robot consists of two co** *in the indeed to connet in the informal model for one* **and the tuning of two-degree-of-freedom (2-DOF) upper limb exoskeleton** human limb. Moreove obsolut This robot consists of two connected body segments to force as le *Abstract***—This paper presents a computational model for one candally to enable the robotic robot. This robot consists of two connected body segments to force as less as possible where obot a minitate the upper shoulder a Abstract—This paper presents a computational model for one usually to enable the robot wind of two-degree-of-freedom (2-DOF) upper limb exoskeleton** human limb. Moreover, the upper shoulder and elbow joints of two smally **Presented Example the internal present are completations are continuous in the distance of the improved beet current (DC) motors are severally used to the improvides the amound drive the upper shoulder and elbow joints algorithm to the difficult and algorithm can relative and the PID gains. The original BAS**<br> **Expection and the original BAS**<br> **Altert CIC Expective the robotic shoulder and elbow joints of human hand.**<br> **Alter the robot** Figure 11 and 1000 counter and the whome the time the time the search and the search and the search of contor is accurately developed to achieve the input current and the actuated force/torque. The distribution between the ITHURE THE SIGNATE THOM THE THE CONDITION THE STONG THE SIGNATE THE SIGNATE THE SIGNATE THE SIGNATE THE CONDIT IS A STATE THE CONDITE AND THE CONDITE AND THE POLE THE SIGNATE PORTER THE SIGNATE PHE DEVIDENT AND THE CONDITE Two sets of ureret current (DC) mootors are severally used to detect and elbow joints. The dynamic of the between the input current and the actuated force/torque. The between the input current and the actuated force/torque From the controlls should are a state of the transmit of the US and state and the HBAS algorithm gains since the HBAS algorithm gains since the HBAS algorithm gains since the HBAS algorithm and the transmit applications, a **Bream the interact of the algorithm** is considered to estable the extractions, and selected to control the whole system in this paper. Therefore, the tunion of PID controller is considered as a standard in technical and M **Solution** Figure The menticurity and an interaction of the extraction of the multiplications, and selected to control the whole system in this paper. Therefore, the tuning of PID gains is extremely equation between the we **FID confident is considered as a standard in technical and that in this paper. Therefore, the tuning of PID gains is extremely** out the inherent difficultion approach of tuning PID gains. This paper and the inherent diffi **The IBAS algorithm is appendixed** to control the whore system<br>
in this paper. Therefore, the tuning of PID gains is extremely<br>
important, while utilizing optimization algorithms to reduce the<br>
tracking error is one approa In this paper. Therefore, the tuning of *FID* gaint<br>important, while utilizing optimization algorithm<br>tracking error is one approach of tuning PID ga<br>presented the improved beetle antennae<br>algorithm to optimize the PID gai Expressed the improved bettle antennae search (IBAS) dynamic relationship be<br> **Index Terms** (orithm to optimize the PID gains. The original BAS the Newton-Euler and I<br>
Intervention can achieve a wide search range, cost low presented and empirical determined search antennae search in the optimize the PID gains. The original BAS the New<br>algorithm can achieve a wide search range, cost low time<br>proposed IBAS algorithm can realize more extensive e a wide search range, cost low time<br>
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Therefore, the<br>
The upper limb exoskeleton robot has recently attracted<br>
considerable interests due to its potential appli I.INTRODUCTION<br> **The upper limb exoskeleton robot has recently a**<br>
considerable interests due to its potential applica<br>
the fields of medicine, industry and military [1-2<br>
devices can provide additional assistance to mili **L** considerable interests due to its p<br>the fields of medicine, industry and<br>devices can provide additional assista:<br>Manuscript received January 9, 2024; revise<br>This work was supported by the National Na<br>China "Research on The upper limb exoskeleton robot has recently attracted<br>considerable interests due to its potential applications in<br>and others, are increased

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human limb. Moreover, the human limb exerts the amount yun He, Lie Yu and Lei Ding<br>engaged in heavy lifting tasks. The purpose of such robots is<br>usually to enable the robotic limb to move in unison with the<br>human limb. Moreover, the human limb exerts the amount of<br>force as les n He, Lie Yu and Lei Ding<br>gaged in heavy lifting tasks. The purpose of such robots is<br>ually to enable the robotic limb to move in unison with the<br>man limb. Moreover, the human limb exerts the amount of<br>roce as less as poss yun He, Lie Yu and Lei Ding<br>engaged in heavy lifting tasks. The purpose of such robots is<br>usually to enable the robotic limb to move in unison with the<br>human limb. Moreover, the human limb exerts the amount of<br>force as les engaged in heavy lifting tasks. The purpose of such robots is<br>usually to enable the robotic limb to move in unison with the<br>human limb. Moreover, the human limb exerts the amount of<br>force as less as possible when lifting a engaged in heavy lifting tasks. The purpose of such robots is<br>usually to enable the robotic limb to move in unison with the<br>human limb. Moreover, the human limb exerts the amount of<br>force as less as possible when lifting a

engaged in heavy lifting tasks. The purpose of such robots is<br>usually to enable the robotic limb to move in unison with the<br>human limb. Moreover, the human limb exerts the amount of<br>force as less as possible when lifting a usually to enable the robotic limb to move in unison with the<br>human limb. Moreover, the human limb exerts the amount of<br>force as less as possible when lifting a load, while the robotic<br>limb provides the amount as bigger as human limb. Moreover, the human limb exerts the amount of<br>force as less as possible when lifting a load, while the robotic<br>limb provides the amount as bigger as possible. As this kind<br>of robot is a wearable device attached force as less as possible when lifting a load, while the robotic<br>limb provides the amount as bigger as possible. As this kind<br>of robot is a wearable device attached closely with the human<br>limbs, the controller design must limb provides the amount as bigger as possible. As<br>of robot is a wearable device attached closely with tl<br>limbs, the controller design must consider the co<br>between the human and robot movements.<br>Many studies have been made robot is a wearable device attached closely with the human<br>abs, the controller design must consider the cooperation<br>tween the human and robot movements.<br>Many studies have been made to establish the dynamic<br>uation between t limbs, the controller design must consider the cooperation<br>between the human and robot movements.<br>Many studies have been made to establish the dynamic<br>equation between the wearer and the robot in order to figure<br>out the in between the human and robot movements.<br>
Many studies have been made to establish the dynamic<br>
equation between the wearer and the robot in order to figure<br>
out the inherent difficulties associated with mathematical<br>
modell Many studies have been made to establish the dynamic<br>equation between the wearer and the robot in order to figure<br>out the inherent difficulties associated with mathematical<br>modelling. The most prevalent methods for develo

Figure 1988 algorithm can realize more extensive search<br>
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e and more precise search compared with the original BAS<br>
mall BAS, PSO and GA algorithms on the basis of e and more precise search compared with the original BAS<br>
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interable interapl controller is widely u<br>
interaction results. The c algorithm. Finally, the IBAS algorithm is compared with the<br>
original BAS, PSO and GA algorithms on the basis of the basis of mplementation<br>
optimization results. The comparison results demonstrated that<br>
determined by th orginal BAS, PSO and GA argorithms on the basis of melomean basistance complemization results. The comparison results demonstrated that determined by three paramometer the IBAS algorithm gains superior performance in addr I. INTRODUCTION<br>
I. INTRODUCTION<br>
The upper limb exoskeleton robot has recently attracted<br>
genetic algorithm (GA) [1<br>
considerable interests due to its potential applications in<br>
Fields of medicine, industry and military [ The upper limb exoskeleton robot has recently attracted<br>
The upper limb exoskeleton robot has recently attracted<br>
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the fields of medicine, industry and military [1-The upper limb exoskeleton robot has recently attracted genetic algorithm (Considerable interests due to its potential applications in expected and others, are increvies can provide additional assistance to military soldie The upper limb exoskeleton robot has recently attracted<br>
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the fields of medicine, industry and military [1-2]. Such<br>
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This work was supported by the National Natural Science Foundation of<br>
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This work was supported by the National Natural Science Foundation of<br>
china "Research on motion pattern recognition of exoskeleton robot equation between the wearer and the robot in order to figure<br>out the inherent difficulties associated with mathematical<br>modelling. The most prevalent methods for developing the<br>dynamic relationship between force/torque and out the inherent difficulties associated with mathematical<br>modelling. The most prevalent methods for developing the<br>dynamic relationship between force/torque and position are<br>the Newton-Euler and Lagrange equations [6-7]. modelling. The most prevalent methods for developing the<br>dynamic relationship between force/torque and position are<br>the Newton-Euler and Lagrange equations [6-7]. Moreover,<br>as the number of degrees of freedom (DOF) increas dynamic relationship between force/torque and position are<br>the Newton-Euler and Lagrange equations [6-7]. Moreover,<br>as the number of degrees of freedom (DOF) increases, the<br>computational cost of creating the dynamic model the Newton-Euler and Lagrange equations [6-7]. Moreover,<br>as the number of degrees of freedom (DOF) increases, the<br>computational cost of creating the dynamic model also<br>dramatically increases.<br>The PID controller is widely u as the number of degrees of freedom (DOF) increases, the computational cost of creating the dynamic model also dramatically increases.<br>The PID controller is widely used due to its simple design and ease of implementation [ computational cost of creating the dynamic model also<br>dramatically increases.<br>The PID controller is widely used due to its simple design<br>and ease of implementation [8]. Its performance is mainly<br>determined by three paramet dramatically increases.<br>The PID controller is widely used due to its simple design<br>and ease of implementation [8]. Its performance is mainly<br>determined by three parameters, such as the proportional gain,<br>the integral gain, The PID controller is widely used due to its simple design<br>and ease of implementation [8]. Its performance is mainly<br>determined by three parameters, such as the proportional gain,<br>the integral gain, and the differential ga and ease of implementation [8]. Its performance is mainly<br>determined by three parameters, such as the proportional gain,<br>the integral gain, and the differential gain [9-10]. It is<br>important to select appropriate gains for determined by three parameters, such as the proportional gain,<br>the integral gain, and the differential gain [9-10]. It is<br>important to select appropriate gains for the PID controller.<br>In the literature, numerous methods fo the integral gain, and the differential gain [9-10]. It is<br>important to select appropriate gains for the PID controller.<br>In the literature, numerous methods for determining the PID<br>gains have been established. The most con important to select appropriate gains for the PID controller.<br>In the literature, numerous methods for determining the PID<br>gains have been established. The most conventional method<br>is the Ziegler-Nichols (Z-N) method [11]. In the literature, numerous methods for determining the PID<br>gains have been established. The most conventional method<br>is the Ziegler-Nichols (Z-N) method [11]. However, this Z-N<br>method would cause large overshoot and cont gains have been established. The most conventional method<br>is the Ziegler-Nichols (Z-N) method [11]. However, this Z-N<br>method would cause large overshoot and control oscillation.<br>Therefore, the artificial intelligence metho is the Ziegler-Nichols (Z-N) method [11]. However, this Z-N<br>method would cause large overshoot and control oscillation.<br>Therefore, the artificial intelligence methods, such as the<br>genetic algorithm (GA) [12-13], particle s method would cause large overshoot and control oscillation.<br>Therefore, the artificial intelligence methods, such as the<br>genetic algorithm (GA) [12-13], particle swarm optimization<br>(PSO) [14-15], whale optimization algorith Therefore, the artificial intelligence methods, such as the genetic algorithm (GA) [12-13], particle swarm optimization (PSO) [14-15], whale optimization algorithm (WOA) [16-17], and others, are increasingly commonly utili netic algorithm (GA) [12-13], particle swarm optimization<br>SO) [14-15], whale optimization algorithm (WOA) [16-17],<br>d others, are increasingly commonly utilized to realize the<br>D gains adjustment. This study employs a simila (PSO) [14-15], whale optimization algorithm (WOA) [16-17],<br>and others, are increasingly commonly utilized to realize the<br>PID gains adjustment. This study employs a similar approach<br>to the Reference [18] using the beetle an and others, are increasingly commonly utilized to realize the<br>PID gains adjustment. This study employs a similar approach<br>to the Reference [18] using the beetle antennae search (BAS)<br>method to tune the PID gains. The BAS a PID gains adjustment. This study employs a similar approach<br>to the Reference [18] using the beetle antennae search (BAS)<br>method to tune the PID gains. The BAS algorithm can<br>achieve effective global optimization, and has be

