A Novel Containment Control Design Scheme for Second-Order Multi-agent Systems with Adjustable Reference Signals

Mengyi Jiang, Chuang Gao, Yonghui Yang, and Anatolii K. Pogodaev

***Abstract*—Due to the flexibility issue in existing containment control schemes for second-order multi-agent systems, we design an innovative control scheme. The proposed scheme involves the generation of a set of adjustable reference signals, and arbitrary adjustments of the reference signals are allowed. The innovative scheme not only enhances the flexibility of multi-agent systems, but also significantly decreases the likelihood of collision among multiple agents during their movements. Furthermore, the effectiveness of the proposed scheme is confirmed through simulation results. The proposed scheme addresses a critical issue in existing containment control schemes and presents a promising solution for improving the overall performance and safety of multi-agent systems.**

***Index Terms*—containment control, observer design, tracking performance, multi-agent systems.**

I. Introduction

With the continuous development of artificial intelligence technology, multi-agent systems (MASs) have been widely used in various fields [1]− [3]. In practical applications, MASs need to be controlled cooperatively to realize the expected tasks. Backstepping, as an effective control method, is widely used in MASs. The backstepping method is a nonlinear control method that designs control laws by constructing Lyapunov functions and utilizing Lyapunov functions to achieve stability and robustness of the system [4]. The applications of backstepping were discussed in [5]− [7] to illustrate its superior advantages. In a MAS, each agent is regarded as a nonlinear dynamical system, therefore the backstepping method can be applied to the control of MASs.

Containment control is a control method in the framework of a multi-leader-multi-follower system that drives all followers to converge into a convex hull formed by all leaders [8] and [9]. In fact, backstepping containment control is of great significance [10] and [11]. Firstly, the stability of a MAS is realized by introducing a backstepping controller, which ensures that the system operates normally under uncertain environments. Secondly, backstepping containment control realizes the cooperative work among multiple agents and improves the overall performance of MASs. For example, in the fields of unmanned vehicle queue control and unmanned aircraft formation flight, backstepping control can ensure and improve the efficiency and safety of MASs. Therefore, how to effectively design a backstepping containment control scheme has become a hot spot and challenge in current research.

In the existing containment control schemes [12]− [14], the reference signal of the follower is obtained by Laplace matrix mapping, the disadvantage of these schemes is that the reference signal is not allowed to be adjusted. For some special environments, collisions between multiple intelligences may occur. To address this problem, we take an alternative approach by generating adjustable reference signals, which arbitrarily adjusts the reference signals while ensuring that the followers enters the convex hull, which not only improves the flexibility of the control scheme, but also greatly reduces the probability of collision among multiple agents in the process of movements. Additionally, observer design is an important area in control theory. The observer is used to estimate the states of a system by making observations of the system output without direct measurements [15]− [17]. Therefore, we also apply an observer design scheme into the containment control. Finally, the proposed method is verified to be effective by simulation.

II. Problem Statements

Consider a class of MASs as follows [18]:

$$\begin{cases} \dot{x}_{h,1} = x_{h,2}, \\ \dot{x}_{h,2} = \frac{1}{M_h}\left(u_h - D_h x_{h,2}\right), \\ y_h = x_{h,1}, \end{cases} \tag{1}$$

where $x_{h,1} \in R$ and $x_{h,2} \in R$ represent the position and velocity of the agent. M_h and D_h are the mass and the damping constant of the agent. $u_h \in R$ is the control law of MAS. $y_h \in R$ denotes the output of MAS. By following the descriptions in [12]− [14], we consider a directed graph communication among H number of follower agents, where a Laplacian matrix of a directed graph $\bar{G}$ is defined by $\bar{L} = \bar{D} - \bar{A}$ with

$$\bar{D} = diag(\textstyle\sum_{j \in H} a_{h,j}) \in R^{H \times H} \tag{2}$$

and

$$\bar{A} = [a_{h,j}] \in R^{H \times H}. \tag{3}$$

The control goal of this paper is to design a group of controllers u_h and observers to realize the containment control of MASs (1) if the velocities of the agents are unmeasurable, so that the containment errors $\varsigma_{h,1} = \sum_{j=1}^{H} a_{h,j}(y_h - y_j) +$

Manuscript received January 2, 2024; revised April 10, 2024.

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$y_h - y_{d,h}$ are bounded with $y_{d,h}$ being reference signals. Furthermore, we need to select appropriate reference signals $y_{d,h} = \sum_{k=1}^{N} \vartheta_k L_k$ within the convex hull according to **Definition 1** in [19]. Note that the reference signal $y_{d,h}$ is differentiable.

