A Fractional-Order SEIDR Network Public Opinion Dissemination Prediction Model Considering the Heterogeneity of Susceptibilities and Dissuasion Mechanism

Qiujuan Tong, Xiaolong Xu, and Jianke Zhang

Abstract—**As the landscape and ecology of network public opinion become increasingly complex and dynamic, the demand for accuracy in the field of public opinion prediction is also increasing, requiring more refined prediction models to describe the trends of public opinion dissemination. This paper considers the heterogeneity among susceptible individuals and constructs a fractional-order SEIDR public opinion dissemination prediction model incorporating a dissuasion mechanism. The basic reproduction number of the new model is calculated, and its sensitivity is examined, along with equilibrium and stability analysis of the model. Finally, numerical simulations are conducted using actual popular opinion cases to verify the model's predictive ability. The simulation results indicate that the predictive capability of the new model is significantly improved compared to the integer-order model and traditional model and its applicability is also stronger. The conclusions of this paper provide a reference for the formulation of public opinion policies, suggesting that targeted intervention measures should be implemented for different susceptible groups. Strengthening social encouragement and propaganda to increase the proportion of persuaders can suppress the spread of false public perception.**

Index Terms—**Susceptibility heterogeneity, fractional order, SEIDR model, stability, public opinion prediction**

I. Introduction

IN the digital age, online public opinion dissemination has become a crucial aspect of social interaction and decision-making processes. Grasping the dynamics of how public opinion spreads is essential in various fields, including public health, politics, marketing, and social sciences [1]. Based on the 53rd Statistical Report on China's Internet Development [2], the number of internet users in China reached 1.092 billion by December 2023, resulting in an internet penetration rate of 77.5%. People are increasingly inclined to express opinions, share information, and participate in discussions online, facilitating the rapid diffusion of public perception in cyberspace. The dissemination of cyber public opinion significantly influences the ideologies, values, and behaviors of the populace. Through the dissemination of online public opinion, governments, institutions, and individuals can guide popular opinion, promote social harmony and stability, and drive social progress. However, the dissemination of online public opinion also poses certain risks. Sudden events, such as rumors and negative popular perception, if not handled properly, may affect social stability and public trust. For example, during the COVID-19 pandemic, false popular opinions such as "vinegar can kill the coronavirus" circulated online, threatening people's health and safety. Therefore, timely detection, analysis, and response to changes in online public opinion are crucial for mitigating public opinion risks and maintaining social stability.

The generation and evolution of online public opinion are influenced by various factors, including media reports, social networks, personal opinions, and government policies, forming a complex information ecosystem [3]. The dissemination of online public opinion is analogous to the transmission of infectious diseases. Since David G. Kendall initially suggested in his 1964 study the application of epidemic models to describe the spread of rumors and public perception [4], many scholars have employed these models to examine the mechanisms behind public opinion dissemination, proposing numerous new models and methods. Applying epidemic models to predict the dissemination of online popular perception provides a systematic and more objective perspective, aiding in understanding of the mechanism and dynamics of popular opinion spread in social networks. Infectious disease models have a rich research history spanning many years. As early as the early 20th century, Hamer, Ross, and others conducted extensive work in establishing deterministic mathematical models of infectious diseases. This effort culminated in 1927 when Kermack and McKendrick proposed the SIR (Susceptible-Infectious-Recovered) compartmental model to investigate the transmission of the Black Death in London,

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describing the diffusion process of infectious diseases in populations [5]. In the mid-20th century, the SEIR (Susceptible-Exposed-Infectious-Recovered) model came into existence, incorporating a latent period to describe the stage where infected individuals are exposed but have not yet infected others, enhancing the model's applicability [6]. These pioneering studies are of significant importance to the development of the field of public opinion dissemination. While traditional infectious disease models are now extensively employed to simulate the dissemination of online public opinion, they also exhibit certain limitations. For instance, it is challenging for these models to account for changes in population behavior, such as the implementation of public opinion policies and the adoption of personal protective measures, which can significantly impact popular opinion dissemination.

Nowadays, the landscape and ecology of public opinion are undergoing profound changes, with the spread and evolution of public opinion on social platforms are becoming increasingly complex, necessitating more sophisticated models to study the dissemination process. With continuous advancements in the study of infectious disease transmission processes, public opinion dissemination models are also continuously evolving and expanding. Researchers have proposed various variants and extended models that considering more realistic factors, such as population mobility, spatial dissemination, and intervention strategies, to better reflect the characteristics of different information dissemination. Factors considered include the opinion leader mechanism [7], forgetting and memory mechanisms [8], countermeasures [9], and interfering opinions [10]. The authors of [11] examined the phenomenon of public opinion reversal based on agent models, while [12] analyzed the causal relationship between popular perception and policy.

In recent years, influenced by the COVID-19 pandemic, scholars have increasingly shown enthusiasm for the field of cyber popular perception dissemination, focusing on enhancing the robustness, applicability, and predictive power of models. Scholars have proposed various models based on the differing situations of different countries facing the COVID-19 pandemic [13][14], providing significant insights into the study of online social perception dissemination. Moreover, an increasing number of scholars recognize that the heterogeneity within susceptible individuals, as an important link in public opinion dissemination, exerts a substantial influence on the transmission of popular perception. Some studies have begun to attempt profiling and categorizing susceptible individuals to more accurately describe the transmission process of diseases. The authors of [15] considered population heterogeneity by categorizing individuals into types such as families and workplaces, modeling susceptible and infected individuals for each type, and analyzing the impact of transmission between different types. [16] integrates opinion polarization with the transmission process, constructing a SEIR-JA integrated model that considers individual heterogeneity elements. The authors of [17] proposed a model that considers the heterogeneity of susceptible individuals in dynamic contact networks, classifying susceptible individuals and analyzing epidemic thresholds under varying heterogeneity conditions. The authors of [18] discussed the importance of considering heterogeneity among susceptible individuals in dynamic models and proposed a dynamic SIR model to characterize the infective process of respiratory diseases. The authors of [19] proposed a bimodal SITR model in the context of COVID-19, incorporating a category of susceptible individuals who are elderly or have other illnesses.

