On Some Topological Structures of Transformation Groups

Keerthana Dhanasekar and V. Visalakshi*

Abstract—**This paper explores fundamental aspects of irresolute maps and investigates separation axioms, specifically semi-T_2 and semi normal properties. The study introduces Irr-topological transformation groups, Irr*-topological transformation groups, I*-topological transformation groups and I-topological transformation groups, providing an in-depth analysis of their interrelations. Through detailed examination, the topological properties of these groups are elucidated, complemented by relevant examples and counterexamples. This comprehensive exploration aims to enhance our understanding of irresolute maps, separation axioms, and the intricate relationships within the introduced transformation groups.**

Index Terms—**Topological groups, Transformation groups, Irresolute functions, Pre semi open, Separation axioms.**

I. Introduction

The exploration of sets possessing both algebraic and topological structures invites an investigation into their interrelation. Examining continuous algebraic operations emerges as a natural approach for such a study. Mathematicians like Andrew Gleason, Deane Montgomery, and Leo Zippin made significant contributions to understand the structure and properties of topological groups. In a topological group structure, the multiplication and inverse mapping exhibit continuity. However, it is equally reasonable to investigate structures in which algebraic operations are endowed with the weaker forms of continuity. This exploration led to the study of semi-topological groups (where the multiplication map is separately continuous), paratopological groups (where multiplication is jointly continuous), and quasi-topological groups (semi-topological groups with a continuous inverse map) between the 1930s and 1950s [1], [18], [15].

Diverse topological groups, including S-topological groups [2], irresolute topological groups [12], almost topological groups [17], and p-topological groups [23], arise when continuity is replaced by semi continuity, irresolute, almost continuous, and pre continuity, respectively, in the definition of a topological group. Numerous authors have examined the properties of these groups. Jinfan Xu et al., [9] investigated various interior and additive closure operators and their relations on quasi-pseudo-BL algebras. Muhammad Arshad et al., [14] analyzed the common fixed point of generalized contractive type mappings in 2011. The mid-20th century saw the formalization and systematic development of the theory of topological transformation groups. The study of topological transformation groups continues to evolve with advancements in mathematics. Researchers explore deeper connections with other areas, such as geometry, algebraic topology, and functional analysis. A topological transformation group introduced by Gleason [3], provides a similar bridge between algebraic and topological structures. When a topological group acts continuously on a topological space, it gives rise to a topological transformation group.

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The concept of semi open sets is a generalization of open sets in topology. The study of semi open sets emerged as part of the broader development of topology, which began in the early 20th century. Specifically in 1963, Levine [10] popularized the term semi open by incorporating closure and interior operators. Sterling Gene Crossley [8] explored semi topological spaces and their characteristics. Subsequently, C. Dorsett [5] defined semi compactness and explored its properties. Maheswari established the separation axioms [11] and the characteristics of semi open sets. The study of semi open sets has found applications in various branches of mathematics, including functional analysis, topology, and set theory. Sandhya S Pai et al., [22] defined soft L-topological spaces and discussed the properties of separation axioms and continuity.

In 2024, Rajapandiyan et al., [20] introduced the innovative concept of S-topological transformation groups—a novel structure encompassing semi-totally continuous actions on topological groups. The study investigates the comprehensive exploration of properties associated with algebraic and topological concepts. Additionally, the definition of fixed points within the S-topological transformation group is elucidated, and their fundamental properties are explored [21].

The paper is organized into distinct sections. Section 2 serves as an introduction, presenting the necessary preliminaries essential for the development of the primary outcomes. In Section 3, an in-depth exploration is conducted on characteristics associated with irresolute functions and separation axioms for semi open sets. Section 4 is dedicated to the comprehensive definition of various topological aspects of transformation groups. This includes Irr-topological transformation groups, Irr*-topological transformation groups, I*-topological transformation groups, and I-topological transformation groups. The section further investigates the interrelationships between these defined transformation groups, substantiated by relevant examples and counterexamples.

II. Preliminaries

In this section, fundamental definitions essential for deriving the main outcomes are presented. These definitions establish the groundwork for a comprehensive understanding of the subsequent discussions and pave the way for the exploration and analysis of the broader context or topic at hand.

$(\mathsf{Y}, \mathcal{T}_\mathsf{Y})$ denotes the topological space. For $\mathsf{B} \subseteq \mathsf{Y}$, $Cl(\mathsf{B})$ denotes the closure of B and $Int(\mathsf{B})$ denotes the interior of B.

Definition 2.1. *[10] A set $\mathsf{B} \subseteq \mathsf{Y}$ is semi open $\iff$ $\mathsf{B} \subseteq Cl(Int(\mathsf{B}))$. The class of all semi open sets in Y is denoted by $SO(\mathsf{Y})$.*

Definition 2.2. *[8] A set $\mathsf{B} \subseteq \mathsf{Y}$ is semi closed $\iff$ $Int(Cl(\mathsf{B})) \subseteq \mathsf{B}$. The class of all semi closed sets in Y is denoted by $SC(\mathsf{Y})$.*

Definition 2.3. *[8] A function $g : \mathsf{Y} \to \mathsf{Z}$ is said to be irresolute $\iff$ for any $D_1 \in SO(\mathsf{Z})$, $g^{-1}(D_1) \in SO(\mathsf{Y})$.*

Definition 2.4. *[8] Let Y and Z be two topological spaces. Y and Z are said to be semi homeomorphic $\iff$ there exists a function $g : \mathsf{Y} \to \mathsf{Z}$, such that g is bijective, pre semi open and irresolute. Such a function g is called semi homeomorphism.*

Definition 2.5. *[6] A space Y is said to be semi normal if for each $\mathsf{B}, \mathsf{C} \in SC(Y)$ such that $\mathsf{B} \cap \mathsf{C} = \emptyset$, there exist a disjoint $\mathsf{D}_1, \mathsf{D}_2 \in SO(\mathsf{Y})$ such that $\mathsf{B} \subset \mathsf{D}_1$ and $\mathsf{C} \subset \mathsf{D}_2$.*