This work was supported by the National Natural Science Foundation of<br>
China "Research on motion pattern recognition of exoskeleton robot based<br>
on curve similarity model" (62106178).<br>
Fenggang Liu is an Associate Professo China "Research on motion pattern recognition of exoskeleton robot based [19-20], robotics [21]<br>
on curve similarity model" (62106178).<br>
fengang Liu is an Associate Professor at the School of Artificial Additionally, the B Fenggang Liu is an Associate Professor at the School of Artificial<br>
elligence, Wuchang University of Technology, Wuhan, 430221, China. modifications, such as<br>
mail: 34834779@qq.com).<br>
Lang Rao is an Associate Professor at Intelligence, Wuchang University of Technology, Wuhan, 430221, China. Inoutincations, such a<br>
(e-mail: 3483479/@qq.com).<br>
Lang Rao is an Associate Professor at the School of Artificial Intelligence,<br>
Wuchang University of to the Reference [18] using the beetle antennae search (BAS)<br>method to tune the PID gains. The BAS algorithm can<br>achieve effective global optimization, and has been applied in<br>several scientific domains, including machine method to tune the PID gains. The BAS algorithm can<br>achieve effective global optimization, and has been applied in<br>several scientific domains, including machine learning<br>[19-20], robotics [21], engineering [22], and financ achieve effective global optimization, and has been applied in<br>several scientific domains, including machine learning<br>[19-20], robotics [21], engineering [22], and finance [23].<br>Additionally, the BAS algorithm has undergon several scientific domains, including machine learning [19-20], robotics [21], engineering [22], and finance [23].<br>Additionally, the BAS algorithm has undergone several modifications, such as binary [24] and semi-integer [ [19-20], robotics [21], engineering [22], and finance [23].<br>Additionally, the BAS algorithm has undergone several<br>modifications, such as binary [24] and semi-integer [25]<br>versions, to more effectively address different pr

IAENG International Journal of Applied Mathematic<br>can realize more uniform traversal and accelerate the more power and the human of<br>convergence. A novel method is proposed to compute the sensor is mounted on the wrist as<br>s IAENG International Journal of Applied Mathematic<br>can realize more uniform traversal and accelerate the more power and the human of<br>convergence. A novel method is proposed to compute the sensor is mounted on the wrist as<br>s **IAENG International Journal of Applied Mathemat**<br>can realize more uniform traversal and accelerate the more power and the human<br>convergence. A novel method is proposed to compute the sensor is mounted on the wris<br>step siz **IAENG International Journal of Applied Mathems**<br>can realize more uniform traversal and accelerate the more power and the humar<br>convergence. A novel method is proposed to compute the sensor is mounted on the wri<br>step size **EXECUTE IDE LARENG International Journal of Applied Mathematics**<br>can realize more uniform traversal and accelerate the more power and the human offer<br>convergence. A novel method is proposed to compute the sensor is mount **IAENG International Journal of Applied Matl**<br>
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convergence. A novel method is proposed to compute the sensor is mounted on the wrist atep size which can implement more extensive search scope contact force between the his in the early stages, and more precise search in step size which can implement more extensive search scope<br>
in the early stages, and more precise search in the later stages.<br>
in the aterd in operational space in<br>
Finally, the control performance based on IBAS algorithm in the early stages, and more precise search in the later stage<br>
Finally, the control performance based on IBAS algorithm<br>
compared with those based on the other optimizati<br>
algorithms.<br>
II. DYNAMICS OF THE UPPER LIMB EXOS







**and Solution Control Mathematics**<br>more power and the human offers less. A multi-axis force<br>sensor is mounted on the wrist as the end effector to detect the<br>contact force between the human and robot. The force<br>detected in **Solution 1 and Solution Mathematics**<br>sensor is mounted on the wrist as the end effector to detect the<br>contact force between the human and robot. The force<br>detected in operational space must be transformed into a<br>torque in **and of Applied Mathematics**<br>more power and the human offers less. A multi-axis force<br>sensor is mounted on the wrist as the end effector to detect the<br>contact force between the human and robot. The force<br>detected in operat **al of Applied Mathematics**<br>more power and the human offers less. A multi-axis force<br>sensor is mounted on the wrist as the end effector to detect the<br>contact force between the human and robot. The force<br>detected in operat **al of Applied Mathematics**<br>more power and the human offers less. A multi-axis force<br>sensor is mounted on the wrist as the end effector to detect the<br>contact force between the human and robot. The force<br>detected in operat **of Applied Mathematics**<br>
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nsor is mounted on the wrist as the end effector to detect the<br>
ntact force between the human and robot. The force<br>
tected in operational more power and the human offers less. A multi-axis force<br>sensor is mounted on the wrist as the end effector to detect the<br>contact force between the human and robot. The force<br>detected in operational space must be transfor more power and the human offers less. A multi-axis force<br>sensor is mounted on the wrist as the end effector to detect the<br>contact force between the human and robot. The force<br>detected in operational space must be transfor more power and the human offers less. A multi-axis force<br>sensor is mounted on the wrist as the end effector to detect the<br>contact force between the human and robot. The force<br>detected in operational space must be transfor

$$
T = J^T F \tag{1}
$$

$$
T = \begin{bmatrix} T_s & T_e \end{bmatrix}^T \tag{2}
$$

$$
J = \begin{bmatrix} L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2}) & L_2 \cos(q_{H1} + q_{H2}) \\ L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2}) & L_2 \sin(q_{H1} + q_{H2}) \end{bmatrix}
$$
(3)

$$
F = \begin{bmatrix} F_x & F_y \end{bmatrix}^T \tag{4}
$$

ort controls, the Equation (1) could be specified below.<br>  $T = [T_s \quad T_e]^T$  (2)<br>  $T = \begin{bmatrix} L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2}) & L_2 \cos(q_{H1} + q_{H2}) \\ L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2}) & L_2 \sin(q_{H1} + q_{H2}) \end{bmatrix}$ <br>
(3)<br>  $F = [F_x \quad F_y]^T$  (4)<br>
where  $T_s$  is  $T = [T_s \quad T_e]^T$  (2)<br>  $J = \begin{bmatrix} L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2}) & L_2 \cos(q_{H1} + q_{H2}) \\ L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2}) & L_2 \sin(q_{H1} + q_{H2}) \end{bmatrix}$ <br>
(3)<br>  $F = [F_x \quad F_y]^T$  (4)<br>
where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>
of  $T = [T_s \quad T_e]^T$  (2)<br>  $J = \begin{bmatrix} L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2}) & L_2 \cos(q_{H1} + q_{H2}) \\ L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2}) & L_2 \sin(q_{H1} + q_{H2}) \end{bmatrix}$ <br>
(3)<br>  $F = [F_x \quad F_y]^T$  (4)<br>
where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>
of  $J = \begin{bmatrix} L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2}) & L_2 \cos(q_{H1} + q_{H2}) \\ L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2}) & L_2 \sin(q_{H1} + q_{H2}) \end{bmatrix}$ <br>
(3)<br>  $F = [F_x \quad F_y]^T$  (4)<br>
where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>
of elbow joint. *J* is  $J = \begin{bmatrix} L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2}) & L_2 \cos(q_{H1} + q_{H2}) \\ L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2}) & L_2 \sin(q_{H1} + q_{H2}) \end{bmatrix}$ <br>
(3)<br>  $F = [F_x \quad F_y]^T$  (4)<br>
where *T<sub>s</sub>* is the torque of shoulder joint, and *T<sub>e</sub>* is the torque<br>
of elbow joint. *J*  $J = \begin{bmatrix} L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2}) & L_2 \cos(q_{H1} + q_{H2}) \\ L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2}) & L_2 \sin(q_{H1} + q_{H2}) \end{bmatrix}$ <br>
(3)<br>
(3)<br>
Where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>
of elbow joint. *J* is a 2×2 Jacob m  $\lfloor L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2}) \rfloor$   $L_2 \sin(q_{H1} + q_{H2})$  (3)<br>
(3)<br>
Where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>
elbow joint. *J* is a 2×2 Jacob matrix.  $L_1$  is the forearm<br>
ngth, and  $L_2$  is the upp (3)<br>  $F = [F_x \quad F_y]^T$  (4)<br>
where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>
of elbow joint. *J* is a 2×2 Jacob matrix. *L*<sub>1</sub> is the forearm<br>
length, and *L*<sub>2</sub> is the upper arm length. *q<sub>H1</sub>* is the huma

 $F = [F_x \quad F_y]^T$  (4)<br>where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>of elbow joint. *J* is a 2×2 Jacob matrix. *L*<sub>1</sub> is the forearm<br>length, and *L*<sub>2</sub> is the upper arm length. *q<sub>H1</sub>* is the human<br>should  $F = [F_x \quad F_y]^T$  (4)<br>where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>of elbow joint. *J* is a 2×2 Jacob matrix.  $L_1$  is the forearm<br>length, and  $L_2$  is the upper arm length.  $q_{H1}$  is the human<br>shoulder  $F = [F_x \ F_y]$  (4)<br>where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>of elbow joint. *J* is a 2×2 Jacob matrix. *L*<sub>1</sub> is the forearm<br>length, and *L*<sub>2</sub> is the upper arm length. *q<sub>H1</sub>* is the human<br>shoulder where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>of elbow joint. *J* is a 2×2 Jacob matrix.  $L_1$  is the forearm<br>length, and  $L_2$  is the upper arm length.  $q_{H1}$  is the human<br>shoulder angle, and  $q_{H2}$ where  $T_s$  is the torque of shoulder joint, and  $T_e$  is the torque<br>of elbow joint. J is a  $2\times2$  Jacob matrix.  $L_1$  is the forearm<br>length, and  $L_2$  is the upper arm length.  $q_{H1}$  is the human<br>shoulder angle, and  $q_{H2$ of elbow joint. *J* is a 2×2 Jacob matrix.  $L_1$  is the forearm<br>length, and  $L_2$  is the upper arm length.  $q_{H1}$  is the human<br>shoulder angle, and  $q_{H2}$  is the human elbow angle. Actually,<br> $F_x$  and  $F_y$  are obtained th length, and  $L_2$  is the upper arm length.  $q_{H1}$  is the human<br>shoulder angle, and  $q_{H2}$  is the human elbow angle. Actually,<br> $F_x$  and  $F_y$  are obtained through the multi-axis force sensor, as<br>shown in Fig. 2 and Fig. 3 can be described as follows.<br>
"=  $J^T F$  (1)<br>
the joint space, and *J* is the Jacob<br>
operational space, and monitored<br>
sor. As this study referred to two<br>
(1) could be specified below.<br>
=  $[T_s \t T_e]^T$  (2)<br>  $(q_{H1} + q_{H2}) \t L_2 \$ formation. In this paper, the deformation can be regarded<br>the error between the human and robot positions. If this<br>or is closed to zero, the robot synchronizes well with the<br>man such that the values of  $F_x$  and  $F_y$  appro as the error between the human and robot positions. If this<br>error is closed to zero, the robot synchronizes well with the<br>human such that the values of  $F_x$  and  $F_y$  approach around zero.<br>If this error is significantly la

$$
\begin{cases}\nF = k_f L \\
L = [L_x \quad L_y]^T\n\end{cases}
$$
\n(5)

below.