The linear characteristics of traditional SEIR and other public opinion prediction models limit their ability to adequately describe nonlinear dissemination processes, such as large-scale media exposure or information explosions triggered by sudden events. Additionally, the models have the drawback of homogenizing the population. For instance, traditional SEIR models assume that everyone's behaviors and responses are identical; whereas in reality, individuals' behaviors and reactions may vary. For example, some individuals may be more prone to believing rumors, while others may approach information more rationally. This variability is not adequately considered in the models.

This paper aims to address the aforementioned deficiencies by proposing a fractional-order SEIDR network public opinion dissemination prediction model that considers the heterogeneity of susceptible individuals. Fractional-order differential equations exhibit nonlocality and nonlinearity, such as memory effects and long-tail effects. Fractional-order differential equations can modify system behavior and enhance prediction accuracy by adjusting the order α, in contrast to integer-order differential equations. They offer richer descriptive capabilities and broader applicability, allowing for more accurate capture of the dynamic characteristics of complex systems. Therefore, we chose fractional-order differential equations to model the system. The consideration of susceptible individuals' heterogeneity involves classifying them and taking into account the attribute structure of populations in social networks. Different attribute structures, such as age, education level, social connections, and psychological characteristics, influence information spread and the shaping of popular perception. Additionally, populations with different attribute structures have varied responses to cyber popular perception. Accordingly, considering the heterogeneity of susceptible individuals is crucial for predicting the transmission of online popular perception. It is also important for formulating popular opinion policies and curbing the diffusion of false information.

In summary, this article offers the following innovative contributions:

- The use of fractional differential equations to establish the model, leveraging the advantageous properties of fractional differentiation;
- Consideration of the heterogeneity among susceptibles, making them non-homogeneous and more representative of real-world conditions;
- Inclusion of a dissuasion mechanism that accounts for the role of discouragers in public opinion dissemination;
- Calculation and analysis of the steady-state of the model, including the basic reproduction number’s sensitivity and the equilibrium point’s stability.

II. Model Development Process

This section introduces the methods involved in the development of the entire model. The first subsection presents the concept and theorems of conformable fractional derivative on which the model is based. The second subsection provides a detailed introduction to the SEIDR network public opinion dissemination prediction model.

A. Conformable Fractional Derivative

In academic research, fractional-order calculus is widely used to describe the dynamic behavior of complex systems, including stationary and non-stationary systems, such as in the fields of viscous fluid dynamics, materials science, and biomedical engineering. There are two common definitions of fractional-order derivative: Riemann-Liouville [20] and Caputo [21], which describe different initial condition problems. However, they also have some shortcomings, such as the sensitivity of Riemann-Liouville fractional-order derivatives to initial conditions and, like Caputo fractional-order derivatives, they may lose the fundamental properties of general derivatives in certain situations, for example, the product rule and chain rule [22]. Therefore, researchers often explore other forms of fractional-order calculus, such as conformable fractional derivative (CFD) [23], to address these shortcomings. CFD is defined through series expansion, which is simpler compared to the former two definitions and can overcome the shortcomings of traditional definitions. Therefore, this paper uses CFD to represent the system. The CFD is defined in the following manner.

Definition 1. [23] Set a function f: $[0, \infty) \rightarrow \mathbb{R}$, the definition of the CFD of f of order α is given as

$$\mathrm{D}_{\alpha}\left(f\right)\left(t\right)=\lim_{\varepsilon\rightarrow 0}\frac{f\left(t+\varepsilon t^{1-\alpha}\right)-f\left(t\right)}{\varepsilon} \tag{1}$$

where $t > 0$, $\alpha \in (0, 1)$. If the function f is α-differentiable within a specified interval (0, a), where $a > 0$, and exists, then get

$$f^{(\alpha)}(0)=\lim_{t\rightarrow 0^{+}} f^{(\alpha)}(t) \tag{2}$$

Theorem 1. [23] Set $\alpha \in (0, 1]$ and the function f be α-differentiable at a point $t > 0$. Furthermore, if f is differentiable, we obtain

$$D_{\alpha}(f)(t)=t^{1-\alpha}\frac{df}{dt}(t) \tag{3}$$

B. Description of the Fractional-Order SEIDR Model

Addressing the shortcomings and limitations of the traditional SEIR model, this model incorporates innovations such as fractional-order calculus, susceptibility heterogeneity, and dissuasion mechanisms. For more complex real-world public perception dissemination scenarios, this model has better applicability and predictive ability, improving the effectiveness of the traditional SEIR network public opinion dissemination prediction model. We profile and classify susceptible individuals, considering variations in potential factors such as age, education level, social connections, and psychological characteristics, summarized as differences in netizen attribute structures. Based on these differences, we categorize the susceptible individuals into two groups with different infection probabilities.

The SEIDR model considered consists of six compartments: S_1: vulnerable susceptible individuals, S_2: general susceptible individuals, E: exposed individuals, I: infected individuals, D: discouraged individuals, and R: recovered individuals. Taking susceptibility heterogeneity into account, the susceptible population is categorized into two distinct groups: S_1 and S_2. The S_1 category represents individuals who are more likely to be infected within the social network. They transition to exposed individuals with a probability of β_1. Their attribute structure makes them more likely to believe false information for the following possible reasons:

- They are situated in denser social networks, characterized by close social connections and relatively high information flow, increasing their likelihood of encountering and spreading rumors or false information.
- Their lower educational level and limited cognitive abilities hinder their capacity to discern and filter false information.
- Their individual characteristics and psychological factors, including emotional state, cognitive biases, and personal values, make them more inclined to trust less reliable information sources.