Definition 1. If the number of dynamic leaders is N, define a class of time-varying functions for a set $\Xi = \{L_1, \cdots, L_N\} \in R^N$ with L_k being a time-varying function and a contant $\vartheta_k \in [0,1]$ $(k = 1, 2, \cdots, N)$, the a convex hull $Co(\Xi)$ can be defined by $Co(\Xi) = \left\{\sum_{k=1}^{N} \vartheta_k L_k \mid L_k \in \Xi, \sum_{k=1}^{N} \vartheta_k = 1\right\}$.

III. Observer Design Scheme

First, we consider the states of MASs are unmeasurable, it is necessary to design state observers. Define a vector $\check{x}_h = [\hat{x}_{h,1}, \hat{x}_{h,2}]^T$ to be an estimated vector of $\breve{x}_h = [x_{h,1}, x_{h,2}]^T$. Then, the state observers are designed as follows:

$$\begin{cases} \dot{\hat{x}}_{h,1} = \hat{x}_{h,2} + \kappa_{h,1} e_{h,1}, \\ \dot{\hat{x}}_{h,2} = \frac{u_h - D_h \hat{x}_{h,2}}{M_h} + \kappa_{h,2} e_{h,1}, \end{cases} \tag{4}$$

where $e_{h,1} = x_{h,1} - \hat{x}_{h,1}$, $\kappa_{h,1}$ and $\kappa_{h,2}$ are positive constants. Next, define an error vector $e_h = [e_{h,1}, e_{h,2}]^T = \breve{x}_h - \check{x}_h$, we obtain

$$\dot{e}_h = \Theta_h e_h - \frac{D_h}{M_h} B e_h, \tag{5}$$

where $\Theta_h = \begin{bmatrix} -\kappa_{h,1} & 1 \\ -\kappa_{h,2} & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. To analyze the stability of the observers, we introduce a Lyapunov function as

$$V_0 = \sum_{h=1}^{H} e_h^T P_h e_h. \tag{6}$$

Combining (5) with (6) yields

$$\dot{V}_0 = \sum_{h=1}^{H} e_h^T \left(\left(\Theta_h^T P_h + P_h^T \Theta_h \right) - 2 \frac{D_h}{M_h} P_h B \right) e_h. \tag{7}$$

By selecting a suitable vector $K_h = [\kappa_{h,1}, \kappa_{h,2}]^T$ to make sure that Θ_h is strictly Huiwitz. If there is an identity matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, there exists a matrix $P_h = P_h^T > 0$ satisfying

$$\Theta_h^T P_h + P_h^T \Theta_h - 2 \frac{D_h}{M_h} P_h B \leq -2I. \tag{8}$$

If the condition (8) is ensured, then the observers (4) are asymptotically stable.

IV. Backstepping Containment Control Scheme

Now, we need to design distributed containment controllers for MASs to realize that the follower agents enter the convex hull, then some errors are defined as follows:

$$\begin{cases} \varsigma_{h,1} = \sum_{j=1}^{H} a_{h,j}(y_h - y_j) + y_h - y_{d,h}, \\ \eta_{h,1} = \varsigma_{h,1} - \xi_{h,1}, \\ \eta_{h,2} = \hat{x}_{h,2} - \gamma_{h,2}. \end{cases}, \tag{9}$$

where $\varsigma_{h,1}$ is a containment error and $\gamma_{h,2}$ is an output of the command filter:

$$a_{h,2} \dot{\gamma}_{h,2} + \gamma_{h,2} = \gamma_{h,1}, \gamma_{h,2}(0) = \gamma_{h,1}(0), \tag{10}$$

where $a_{h,2} > 0$ is a constant and $\gamma_{h,1}$ is a designed virtual control law. $\eta_{h,1}$ presents a compensation error, and $\xi_{h,1}$ denotes a compensation signal satisfying

$$\dot{\xi}_{h,1} = -b_{h,1} \xi_{h,1} + \gamma_{h,2} - \gamma_{h,1}, \quad \xi_{h,1}(0) = 0 \tag{11}$$

where $b_{h,1} > 0$ is a constant. To ensure the stability of MASs (1), establish a Lyapunov candidate function as

$$V_1 = \sum_{h=1}^{H} \frac{1}{2} \left(\eta_{h,1}^2 + \eta_{h,2}^2 \right). \tag{12}$$