$$
\begin{cases}\nL_x = [L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2})] \\
-[L_1 \cos(q_{M1}) + L_2 \cos(q_{M1} + q_{M2})] \\
L_y = [L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2})] \\
-[L_1 \sin(q_{M1}) + L_2 \sin(q_{M1} + q_{M2})]\n\end{cases} (6)
$$

where  $k_f$  is the elastic coefficient, and *L* is the error between<br>
e human and robot positions.  $L_x$  and  $L_y$  can be specified<br>
low.<br>  $\begin{cases}\nL_x = [L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2})] \\
-L_1 \cos(q_{M1}) + L_2 \cos(q_{M1} + q_{M2})] \\
L_y = [L_1 \sin(q_{H1}) + L$ where  $k_f$  is the elastic coefficient, and L is the error between<br>the human and robot positions.  $L_x$  and  $L_y$  can be specified<br>below.<br> $\begin{cases} L_x = [L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2})] \\ -[L_1 \cos(q_{M1}) + L_2 \sin(q_{H1} + q_{H2})] \\ L_y = [L_1 \sin(q_{H1}) + L$ where  $k_f$  is the ensure coefficient, and L is the error between<br>the human and robot positions.  $L_x$  and  $L_y$  can be specified<br>below.<br> $\begin{cases} L_x = [L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2})] \\ -[L_1 \cos(q_{M1}) + L_2 \sin(q_{H1} + q_{H2})] \\ L_y = [L_1 \sin(q_{H1}) + L_$ the numan and root positions.  $L_x$  and  $L_y$  can be specified<br>below.<br>  $\begin{cases} L_x = [L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2})] \\ -[L_1 \cos(q_{M1}) + L_2 \sin(q_{H1} + q_{H2})] \\ L_y = [L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2})] \end{cases}$  (6)<br>  $-[L_1 \sin(q_{M1}) + L_2 \sin(q_{M1} + q_{M2})]$ <br>
where below.<br>  $\left[ L_x = [L_1 \cos(q_{H1}) + L_2 \cos(q_{H1} + q_{H2})] \right]$ <br>  $-[L_1 \cos(q_{M1}) + L_2 \cos(q_{M1} + q_{M2})]$ <br>  $\left[ L_y = [L_1 \sin(q_{H1}) + L_2 \sin(q_{H1} + q_{H2})] \right]$ <br>  $-[L_1 \sin(q_{M1}) + L_2 \sin(q_{M1} + q_{M2})]$ <br>
where  $q_M$  is the robot shoulder angle, and  $q_{M2}$  is the robo

**IAENG International Journal of Applied Mathematical**  
\n
$$
\begin{bmatrix}\nT_L + T = M(q_M)\ddot{q}_M + C(q_M, \dot{q}_M) + G(q_M) & \text{where } M^1 \text{ is the inverse ma} \\
T_L = [T_{L1} \quad T_{L2}]^T & (7) \\
q_M = [q_{M1} \quad q_{M2}]^T & M^{-1} = \text{where } T_{L1} \text{ is the driving torque generated by the DC motor} \\
\text{the DC motor to drive the upper arm. } M \text{ is a } 2 \times 2 \text{ matrix} \\
\text{at inertia torque } C \text{ is a } 2 \times 2 \text{ matrix about the torque}\n\end{bmatrix}
$$

**IAENG International Journal of Applied Mather**<br>  $T_L + T = M(q_M) \ddot{q}_M + C(q_M, \dot{q}_M) + G(q_M)$  where  $M^1$  is the inverse  $T_L = [T_{L1} \quad T_{L2}]^T$  (7)  $M^1$  can be written as:<br>  $q_M = [q_{M1} \quad q_{M2}]^T$   $M$ <br>
there  $T_{L1}$  is the driving tor **IAENG International Journal of Applied Mathema<br>**  $T_L + T = M(q_M)\ddot{q}_M + C(q_M, \dot{q}_M) + G(q_M)$  **where**  $M^1$  **is the inverse m:<br>**  $T_L = [T_{L1} \quad T_{L2}]^T$  **(7)**  $M^1$  **can be written as:<br>
where**  $T_{L1}$  **is the driving torque generated by the DC IAENG International Journal of Applied Mather<br>**  $T_L + T = M(q_M) \ddot{q}_M + C(q_M, \dot{q}_M) + G(q_M)$  **where**  $M^1$  **is the invers<br>**  $T_L = [T_{L1} \quad T_{L2}]^T$  **(7) Where**  $T_{L1}$  **is the driving torque generated by the DC motor<br>
to drive the forearm, IAENG International Journal of Applied Mathema<br>**  $T_L + T = M(q_M)\ddot{q}_M + C(q_M, \dot{q}_M) + G(q_M)$  **where**  $M^1$  **is the inverse m<br>**  $T_L = [T_{L1} \ T_{L2}]^T$  **(7)**  $M^1$  **can be written as:<br>**  $q_M = [q_{M1} \ q_{M2}]^T$  $M^{-1}$  **=<br>
where**  $T_{L1}$  **is the drivin IAENG International Journal of Applied Mathema**<br>  $\begin{cases} T_L + T = M(q_M) \ddot{q}_M + C(q_M, \dot{q}_M) + G(q_M) \end{cases}$  where  $M^{\text{-}1}$  is the inverse m<br>  $\begin{cases} T_L = [T_{L1} \quad T_{L2}]^T & (7) \end{cases}$  where  $T_{L1}$  is the driving torque generated by the DC **produced by the centripetal force, and** *G* is a 2×1 matrix about<br>
T<sub>L</sub> =  $[T_{L1}$   $T_{L2}]^T$  (7)  $M^{-1}$  can be written as:<br>  $q_M = [q_{M1} \ q_{M2}]^T$   $M^{-1}$  =  $M^{-1}$  is the driving torque generated by the DC motor<br>
to drive the  $\begin{cases} T_L + T = M(q_M) \ddot{q}_M + C(q_M, \dot{q}_M) + G(q_M) \end{cases}$  where  $M^1$  is the inverse matrix  $T_L = [T_{L1} \ T_{L2}]^T$  (7)  $M^1$  can be written as:<br>  $\begin{cases} q_M = [q_{M1} \ q_{M2}]^T \end{cases}$  (7)  $M^{-1} = \begin{bmatrix} H \\ H \end{bmatrix}$ <br>
where  $T_{L1}$  is the driving torq  $\begin{cases} T_L + T = M(q_M) \ddot{q}_M + C(q_M, \dot{q}_M) + G(q_M) \end{cases}$  where *M*<br>  $T_L = [T_{L1} \quad T_{L2}]^T$  (7)  $M^{-1}$  can be<br>  $q_M = [q_{M1} \quad q_{M2}]^T$ <br>
where  $T_{L1}$  is the driving torque generated by the DC motor<br>
to drive the forearm, and  $T_{L2}$  is th

$$
M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}
$$
 (8)  $H_{R12} = -$ 

$$
C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_{M1}^2 \\ \dot{q}_{M2}^2 \end{bmatrix} + \begin{bmatrix} C_{13} & C_{14} \\ C_{23} & C_{24} \end{bmatrix} \begin{bmatrix} \dot{q}_{M1}\dot{q}_{M2} \\ \dot{q}_{M2}\dot{q}_{M1} \end{bmatrix}
$$
 (9)  $H_{R21} = \frac{-4(4I_2 + 2m_2)}{4(4I_2 + 2m_2)}$ 

$$
G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \tag{10} H_{R22} =
$$

$$
M_{11} = m_1 L_{g1}^2 + m_2 (L_1^2 + L_{g1}^2) + 2 m_2 L_{g2} L_1 \cos(q_{M2}) + I_1 + I_2
$$
  
(11)

$$
M_{12} = M_{21} = m_2 L_{g2}^2 + m_2 L_{g2} L_1 \cos(q_{M2}) + I_2
$$
\n
$$
H_{\text{note}} = 4I_1 I_2^2 + I_2^2 + I_1^2
$$
\n
$$
+ m_1 I_2 I_2^2 + 4m_1 I_2^2
$$

$$
M_{22} = m_2 L_{g2}^2 + I_2
$$
 (13) The DC

$$
C_{11} = C_{22} = C_{23} = C_{24} = 0 \tag{14}
$$

$$
C_{12} = C_{13} = C_{14} = -m_2 L_{g2} L_1 \cos(q_{M2})
$$
\n(15)

$$
C_{21} = m_2 L_{g2} L_1 \cos(q_{M2})
$$
 (16)

$$
G_1 = m_2 g L_{g2} \sin(q_{M1} + q_{M2}) + m_1 g L_{g1} \sin(q_{M1})
$$
 where  $T_e$  is the electromagnetic

$$
+ m_2 g L_1 \sin(q_{M1})
$$

$$
G_2 = m_2 g L_{g2} \sin(q_{M1} + q_{M2})
$$
\n(18)

 $C_{12} = C_{13} = C_{14} = -m_2 L_{g2} L_1 \cos(q_{M2})$  (15) determined by the rotor position<br>  $C_{21} = m_2 L_{g2} L_1 \cos(q_{M2})$  (16)<br>  $T_e = J_{\Omega} \ddot{q}_M + H_{G1} = m_2 g L_{g2} \sin(q_{M1} + q_{M2}) + m_1 g L_{g1} \sin(q_{M1})$ <br>  $+ m_2 g L_1 \sin(q_{M1})$  (17)<br>  $+ m_2 g L_1 \sin(q_{M1})$ <br>  $C_{21} = m_2 L_{g2} L_1 \cos(q_{M2})$  (16)<br>  $T_e = J_{\Omega} \ddot{q}_M + I$ <br>  $G_1 = m_2 g L_{g2} \sin(q_{M1} + q_{M2}) + m_1 g L_{g1} \sin(q_{M1})$  (17) where  $T_e$  is the electromagnetic<br>  $+ m_2 g L_1 \sin(q_{M1})$  (17) where  $T_e$  is the electromagnetic<br>  $G_2 = m_2 g L_{g2} \sin(q_{M1} + q$ for the DC motor<br>  $C_{21} = m_2 L_{g2} L_1 \cos(q_{M2})$  (16)<br>  $T_e = J_{\Omega}$ <br>  $G_1 = m_2 g L_{g2} \sin(q_{M1} + q_{M2}) + m_1 g L_{g1} \sin(q_{M1})$ <br>  $+ m_2 g L_1 \sin(q_{M1})$ <br>  $+ m_2 g L_2 \sin(q_{M1} + q_{M2})$ <br>
(17) where  $T_e$  is the electrom<br>
inertia of DC motor, *q* is the<br>
c  $C_{21} = m_2 L_{g2} L_1 \cos(q_{M2})$  (16)<br>  $T_e = J_{\Omega} \ddot{q}_M +$ <br>  $G_1 = m_2 g L_{g2} \sin(q_{M1})$  (17) where  $T_e$  is the electromagnet<br>  $+ m_2 g L_1 \sin(q_{M1})$  (17) where  $T_e$  is the electromagnet<br>  $+ m_2 g L_1 \sin(q_{M1})$  (17) where  $T_e$  is the electromag  $I_e = J_{\Omega}q_M +$ <br>  $G_1 = m_2 g L_{g2} \sin(q_{M1} + q_{M2}) + m_1 g L_{g1} \sin(q_{M1})$  (17) where  $T_e$  is the electromagnet<br>  $+ m_2 g L_1 \sin(q_{M1})$  (17) where  $T_e$  is the electromagnet<br>
inertia of DC motor, *q* is the roto<br>  $G_2 = m_2 g L_{g2} \sin(q_{M1} + q_{M2})$  $G_1 = m_2 g L_{g2} \sin(q_{M1} + q_{M2}) + m_1 g L_{g1} \sin(q_{M1})$  (17) where  $T_e$  is the electromagne<br>  $+ m_2 g L_1 \sin(q_{M1})$  (17) where  $T_e$  is the electromagne<br>  $G_2 = m_2 g L_{g2} \sin(q_{M1} + q_{M2})$  (18) electromagnetic torque is cons<br>
where  $m_1$  is th  $G_1 = m_2 g L_g$  sin(q<sub>M1</sub>)<br>  $G_2 = m_2 g L_{g2} \sin(q_{M1} + q_{M2})$  (17) where  $T_e$  is the electromagnet<br>
inertia of DC motor, q is the rot<br>
coefficient, and  $T_L$  is the load<br>
input current, which can be descended to rope is considerab 4  $m_2 g L_1 \sin(q_{M1})$  inertia of DC motor, *q* is the roto<br>  $G_2 = m_2 g L_{g2} \sin(q_{M1} + q_{M2})$  (18) electromagnetic torque is considered in the base<br>
where  $m_1$  is the mass of forearm, and  $m_2$  is the mass of<br>
upper arm.  $L_1$  i following.