The S_2 category represents general susceptible individuals. Compared to the S_1 category, they become exposed with a lower probability after encountering an infected person. This group represents the most prevalent type of susceptible individuals within social networks. Both types of susceptible individuals will gradually saturate at different netizen influx rates (Λ), while the model assumes an outflow rate (d) across all populations. The E category represents exposed individuals. They have been influenced by popular opinion but have not yet disseminated it. They may be considering or waiting for the optimal time to spread the opinion. After a latency period of 1/σ, they transition into infected individuals who actively disseminate the opinion. The I category represents infected individuals who have been influenced by public opinion and have begun to actively spread it, enhancing the reach of the message in social networks through likes, shares, and comments. The D category represents discouraged individuals who use intervention methods such as advocacy and resistance to exert a certain dampening influence on the dissemination of public perception, thereby influencing its transmission and maintaining rational and healthy public discourse. Their specific ways of influencing the transmission of popular perception are as follows:

- Information rebuttal: Discouragers resist the diffusion of popular perception by providing opposing views, fact-checking, and logical reasoning. They can post comments on social media, forums, and other platforms to point out errors or biases in the popular opinion, thereby slowing its spread.
- Public opinion guidance: Discouragers endeavor to guide the direction of popular perception by providing different perspectives, sources of information, or explanations to influence public perception. They may urge the public to think critically and avoid blind conformity, thereby reducing the spread of public opinion.
- Organized resistance: Discouragers organize resistance activities, such as boycotting certain products, services, or events, to express dissatisfaction with or opposition to public opinion content. This resistance behavior can affect the

dissemination of popular perception and diminish its intended impact.

● Leveraging social influence: Some discouragers possess significant social influence, such as government officials, experts, and opinion leaders. Their statements and actions significantly impact public perception transmission. They may resist the spread of public perception by issuing statements or holding public lectures, guiding public attitudes and behaviors.

● Public opinion monitoring and feedback: Discouragers engage in public opinion monitoring, promptly identifying and addressing issues and misinformation to prevent further spread or public misguidance.

R class represents the recovered individuals who were previously infected and recovered at a rate of γ, gaining temporary immunity for a limited duration. However, they only possess limited immunity and cannot remain immune permanently. After a limited duration, they become susceptible again and resume participation in public opinion dissemination. Due to the temporary immunity of recovered individuals and the time delay in the reinfection process, the transition from recovered to susceptible does not occur immediately but after a delay. Consequently, there is no direct transmission link between the recovered category R and both susceptible category S.

The bidirectional transmission between the recovered category R and the discourager category D indicates that some recovered individuals, after experiencing the transmission of popular perception, recognize the harm of false information and actively become discouragers. They inhibit the diffusion of false popular perception through their words and actions. They are enthusiastic positive influencers in social networks and may be part of government departments, experts, social welfare organizations, or ordinary citizens, all contributing to social stability and public interest.

We assume that the initial states of each category in the model, namely $S_1(0)$, $S_2(0)$, E(0), I(0), D(0), and R(0), are known. We also assume that the infection rate, incubation period, and recovery rate among the population are fixed, with each node transitioning at a fixed rate. The SEIDR model is encompassed by the differential equations given below.

$$\begin{cases} D_\alpha(S_1)(t) = \Lambda_1 - \Psi_1(t) - dS_1(t) \\ D_\alpha(S_2)(t) = \Lambda_2 - \Psi_2(t) - dS_2(t) \\ D_\alpha(E)(t) = \Psi_1(t) + \Psi_2(t) - \eta E(t)D(t) - (\sigma + d)E(t) \\ D_\alpha(I)(t) = \sigma E(t) - \mu I(t)D(t) - (\gamma + d)I(t) \\ D_\alpha(D)(t) = \left[\eta E(t) + \mu I(t)\right]D(t) + \rho R(t) - (\delta + d)D(t) \\ D_\alpha(R)(t) = \gamma I(t) + \delta D(t) - (\rho + d)R(t) \end{cases} \quad (4)$$

where, $\Psi_1(t) = \beta_1 S_1(t)I(t)$, $\Psi_2(t) = \beta_2 S_2(t)I(t)$.

Based on the above rules and explanations, the dynamic dissemination of the SEIDR public opinion prediction model among the population is shown in Fig. 1.

The system is constrained by the following initial condition system:

$$\begin{cases} S_1(t), S_2(t), E(t), I(t), D(t), R(t) \geq 0 \\ \Lambda_1, \Lambda_2, \beta_1, \beta_2, \sigma, \gamma, \eta, \mu, \delta, \rho, d \in (0,1) \end{cases} \quad (5)$$

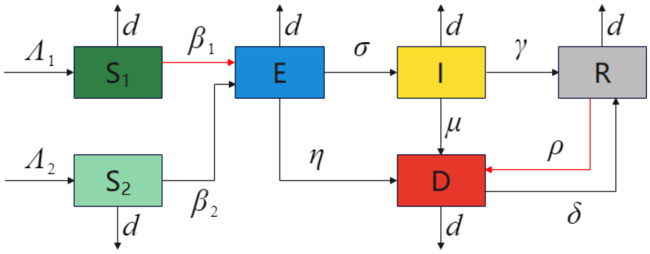

Fig. 1. The framework of the fractional order SEIDR model.