Definition 2.6. *[11] A space Y is semi-T_2 if $\forall\ y_1 \neq y_2$ of Y, there exist $\mathsf{D}_1, \mathsf{D}_2 \in SO(\mathsf{Y})$ such that $\mathsf{D}_1 \cap \mathsf{D}_2 = \emptyset$ containing y_1 and y_2, respectively.*

Definition 2.7. *[25] A space Y is called extremally disconnected if for each $\mathsf{D}_1 \in \mathcal{T}_\mathsf{Y}$, $Cl(\mathsf{D}_1) \in \mathcal{T}_\mathsf{Y}$.*

Definition 2.8. *[5] A space Y is called semi compact if every semi open cover of Y has a finite subcover. A set $\mathsf{B} \subseteq \mathsf{Y}$ is said to be semi compact if it is semi compact as a subspace.*

Definition 2.9. *[8] A function $g : \mathsf{Y} \to \mathsf{Z}$ is said to be pre semi open $\iff$ for all $\mathsf{B} \in SO(\mathsf{Y})$, $g(\mathsf{B}) \in SO(\mathsf{Z})$.*

Definition 2.10. *[10] Let $g : \mathsf{Y} \to \mathsf{Z}$ be a function such that $\forall\ \mathsf{D}_1 \in \mathcal{T}_\mathsf{Z}$, $g^{-1}(\mathsf{D}_1) \in SO(\mathsf{Y})$ then g is said to be semi continuous.*

Proposition 2.1. *[4] Let $g : \mathsf{Y} \to \mathsf{Z}$ be an irresolute function. Let B be a subset of Y. If B is semi compact in Y, then $g(\mathsf{B})$ is semi compact in Z.*

Definition 2.11. *[19] A space Y is said to be SCS if $\mathsf{B} \subseteq \mathsf{Y}$ which is semi compact in Y is semi closed.*

Theorem 2.1. *[19] Let Y be a semi-T_2 extremally disconnected space. Then Y is SCS.*

Corollary 2.1. *[19] For a semi-T_2 semi compact space, the following are equivalent:*

1) *Y is SCS.*
2) *Y is extremally disconnected.*

Definition 2.12. *[24] A function $g : \mathsf{Y} \to \mathsf{Z}$ is said to be pre semi closed if $g(F) \in SC(\mathsf{Z})$ for every $F \in SC(\mathsf{Y})$.*

Definition 2.13. *[16] Let $(\mathsf{H}, \circledast)$ be a group and, $\mathcal{T}_\mathsf{H}$ be a topology on H. Then $(\mathsf{H}, \circledast, \mathcal{T}_\mathsf{Y})$ is said to be a topological group if the multiplication map*

$$\mathfrak{m} : \mathsf{H} \times \mathsf{H} \to \mathsf{H} \ni \mathfrak{m}(h_1, h_2) = h_1 h_2$$

and the inverse map

$$\mathfrak{i} : \mathsf{H} \to \mathsf{H} \ni \mathfrak{i}(h) = h^{-1}$$

are continuous.

Definition 2.14. *[12] Let $(\mathsf{H}, \circledast)$ be a group and $\mathcal{T}_\mathsf{H}$ be a topology on H. Then $(\mathsf{H}, \circledast, \mathcal{T}_\mathsf{H})$ is said to be an Irr-topological group if the multiplication map*

$$\mathfrak{m} : \mathsf{H} \times \mathsf{H} \to \mathsf{H} \ni \mathfrak{m}(h_1, h_2) = h_1 h_2$$

and the inverse map

$$\mathfrak{i} : \mathsf{H} \to \mathsf{H} \ni \mathfrak{i}(h) = h^{-1}$$

are irresolute.

Definition 2.15. *[12] Let $(\mathsf{H}, \circledast)$ be a group and, $\mathcal{T}_\mathsf{H}$ be a topology on H. Then $(\mathsf{H}, \circledast, \mathcal{T}_\mathsf{H})$ is said to be an irresolute topological group if for all $h_1, h_2 \in \mathsf{H}$ and for each $\mathsf{D}_3 \in SO(\mathsf{H})$ of $h_1 \circledast h_2^{-1}$, there exist $\mathsf{D}_1 \in SO(\mathsf{H})$ of h_1 and $\mathsf{D}_2 \in SO(\mathsf{H})$ of h_2 such that*

$$\mathsf{D}_1 \circledast \mathsf{D}_2^{-1} \subseteq \mathsf{D}_3.$$

Definition 2.16. *[3] Let $(\mathsf{H}, \circledast)$ be a group and Y be a set. Then a map*

$$\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y} \ni \psi(h, y) = hy$$

satisfying the following conditions,

1) *$\psi(e, y) = y$, $\forall\ y \in \mathsf{Y}$, where e is the identity of H;*
2) *$\psi(h_2, \psi(h_1, y)) = \psi(h_2 h_1, y)$ for all $h_1, h_2 \in \mathsf{H}$ and $y \in \mathsf{Y}$.*

The triple $(\mathsf{H}, \mathsf{Y}, \psi)$ is called a transformation group or H-action on Y and Y is called a H-set.

Definition 2.17. *[3] A triplet $(\mathsf{H}, \mathsf{Y}, \psi)$ is called a topological transformation group (TTG) in which H is a topological group, Y is a topological space and*

$$\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y} \ni \psi(h, y) = hy$$

is a continuous map satisfying the following conditions,

1) *$\psi(e, y) = y$, for all $y \in \mathsf{Y}$, where e is the identity element of H.*
2) *$\psi(h_2, (h_1, y)) = \psi(h_2 h_1, y)$, for every $h_1, h_2 \in \mathsf{H}$ and $y \in \mathsf{Y}$ The space Y, along with a given action ψ of H, is called a H-space.*

Definition 2.18. *[13] A map $g : \mathsf{Y} \to \mathsf{Z}$ is said to be semi quotient, for any subset D_1 of Z is open in Z if and only if $g^{-1}(\mathsf{D}_1)$ is semi open in Y.*

Theorem 2.2. *[7] Let $(\mathsf{Y}, \mathcal{T}_\mathsf{Y})$ be a topological space. Then the following statements are equivalent.*