$$
\ddot{q}_M = M^{-1}(T_L + T - C - G) \tag{19}
$$

 $(7)$ **is the inverse matrix of** *M*, and the expression of<br>ritten as:<br> $M^{-1} = \begin{bmatrix} H_{R11} & H_{R12} \end{bmatrix}$  (20) **al of Applied Mathematics**<br>where  $M^1$  is the inverse matrix of M, and the expression<br> $M^1$  can be written as:<br> $M^{-1} = \begin{bmatrix} H_{R11} & H_{R12} \\ H_{R21} & H_{R22} \end{bmatrix}$  (

$$
M^{-1} = \begin{bmatrix} H_{R11} & H_{R12} \\ H_{R21} & H_{R22} \end{bmatrix}
$$
 (20)

of Applied Mathematics<br>
where  $M^1$  is the inverse matrix of *M*, and the expression of<br>
<sup>1</sup> can be written as:<br>  $M^{-1} = \begin{bmatrix} H_{R11} & H_{R12} \\ H_{R21} & H_{R22} \end{bmatrix}$  (20)<br>
where the elements of matrix  $M^1$  can be described as<br> follows.

$$
H_{R11} = \frac{4(m_2 L_2^2 + 4I_2)}{H_{note}} \tag{21}
$$

$$
H_{R12} = \frac{-4(4I_2 + 2m_2L_1L_2\cos(q_{M2}) + m_2L_2^2)}{H_{\text{note}}}
$$
(22)

$$
\begin{aligned}\nM_1 \dot{q}_{M2} \\
M_2 \dot{q}_{M1}\n\end{aligned}\n\tag{23}
$$
\n
$$
H_{R21} = \frac{-4(4I_2 + 2m_2L_1L_2 \cos(q_{M2}) + m_2L_2^2)}{H_{\text{note}}}\n\tag{23}
$$

$$
H_{R22} = \frac{4(4I_1 + 4m_2L_1^2 + 4m_2L_1L_2\cos(q_{M2}))}{H_{note}}
$$
\n
$$
+ \frac{4(m_2L_2^2 + m_1L_1^2 + 4I_2)}{H_{note}}
$$
\n
$$
(24)
$$
\n
$$
+ \frac{4(m_2L_2^2 + m_1L_1^2 + 4I_2)}{H_{note}}
$$
\n
$$
H_{note} = 4I_1m_2L_2^2 + 16I_1I_2 + 4m_2^2L_1^2L_2^2 + 16m_2L_1^2I_2
$$
\n
$$
+ m_1L_1^2m_2L_2^2 + 4m_1L_1^2I_2 - 4m_2^2L_1^2L_2^2(\cos(q_{M2}))^2
$$
\nThe DC motors selected in this system are permanent agent synchronous motors, which are driven by DC voltage.  
\nne current commutation of DC motors is achieved by lid-state switches, while the commutation instant is terminated by the rotor position, which is detected by on

$$
H_{note} = 4I_1m_2L_2^2 + 16I_1I_2 + 4m_2^2L_1^2L_2^2 + 16m_2L_1^2I_2
$$
  
+  $m_1L_1^2m_2L_2^2 + 4m_1L_1^2I_2 - 4m_2^2L_1^2L_2^2(\cos(q_{\text{M2}}))^2$  (25)

The current commutation of DC motors is achieved by determined by the rotor position, which is detected by an encoder mounted inside the robot joint. Consequently, the  $H_{R22} = \frac{4(4I_1 + 4m_2L_1^2 + 4m_2L_1L_2\cos(q_{M2})))}{H_{note}}$  (24)<br>  $+\frac{4(m_2L_2^2 + m_1L_1^2 + 4I_2)}{H_{note}}$  (24)<br>  $H_{note} = 4I_1m_2L_2^2 + 16I_1I_2 + 4m_2^2L_1^2L_2^2 + 16m_2L_1^2I_2$  (25)<br>  $+m_1L_1^2m_2L_2^2 + 4m_1L_1^2I_2 - 4m_2^2L_1^2L_2^$  $H_{R22} = \frac{4(4t_1 + 4t_2t_2 + 4t_1t_2t_1 + 2t_2t_2t_2t_1t_2t_2t_1t_2t_2t_1$  $H_{note}$  (24)<br>  $+\frac{4(m_2L_2^2 + m_1L_1^2 + 4I_2)}{H_{note}}$  (24)<br>  $H_{note} = 4I_1m_2L_2^2 + 16I_1I_2 + 4m_2^2L_1^2L_2^2 + 16m_2L_1^2I_2$  (25)<br>  $+m_1L_1^2m_2L_2^2 + 4m_1L_1^2I_2 - 4m_2^2L_1^2L_2^2(\cos(q_{M2}))^2$ <br>
The DC motors selected in this syst  $+\frac{4(m_2L_2^2 + m_1L_1^2 + 4I_2)}{H_{note}}$ <br>  $H_{note} = 4I_1m_2L_2^2 + 16I_1I_2 + 4m_2^2L_1^2L_2^2 + 16m_2L_1^2I_2$  (25)<br>  $+m_1L_1^2m_2L_2^2 + 4m_1L_1^2I_2 - 4m_2^2L_1^2L_2^2(\cos(q_{M2}))^2$ <br>
The DC motors selected in this system are permanent<br>  $H_{note}$ <br>  $H_{note}$ <br>  $H_{note}$  = 4I<sub>1</sub>m<sub>2</sub>L<sub>2</sub><sup>2</sup> + 16I<sub>1</sub>I<sub>2</sub> + 4m<sub>2</sub><sup>2</sup>L<sub>1</sub><sup>2</sup><sub>2</sub><sup>2</sup> + 16m<sub>2</sub>L<sub>1</sub><sup>2</sup><sub>1</sub><sub>2</sub><sup>2</sup> (25)<br>
+m<sub>1</sub>L<sub>1</sub><sup>2</sup>m<sub>2</sub>L<sub>2</sub><sup>2</sup> + 4m<sub>1</sub>L<sub>1</sub><sup>2</sup><sub>1</sub><sub>2</sub><sup>2</sup> + 4m<sub>2</sub><sup>2</sup>L<sub>1</sub><sup>2</sup><sub>2</sub><sup>2</sup> (cos(*q<sub>M2</sub>*)<sup>2</sup><br>
The DC motors selecte  $H_{\text{note}} = 4I_1m_2L_2^2 + 16I_1I_2 + 4m_2^2L_1^2L_2^2 + 16m_2L_1^2I_2$  (25)<br>  $+m_1L_1^2m_2L_2^2 + 4m_1L_1^2I_2 - 4m_2^2L_1^2L_2^2(\cos(q_{\text{M2}}))^2$ <br>
The DC motors selected in this system are permanent<br>
magnet synchronous motors, whic The DC motors selected in this system are permanent<br>The DC motors selected in this system are permanent<br>gnet synchronous motors, which are driven by DC voltage.<br>e current commutation of DC motors is achieved by<br>lid-state The DC motors selected in this system are permanent<br>magnet synchronous motors, which are driven by DC voltage.<br>The current commutation of DC motors is achieved by<br>solid-state switches, while the commutation instant is<br>det The DC motors selected in this system are permanent<br>magnet synchronous motors, which are driven by DC voltage.<br>The current commutation of DC motors is achieved by<br>solid-state switches, while the commutation instant is<br>det magnet synchronous motors, which are driven by DC voltage.<br>The current commutation of DC motors is achieved by<br>solid-state switches, while the commutation instant is<br>determined by the rotor position, which is detected by

$$
T_e = J_\Omega \ddot{q}_M + R_\Omega \dot{q}_M + T_L \tag{26}
$$

electromagnetic torque is considered to be proportional to the input current, which can be described as: The current commutation of DC motors is a<br>cnieved by<br>solid-state switches, while the commutation instant is<br>determined by the rotor position, which is detected by an<br>encoder mounted inside the robot joint. Consequently, t  $T_e = J_{\Omega} \ddot{q}_M + R_{\Omega} \dot{q}_M + T_L$  (26)<br>where  $T_e$  is the electromagnetic torque,  $J_{\Omega}$  is the moment of<br>ertia of DC motor,  $q$  is the rotor position,  $R_{\Omega}$  is the damping<br>efficient, and  $T_L$  is the load torque. In this  $T_e = J_{\Omega} \ddot{q}_M + R_{\Omega} \dot{q}_M + T_L$  (26)<br>where  $T_e$  is the electromagnetic torque,  $J_{\Omega}$  is the moment of<br>inertia of DC motor,  $q$  is the rotor position,  $R_{\Omega}$  is the damping<br>coefficient, and  $T_L$  is the load torque. In  $r_e - r_{\Omega}q_M + r_{\Omega}q_M + r_L$  (20)<br>where  $T_e$  is the electromagnetic torque,  $J_Q$  is the moment of<br>inertia of DC motor, *q* is the rotor position,  $R_{\Omega}$  is the damping<br>coefficient, and  $T_L$  is the load torque. In this paper,

$$
T_e = K_t i \tag{27}
$$

$$
u = R_A i + L_A \frac{di}{dt}
$$
 (28)

where  $R_A$  is the armature winding resistance, and  $L_A$  is the armature inductance. Substituting the Equations (13) and (14) both current, which can be described as:<br>  $T_e = K_i i$  (27)<br>
where  $K_t$  is the motor torque constant and *i* is the input<br>
rect current. For DC motor, the relationship between the<br>
rent (i.e., *i*) and the voltage (i.e., *u*) area in the direct and the discrete as.<br>  $T_e = K_t i$  (27)<br>
where  $K_t$  is the motor torque constant and t is the input<br>
direct current. For DC motor, the relationship between the<br>
current (i.e., t) and the voltage (i.e., u) c  $T_e = K_t i$  (27)<br>where  $K_t$  is the motor torque constant and *i* is the input<br>direct current. For DC motor, the relationship between the<br>current (i.e., *i*) and the voltage (i.e., *u*) can be written as<br> $u = R_A i + L_A \frac{di}{dt}$  (28

# **Volume 54, Issue 12, December 2024, Pages 2758-2765**

$$
\begin{cases}\nT_{L1} = K_t i_1 - J \ddot{q}_{M1} - R_{\Omega} \dot{q}_{M1} & \text{human-machine position} \\
\frac{di_1}{dt} = \frac{u_1}{L_A} - \frac{R_A}{L_A} i_1 & \text{(29)}\n\end{cases}
$$
\nthan the described in detail

\nthan the desired in detail.

$$
\left\{\frac{u_{1}}{dt} = \frac{u_{1}}{L_{A}} - \frac{K_{A}}{L_{A}}i_{1}\right\}
$$
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\left\{\frac{d_{2}}{dt} = \frac{u_{2}}{L_{A}} - \frac{R_{A}}{L_{A}}i_{2}\right\}
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\left\{\frac{di_{2}}{dt} = \frac{u_{2}}{L_{A}} - \frac{R_{A}}{L_{A}}i_{2}\right\}
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u_{1}
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\nare the input direct current and voltage into  
\ncounted on the upper arm, while  $i_{2}$  and  $u_{2}$  are  
\nt current and voltage into the DC motor  
\nfore 1.11. CONTROLLER DESIGN  
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**IAENG International Journal of Applied Mathemation**<br>  $\begin{cases}\nT_{L1} = K_i \dot{i}_1 - J \ddot{q}_{M1} - R_{\Omega} \dot{q}_{M1} & \text{human-machine positions. In a  
\n $\frac{di_1}{dt} = \frac{u_1}{L_A} - \frac{R_A}{L_A} i_1 \\
\frac{di_2}{dt} = \frac{u_2}{L_A} - \frac{R_A}{L_A} i_2\n\end{cases}$ \n(29) can be described in detail as for  
\nmin  $f = \frac{1}{N} \sum_{i=$$  $\begin{cases}\nT_{L1} = K_t i_1 - J \ddot{q}_{M1} - R_{\Omega} \dot{q}_{M1} & \text{human-machine plane} \\
\frac{di_1}{dt} = \frac{u_1}{L_A} - \frac{R_A}{L_A} i_1 & (29) \\
\frac{di_2}{dt} = \frac{u_2}{L_A} - \frac{R_A}{L_A} i_2 & (30) \\
\frac{di_2}{dt} = \frac{u_2}{L_A} - \frac{R_A}{L_A} i_2 & (30) \\
\text{where } i_1 \text{ and } u_1 \text{ are the input direct current and voltage into} \\
\text{the DC motor mounted on the upper arm, while } i_2 \text{ and } u$  $\begin{cases} T_{L1} = K_t i_1 - J \ddot{q}_{M1} - R_{\Omega} \dot{q}_{M1} & \text{human-machine positions.} \text{ In} \\ \frac{di_1}{dt} = \frac{u_1}{L_A} - \frac{R_A}{L_A} i_1 & (29) \end{cases}$  can be described in detail as  $\min f = \frac{1}{N}$ <br>  $\begin{cases} T_{L2} = K_t i_2 - J \ddot{q}_{M2} - R_{\Omega} \dot{q}_{M2} & \text{in } T = \frac{1}{N} \\ \frac{di_2}{dt} = \frac{u_2$  $\begin{cases}\nT_{L1} = K_t i_1 - J \ddot{q}_{M1} - R_{\Omega} \dot{q}_{M1} \\
\frac{di_1}{dt} = \frac{u_1}{L_A} - \frac{R_A}{L_A} i_1 \\
\int T_{L2} = K_t i_2 - J \ddot{q}_{M2} - R_{\Omega} \dot{q}_{M2} \\
\frac{di_2}{dt} = \frac{u_2}{L_A} - \frac{R_A}{L_A} i_2\n\end{cases}$ where  $i_1$  and  $u_1$  are the input direct current and voltag<br>t *A. Design of Traditional PID Controller*<br> *A. Design of Traditional PID Controller*<br> *A. Design of Traditional PID Controller*<br> **A.** Design of Traditional PID Controller<br>
The human rotary angles  $(q_{n1}$  and  $q_{n2}$  are t