TABLE I
MODEL PARAMETER INTERPRETATION

Symbol	Description
Λ_1	Inflow rate of S_1 in vulnerable susceptible population
Λ_2	Inflow rate of S_2 in general susceptible population
β_1	Infection rate of S_1 in vulnerable susceptible population
β_2	Infection rate of S_2 in general susceptible population
σ	Latency conversion rate, that is, it signifies the probability of an exposed individual becoming infected
γ	The recovery rate, that is, it signifies the probability of an infected individual becoming recovered
η	The disenchantment rate denotes the probability of an exposed individual transitioning to become a dissuader
μ	The dissuasion rate denotes the probability of an infected individual transitioning to become a dissuader
δ	The return rate denotes the probability of the dissuader transitioning to become a recovered
ρ	The positive energy rate denotes the probability of a recovered transitioning to become a dissuader
d	The exodus rate of the crowd

The system parameters are explained in detail in Table I.

The feasible region of the system is a positively invariant set defined as:

$$X_f = \left\{ S_1(t), S_2(t), E(t), I(t), D(t), R(t) \in R_+^6 : N(t) \leq \frac{\Lambda}{d} \right\} \quad (6)$$

Assuming the system is dynamic, then

$$N(t) = S_1(t) + S_2(t) + E(t) + I(t) + D(t) + R(t) \quad (7)$$

Combining the above formula with system (4), and let $\Lambda_1 + \Lambda_2 = \Lambda$, then

$$D_\alpha(N)(t) = \Lambda_1 + \Lambda_2 - dN(t) = \Lambda - dN(t) \quad (8)$$

Upon integrating over the interval [0, t], then we get:

$$N(t) = \frac{\Lambda}{d}(1 - e^{-dt}) + N(0)e^{-dt} \quad (9)$$

$$\lim_{t \to \infty} N(t) = \frac{\Lambda}{d} \quad (10)$$

The results show that all solutions of system (4) fall in the feasible domain X_f , and the positive invariance of the feasible domain is assured by its positive boundedness.

III. THE EQUILIBRIUM AND STABILITY ANALYSIS

This part calculates the basic reproduction number and the equilibrium point of the model, conducts sensitivity analysis on the model's basic reproduction number, and analyzes the equilibrium point, providing a profound understanding of public opinion dissemination and control while laying a theoretical foundation for further research.

A. Equilibrium Point and Basic Reproduction Number

The Disease-Free Equilibrium (DFE) refers to a state of equilibrium in a system where no disease exists. At this equilibrium point, all individuals are healthy, and no infections are present. Under the conditions $I^0 = 0$ and $D^0 = 0$, the right-hand side of each equation in system (4) is set to zero, then the DFE of the system is obtained as:

$$E_0 = \left(S_1^{\,0}, S_2^{\,0}, E^0, I^0, D^0, R^0\right) = \left(\frac{\Lambda_1}{d}, \frac{\Lambda_2}{d}, 0, 0, 0, 0\right) \quad (11)$$

The basic reproduction number (R_0) is a key indicator of the potential for public opinion dissemination. It represents the average number of susceptible individuals directly infected by a representative infectious individual throughout their whole infectious period, serving as an important reference for formulating public opinion policies and intervention measures [24]. If $R_0 > 1$, popular opinion will spread and lead to an epidemic; if $R_0 = 1$, popular opinion will remain stable within the community, serving as the threshold for whether the opinion spreads; if $R_0 < 1$, popular opinion will gradually decline. To control the propagation of false popular perception, the goal of public opinion measures is to reduce R_0 to less than 1. Here, we consider three categories: exposed individuals, infectious individuals, and discouraged individuals, employing the next-generation matrix method to calculate the basic reproduction number [25][26].

Let $X(t) = [E(t), I(t), D(t)]^T$ then

$$D_\alpha(X)(t) = F(t) - V(t) \ , \quad (12)$$

where F(t) is the infection matrix and V(t) is the transfer matrix, as shown below.

$$F(t) = \begin{bmatrix} [\beta_1 S_1(t) + \beta_2 S_2(t)]I(t) \\ 0 \\ 0 \end{bmatrix},$$

$$V(t) = \begin{bmatrix} \eta E(t)D(t) + (\sigma + d)E(t) \\ -\sigma E(t) + \mu I(t)D(t) + (\gamma + d)I(t) \\ -[\eta E(t) + \mu I(t)]D(t) - \rho R(t) + (\delta + d)D(t) \end{bmatrix}.$$

Compute the Jacobian matrix of F(t) and V(t) at the point E_0 represented by J as follows.

$$J(V \mid E_0) = \begin{bmatrix} \sigma + d & 0 & 0 \\ -\sigma & \gamma + d & 0 \\ 0 & 0 & \delta + d \end{bmatrix} = \begin{bmatrix} V_0 & 0 \\ 0 & \delta + d \end{bmatrix},$$

$$J(V \mid E_0) = \begin{bmatrix} \sigma + d & 0 & 0 \\ -\sigma & \gamma + d & 0 \\ 0 & 0 & \delta + d \end{bmatrix} = \begin{bmatrix} V_0 & 0 \\ 0 & \delta + d \end{bmatrix},$$

where,

$$F_0 = \begin{bmatrix} 0 & \dfrac{\beta_1\Lambda_1 + \beta_2\Lambda_2}{d} \\ 0 & 0 \end{bmatrix}, V_0 = \begin{bmatrix} \sigma + d & 0 \\ -\sigma & \gamma + d \end{bmatrix}.$$