1) *$(\mathsf{Y}, \mathcal{T}_\mathsf{Y})$ is extremally disconnected*
2) *For each $\mathsf{D}_1, \mathsf{D}_2 \in SO(\mathsf{Y})$, $\mathsf{D}_1 \cap \mathsf{D}_2 \in SO(\mathsf{Y})$.*

Lemma 2.1. *Let H, K be two groups and g be a homomorphism of H into K. Then*

1) *for any subset C and D of $\mathsf{H}, g(\mathsf{CD}) = g(\mathsf{C})g(\mathsf{D})$*

2) *for any subset* E *and* F *of* $\mathsf{K}, g^{-1}(\mathsf{E})g^{-1}(\mathsf{F}) \subset g^{-1}(\mathsf{EF})$.
3) *for any symmetric subset* C *of* $\mathsf{H}, g(\mathsf{C})$ *is symmetric in* K
4) *for any symmetric subset* D *of* $\mathsf{K}, g^{-1}(\mathsf{D})$ *is symmetric in* H

III. Basic properties of irresolute functions and Separation axioms

This section outlines the characteristics of an irresolute functions and the separation axioms. It elucidates the properties that are necessary for providing a foundational understanding of subsequent discussions.

Lemma 3.1. *If the maps* $\psi_i : \mathsf{Y}_i \to \mathsf{Z}_i$ *are pre semi open for* $i = 1, 2$ *then*

$$\psi_1 \times \psi_2 : \mathsf{Y}_1 \times \mathsf{Y}_2 \to \mathsf{Z}_1 \times \mathsf{Z}_2,$$

$\forall\ \mathsf{D}_1 \in SO(\mathsf{Y}_1), \mathsf{D}_2 \in SO(\mathsf{Y}_2), \exists\ \mathsf{E}_1 \in SO(\mathsf{Z}_1), \mathsf{E}_2 \in SO(\mathsf{Z}_2)$ *such that*

$$(\psi_1 \times \psi_2)(\mathsf{D}_1 \times \mathsf{D}_2) \subseteq \mathsf{E}_1 \times \mathsf{E}_2.$$

Proof: Let ψ_1, ψ_2 be a pre semi open maps, $\mathsf{D}_1 \in SO(\mathsf{Y}_1), \mathsf{D}_2 \in SO(\mathsf{Y}_2)$ then there exist $\mathsf{E}_1 \in SO(\mathsf{Z}_1), \mathsf{E}_2 \in SO(\mathsf{Z}_2) \ni \psi_1(\mathsf{D}_1) \subseteq \mathsf{E}_1$ and $\psi_2(\mathsf{D}_2) \subseteq \mathsf{E}_2$ by assumption. Thus

$$(\psi_1 \times \psi_2)(\mathsf{D}_1 \times \mathsf{D}_2) \subseteq \mathsf{E}_1 \times \mathsf{E}_2.$$

■

Remark 3.1. *In general, product of pre semi open maps need not be pre semi open*

Theorem 3.1. *The projection map*

$$\mathfrak{p} : \mathsf{Y} \times \mathsf{Z} \to \mathsf{Y} \ni \mathfrak{p}(y, z) = y$$

is irresolute and pre semi open.

Proof: Let $\mathfrak{p} : \mathsf{Y} \times \mathsf{Z} \to \mathsf{Y}$ such that $\mathfrak{p}(y, z) = y$. To prove $\mathfrak{p}$ is both irresolute and pre semi open. Let $\mathsf{D} \in SO(\mathsf{Y} \times \mathsf{Z})$, there exists $\mathsf{D}_1 \in \mathcal{T}_{\mathsf{Y}\times\mathsf{Z}}$ such that $\mathsf{D}_1 \subseteq \mathsf{D} \subseteq Cl(\mathsf{D}_1)$ then

$$\mathfrak{p}(\mathsf{D}_1) \subseteq \mathfrak{p}(\mathsf{D}) \subseteq \mathfrak{p}(Cl(\mathsf{D}_1)) \subseteq Cl(\mathfrak{p}(\mathsf{D}_1)),$$

since $\mathfrak{p}$ is continuous and open, $\mathfrak{p}(\mathsf{D}) \in SO(\mathsf{Y})$. Thus $\mathfrak{p}$ is pre semi open. Similarly $\mathfrak{p}$ is irresolute. ■

Example 3.1. *Let* $\mathsf{Y} = \{1, 2\}$ *be the set,* $\mathcal{T}_{\mathsf{Y}} = \{\emptyset, \{1\}, \{1, 2\}\}$ *be a topology on* Y, $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *be a topological space. Then the projection map*

$$\mathfrak{p} : \mathsf{Y} \times \mathsf{Y} \to \mathsf{Y} \ni \mathfrak{p}(y_1, y_2) = y_1$$

is irresolute and pre semi open.

Lemma 3.2. *Let* $\mathsf{Y}, \mathsf{Z},$ *and* W *be a topological spaces. In below commutative diagram,* ψ_3 *is an irresolute map, if* ψ_1 *is an irresolute map and* ψ_2 *is a surjective pre semi open map.*

$$\begin{tikzcd} \mathsf{Y} \arrow[d, "\psi_2"'] \arrow[dr, "\psi_1"] & \\ \mathsf{Z} \arrow[r, "\psi_3"'] & \mathsf{W} \end{tikzcd}$$

Fig. 1. Commutative diagram of functions

Proof: Let $\mathsf{D}_1 \in SO(\mathsf{W})$. Since ψ_2 is surjective,

$$\psi_3^{-1}(\mathsf{D}_1) = \psi_2 \circ \psi_2^{-1}(\psi_3^{-1}(\mathsf{D}_1)) = \psi_2(\psi_1^{-1}(\mathsf{D}_1)).$$

Since ψ_1 is irresolute and ψ_2 is pre semi open, $\psi_3^{-1}(\mathsf{D}_1) \in SO(\mathsf{Z})$, which implies that ψ_3 is irresolute. ■