The outcome of Equal<br>  $\frac{di_2}{dt} = \frac{u_2}{L_A} - \frac{R_A}{L_A}i_2$  (30) The outcome of Equal<br>
optimal synchronization p<br>
where *i*<sub>1</sub> and *u*<sub>1</sub> are the input direct current and voltage into<br>
the DC motor mounted on the upper arm,  $\left\{\begin{array}{ll}\n\frac{du_2}{dt} = \frac{u_2}{L_4} - \frac{K_4}{L_4}i_2\n\end{array}\right\}$ The outcome of Equal optimal synchronization p<br>
then DC motor mounted on the upper arm, while *i*<sub>2</sub> and *u*<sub>2</sub> are<br>
the input direct current and voltage into the (*dt*  $L_A$   $L_A$  optimal synchronization performation.<br>
the DC motor mounted on the upper arm, while *i*<sub>2</sub> and *u*<sub>2</sub> are<br>
the input direct current and voltage into the DC motor<br>
the input direct current and voltage into where  $i_1$  and  $u_1$  are the input direct current and voltage into<br>the DC motor mounted on the upper arm, while  $i_2$  and  $u_2$  are<br>the input direct current and voltage into the DC motor<br>mounted on the forearm.<br>
The cont where  $i_1$  and  $u_1$  are the input direct current and volt<br>the DC motor mounted on the upper arm, while  $i_2$  and<br>the input direct current and voltage into the DC<br>mounted on the forearm.<br>III. CONTROLLER DESIGN<br>A. Design



$$
T_{\text{err}} = T_d - T = 0 - T \tag{31}
$$

Fig. 4. The block diagram of the normal PID feedback control system.<br>
Fig. 4. The block diagram of the normal PID feedback control system.<br>
The second step is to update and the fitness is written as  $d\theta_0$ , and the fitne Fig. 4. The block diagram of the normal PID feedback control system. The second step is to up<br>
antennae, which can be described to the fit<br>
second step is to up<br>
antennae, which can be described to the fit<br>
control aim is aim.

$$
i = K_p T_{err} + K_i \int T_{err} dt + K_d \frac{dT_{err}}{dt}
$$
 (32)

where *K*<sub>p</sub>, *K<sub>i</sub>* and *Ki* are so the best of the best of the particle of the proportion.<br>
where  $x^i$  is the optimization<br>
where  $x^i$  is the optimization<br>
tween the desired and actual torques. Then, the PID  $d^t$  r  $T_{\text{err}} = T_d - T = 0 - T$  (31)<br>
where  $x^i$  is the optimiz<br>
between the desired and actual torques. Then, the PID  $d^i$  represents the searching<br>
controller is selected and designed to implement this control<br>
aim.<br>  $i = K_p T_{err} + K_i$ where *T* has been constructed in Eq. (1)-(6).  $T_{err}$  is the error and *x<sub>i</sub>* are severally the positive<br>between the desired and actual torques. Then, the PID  $d^i$  represents the searching di<br>controller is selected and d where *T* has been constructed in Eq. (1)-(6).  $T_{err}$  is the error and *x*<sub>*I*</sub> are severally the position between the desired and actual torques. Then, the PID  $d^t$  represents the searching controller is selected and de where *I* has been constructed in Eq. (1)-(6). *I*<sub>err</sub> is the error and *X* at essertany the bost<br>between the desired and actual torques. Then, the PID d<sup>*i*</sup> represents the searching<br>controller is selected and designed between the desired and actual torques. Then, the PID<br>controller is selected and designed to implement this control<br>aim.<br> $i = K_p T_{err} + K_i \int T_{err} dt + K_d \frac{dT_{err}}{dt}$  (32)<br>where  $K_p$ ,  $K_i$  and  $K_d$  are the proportional, integral and<br> *B.*  $i = K_p T_{err} + K_i \int T_{err} dt + K_d \frac{dT_{err}}{dt}$  (32)<br>
where  $K_p$ ,  $K_i$  and  $K_d$  are the proportional, integral and<br>
differential gains, respectively. The output is the current *i*<br>
which controls the DC motor to generate the drivi  $i = K_p T_{err} + K_i \int T_{err} dt + K_d \frac{dT_{err}}{dt}$  (32)<br>
where  $K_p$ ,  $K_i$  and  $K_d$  are the proportional, integral and<br>
ferential gains, respectively. The output is the current *i* where  $K_p$ ,  $K_i$  and  $K_d$  are thich controls the DC motor  $i = K_p T_{err} + K_i \int T_{err} dt + K_d \frac{dT_{err}}{dt}$  (32)<br>
where  $K_p$ ,  $K_i$  and  $K_d$  are the proportional, integral and<br>
differential gains, respectively. The output is the current *i* where  $K_p$ ,  $K_i$  and  $K_d$  are the<br>
which controls the D

where  $K_p$ ,  $K_i$  and  $K_d$  are the proportional, integral and<br>differential gains, respectively. The output is the current *i* where  $K_p^i$ ,  $K_i^i$  and  $K_d$  and<br>which controls the DC motor to generate the driving torque *i* where  $K_p$ ,  $K_i$  and  $K_d$  are the proportional, integral and<br>differential gains, respectively. The output is the current *i* where  $K_p^i$ ,  $K_i^j$  and  $K_d^j$ <br>which controls the DC motor to generate the driving torque *i-th* where  $K_p$ ,  $K_i$  and  $K_d$  are the proportional, integral and<br>differential gains, respectively. The output is the current *i*<br>which controls the DC motor to generate the driving torque *i*-th iteration. rnd() is a functi<br>T differential gains, respectively. The output is the current *i* where  $K_p^i$ ,  $K_i^i$  and  $K_d^i$  are the which controls the DC motor to generate the driving torque *T<sub>L</sub>* ration.  $rnd()$  is a function  $T_L$  according to Equati which controls the DC motor to generate the driving torque *i-th* iteration.  $rnd()$  is a fit *T<sub>L</sub>* according to Equation (29). Then, the driving torque *T<sub>L</sub>* ranging from [0, 1], which the DC motor and the joint torque *T* T<sub>L</sub> according to Equation (29). Then, the driving torque T<sub>L</sub> ranging from [0, 1], which obe<br>from the DC motor and the joint torque T would together The third step is to establish the<br>drive the robot arm to move.<br>B. Desi

**al of Applied Mathematics**<br>human-machine positions. In summary, the fitness function<br>can be described in detail as follows.<br> $\min f = \frac{1}{N} \sum_{i=1}^{N} (q_{Mi} - q_{Hi})^2$  (33) **al of Applied Mathematics**<br>human-machine positions. In summary, the fitness function<br>can be described in detail as follows.<br> $\min f = \frac{1}{N} \sum_{i=1}^{N} (q_{Mi} - q_{Hi})^2$  (33)

$$
\min f = \frac{1}{N} \sum_{i=1}^{N} (q_{Mi} - q_{Hi})^2
$$
 (33)

optimal synchronization performance is achieved when the The outcome of Equation (33) demonstrated that the **of Applied Mathematics**<br>
man-machine positions. In summary, the fitness function<br>
n be described in detail as follows.<br>
min  $f = \frac{1}{N} \sum_{i=1}^{N} (q_{Mi} - q_{Hi})^2$  (33)<br>
The outcome of Equation (33) demonstrated that the<br>
tima **and of Applied Mathematics**<br>human-machine positions. In summary, the fitness function<br>can be described in detail as follows.<br> $\min f = \frac{1}{N} \sum_{i=1}^{N} (q_{Mi} - q_{Hi})^2$  (33)<br>The outcome of Equation (33) demonstrated that the<br>opt **al of Applied Mathematics**<br>human-machine positions. In summary, the fitness function<br>can be described in detail as follows.<br> $\min f = \frac{1}{N} \sum_{i=1}^{N} (q_{Mi} - q_{Hi})^2$  (33)<br>The outcome of Equation (33) demonstrated that the<br>opti



Fig. 5. The block diagram of the BAS-PID feedback control system.<br>
Fig. 5. The block diagram of the BAS-PID feedback control system.<br>
Fig. 5. The block diagram of the BAS-PID feedback control system.<br>
For the BAS algorith The specifically, the dimensions of the search space is indicated<br>in the dimension of the searching process is divided<br>into four steps, such as parameter initialization, direction<br>updating, position updating, and other pa as *<sup>n</sup>*, the initial step size is described as *<sup>δ</sup>*0, the maximum T<sub>a</sub>  $+$   $\bigcup_{\text{Controller}}$  For Motor  $\bigcup_{\text{Pymanic}}$  For Motor  $\bigcup_{\text{Pymanic}}$  For the BAS algorithm, the searching process is divided into four steps, such as parameter initialization, direction updating, position updating, and o Fig. 5. The block diagram of the BAS-PID feedback control system.<br>
Fig. 5. The block diagram of the BAS-PID feedback control system.<br>
For the BAS algorithm, the searching process is divided<br>
into four steps, such as param Fig. 5. The block diagram of the BAS-PID feedback control system.<br>
Fig. 5. The block diagram of the BAS-PID feedback control system.<br>
For the BAS algorithm, the searching process is divided<br>
into four steps, such as param Fig. 5. The block diagram of the BAS-PID feedback control system.<br>
Fig. 5. The block diagram of the BAS-PID feedback control system.<br>
For the BAS algorithm, the searching process is divided<br>
into four steps, such as param *Firstly,* some basic parameters should be initialized.<br>Specifically, the dimensions of the search space is indicated<br>as *n*, the initial step size is described as  $\delta_0$ , the maximum<br>number of iterations is written as *K s n*, the initial step size is described as  $\delta_0$ , the maximum<br>umber of iterations is written as *K*, the initial search distance<br>s defined as  $d_0$ , and the fitness function is expressed as  $f()$ .<br>he second step is to

$$
\begin{cases} x_r = x^i + d^i b \\ x_l = x^i - d^i b \end{cases}
$$
 (34)

where  $x^i$  is the optimization target at the *i-th* iteration.  $x_r$  $d<sup>i</sup>$  represents the searching distance at the *i-th* iteration, and *b* number of iterations is written as *K*, the initial search distance<br>is defined as  $d_0$ , and the fitness function is expressed as  $f()$ .<br>The second step is to update the directions of the two<br>antennae, which can be describ as *K*, the initial search distance<br>
is function is expressed as *f*().<br>
i.e the directions of the two<br>
ord as follows.<br>  $x^{i} + d^{i}b$  (34)<br>  $x^{i} - d^{i}b$  (34)<br>
i.target at the *i*-th iteration.  $x_r$ <br>
is of the right and le

$$
\begin{cases}\n x^{i} = [K_{p}^{i} & K_{d}^{i}] \\
 b = \frac{rnd(n,1)}{\|rnd(n,1)\|}\n\end{cases}
$$
\n(35)