Then, it follows from (12) that

$$\dot{V}_1 = \sum_{h=1}^{H} \left(\eta_{h,1} \dot{\eta}_{h,1} + \eta_{h,2} \dot{\eta}_{h,2} \right). \tag{13}$$

According to $\eta_{h,1} = \varsigma_{h,1} - \xi_{h,1}$, it produces

$$\dot{V}_1 = \sum_{h=1}^{h} \left(\eta_{h,1} (\dot{\varsigma}_{h,1} - \dot{\xi}_{h,1}) + \eta_{h,2} \dot{\eta}_{h,2} \right). \tag{14}$$

Substituting (9) and (11) into (14) yields

$$\begin{aligned} \dot{V}_1 &= \sum_{h=1}^{H} \eta_{h,1} \left(\sum_{j=1}^{H} a_{h,j} (\dot{y}_h - \dot{y}_j) + \dot{y}_h - \dot{y}_{d,h} \right) \\ &\quad + \sum_{h=1}^{H} \eta_{h,1} \left(b_{h,1} \xi_{h,1} - \gamma_{h,2} + \gamma_{h,1} \right) + \eta_{h,2} \dot{\eta}_{h,2} \\ &= \sum_{h=1}^{H} \eta_{h,1} \left(\sum_{j=1}^{H} a_{h,j} (x_{h,2} - x_{j,2}) + x_{h,2} \right) \\ &\quad + \sum_{h=1}^{H} \eta_{h,1} \left(b_{h,1} \xi_{h,1} - \dot{y}_{d,h} \right) \\ &\quad + \sum_{h=1}^{H} \eta_{h,1} \left(-\gamma_{h,2} + \gamma_{h,1} \right) + \eta_{h,2} \dot{\eta}_{h,2}. \end{aligned} \tag{15}$$

From $e_{h,2} = x_{h,2} - \hat{x}_{h,2}$, one has

$$\begin{aligned} \dot{V}_1 &= \sum_{h=1}^{H} \eta_{h,1} \left(\sum_{j=1}^{H} a_{h,j} (e_{h,2} + \hat{x}_{h,2} - e_{j,2} - \hat{x}_{j,2}) \right) \\ &\quad + \sum_{h=1}^{H} \eta_{h,1} \left(e_{h,2} + \hat{x}_{h,2} - \dot{y}_{d,h} + b_{h,1} \xi_{h,1} \right) \\ &\quad + \sum_{h=1}^{h} \eta_{h,1} \left(\gamma_{h,1} - \gamma_{h,2} \right) + \eta_{h,2} \dot{\eta}_{h,2}. \\ &= \sum_{h=1}^{H} \eta_{h,1} \left(\sum_{j=1}^{H} a_{h,j} (\hat{x}_{h,2} - \hat{x}_{j,2}) + \hat{x}_{h,2} - \dot{y}_{d,h} \right) \\ &\quad + \sum_{h=1}^{H} \eta_{h,1} \left(b_{h,1} \xi_{h,1} - \gamma_{h,2} + \gamma_{h,1} \right) \\ &\quad + \eta_{h,2} \dot{\eta}_{h,2} + \sum_{h=1}^{H} \sum_{j=1}^{H} a_{h,j} \eta_{h,1} \left(e_{h,2} - e_{j,2} \right) \\ &\quad + \sum_{h=1}^{H} \eta_{h,1} e_{h,2}. \end{aligned} \tag{16}$$

Next, we design $\gamma_{h,1}$ as follows:

$$\gamma_{h,1} = -b_{h,1}\varsigma_{h,1} - \sum_{j=1}^{H} a_{h,j}(\hat{x}_{h,2} - \hat{x}_{j,2}) - \hat{x}_{h,2} + \dot{y}_{d,h}. \quad (17)$$

where $b_{h,1} > 0.75$ is a design constant. Then, it follows from (15) that

$$\begin{aligned}\dot{V}_1 \leq & -\sum_{j=1}^{H} b_{h,1}\eta_{h,1}^2 + \sum_{j=1}^{H}\eta_{h,2}\left(\dot{\eta}_{h,2} + \eta_{h,1}\right) \\ & + \sum_{h=1}^{H}\sum_{j=1}^{H} a_{h,j}\eta_{h,1}\left(e_{h,2} - e_{j,2}\right) \\ & + \sum_{j=1}^{H}\eta_{h,1}e_{h,2}. \end{aligned} \quad (18)$$