Example 3.2. *Let* $\mathsf{Y} = \{0, 1, 2, 3\} = \mathbb{Z}_4$ *be the set,*

$$\mathcal{T}_{\mathsf{Y}} = \{\emptyset, \{0, 2\}, \{1, 3\}, \mathsf{Y}\}$$

be a topology on Y, *the map*

$$\psi_1, \psi_2, \psi_3 : \mathbb{Z}_4 \to \mathbb{Z}_4$$

such that $\psi_1(0) = \psi_2(0) = 1, \psi_1(1) = \psi_2(1) = 2, \psi_1(2) = \psi_2(2) = 3, \psi_1(3) = \psi_2(3) = 0$ *and*

$$\psi_3(0) = 0, \psi_3(1) = 1, \psi_3(2) = 2, \psi_3(3) = 3.$$

Then ψ_3 *is an irresolute map, since* ψ_1 *is an irresolute map and* ψ_2 *is a surjective pre semi open map.*

Lemma 3.3. *Let* Y *be a semi compact space and* Z *be a semi-*T_2 *extremally disconnected space.*

1) *Then any irresolute map* $\psi : \mathsf{Y} \to \mathsf{Z}$ *is pre semi closed.*
2) *If* ψ *is bijective and irresolute, then* ψ *is a semi homeomorphism.*

Proof: Any subset $\mathsf{F} \in SC(\mathsf{Y})$ is semi compact, since Y is semi compact. Hence $\psi(\mathsf{F})$ is semi compact by Proposition 2.1. Similarly $\psi(\mathsf{F}) \in SC(\mathsf{Z})$, since Z is SCS by Theorem 2.1. Thus ψ is pre semi closed.

If ψ is bijective and irresolute, ψ^{-1} is irresolute, hence ψ is a semi homeomorphism, since bijective, irresolute pre semi closed map is a semi homeomorphism. ■

Example 3.3. *Let* $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ *be the set,*

$$\mathcal{T}_{\mathbb{Z}_4} = \{\emptyset, \{0, 2\}, \{1, 3\}, \mathbb{Z}_4\},$$

be a topology on $\mathbb{Z}_4$, $(\mathbb{Z}_4, \mathcal{T}_{\mathbb{Z}_4})$ *be a topological space and*

$$\mathbb{Z}_2 = \{0, 1\}, \mathcal{T}_{\mathbb{Z}_2} = \{\emptyset, \{0\}, \mathbb{Z}_2\},$$

$(\mathbb{Z}_2, \mathcal{T}_{\mathbb{Z}_2})$ *be a topological space then the irresolute map*

$$\psi : \mathbb{Z}_4 \to \mathbb{Z}_2 \ni \psi(0) = 0, \psi(1) = 1, \psi(2) = 0, \psi(3) = 1$$

is not pre semi closed, since $\mathbb{Z}_2$ *is not a semi-*T_2 *extremally disconnected space.*

Lemma 3.4. *In the commutative diagram of Lemma* 3.2, *if* ψ_2 *is surjective and* Z *is endowed with semi quotient topology induced by the map* ψ_2. *If* ψ_1 *is irresolute, then* ψ_3 *is semi continuous.*

Proof: Let S be any semi open subset of W, the set

$$\psi_2^{-1}(\psi_3^{-1}(S)) = \psi_1^{-1}(S)$$

is semi open. Thus $\psi_3^{-1}(S)$ is semi open from the definition of the semi quotient topology. ■

Lemma 3.5. *In the commutative diagram of Lemma* 3.2*, if* ψ_2 *is a surjective irresolute map. If* ψ_1 *is pre semi open, then* ψ_3 *is pre semi open.*

Proof: Let $\mathsf{D}_1 \in SO(\mathsf{Z})$. Since ψ_2 is surjective,

$$\psi_3(\mathsf{D}_1) = \psi_3 \circ \psi_2(\psi_2^{-1}(\mathsf{D}_1)) = \psi_1(\psi_2^{-1}(\mathsf{D}_1)).$$

Hence $\psi_3(\mathsf{D}_1)$ is semi open by assumption. ■

Example 3.4. *In Example* 3.2 ψ_3 *is pre semi open since* ψ_2 *is a surjective irresolute map and* ψ_1 *is pre semi open.*

Proposition 3.1. *Let* Y *be a semi-*T_2 *semi compact SCS, then* $SO(\mathsf{Y})$ *forms a topology.*

Proof: Let Y be a semi-T_2 semi compact SCS, then by Corollary 2.1, Y is extremally disconnected, by Definition 2.1 $SO(\mathsf{Y})$ is closed under arbitrary union and by Theorem 2.2 $SO(\mathsf{Y})$ is closed under finite intersection. Thus $SO(\mathsf{Y})$ forms a topology on Y. ■

Remark 3.2. *If* Y *is semi compact semi-*T_2 *space then Proposition* 3.1 *need not be true.*

Example 3.5. *Let* $\mathsf{Y} = \{1,2,3,4\}$*, be the set,*

$$\mathcal{T}_{\mathsf{Y}} = \{\emptyset, \{1\}, \{3\}, \{1,3\}, \{1,2,3,4\}\},$$

be a topology on Y*,* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *be a topological space and* $SO(\mathsf{Y}) = \{\emptyset, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\}$*,* $SO(\mathsf{Y})$ *is not closed under finite intersection, since semi compact semi-*T_2 *space not a SCS, since* $\{1,3,4\}$ *is semi compact but not semi closed.*

Lemma 3.6. *If* Z *is a semi compact subspace of semi-*T_2 *extremally disconnected space* Y *and* y *is not in* Z*, then there exist* $\mathsf{D}, \mathsf{E} \in SO(\mathsf{Y})$ *containing* y *and* Z *respectively such that* $\mathsf{D} \cap \mathsf{E} = \emptyset$*.*

Proof: Let Z be a semi compact subspace of a semi-T_2 extremally disconnected space Y and $y \notin \mathsf{Z}$. Let $z \in \mathsf{Z}$, then $y \neq z$. Since Y is semi-T_2 then there exists two disjoint semi open sets D_z and E_z of y and z respectively. Since for each $z \in \mathsf{Z}$, there exist $\mathsf{E}_z \in SO(\mathsf{Y})$ containing z. So

$$\mathsf{Z} \subset \bigcup_{z \in \mathsf{Z}} \mathsf{E}_z.$$

Thus the collection $\{\mathsf{E}_z | z \in \mathsf{Z}\}$ is a semi open cover of Z. Since Z is semi compact there exist a finite semi open subcover say $\mathsf{E}_{z_1}, \mathsf{E}_{z_2}, \cdots, \mathsf{E}_{z_n}$ of $\{\mathsf{E}_z | z \in \mathsf{Z}\}$. Consider

$$\mathsf{E} = \mathsf{E}_{z_1} \cup \mathsf{E}_{z_2} \cup \cdots \cup \mathsf{E}_{z_n}.$$