Hence, the calculation result of the next generation matrix FV^{-1} is as follows:

$$FV^{-1} = F_0 V_0^{-1} = \begin{bmatrix} \dfrac{\sigma(\beta_1\Lambda_1 + \beta_2\Lambda_2)}{d(\sigma + d)(\gamma + d)} & \dfrac{\beta_1\Lambda_1 + \beta_2\Lambda_2}{d(\gamma + d)} \\ 0 & 0 \end{bmatrix} \quad (13)$$

The spectral radius, which is the largest eigenvalue of the next-generation matrix FV^{-1}, represents the basic reproduction number R_0, so we get the following R_0 values.

$$R_0 = \frac{\sigma(\beta_1\Lambda_1 + \beta_2\Lambda_2)}{d(\sigma + d)(\gamma + d)} \quad (14)$$

B. Sensitivity Analysis of Basic Reproduction Number

Analyzing the sensitivity of the basic reproduction number R_0 helps identify which parameter changes most significantly influence the model output, thereby enhancing understanding of R_0. The Normalized Forward Sensitivity Index (NFSI) [27] is an indicator employed to measure the impact of model parameters on the model output. This is achieved by normalizing the model parameters, calculating the forward sensitivity index using (15), and further normalizing the sensitivity with (16) to assess how these parameter changes affect the model output.

$$S_{x_i} = \frac{\partial R_0}{\partial x_i} \cdot \frac{x_i}{R_0} \quad (15)$$

where, $\frac{\partial R_0}{\partial x_i}$ represents the partial derivative of the model output concerning the parameter x_i, and $\frac{x_i}{R_0}$ is the normalization factor.

$$NFSI_{x_i} = \frac{S_{x_i}}{\max(|S_{x_i}|)} \quad (16)$$

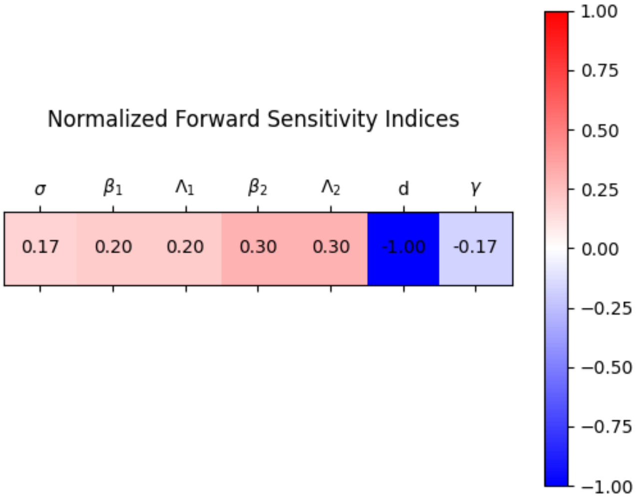

Fig. 2. NFSI heat map of basic reproduction number R_0.

The calculation of the forward sensitivity index for each parameter within the basic reproduction number is displayed below.

$$S_\sigma = \left[\frac{\beta_1\Lambda_1 + \beta_2\Lambda_2}{d(\sigma + d)(\gamma + d)} - \frac{\sigma(\beta_1\Lambda_1 + \beta_2\Lambda_2)}{d(\sigma + d)^2(\gamma + d)}\right] \cdot \frac{\sigma}{R_0} \ ,$$

$$S_{\beta_1} = \frac{\sigma\Lambda_1}{d(\sigma + d)(\gamma + d)} \cdot \frac{\beta_1}{R_0} \ ,$$

$$S_{\Lambda_1} = \frac{\sigma\beta_1}{d(\sigma + d)(\gamma + d)} \cdot \frac{\Lambda_1}{R_0} \ ,$$

$$S_{\beta_2} = \frac{\sigma\Lambda_2}{d(\sigma + d)(\gamma + d)} \cdot \frac{\beta_2}{R_0} \ ,$$

$$S_{\Lambda_2} = \frac{\sigma\beta_2}{d(\sigma + d)(\gamma + d)} \cdot \frac{\Lambda_2}{R_0} \ ,$$

$$S_d = -\frac{\sigma(\beta_1\Lambda_1+\beta_2\Lambda_2)}{(\sigma+d)(d(\sigma+d)(\gamma+d)+d(\sigma+d)^2)}\cdot\frac{d}{R_0},$$

$$S_\gamma = -\frac{\sigma(\beta_1\Lambda_1+\beta_2\Lambda_2)}{d(\sigma+d)(\gamma+d)^2}\cdot\frac{\gamma}{R_0}.$$

It can be seen that increases in σ, β_1, Λ_1, β_2, and Λ_2 will increase R_0, while increases in d and γ will decrease R_0. As can be seen from Fig. 2, R_0 is most sensitive to parameter d, that is, d has the greatest influence on the output of R_0.

C. Stability Analysis

In this section, the local stability of the model at the DFE point is assessed using Lyapunov stability theorem [28].

Theorem 2. The DFE point of system (4) is locally asymptotically stable if and only if $R_0 < 1$.

Proof. The Jacobian matrix of system (4) at the DFE point is as follows:

$$J(E_0)=\begin{bmatrix} -d & 0 & 0 & 0 & 0 & 0\\ -d & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & -\sigma-d & \frac{\beta_1\Lambda_1+\beta_2\Lambda_2}{d} & 0 & 0\\ 0 & 0 & \sigma & -\gamma-d & 0 & 0\\ 0 & 0 & 0 & 0 & -\delta-d & \rho\\ 0 & 0 & 0 & \gamma & \delta & -\rho-d \end{bmatrix}$$

The six eigenvalues of the matrix result from the following calculations:

$\lambda_1=0$, $\lambda_2=\lambda_3=-d<0$, $\lambda_4=-\delta-\rho-d<0$,

$\lambda_5=\frac{-2d^2-\sigma d-\gamma d-\Phi}{2d}<0$, $\lambda_6=\frac{-2d^2-\sigma d-\gamma d+\Phi}{2d}$,

where, $\Phi=[\sigma^2d^2+\gamma^2d^2-2\sigma\gamma d^2+4\sigma d(\beta_1\Lambda_1+\beta_2\Lambda_2)]^{\frac{1}{2}}$.