where  $K_p^i$ ,  $K_i^i$  and  $K_d^i$  are the optimized *i* are the *i*-*th* iteration.  $x_r$ <br>
ization target at the *i*-*th* iteration,  $x_r$ <br>
ositions of the right and left antennae,<br>
aper,  $x^i$  and *b* can be specified as:<br>  $=[K_p^i \t K_i^i \t K_d^i]$ <br>  $=\frac{rnd(n,1)}{Vert(rn,1)||}$  (35)<br>  $\downarrow$ where  $x^i$  is the optimization target at the *i*-*th* iteration.  $x_r$ <br>and  $x_l$  are severally the positions of the right and left antennae,<br>*d'* represents the searching distance at the *i*-*th* iteration, and *b*<br>is a un where  $x^i$  is the optimization target at the *i*-*th* iteration.  $x_r$ <br>and  $x_l$  are severally the positions of the right and left antennae,<br>*d'* represents the searching distance at the *i*-*th* iteration, and *b*<br>is a un and  $x_i$  are severally the positions of the right and left antennae,<br>
d' represents the searching distance at the *i*-*th* iteration, and *b*<br>
is a unit vector. In this paper,  $x^i$  and *b* can be specified as:<br>  $\begin{cases} x^i$ d' represents the searching distance at the *i*-*th* iteration, and *b*<br>is a unit vector. In this paper,  $x^i$  and *b* can be specified as:<br> $\begin{cases} x^i = [K_p^i \quad K_i^i \quad K_d^i] \\ b = \frac{rnd(n,1)}{||rnd(n,1)||} \end{cases}$  (35)<br>where  $K_p^i$ ,  $K_i^j$  a  $i \int_{p}^{i} K_{p}^{i} K_{p}^{i}$  and  $K_{d}^{j}$  are the optimized PID gains at the<br>on. *rnd*() is a function to produce a random value<br>om [0, 1], which obeys the uniform distribution.<br>step is to establish the iterative model and where  $K_p^i$ ,  $K_i^i$  and  $K_d^j$  are the optimized PID gains at the *i*-*th* iteration. *rnd*() is a function to produce a random value ranging from [0, 1], which obeys the uniform distribution. The third step is to establ where  $K_p^i$ ,  $K_i^j$  and  $K_d^j$  are the optimized PID gains at the *i*-*th* iteration. *rnd*() is a function to produce a random value ranging from [0, 1], which obeys the uniform distribution. The third step is to establ

$$
x^{i+1} = \begin{cases} x^i - \delta^i \cdot b, f(x_r^i) > f(x_l^i) \\ x^i + \delta^i \cdot b, f(x_r^i) \le f(x_l^i) \end{cases}
$$
 (36)

where  $\delta^i$  indicates the step size at the *i*-th iteration, which is update the other parameters, such as  $\delta^i$  and  $d^i$ . *<sup>i</sup>*.

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$$
\begin{cases}\n d^{i+1} = c_1 d^i + c_2 & \text{parameters} \\
 \delta^{i+1} = c_3 \delta^i & \text{intess defi} \\
 \text{and } \delta \text{ show}\n\end{cases}
$$

**IAENG International Journal of Applied Mathematic<br>**  $\begin{cases}\nd^{i+1} = c_1 d^i + c_2 \\
\delta^{i+1} = c_3 \delta^i\n\end{cases}$  **parameters in** *x* **defined in Equation (33).<br>**  $\delta^{i+1} = c_3 \delta^i$  **(37) and**  $\delta$  **should be updated in order the optimal gains IAENG International Journal of Applied Mathematics**<br>  $\begin{cases}\nd^{i+1} = c_1 d^i + c_2 \\
\delta^{i+1} = c_3 \delta^i\n\end{cases}$  parameters in *x* defined in Equation (33).<br>
Then sa do should be updated in order<br>
where  $c_1$ ,  $c_2$  and  $c_3$  are con **IAENG International Journal of Applied Mathematics**<br>  $d^{i+1} = c_1 d^i + c_2$  parameters in *x* defined in Equation (33). C<br>  $s^{i+1} = c_3 s^i$  (37) fitness defined in Equation (33). And the optimization process is<br>  $c_1 = c_2 = 0.$ **IAENG International Journal of Applied Mathemation**<br>  $\begin{cases}\na^{t+1} = c_1 a^i + c_2$  parameters in *x* defined in Equation (33)<br>  $\delta^{t+1} = c_3 \delta^i$  (37) finess defined in Equation (33)<br>
where *c*<sub>1</sub>, *c*<sub>2</sub> and *c*<sub>3</sub> are consta **IAENG International Journal of Applied Mathematic<br>**  $\begin{cases}\n d^{i+1} = c_1 d^i + c_2 \\
 \delta^{i+1} = c_3 \delta^i\n \end{cases}$ **Then the equation (33).<br>
The same of the initial conditions of the optimal gains are obtained.<br>
Where**  $c_1$ **,**  $c_2$  **and c EXERTS INTERTATION INTERTATION INTERTATION AND SURFATHEM VALUE IN A SURFATHEM (37)** and parameters in *x* defined in Equation (33), and  $\delta$  should be updated in order the optimal gains are obtained.<br>
where  $c_1$ ,  $c_2$   $a^{\int d^{i+1} = c_1 d^i + c_2}$  (37) fitness defined in Equation (33),<br>  $\delta^{i+1} = c_3 \delta^i$  (37) fitness defined in Equation (33),<br>
where  $c_1$ ,  $c_2$  and  $c_3$  are constant coefficients to realize the<br>
updating in Equation (37),  $\begin{cases} d^{i+1} = c_1 d^i + c_2 \\ \delta^{i+1} = c_3 \delta^i \end{cases}$  (37)<br>where  $c_1$ ,  $c_2$  and  $c_3$  are constant coefficients to realize the<br>updating in Equation (37), and their settings are listed as<br> $c_1 = c_3 = 0.95$ , and  $c_2 = 0.01$ . The wh  $c_1^{i+1} = c_1d^i + c_2$  (37) parameters in *x* defined in Equation (3<br>  $\frac{1}{2}$ ,  $\frac{1}{2}$  $\begin{cases}\n d^{i+1} = c_1 d^i + c_2\n \end{cases}$ (37) parame<br>
the optimum of the set of the optimization (37), and their settings are listed as<br>  $c_1 = c_3 = 0.95$ , and  $c_2 = 0.01$ . The whole optimization process is<br>
the optimization coeffic

inputs should be given, and we define that UB and LB are the upper and low limits of 
$$
x^i
$$
, respectively. Thus, the initial  
\nupper and low limits of  $x^i$ , respectively. Thus, the initial  
\ncondition could be expressed as follows.

\n1CMIC mapping is a chaotic m  
\nof mapping folds, offering the  
\nand rapid convergence. The  
\nICMIC mapping *b* value can be

\n1CMIC mapping *c* and *c* value can be

\n1CMIC mapping *c* and *c* value.

\n1-*i* max

\n1-*i* max

\n2

\n38

\n40° =  $\frac{UB + LB}{2}$ 

\n410° =  $\frac{UB + LB}{2}$ 

\n54

\n62

\n73

\n83

\n94

\n1-*i* max

\n1

*<sup>i</sup>*.





parameters in  $x$  defined in Equation (36), and the output is the fitness defined in Equation (33). Concurrently, the values of  $d$ **al of Applied Mathematics**<br>parameters in *x* defined in Equation (36), and the output is the<br>fitness defined in Equation (33). Concurrently, the values of *d*<br>and  $\delta$  should be updated in order to adjust the input. Fina **al of Applied Mathematics**<br>parameters in *x* defined in Equation (36), and the output is the<br>fitness defined in Equation (33). Concurrently, the values of *d*<br>and  $\delta$  should be updated in order to adjust the input. Fina **and Solution Mathematics**<br>parameters in *x* defined in Equation (36), and the output is the<br>fitness defined in Equation (33). Concurrently, the values of *d*<br>and  $\delta$  should be updated in order to adjust the input. Final **al of Applied Mathematics**<br>parameters in *x* defined in Equation (36), and the output is the<br>fitness defined in Equation (33). Concurrently, the values of<br>and  $\delta$  should be updated in order to adjust the input. Finally<br> **Example 10 CAPPIDE Mathematics**<br>
parameters in x defined in Equation (36), and the output is the<br>
fitness defined in Equation (33). Concurrently, the values of d<br>
and  $\delta$  should be updated in order to adjust the input

ICMIC mapping *b* value can be described as follows. **In order to end in the performance of the performance the performance the performance of the BAS algorithm, be improved BAS algorithm, in order al of Applied Mathematics**<br>parameters in *x* defined in Equation (36), and the output is the<br>fitness defined in Equation (33). Concurrently, the values of *d*<br>and  $\delta$  should be updated in order to adjust the input. Fina **al of Applied Mathematics**<br>parameters in *x* defined in Equation (36), and the output is the<br>fitness defined in Equation (33). Concurrently, the values of *d*<br>and  $\delta$  should be updated in order to adjust the input. Fina **Example Function** Function is a defined in Equation (36), and the output is the fitness defined in Equation (33). Concurrently, the values of *d* and  $\delta$  should be updated in order to adjust the input. Finally, the opti parameters in *x* defined in Equation (36), and the output is the fitness defined in Equation (33). Concurrently, the values of *d* and  $\delta$  should be updated in order to adjust the input. Finally, the optimal gains are o parameters in *x* defined in Equation (36), and the output is the fitness defined in Equation (33). Concurrently, the values of *d* and  $\delta$  should be updated in order to adjust the input. Finally, the optimal gains are o fitness defined in Equation (33). Concurrently, the values of *d* and  $\delta$  should be updated in order to adjust the input. Finally, the optimal gains are obtained.<br> *C. Design of IBAS-PID Controller* In order to enhance t and  $\delta$  should be updated in order to adjust the input. Finally,<br>the optimal gains are obtained.<br>C. Design of IBAS-PID Controller<br>In order to enhance the performance of the BAS algorithm,<br>the improved BAS algorithm (IBAS the optimal gains are obtained.<br> *C. Design of IBAS-PID Controller*<br>
In order to enhance the performance of the BAS algorithm,<br>
the improved BAS algorithm (IBAS) is developed by<br>
incorporating the following enhancements. corporating the rollowing enhancements. In Equation (35),<br>
r andom function is selected to generate the value for *b*.<br>
owever, the iterative chaotic map with infinite collapses<br>
CMIC) method is used to produce the *b* pa the random function is selected to generate the value for *b*.<br>However, the iterative chaotic map with infinite collapses<br>(ICMIC) method is used to produce the *b* parameter. The<br>ICMIC mapping is a chaotic model with an i However, the iterative chaotic map with infinite collapses<br>(ICMIC) method is used to produce the *b* parameter. The<br>ICMIC mapping is a chaotic model with an infinite number<br>of mapping folds, offering the benefits of unifo

$$
b^{i+1} = \sin(\frac{a_x \pi}{b^i})
$$
 (39)

from zero to positive infinity. Meanwhile, the step size  $\delta^i$  in

$$
\delta^i = \frac{(\delta_{\text{max}} - \delta_{\text{min}})(i - i_{\text{max}})}{1 - i_{\text{max}}} + \delta_{\text{min}}
$$
(40)