Since $\mathsf{E}_{z_1}, \mathsf{E}_{z_2}, \cdots, \mathsf{E}_{z_n}$ covers Z, so $\mathsf{Z} \subset \mathsf{E}$. Consider

$$\mathsf{D} = \mathsf{D}_{z_1} \cap \mathsf{D}_{z_2} \cap \cdots \cap \mathsf{D}_{z_n},$$

where $\mathsf{D}_{z_1}, \mathsf{D}_{z_2}, \cdots, \mathsf{D}_{z_n} \in SO(\mathsf{Y})$ corresponding to $\mathsf{E}_{z_1}, \mathsf{E}_{z_2}, \cdots, \mathsf{E}_{z_n} \in SO(\mathsf{Y})$. Since for any $z \in \mathsf{E}$, then $z \in \mathsf{E}_{z_i}$ for some $i \in \{1, 2, \cdots, n\}$, D_{z_i} and E_{z_i} are disjoint. Thus $z \notin \mathsf{D}$. Therefore $\mathsf{D} \cap \mathsf{E} = \emptyset$. Thus $\mathsf{D}, \mathsf{E} \in SO(\mathsf{Y})$ containing y and z respectively such that $\mathsf{D} \cap \mathsf{E} = \emptyset$. ■

Theorem 3.2. *Every semi-*T_2 *semi compact extremally disconnected space is semi normal.*

Proof: Let Y be a semi-T_2 semi compact extremally disconnected space, $y \in \mathsf{Y}$ and F is a semi closed set in Y not containing y then F is semi compact, from Lemma 3.6, there exists disjoint semi open sets containing y and F respectively. Then given disjoint semi closed set F and C in Y, choose for each point f of F, disjoint semi open sets D_f and E_f containing f and C, respectively. The collection $\{\mathsf{D}_f\}$ covers F because F is semi compact, F can be covered by finitely many semi open sets $\mathsf{D}_{f_1}, \mathsf{D}_{f_2}, \cdots, \mathsf{D}_{f_m}$. Then

$$\mathsf{D} = \mathsf{D}_{f_1} \cup \mathsf{D}_{f_2} \cup \cdots \cup \mathsf{D}_{f_m} \text{ and}$$

$$\mathsf{E} = \mathsf{E}_{f_1} \cap \mathsf{E}_{f_2} \cap \cdots \cap \mathsf{E}_{f_m}$$

are disjoint semi open sets containing F and C respectively. ■

Remark 3.3. *Theorem* 3.2 *need not be true for a semi compact semi-*T_2 *space.*

Theorem 3.3. *Let* Y *be a semi-*T_2 *semi compact extremally disconnected space then for a given* y *in* Y *and given* $\mathsf{D}_1 \in SO(\mathsf{Y})$ *of* y*, there exists* $\mathsf{D}_2 \in SO(\mathsf{Y})$ *of* y *such that* $Cl(\mathsf{D}_2)$ *is semi compact and* $Cl(\mathsf{D}_2) \subseteq \mathsf{D}_1$*.*

Proof: Let Y a semi-T_2 semi compact extremally disconnected space, $y \in \mathsf{Y}$ and $\mathsf{D}_1 \in SO(\mathsf{Y})$ and C be the set $\mathsf{Y} - \mathsf{D}_1$. Then $\mathsf{C} \in SC(\mathsf{Y})$, thus C is a semi compact subspace of Y.

By Lemma 3.6 there exist disjoint semi open sets D_2 and D_3 containing y and C, respectively. Then $Cl(\mathsf{D}_2)$ of D_2 in Y is semi compact, $Cl(\mathsf{D}_2)$ is disjoint from C, so that $Cl(\mathsf{D}_2) \subset \mathsf{D}_1$, as desired. ■

Remark 3.4. *Theorem* 3.3 *need not be true for a semi compact semi-*T_2 *space.*

Example 3.6. *Let* $\mathsf{Y} = \{a,b,c,d\}$*, be the set,*

$$\mathcal{T}_{\mathsf{Y}} = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c,d\}\},$$

be a topology on Y*,* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *be a semi compact semi-*T_2 *space and* $SO(\mathsf{Y}) = \emptyset$*,* $\{a\}$*,* $\{b\}$*,* $\{a,b\}$*,* $\{a,c\}$*,* $\{a,d\}$*,* $\{b,c\}$*,* $\{b,d\}$*,* $\{a,b,c\}$*,* $\{a,b,d\}$*,* $\{a,c,d\}$*,* $\{b,c,d\}$*,* $\{a,b,c,d\}$*,* $c \in \{b,c\} \in SO(\mathsf{Y})$*, but* $\nexists \mathsf{D} \in SO(\mathsf{Y})$ *such that* $Cl(\mathsf{D}_1) \subseteq \{b,c\}$*.*

Lemma 3.7. *Let* H, K *be two irresolute topological groups and* g *be a homomorphism of* H *into* K*. Then*

1) *for any subset* C *and* D *of* H*,* $\overline{g(\mathsf{C})}\ \overline{g(\mathsf{D})} \subset \overline{g(\mathsf{CD})}$
2) *for any subset* E *and* F *of* K*,* $\overline{g^{-1}(\mathsf{E})}\ \overline{g^{-1}(\mathsf{F})} \subset \overline{g^{-1}(\mathsf{EF})}$*.*
3) *for any symmetric subset* C *of* H*,* $\overline{g(\mathsf{C})}$ *is symmetric in* K *and hence* $\overline{g(C^{-1})} = \left[\overline{g(C)}\right]^{-1}$*.*
4) *for any symmetric subset* D *of* K*,* $\overline{g^{-1}(\mathsf{D})}$ *is symmetric in* H *and hence* $\overline{g^{-1}(D)} = \left[\overline{g^{-1}(D)}\right]^{-1}$*.*

Proof: Statement (1) and (2) follows from Statement (1) and (2) of Lemma 2.1 and for any subset C and D of topological group H, $\overline{\mathsf{C}}\ \overline{\mathsf{D}} \subset \overline{\mathsf{CD}}$ since the multiplication is irresolute. For (3), $g(\mathsf{C})$ is symmetric by condition (3) of Lemma 2.1.