The first five eigenvalues are all less than zero, and for λ_6, we assume that $\lambda_6 < 0$, then obtain

$$[\sigma^2d^2+\gamma^2d^2-2\sigma\gamma d^2+4\sigma d(\beta_1\Lambda_1+\beta_2\Lambda_2)]^{\frac{1}{2}} < 2d^2+\sigma d+\gamma d,$$

if we expand and simplify, we get

$$\sigma(\beta_1\Lambda_1+\beta_2\Lambda_2)<d(\sigma+d)(\gamma+d),$$

$$\frac{\sigma(\beta_1\Lambda_1+\beta_2\Lambda_2)}{d(\sigma+d)(\gamma+d)}<1.$$

That is, $R_0 < 1$. The above derivation is reversible, all eigenvalues exhibit negative real parts, and according to the Lyapunov stability theorem, the DFE point E_0 is locally asymptotically stable.

IV. Numerical Simulation

This section conducts experimental simulations on the system using the MATLAB platform to confirm the stability of the DFE point proposed in Theorem 2. It discusses the influence of diverse parameters on model performance and specific properties of the model itself. Finally, it tests the predictive ability of the model with real-time public opinion from social networks, exemplified by the question: "Is Guizhou's 'China Sky Eye' turning into a 'garbage dump'?" The results indicate that the proposed SEIDR model significantly reduces prediction errors and demonstrates better predictive ability and applicability than the SEIR model in more complex real-world public opinion dissemination scenarios.

A. Stability Simulation and Analysis of DFE Point

Based on the SEIDR model and Theorem 2 presented in this paper, we simulated real-world social network public opinion dissemination scenarios. The model's parameter settings conform to actual regulations, and the results confirm the correctness of the system's stability at the DFE point. The model's simulation parameters are presented in Table II, with corresponding results derived from these parameter settings are depicted in Fig. 3.

TABLE II
DFE Point Parameter Settings

Λ_1	Λ_2	β_1	β_2	σ	γ	η	μ	δ	ρ	d
0.05	0.15	0.6	0.3	0.4	0.1	0.4	0.1	0.6	0.4	0.2

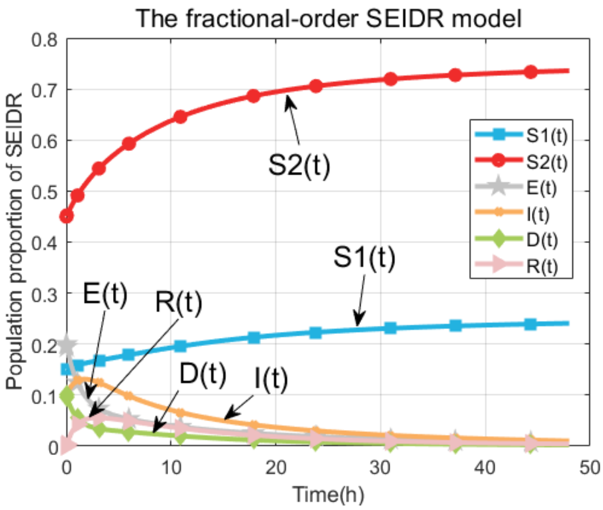

Fig. 3. Model dynamics at DFE point.

According to the data in the table above and the basic reproduction number (14), it was calculated that the basic reproduction number $R_0 = 0.833 < 1$, and it can be seen from Fig. 3 that the system finally converges to the DFE point E_0 = (0.25, 0.75, 0, 0, 0, 0), which indicates that the DFE point E_0 is asymptotically stable. These results are consistent with Theorem 2, which verifies the stability of the system.

Throughout the entire process of public perception dissemination, there exist stages encompassing incubation, development, outbreak, decline, and settling. The influence of public opinion varies in characteristics and patterns at each stage. Timely resolution of the risks posed by false popular opinion and rumors is crucial. The graph shows that the curves of the two susceptible groups initially rise over time, indicating the continuous spread of false popular perception content. The influence of popular perception is still spreading to a certain extent, affecting more people. However, due to the strong ability of most people in social networks to discern false popular opinion and the low proportion and small base of vulnerable susceptible groups in the population, the growth rate of general susceptible groups in the early stage is faster than that of vulnerable susceptible groups. This will lead to more people becoming general susceptible groups when the next similar false public opinion impact arrives, thereby enhancing the overall resistance of the population to false public opinion. Over time, as public opinion enters the decline and settling stages, its influence gradually weakens, and the two susceptible groups stabilize.

In Fig. 3, both the E and D categories show a general decline. For category E, this decline occurs as people gradually realize the true situation of public opinion, the impact diminishes, leading to a slowdown in the dissemination of public opinion. As for category D, although they don't represent a large proportion in social networks, its influence remains significant. The rapid decline in the early stage is attributed to the restraining behavior of dissuaders, which slow the dissemination of popular perception.

B. The Impact of Model Parameters on the Model

Fractional calculus enables the use of differential operators with non-integer orders, providing advantages in capturing more complex dynamical behaviors compared to integer-order calculus. Thus, we investigate the impact of the differential operator α within the model, focusing on how changes in α affect the proportions of type I and type D populations.