 $b^{i+1} = \sin(\frac{a_x \pi}{b^i})$  (39)<br>where  $a_x$  is a fixed parameter, and has a range extending<br>om zero to positive infinity. Meanwhile, the step size  $\delta^i$  in<br>quation (37) is rewritten in the following.<br> $\delta^i = \frac{(\delta_{\text{max}} - \delta_{\text{$  $b^{i+1} = \sin(\frac{a_x \pi}{b^i})$  (39)<br>where  $a_x$  is a fixed parameter, and has a range extending<br>from zero to positive infinity. Meanwhile, the step size  $\delta^i$  in<br>Equation (37) is rewritten in the following.<br> $\delta^i = \frac{(\delta_{\text{max}} - \delta_{$  $b^{111} = \sin(\frac{x}{b^i})$  (39)<br>
where  $a_x$  is a fixed parameter, and has a range extending<br>
from zero to positive infinity. Meanwhile, the step size  $\delta^i$  in<br>
Equation (37) is rewritten in the following.<br>  $\delta^i = \frac{(\delta_{\text{max}} - \delta_{$ where  $a_x$  is a fixed parameter, and has a range extending<br>from zero to positive infinity. Meanwhile, the step size  $\delta^i$  in<br>Equation (37) is rewritten in the following.<br> $\delta^i = \frac{(\delta_{\text{max}} - \delta_{\text{min}})(i - i_{\text{max}})}{1 - i_{\text{max}}} + \delta$ barancett, and has a range extending<br>
in finity. Meanwhile, the step size  $\delta^i$  in<br>
en in the following.<br>  $\delta_{min}$   $\left(i - i_{max}\right)$  +  $\delta_{min}$  (40)<br>  $-i_{max}$ <br>
estrict the range of  $\delta$  value assigned to<br>
the  $\delta$  value is decrea *A. Parameter Selection*<br>*A. Parameter Selection*<br>*A. Parameter Selection*<br>*A. Parameter Selection*<br>*A. Parameter Selection*<br>*A. Parameter Selection*<br>*A. Parameter Selection*<br>*We* established the dynamics of robotic arm a

(altronomyon)  $\delta^i = \frac{(\delta_{\text{max}} - \delta_{\text{min}})(i - i_{\text{max}})}{1 - i_{\text{max}}} + \delta_{\text{min}}$  (40)<br>
where  $\delta_{\text{max}}$  and  $\delta_{\text{min}}$  restrict the range of  $\delta$  value assigned to<br>
novel step factor. The  $\delta$  value is decreasing which could<br>
quir  $\delta^i = \frac{(\delta_{\text{max}} - \delta_{\text{min}})(i - i_{\text{max}})}{1 - i_{\text{max}}} + \delta_{\text{min}}$  (40)<br>
where  $\delta_{\text{max}}$  and  $\delta_{\text{min}}$  restrict the range of  $\delta$  value assigned to<br>
the novel step factor. The  $\delta$  value is decreasing which could<br>
acquire more e  $\delta^i = \frac{(O_{\text{max}} - O_{\text{min}})(t - t_{\text{max}})}{1 - i_{\text{max}}} + \delta_{\text{min}}$  (40)<br>
where  $\delta_{\text{max}}$  and  $\delta_{\text{min}}$  restrict the range of  $\delta$  value assigned to<br>
the novel step factor. The  $\delta$  value is decreasing which could<br>
acquire more ext 1-*i*<sub>max</sub> mand<br>where  $\delta_{max}$  and  $\delta_{min}$  restrict the range of  $\delta$  value assigned to<br>the novel step factor. The  $\delta$  value is decreasing which could<br>acquire more extensive search scope in the early stages, and<br>implement where  $\delta_{max}$  and  $\delta_{min}$  restrict the range of  $\delta$  value assigned to<br>the novel step factor. The  $\delta$  value is decreasing which could<br>acquire more extensive search scope in the early stages, and<br>implement more precise se where  $\delta_{max}$  and  $\delta_{min}$  restrict the range of  $\delta$  value assigned to<br>the novel step factor. The  $\delta$  value is decreasing which could<br>acquire more extensive search scope in the early stages, and<br>implement more precise se the novel step factor. The  $\delta$  value is decreasing which could<br>acquire more extensive search scope in the early stages, and<br>implement more precise search in the later stages.<br>*K*. *Parameter Selection*<br>We established the acquire more extensive search scope in the early stages, and<br>implement more precise search in the later stages.<br>IV. RESULTS<br>A. Parameter Selection<br>We established the dynamics of robotic arm and DC motor<br>with many variable implement more precise search in the later stages.<br>
IV. RESULTS<br>
A. Parameter Selection<br>
We established the dynamics of robotic arm and DC motor<br>
with many variables and parameters. Some of the parameters<br>
are selected to IV. RESULTS<br> *A. Parameter Selection*<br>
We established the dynamics of robotic arm and DC motor<br>
with many variables and parameters. Some of the parameters<br>
are selected to be optimized, while the others should be<br> *APID* gains. As a result, the best parameters are figured out that  $K_p=30.5$ ,  $K_i=1.2$ , and  $K_d=0.98$  for shoulder joint, and  $K_p=25.6$ ,  $K_i=1.26$ , and  $K_d=0.118$  for elbow joint. The LB and UB are given that LB = [0.01 0.01 0. FID controller, the Z-IN include is chosen to acquire the FID<br>gains. As a result, the best parameters are figured out that<br> $K_p=30.5$ ,  $K_f=1.2$ , and  $K_d=0.98$  for shoulder joint, and  $K_p=25.6$ ,<br> $K_f=1.26$ , and  $K_d=0.118$  fo



**IAENG International**<br>*B. Comparison Results*<br>The experiments were carried out to prove the super<br>of the proposed IBAS algorithm by comparing it with<br>optimization algorithms, such as PSO, GA and the ori<br>RAS The PID gains a **THENG International Journal of Applied Mathematics**<br>
The experiments were carried out to prove the superiority<br>
the proposed IBAS algorithm by comparing it with other<br>
timization algorithms, such as PSO, GA and the origi **IAENG International Journal of Applied Mathemat**<br> *B. Comparison Results*<br>
The experiments were carried out to prove the superiority<br>
of the proposed IBAS algorithm by comparing it with other<br>
optimization algorithms, su **IAENG International Journal of Applied Mathem**<br> *B. Comparison Results*<br>
The experiments were carried out to prove the superiority<br>
of the proposed IBAS algorithm by comparing it with other<br>
optimization algorithms, such **EXERC International Journal of Applied Mathemateurs**<br>
B. Comparison Results<br>
The experiments were carried out to prove the superiority<br>
of the proposed IBAS algorithm by comparing it with other<br>
optimization algorithms, **IAENG International Journal of Applied Mathema**<br> *B. Comparison Results*<br>
The experiments were carried out to prove the superiority<br>
of the proposed IBAS algorithm by comparing it with other<br>
optimization algorithms, suc **Example 12**<br> **Example 10**<br> **Example 10 EVALUATE SET ASSET ASSET ASSET ASSETTED**<br> **EVALUATE ASSET AND THE EXPRESENTATION OF the proposed IBAS algorithm by comparing it with other optimization al 14 IABNG International Journal of Applied Mathema**<br>
18. Comparison Results<br>
The experiments were carried out to prove the superiority<br>
of the proposed IBAS algorithm by comparing it with other<br>
optimization algorithms, s FING INTERNATIONAL JOUTHAI OF APPIDED MATHEMA<br>
The experiments were carried out to prove the superiority<br>
of the proposed IBAS algorithm by comparing it with other<br>
optimization algorithms, such as PSO, GA and the origina B. Comparison Results<br>
The experiments were carried out to prove the superiority<br>
of the proposed IBAS algorithm by comparing it with other<br>
optimization algorithms, such as PSO, GA and the original<br>
IBAS. The PID gains a *B. Comparison Results*<br>
The experiments were carried out to prove the superiority<br>
of the proposed IBAS algorithm by comparing it with other<br>
optimization algorithms, such as PSO, GA and the original<br>
BAS. The PID gains B. Comparison Results<br>
The experiments were carried out to prove the superiority<br>
of the proposed IBAS algorithm by comparing it with other<br>
optimization algorithms, such as PSO, GA and the original<br>
BAS. The PID gains af The experiments were carried out to prove the superiority<br>of the proposed IBAS algorithm by comparing it with other<br>optimization algorithms, such as PSO, GA and the original<br>BAS. The PID gians after optimization are descr of the proposed IBAS algorithm by comparing it with other<br>optimization algorithms, such as PSO, GA and the original<br>BAS. The PID gains after optimization are described in Table<br>II. The maximum iteration is set to be 100, optimization algorithms, such as PSO, GA and the original<br>
BAS. The PID gains after optimization are described in Table<br>
II. The maximum iteration is set to be 100, and the fitness<br>
results of four methods are shown in Fi BAS. The PID gains after optimization are described in Table  $\frac{1}{2}$ <br>
II. The maximum iteration is set to be 100, and the fitness<br>
results of four methods are shown in Fig. 7. The performance<br>
evaluation is inversely pr II. The maximum iteration is set to be 100, and the results of four methods are shown in Fig. 7. The percellulation is inversely proportional to the fitness value. The evaluation is inversely proportional to the fitness v SO-PID requires approximately 73 iterations for<br>
1. The proposed IBAS-PID shows only a little<br>
trmance than the original BAS-PID which takes<br>
s to reach the stable goal. However, the GA-PID<br>
0 iterations for stabilization





other side, the GA-PID and normal-PID algorithm achieve The COLANTITATIVE COMPAR<br>
The COLANTITATIVE COMPAR<br>
Tig. 7. Fitness of four methods after 100 iterations.<br>
The Specifically, the proposed BAS-PID algorithm requires the introducer of the BAS-PID and the control performance **EVALUATE CONFERCEMENT CONFERCEMENT CONFERCED (1.48)**<br>
Fig. 7. Fitness of four methods after 100 iterations.<br>
Fig. 7. Fitness of four methods after 100 iterations.<br>
Fig. 7. Fitness of four methods after 100 iterations.<br>
A **Example 1988**<br> **Parts Methods CONSTITATIVE CONSTRAINTS (CONSTRAINTS)**<br>
Fig. 7. Fitness of four methods after 100 iterations.<br>
As pictured in Fig. 8, the step response is selected to BAS-PID at<br>
evaluate the control perfo **Parts Methods**<br>
Fig. 7. Fitness of four methods after 100 iteration<br>
Fig. 7. Fitness of four methods after 100 iterations.<br>
As pictured in Fig. 8, the step response is selected to<br>
evaluate the control performance. Speci  $\frac{1}{6}$  and  $\frac{1}{20}$  and  $\frac{1}{30}$  and  $\frac{1}{30}$  in the interactions.<br>
Fig. 7. Fitness of four methods after 100 iterations.<br>
Example 100 iterations.<br>
Example 100 iterations.<br>
Example 100 iterations.<br>
EXAS-PID algor Fig. 7. Fitness of four methods after 100 iterations.<br>
As pictured in Fig. 8, the step response is selected to<br>
evaluate the control performance. Specifically, the proposed<br>
IBAS-PID algorithm gains the lowest rise time, From the least overshoot. The BRS-PID algorithm acquires the control performance. Specifically, the proposed is selected to the BRS-PID algorithm acquires the biggest rise time. On the GA-PID algorithm acquires the biggest As pictured in Fig. 8, the step response is selected to<br>
BAS-PID algorithm gains the lowest rise time, while the<br>
normal-PID algorithm acquires the biggest rise time, while the<br>
normal-PID algorithm acquires the biggest r evaluate the control performance. Specifically, the proposed<br>
IBAS-PID algorithm agains the lowest rise time, while the<br>
normal-PID algorithm acquires the biggest rise time. On the<br>
cold-PID and normal-PID algorithm calies TBAS-PID algorithm gains the lowest rise time, while the<br>normal-PID algorithm acquires the biggest rise time. On the<br>other side, the GA-PID and normal-PID algorithm achieve<br>Poolen SAS-PID algorithm calizes the<br>biggest over normal-PID algorithm acquires the biggest rise time. On the<br>
other side, the GA-PID and normal-PID algorithm achieve<br>
zero overshoot, while the BAS-PID algorithm realizes the<br>
binks<br>
binks<br>
linked and affected, the paramet other side, the GA-PID and normal-PID algorithm achieve<br>zero overshoot, while the BAS-PID algorithm realizes the<br>biggest overshoot. As the shoulder and elbow joints are<br>linked and affected, the parameters would show differ







both BAS-PID 9.15% 0.045 0.0226 0.0611<br>
BAS-PID 8.26% 0.041 0.0248 0.0607<br>
Normal-PID 41% 0.048 0.0504 0.1176<br>
Elbow GA-PID 38% 0.038 0.0416 0.0955<br>
Joints PSO-PID 34% 0.042 0.0450 0.1082<br>
BAS-PID 42% 0.032 0.0354 0.0932<br> BAS-PID 8.26% 0.041 0.0248 0.0607<br>
Normal-PID 41% 0.048 0.0504 0.1176<br>
Elbow GA-PID 38% 0.038 0.0416 0.0955<br>
Joints PSO-PID 34% 0.042 0.0450 0.1082<br>
BAS-PID 42% 0.032 0.0354 0.0932<br>
BAS-PID 42% 0.021 0.0338 0.0897<br>
As des Normal-PID 41% 0.048 0.0504 0.1176<br>
Elbow GA-PID 38% 0.038 0.0416 0.0955<br>
Joints PSO-PID 34% 0.042 0.0450 0.1082<br>
BAS-PID 42% 0.032 0.0354 0.0932<br>
IBAS-PID 39% 0.021 0.0338 0.0897<br>
As described in Table III, the normal-PID Elbow GA-PID 38% 0.038 0.0416 0.0955<br>
PSO-PID 34% 0.042 0.0450 0.1082<br>
BAS-PID 42% 0.032 0.0354 0.0932<br>
BAS-PID 39% 0.021 0.0338 0.0897<br>
As described in Table III, the normal-PID controller<br>
exhibits zero overshoot, but g Joints BSO-PID 34% 0.042 0.0450 0.1082<br>BAS-PID 42% 0.032 0.0354 0.0932<br>BAS-PID 39% 0.021 0.0338 0.0897<br>As described in Table III, the normal-PID controller<br>exhibits zero overshoot, but gains the greatest rise time<br>(0.13s),