Moreover, the inversion mapping in an irresolute topological group being a semi homeomorphism, for each subset C, $\overline{\mathsf{C}^{-1}} = (\overline{\mathsf{C}})^{-1}$. From this (3) and (4) follows directly. ■

Proposition 3.2. *Let* $(\mathsf{H}, \circledast)$ *be a group,* $\mathcal{T}_{\mathsf{H}}$ *be the topology on* H *and* $SO(\mathsf{H})$ *forms the topology.* $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms an irresolute topological group if and only if* $(\mathsf{H}, \circledast, SO(\mathsf{H}))$ *forms a topological group.*

Proof: Let $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ be an irresolute topological group which implies that for each $h_1h_2^{-1}$, for all $\mathsf{D}_3 \in SO(\mathsf{H})$ containing $h_1h_2^{-1}$, there exist $\mathsf{D}_1 \in SO(\mathsf{H})$ containing h_1 and $\mathsf{D}_2 \in SO(\mathsf{H})$ containing h_2 such that $\mathsf{D}_1\mathsf{D}_2^{-1} \subset \mathsf{D}_3$.

Thus the multiplication map and inverse map are continuous with respect to $SO(\mathsf{H})$. Hence $(\mathsf{H}, \circledast, SO(\mathsf{H}))$ forms a topological group. The proof of converse follows similarly. ■

Corollary 3.1. *Let* $(\mathsf{H}, \circledast)$ *be a group and* $(\mathsf{H}, \mathcal{T}_{\mathsf{H}})$ *is extremally disconnected,* $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms an irresolute topological group if and only if* $(\mathsf{H}, \circledast, SO(\mathsf{H}))$ *forms a topological group.*

Proof: From Proposition 3.2 and Theorem 2.2 the proof of corollary follows. ■

IV. Comparison of topological structures of transformation groups

In this section Irr-topological transformation groups, Irr*-topological transformation groups, I*-topological transformation groups, and I-topological transformation groups are introduced and their interrelations are explored through pertinent examples and counterexamples.

Definition 4.1. *A transformation group* $(\mathsf{H}, \mathsf{Y}, \psi)$ *on* Y *is said to be Irr-topological transformation group (Irr-TTG) if* H *is an Irr-topological group,* Y *is a topological space, and the map* $\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y}$ *is irresolute.*

Example 4.1. *Any Irr-topological group acting on itself forms an Irr-TTG.*

Example 4.2. *Let* $\mathsf{H} = \mathbb{Z}_2 = \{0, 1\}$ *be the group under usual addition modulo 2. Equip* H *with the Sierpinski topology* $\mathcal{T}_{\mathsf{H}} = \{\emptyset, \{0\}, \mathsf{H}\}$.

$$SO(\mathsf{H} \times \mathsf{H}) = \Big\{\emptyset, \{(0,0)\}, \{(0,0),(0,1)\}, \{(0,0),(1,0)\}, \{(0,0),(0,1),(1,0)\}, \{(0,0),(1,0),(0,1),(1,1)\}, \{(0,0),(1,1)\}, \{(0,0),(0,1),(1,1)\}, \{(0,0),(1,0),(1,1)\}\Big\},$$

H *acting on itself,* $(\mathsf{H}, \mathsf{H}, \psi)$ *is an Irr-TTG. But it is not a TTG since* H *is not a topological group and also the map* ψ *is not continuous.*

Example 4.3. *Let* $\mathsf{H} = \{e, (12)(34), (13)(24), (14)(23)\}$ *be the group under composition, and the topology on* H *be*

$$\mathcal{T}_{\mathsf{H}} = \{\emptyset, \mathsf{H}, \{e\}, \{e, (12)(34), (13)(24)\}\},$$

let $\mathsf{Y} = \{1, 2, 3, 4\}$ *be the set and the topology on* Y *be*

$$\mathcal{T}_{\mathsf{Y}} = \{\emptyset, \mathsf{Y}, \{1\}, \{1, 2, 3\}\}$$

such that $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms an Irr-topological group,* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *forms a topological space and* $\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y}$ *such that* $\psi(\sigma, y) = \sigma(y)$ *is irresolute. Thus* $(\mathsf{H}, \mathsf{Y}, \psi)$ *forms an Irr-TTG.*

Definition 4.2. *A transformation group* $(\mathsf{H}, \mathsf{Y}, \psi)$ *on* Y *is said to be Irr*-topological transformation group (Irr*-TTG) if* H *is an Irr-topological group,* Y *is a topological space, and the map* $\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y}$ *such that* $\forall\ h \in \mathsf{H}, y \in \mathsf{Y}, \forall$ *semi open set* D_3 *containing* $hy \in \mathsf{Y}$*, there exist* D_1 *and* D_2 *containing* h *and* y *respectively such that* $\mathsf{D}_1\mathsf{D}_2 \subseteq \mathsf{D}_3$.