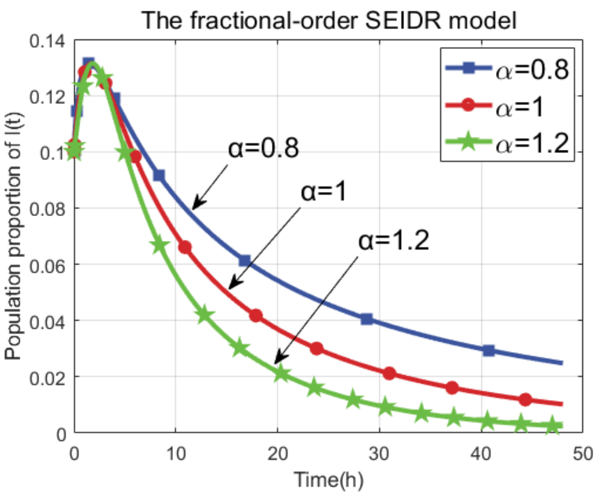

Fig. 4. The effect of differential order α on infected person I(t).

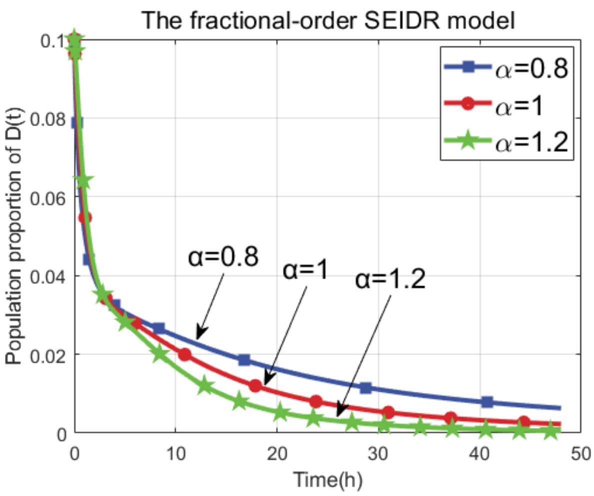

Fig. 5. The effect of the differential order α on dissuader D(t).

This paper examines the effects of three different values of the fractional order: α = 0.8, α = 1, and α = 1.2 on the proportions of susceptibles and dissuaders over time. As the fractional order α increases, the proportions of infectives I and dissuaders D decrease more rapidly, and the convergence of the curves accelerates. This indicates that higher fractional orders of α lead to faster reductions in both infectives and dissuaders. Conversely, lower fractional orders of α result in higher proportions of infectives and dissuaders being maintained over a longer period. See Fig. 4 and Fig. 5.

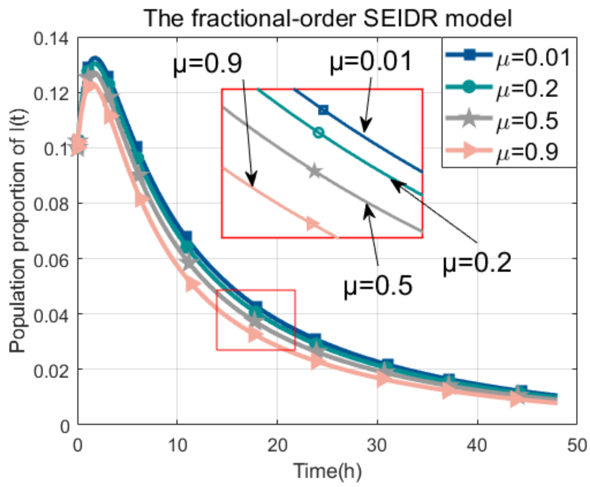

Fig. 6. The influence of different dissuasion rates μ on infected persons.

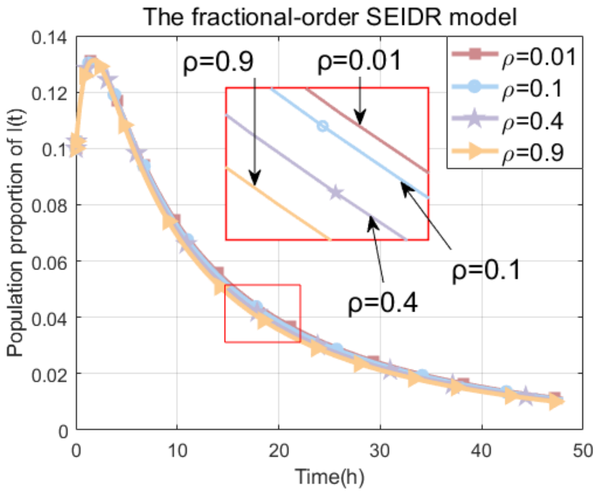

Fig. 7. The influence of different positive energy rate ρ on infected persons.

Fig. 6 and Fig. 7 illustrate the fluctuations in the proportion of infectives under varying dissuasion rates μ and positive energy rates ρ. From Fig. 6, as the dissuasion rate μ increases, the proportion of infectives decreases, flowing towards the dissuaders. This results in an increase in the number of dissuaders and a further decrease in the proportion of infectives. From Fig. 7, it becomes apparent that with the increase in the positive energy rate ρ, the number of individuals actively becoming dissuaders rises, and the dissuasion by dissuaders leads to a decrease in the proportion of infectives. Overall, the increase in both rates enhances the crucial role of dissuaders in the diffusion of false popular opinion, resulting in a weakening of the spread of false public opinion.

Fig. 8 illustrates the correlation between infectives and vulnerable susceptibles $S_1(t)$ at different infection rates. The figure indicates that at low infection rate ($\beta_1 = 0.01$ and $\beta_1 = 0.2$), the proportion of vulnerable susceptibles $S_1(t)$ remains relatively high, decreasing gradually, while the proportion of infectives I(t) increases slowly. At high infection rate (β_1 = 0.6 and β_1 = 0.9), the proportion of vulnerable susceptibles $S_1(t)$ decreases rapidly, while the proportion of infectives I(t) increases quickly. Understanding this relationship aids in comprehending the dynamic changes of susceptibles and infectives across different rates of public opinion dissemination. Therefore, in response to the characteristics of vulnerable susceptible groups, popular opinion measures should promptly focus on their dynamics and implement targeted protection and dissuasion efforts.