**IAENG International Journal of Applied Mathemati**<br>from the metric of overshoot, the proposed IBAS-PID comparison results show that the<br>controller gains the least values (i.e., rise time is 0.021 s, gains the best performa **IAENG International Journal of Applied Mathematics**<br>from the metric of overshoot, the proposed IBAS-PID comparison results show that the controller gains the least values (i.e., rise time is 0.021 s, gains the best perfo IAENG International Journal of Applied Mathemat<br>
from the metric of overshoot, the proposed IBAS-PID comparison results show that<br>
controller gains the least values (i.e., rise time is 0.021 s, gains the best performance i **IAENG International Journal of Applied Mather**<br>from the metric of overshoot, the proposed IBAS-PID comparison results show t<br>controller gains the least values (i.e., rise time is 0.021 s, gains the best performan<br>MAE is **IAENG International Journal of Applied Mathemat**<br>from the metric of overshoot, the proposed IBAS-PID comparison results show that<br>controller gains the least values (i.e., rise time is 0.021 s, gains the best performance **IAENG International Journal of**<br>from the metric of overshoot, the proposed IBAS-PID con<br>controller gains the least values (i.e., rise time is 0.021 s, gaint<br>MAE is 0.0338 rad and RMSE is 0.0897 rad) in other three<br>metric **SUBSE INTERT SUBSET IN THE SUBSET OF A SUBSET AND THE SUBSET ON A SUBSET AND THE PARALLET SUBSET ARE ALLES (i.e., rise time is 0.021 s, gains the best performance ALLES in the separation and RMSE is 0.0897 rad) in other IAENG International Journal of Applied Mathematic**<br>from the metric of overshoot, the proposed IBAS-PID comparison results show that the<br>controller gains the least values (i.e., rise time is 0.021 s, gains the best perfor

**IAENG International Journal of Applied Mathen**<br>from the metric of overshoot, the proposed IBAS-PID comparison results show the<br>controller gains the least values (i.e., rise time is 0.021 s, gains the best performance<br>MAE **THETNG INTERTATION JOUTHAT OF Applied MAD**<br>from the metric of overshoot, the proposed IBAS-PID comparison results sho<br>controller gains the least values (i.e., rise time is 0.021 s, gains the best performed<br>MAE is 0.0338 from the metric of overshoot, the proposed IBAS-PID comparison results show that the controller gains the least values (i.e., rise time is 0.021 s, gains the best performance in MAE is 0.0338 rad and RMSE is 0.0897 rad) i from the metric of overshoot, the proposed IBAS-PID comparison results show that the protontoller gains the least values (i.e., rise time is 0.021 s, gains the best performance in the MAE is 0.0338 rad and RMSE is 0.0897 from the metric of overshoot, the proposed IBAS-PID comparison results show that<br>controller gains the least values (i.e., rise time is 0.021 s, gains the best performance<br>MAE is 0.0338 rad and RMSE is 0.0897 rad) in other controller gains the least values (i.e., rise time is 0.021 s, gains the best performance MAE is 0.0338 rad and RMSE is 0.0897 rad) in other three tuning PID gains.<br>
metrics. The experimental results show that the propose MAE is 0.0338 rad and RMSE is 0.0897 rad) in other three tuning PID gains.<br>
metrics. The experimental results show that the proposed<br>
method gains the most optimal performance for the robot<br>
control compared with other me metrics. The experimental results show that the proposed<br>
method gains the most optimal performance for the robot<br>
control compared with other methods.<br>
Subsequently, the tracking effect of sinusoidal signal<br>
tracking was method gains the most optimal performance for the robot<br>
control compared with other methods.<br>
Subsequently, the tracking effect of sinusoidal signal<br>
tracking was also tested. As depriced in Fig. 10, all the<br>
methods show closely.



From the most optimal control performance of the most optimal control is selected<br>
and and shows the most optimal control performance of the most optimal control performance.<br>
The most optimal control performance for the  $\frac{2}{3}$  of  $\frac{2}{3}$  of  $\frac{1}{3}$  and show angle (Sommal PID)<br>  $\frac{1}{3}$  and  $\$ From the most closely (Namal PID)<br>
Absolute (NAME)<br>
Absolute (NAPID)<br>
IGNOV GA-PID 0.023<br>
Fig. FROM THE TRISTANT CONTECT THE FRO-PID CROW Angle (BAS-PID)<br>
FRO-PID BAS-PID 0.028<br>
Fig. 10. Comparison of sinusoidal response for shoulder joint<br>
Figure 11 demonstrates the tracking performance for the<br>
Figure 11 demonstrates the tracking performance  $^{1.5}$   $^{1.5}$   $^{0.2}$   $^{0.4}$   $^{0.6}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.8}$   $^{0.9}$   $^{0.9}$   $^{0.9}$   $^{0.9}$   $^{0.9}$   $^{0.9}$   $^{0.9}$   $^{0.9$ <sup>26</sup> 0<sup>2</sup> 04 06 <sup>08</sup> time(s)<sup>12</sup> <sup>14</sup> <sup>1.6</sup> <sup>1.8</sup> <sup>2</sup> **IBAS-PID** 0.<br>
10. 10. Comparison of sinusoidal response for shoulder joint<br>
Figure 11 demonstrates the tracking performance for the A mathematical and composed IBAS-P Fig. 10. Comparison of sinusoidal response for shoulder joint<br>
Figure 11 demonstrates the tracking performance for the<br>
elbow joint. From the amplifying part, the robot angle from<br>
the proposed IBAS-PID control is the clos Figure 11 demonstrates the tracking performance for the<br>
and RMSE is 0.025<br>
RMSE is 0.025 rad, and Explorectic in the selected of<br>
the proposed IBAS-PID control is the closest to the human<br>
been successfully constructed,<br> Figure 11 demonstrates the tracking performance for the<br>
elbow joint. From the amplifying part, the robot angle from<br>
the proposed IBAS-PID control is the closest to the human<br>
been successfully constructe<br>
angle, and sho

elbow joint. From the amplifying part, the robot angle from upper exoskeleton robc<br>the proposed IBAS-PID control is the closest to the human been successfully con<br>angle, and shows the most optimal control performance. PID the proposed IBAS-PID control is the closest to the human<br>molecular postical angle, and shows the most optimal control performance. PID control is selected to c<br>However, at most of the time, the robot angle tracks the gain angle, and shows the most optimal control performance. PID control is selected to cont<br>However, at most of the time, the robot angle tracks the gains are optimized through th<br>human angle closely for all methods. Obviously, However, at most of the time, the robot angle tracks the gains are optimized through th<br>human angle closely for all methods. Obviously, it is hard to Specifically, the ICMIC meth<br>evaluate the control performance precisely. human angle closely for all methods. Obviously, it is hard to Specifically, the ICMIC mevaluate the control performance precisely. The specific and random value, which would quantitative results are figured out and reporte evaluate the control performance precisely. The specific and<br>
quantitative results are figured out and reported in Table IV<br>
accelerate the convergence. A<br>
As described in Table IV, the normal-PID controller gains<br>
more pr

**and of Applied Mathematics**<br>
comparison results show that the proposed IBAS-PID control<br>
gains the best performance in the optimization process of<br>
tuning PID gains. **and of Applied Mathematics**<br>
comparison results show that the proposed IBAS-PID control<br>
gains the best performance in the optimization process of<br>
tuning PID gains.<br>  $0.3$  Human Angle<br>  $\bigotimes$  Robot Angle (Normal PID)





TABLE IV

FRID 0.010 0.021 0.63<br>
IBAS-PID 0.010 0.021 0.61<br>
Normal-PID 0.028 0.067 0.90<br>
Elbow GA-PID 0.028 0.067 0.90<br>
FIDOW GA-PID 0.028 0.068 0.76<br>
ISBAS-PID 0.023 0.068 0.76<br>
IBAS-PID 0.023 0.052 0.55<br>
IBAS-PID 0.023 0.052 0.55<br> Ebow GA-PID 0.009 0.021 0.61<br>
Normal-PID 0.028 0.067 0.90<br>
Ebow GA-PID 0.028 0.067 0.90<br>
Joints PSO-PID 0.028 0.068 0.76<br>
BAS-PID 0.024 0.054 0.57<br>
IBAS-PID 0.023 0.052 0.55<br>
V. CONCLUSION<br>
A mathematical and computational Normal-PID 0.028 0.067 0.90<br>
Elbow GA-PID 0.028 0.068 0.68<br>
PSO-PID 0.028 0.068 0.76<br>
BAS-PID 0.024 0.054 0.57<br>
IBAS-PID 0.023 0.052 0.55<br>
V. CONCLUSION<br>
A mathematical and computational model of the 2-DOF<br>
upper exoskelet Elbow GA-PID 0.025 0.058 0.68<br>
PSO-PID 0.028 0.068 0.76<br>
BAS-PID 0.024 0.054 0.57<br>
IBAS-PID 0.023 0.052 0.55<br>
V. CONCLUSION<br>
A mathematical and computational model of the 2-DOF<br>
upper exoskeleton robot, driven by direct cu FRIE 20028 0.068 0.76<br>
BAS-PID 0.028 0.068 0.76<br>
BAS-PID 0.024 0.054 0.57<br>
IBAS-PID 0.023 0.052 0.55<br>
V. CONCLUSION<br>
A mathematical and computational model of the 2-DOF<br>
upper exoskeleton robot, driven by direct current mo BAS-PID 0.024 0.054 0.57<br>
IBAS-PID 0.023 0.052 0.55<br>
V. CONCLUSION<br>
A mathematical and computational model of the 2-DOF<br>
upper exoskeleton robot, driven by direct current motors, has<br>
been successfully constructed, simulat BAS-PID 0.023 0.052 0.55 0.55<br>V. CONCLUSION<br>CONCLUSION<br>A mathematical and computational model of the 2-DOF<br>upper exoskeleton robot, driven by direct current motors, has<br>been successfully constructed, simulated, and tested. V. CONCLUSION<br>
A mathematical and computational model of the 2-DOF<br>
upper exoskeleton robot, driven by direct current motors, has<br>
been successfully constructed, simulated, and tested. The<br>
PID control is selected to contr V. CONCLUSION<br>A mathematical and computational model of the 2-DOF<br>upper exoskeleton robot, driven by direct current motors, has<br>been successfully constructed, simulated, and tested. The<br>PID control is selected to control t A mathematical and computational model of the 2-DOF<br>upper exoskeleton robot, driven by direct current motors, has<br>been successfully constructed, simulated, and tested. The<br>PID control is selected to control the whole syste upper exoskeleton robot, driven by direct current motors, has<br>been successfully constructed, simulated, and tested. The<br>PID control is selected to control the whole system, and its<br>gains are optimized through the proposed Final Controlline and Controlline and Controlline and accelerate the convergence. A novel step size is proposed to realize more extensive search scope in the early stages, and more precise search in the later stages. The c lerate the convergence. A novel step size is proposed to<br>ze more extensive search scope in the early stages, and<br>e precise search in the later stages. The control<br>ormance is tested in terms of step response and sinusoidal<br>

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