Example 4.4. *Let* $\mathsf{H} = \{e, (12)(34), (13)(24), (14)(23)\}$ *be the group under composition, and the topology on* H *be*

$$\mathcal{T}_{\mathsf{H}} = \{\emptyset, \mathsf{H}, \{e\}, \{e, (12)(34), (13)(24)\}\},$$

let $\mathsf{Y} = \{1, 2, 3, 4\}$ *be the set and the topology on* Y *be* $\mathcal{T}_{\mathsf{Y}} = \{\emptyset, \mathsf{Y}\}$ *such that* $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms an Irr-topological group and* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *forms a topological space and* $\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y}$ *such that* $\psi(\sigma, y) = \sigma(y)$*. Thus* $(\mathsf{H}, \mathsf{Y}, \psi)$ *forms an Irr*-TTG.*

Definition 4.3. *A transformation group* $(\mathsf{H}, \mathsf{Y}, \psi)$ *on* Y *is said to be I*-topological transformation group (I*-TTG) if* H *is an irresolute topological group,* Y *is a topological space, and the map* $\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y}$ *is irresolute.*

Example 4.5. *Let* $\mathsf{H} = \{e, (12)(34), (13)(24), (14)(23)\}$ *be the group under composition, and the topology on* H *be*

$$\mathcal{T}_{\mathsf{H}} = \{\emptyset, \mathsf{H}, \{e, (12)(34)\}, \{(14)(23), (13)(24)\}\},$$

$(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms an irresolute topological group and* H *acting on itself,* $(\mathsf{H}, \mathsf{H}, \psi)$ *forms an I*-TTG.*

Definition 4.4. *A transformation group* $(\mathsf{H}, \mathsf{Y}, \psi)$ *on* Y *is said to be I-topological transformation group (I-TTG) if* H *is an irresolute topological group,* Y *is a topological space, and the map* $\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y}$ *such that* $\forall\ h \in \mathsf{H}, y \in \mathsf{Y}, \forall$ *semi open set* D_3 *containing* $hy \in \mathsf{Y}$*, there exist* D_1 *and* D_2 *containing* h *and* y *respectively such that* $\mathsf{D}_1\mathsf{D}_2 \subseteq \mathsf{D}_3$.

Example 4.6. *Any irresolute topological group acting on itself is an I-TTG.*

Example 4.7. *Let* $\mathsf{H} = \{0, 1, 2, 3\}$ *be the group under addition modulo 4,*

$$\mathcal{T}_{\mathsf{H}} = \{\emptyset, \mathsf{H}, \{0, 2\}, \{1, 3\}\}.$$

$(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms an irresolute topological group and* H *acting on* $\mathsf{H}/\mathsf{K} = \{0\mathsf{K}, 1\mathsf{K}\}, \mathsf{K} = \{0, 2\}$ *be the cosets,* $(\mathsf{H}, \mathsf{H}/\mathsf{K}, \psi)$ *forms an I-TTG.*

Proposition 4.1. *Let* $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *be an irresolute topological group,* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *be a topological space and* $SO(\mathsf{H}), SO(\mathsf{Y})$ *forms a topology.* $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *acting on* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$*,* $(\mathsf{H}, \mathsf{Y}, \psi)$ *is an I-TTG if and only if* $(\mathsf{H}, \circledast, SO(\mathsf{H}))$ *acting on* $(\mathsf{Y}, SO(\mathsf{H}))$*,* $(\mathsf{H}, \mathsf{Y}, \psi)$ *is a TTG.*

Proof: Let$(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ acting on $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$, $(\mathsf{H}, \mathsf{Y}, \psi)$ is an I-TTG which implies that for each hy, for all $\mathsf{D}_3 \in SO(\mathsf{Y})$ containing gh there exist $\mathsf{D}_1 \in SO(\mathsf{H})$ containing h and $\mathsf{D}_2 \in SO(\mathsf{Y})$ containing y such that $\mathsf{D}_1\mathsf{D}_2^{-1} \subset \mathsf{D}_3$.

Thus $(\mathsf{H}, \circledast, SO(\mathsf{H}))$ acting on $(\mathsf{Y}, SO(\mathsf{H}))$ is continuous. Hence $(\mathsf{H}, \mathsf{Y}, \psi)$ forms a TTG. Similarly the converse is proved. ■

Corollary 4.1. *Let* $(\mathsf{H}, \mathcal{T}_{\mathsf{H}}), (\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *are extremally disconnected spaces,* $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *be an irresolute topological group and* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *be a topological space.* $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *acting on* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$*,* $(\mathsf{H}, \mathsf{Y}, \psi)$ *is an I-TTG if and only if* $(\mathsf{H}, \circledast, SO(\mathsf{H}))$ *acting on* $(\mathsf{Y}, SO(\mathsf{H}))$*,* $(\mathsf{H}, \mathsf{Y}, \psi)$ *is a TTG.*

Proof: Proof follows from Proposition 3.2 and 4.1, Theorem 2.2 ■

Remark 4.1.

1) *There exist an Irr-TTG which is neither an Irr*-TTG nor a TTG.*
2) *Any Irr-topological group acting on itself will be an Irr-TTG but it need not be an Irr*-TTG.*
3) *Let* $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *be an Irr-TTG,* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *be a discrete space then* H *need not be an Irr*-TTG for a group action* ψ *of* H *on* Y.
4) *If* $\mathcal{T}_{\mathsf{H}} =$ *co-finite topology on* H, H *acting on itself then* H *is TTG if and only if* H *is an I-TTG. If* H *is finite then* $\mathcal{T}_{\mathsf{H}}$ *is discrete which is trivial. If* H *is infinite then* $\mathcal{T}_{\mathsf{H}} = SO(\mathsf{H})$, *thus from Definition* 2.13, 2.15 *its true.*
5) *Any Irr-topological group* H *acting on singleton set* $\{e\}$, e *is the identity of* H, *then it will be both an Irr-TTG and an Irr*-TTG, since topology on singleton set is discrete.*
6) *Any irresolute topological group* H *acting on singleton set* $\{e\}$, e *is the identity of* H, *then it will be both an I-TTG and an I*-TTG, since topology on singleton set is discrete.*
7) *There exist a TTG which is an Irr-TTG, an Irr*-TTG, an I-TTG and an I*-TTG.*

Example 4.8. *Let* $\mathsf{H} = \{e, (123), (132)\}$ *be the group under composition, and the topology on* H *be*

$$\mathcal{T}_{\mathsf{H}} = \{\emptyset, \mathsf{H}, \{e\}, \{e, (123)\}\},$$

let $\mathsf{Y} = \{1, 2, 3\}$ *be the set and the topology on* Y *be*

$$\mathcal{T}_{\mathsf{Y}} = \{\emptyset, \mathsf{Y}, \{1\}, \{1, 2\}\}$$

such that $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms an Irr-topological group and* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *forms a topological space and* $\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y}$ *such that* $\psi(\sigma, y) = \sigma(y)$, $(\mathsf{H}, \mathsf{Y}, \psi)$ *forms an Irr-TTG but it is neither an Irr*-TTG nor a TTG.*

Example 4.9. *Let* $\mathsf{H} = \{e, a, b, c\}$ *be the Klein four group and a topology on* H *be*

$$\mathcal{T}_{\mathsf{H}} = \{\emptyset, \mathsf{H}, \{e\}, \{e, a, b\}\}.$$

H *acting on itself,* $(\mathsf{H}, \mathsf{H}, \psi)$ *is an Irr-TTG. But it is not an Irr*-TTG since* $\mathfrak{m}^{-1}(e)$ *cannot be written as the product of semi open sets of* H.