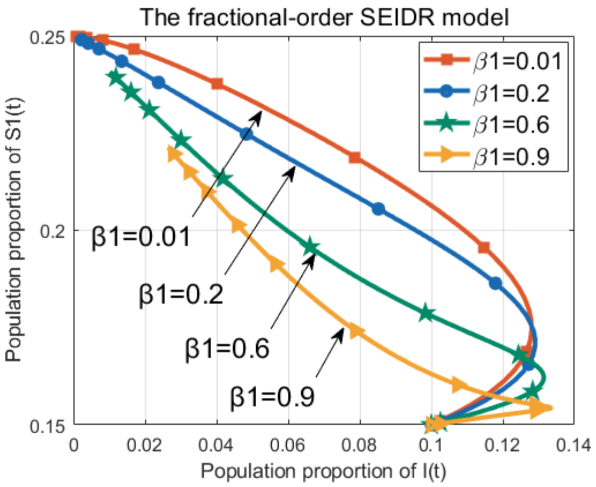

Fig. 8. Correlation between infected person I and vulnerable susceptible person $S_1(t)$.

C. Real Data Simulation

To evaluate the predictive performance of the model in disseminating public opinion within actual social networks, we integrated it with a false popular perception event, comparing the proposed fractional-order SEIDR model with the integer-order SEIDR model and the traditional SEIR model to assess their predictive effects and conduct error analysis. Ultimately, we concluded that the model presented in this article is superior. Weibo, a popular social media platform, attracts numerous users and frequently generates various trending events. Weibo provides real-time and extensive information, enabling the tracking of the development and evolution of trending events and understanding public attention and reactions. We collected interaction data regarding the false popular opinion related to the trending event "Has the 'Chinese Eye' in Guizhou turned into a 'garbage dump'?" from Weibo's trending topics list between April 6th and April 8th, 2024. We used this data as the propagation data for the event and simulated it using the proposed SEIDR model. The results are presented in Fig. 9.

Fig. 9 compares the predictive performance of the three models. The figure shows that during the 'golden first four hours' proposed for public opinion monitoring, the performance differences among the three models are minimal, with the SEIDR model still slightly outperforming both the integer-order SEIDR model and the SEIR model. However, after four hours, the SEIDR model demonstrates significant advantages due to the flexibility of fractional-order modeling, outperforming the other two models and providing more accurate predictions for the later stages of public opinion dissemination. This finding has important implications for establishing an effective long-term mechanism for public opinion monitoring.

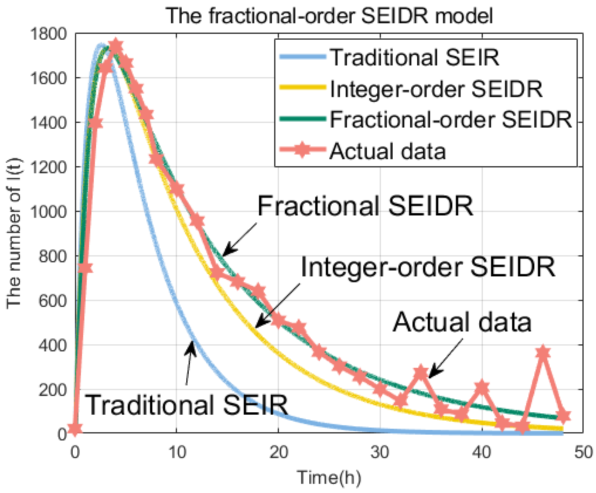

Fig. 9. Comparison of model prediction results.

Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) are two important error evaluation metrics that play a crucial role in performance assessment. Table III presents the error calculations for the predictive performance of the three models. The errors of the fractional-order SEIDR model are substantially lower than those of the integer-order SEIDR model and the SEIR model, as evident from the results. This indicates that the SEIDR model has higher prediction accuracy and can better captures the dynamics of actual social network public opinion dissemination.

TABLE III
MODEL ERROR COMPARISON

Model	SEIR	Integer-order SEIDR	Fractional-order SEIDR
MAE	282.07	108.14	65.52
RMSE	326.58	140.09	100.32

V. CONCLUSIONS

This article builds upon the traditional SEIR model, considering the heterogeneity of susceptibles, and propose a fractional-order SEIDR network public opinion dissemination prediction model that incorporates a dissuasion mechanism D, thereby constructing a more complex and refined model. We establish a mathematical model, determine its basic reproduction number, perform sensitivity analysis, and solve for the model's equilibrium points while conducting a balance analysis. The model's stability is assessed using Lyapunov stability theory. Finally, we validate the model's prediction capability through simulations based on real public opinion dissemination cases. The results demonstrate that the proposed SEIDR model significantly outperforms the integer-order model and the traditional SEIR model, capturing more details of the network public opinion dissemination process. Based on the simulation results, important practical guidance emerges: vulnerable susceptible individuals should be better protected, and the proportion of discouragers can be increased through external social incentives or publicity to enhance popular perception control. This holds great significance for formulating popular perception policies and controlling the spread of false popular perception. Future research should further optimize model parameters by considering more realistic factors, such as spatial heterogeneity and random interference, to enhance the practicality and predictive accuracy of the model. Additionally, the characteristics and control strategies of different public opinions may vary; thus, the new model can be adjusted and applied to specific situations to support broader public opinion decision-making.

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