Example 4.10. *Any abstract group* H *with discrete topology* $\mathcal{T}_{\mathsf{H}} = P(\mathsf{H})$ *where* $P(\mathsf{H})$ *denotes the set of all power sets of* H, H *acting on itself,* $(\mathsf{H}, \mathsf{H}, \psi)$ *forms a TTG, an Irr-TTG, an Irr*-TTG, an I-TTG and an I*-TTG.*

Example 4.11. *Let* $\mathsf{H} = \{e, (12)(34), (13)(24), (14)(23)\}$ *be the group under composition,*

$$\mathcal{T}_{\mathsf{H}} = \emptyset, \mathsf{H}, \{e, (12)(34)\}, \{(14)(23), (13)(24)\}$$

be a topology on H *and* $\mathsf{Y} = \{1, 2, 3, 4\}$ *be the set and the topology on* Y *be*

$$\mathcal{T}_{\mathsf{Y}} = \{\emptyset, \{1, 2\}, \{3, 4\}, \mathsf{Y}\}$$

such that $(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms topological group and an irresolute topological group and* $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *forms a topological space and* $\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y}$ *such that* $\psi(\sigma, y) = \sigma(y)$, $(\mathsf{H}, \mathsf{H}, \psi)$ *forms a TTG, an Irr-TTG, an Irr*-TTG, an I-TTG and an I*-TTG.*

Theorem 4.1. *Let* $(\mathsf{H}, \circledast)$ *be a group,* $\mathcal{T}_{\mathsf{H}}$ *be a topology on* H, $(\mathsf{Y}, \mathcal{T}_{\mathsf{Y}})$ *be a topological space and* $\psi : \mathsf{H} \times \mathsf{Y} \to \mathsf{Y}$.

1) *If* $(\mathsf{H}, \mathsf{Y}, \psi)$ *is an I-TTG, then it is an Irr-TTG, an Irr*-TTG, an I*-TTG.*
2) *If* $(\mathsf{H}, \mathsf{Y}, \psi)$ *is an Irr*-TTG, then it is an Irr-TTG.*
3) *If* $(\mathsf{H}, \mathsf{Y}, \psi)$ *is an I*-TTG, then it is an Irr-TTG.*

Proof:

1) Since every irresolute topological group is an Irr-topological group and product $\mathsf{D}_1 \times \mathsf{D}_2$ of semi open sets D_1 of H and D_2 of Y is semi open in $\mathsf{H} \times \mathsf{Y}$. Hence (1) is true.
2) Every Irr*-TTG is an Irr-TTG, since product $\mathsf{D}_1 \times \mathsf{D}_2$ of semi open sets D_1 of H and D_2 of Y will be a semi open in $\mathsf{H} \times \mathsf{Y}$.
3) Every I*-TTG is an Irr-TTG, since every irresolute topological group is an Irr-topological group.

■

Remark 4.2. *The converse of above statements need not be true.*

Example 4.12. *Let* $\mathsf{H} = \{1, 3, 5, 7\}$ *be the group under multiplication modulo 8,*

$$\mathcal{T}_{\mathsf{H}} = \{\emptyset, \mathsf{H}, \{1\}, \{1, 3, 5\}\}.$$

$(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms an Irr-topological group and* H *acting on itself, forms an Irr-TTG, but it is not an Irr*-TTG, an I*-TTG, an I-TTG.*

Example 4.13. *Let* $\mathsf{H} = \{0, 1, 2\}$ *be the group under addition modulo 3,*

$$\mathcal{T}_{\mathsf{H}} = \{\emptyset, \mathsf{H}, \{0\}, \{0, 1\}\}.$$

$(\mathsf{H}, \circledast, \mathcal{T}_{\mathsf{H}})$ *forms a Irr-topological group and* H *acting on* $\mathsf{K} = \{0\}$, $(\mathsf{H}, \mathsf{H}/\mathsf{K}, \psi)$ *forms a Irr*-TTG, but it is neither an I*-TTG nor an I-TTG.*

Theorem 4.1 can be pictured as follows,

$$\begin{tikzcd}
I-TTG \arrow[r, Rightarrow] \arrow[d, Rightarrow] \arrow[dr, Rightarrow] & I^*-TTG \arrow[d, Rightarrow] \\
Irr^*-TTG \arrow[r, Rightarrow] & Irr-TTG
\end{tikzcd}$$

Fig. 2. Interrelations among various topological transformation groups

V. Conclusion

This paper analyses the characteristics of semi continuous and irresolute functions, along with exploring separation axioms such as semi-T_2 and semi normality. The introduction of novel concepts, namely Irr-topological transformation group, Irr*-topological transformation group, an I*-topological transformation group, and I-topological transformation group, adds depth to the analysis. Through a systematic examination of their interrelations, the paper establishes a comprehensive understanding, supported by examples and counterexamples. This contributes valuable insights into the aspects of a topological transformation groups in the context of an irresolute functions, offering a good understanding of their interplay within the given mathematical framework.

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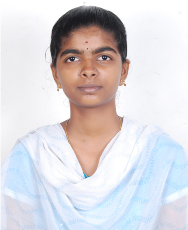

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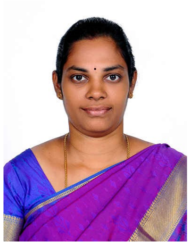

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