For the last two terms in (18), based on Young's inequality, we have

$$\begin{cases} \sum\limits_{h=1}^{H}\sum\limits_{j=1}^{H} a_{h,j}\eta_{h,1}\left(e_{h,2} - e_{j,2}\right) \\ \leq \sum\limits_{h=1}^{H} \frac{1}{2}\eta_{h,1}^2 + \sum\limits_{h=1}^{H}\sum\limits_{j=1}^{H} a_{h,j}e_h^T e_h \\ + \sum\limits_{h=1}^{H}\sum\limits_{j=1}^{H} a_{h,j}e_j^T e_j \\ \sum\limits_{j=1}^{H}\eta_{h,1}e_{h,2} \leq \sum\limits_{j=1}^{H}\frac{1}{4}\eta_{h,1}^2 + \sum\limits_{j=1}^{H} e_h^T e_h \end{cases} \quad (19)$$

Next, define

$$\begin{aligned}\Delta_h = & \sum_{h=1}^{H}\sum_{j=1}^{H} a_{h,j}e_h^T e_h + \sum_{j=1}^{H} e_h^T e_h \\ & + \sum_{h=1}^{H}\sum_{j=1}^{H} a_{h,j}e_j^T e_j. \end{aligned} \quad (20)$$

From (4), (19) and (18), it gives

$$\begin{aligned}\dot{V}_1 \leq & -\sum_{h=1}^{h}\left(b_{h,1} - 0.75\right)\eta_{h,1}^2 + \Delta_h \\ & + \sum_{h=1}^{h}\eta_{h,2}\left(\dot{\hat{x}}_{h,2} - \dot{\gamma}_{h,2} + \eta_{h,1}\right) \\ = & -\sum_{h=1}^{h}\left(b_{h,1} - 0.75\right)\eta_{h,1}^2 + \sum_{h=1}^{h}\eta_{h,2}\frac{u_h}{M_h} \\ & + \Delta_h + \sum_{h=1}^{h}\eta_{h,2}\left(\eta_{h,1} - \frac{D_h}{M_h}\hat{x}_{h,2}\right) \\ & + \sum_{h=1}^{h}\eta_{h,2}(\kappa_{h,2}e_{h,1} - \dot{\gamma}_{h,2}). \end{aligned} \quad (21)$$

Design an actual control input u_h as

$$u_h = -M_h\left(b_{h,2}\eta_{h,2} - \frac{D_h}{M_h}\hat{x}_{h,2} + \kappa_{h,2}e_{h,1} - \dot{\gamma}_{h,2} + \eta_{h,1}\right), \quad (22)$$

where $b_{h,2} > 0$ is a design constant. Then, we obtain

$$\dot{V}_1 \leq -\sum_{h=1}^{H}\left(b_{h,1} - 0.75\right)\eta_{h,1}^2 - \sum_{h=1}^{H} b_{h,2}\eta_{h,2}^2 + \Delta_h. \quad (23)$$

Recall (7) and (23), we construct a Lyapunov function $V = V_0 + V_1$ such that

$$\begin{aligned}\dot{V} \leq & \sum_{h=1}^{H} e_h^T\left(\left(\Theta_h^T P_h + P_h^T\Theta_h\right) - 2\frac{D_h}{M_h}P_h B\right)e_h \\ & -\sum_{h=1}^{H}\left(b_{h,1} - 0.75\right)\eta_{h,1}^2 - \sum_{h=1}^{H} b_{h,2}\eta_{h,2}^2 + \Delta_h \\ \leq & \sum_{h=1}^{H} e_h^T\left(\left(\Theta_h^T P_h + P_h^T\Theta_h\right) - 2\frac{D_h}{M_h}P_h B\right)e_h \\ & + \sum_{h=1}^{H}\left(1 + \sum_{j=1}^{H} a_{h,j} + \sum_{j=1}^{H} a_{j,h}\right)e_h^T e_h \\ & -\sum_{h=1}^{H}\left(b_{h,1} - 0.75\right)\eta_{h,1}^2 - \sum_{h=1}^{H} b_{h,2}\eta_{h,2}^2 \end{aligned} \quad (24)$$

By solving the linear matrix inequality

$$\Theta_h^T P_h + P_h^T\Theta_h - 2\frac{D_h}{M_h}P_h B + I + \sum_{j=1}^{H}\left(a_{h,j} + a_{j,h}\right)I \leq -2I, \quad (25)$$

it is easy to obtain that $\dot{V} \leq 0$, which implies

$$\begin{cases} \lim\limits_{t\to\infty}\|e_h\| = 0, \\ \lim\limits_{t\to\infty}\eta_{h,1} = 0, \\ \lim\limits_{t\to\infty}\eta_{h,2} = 0. \end{cases} \quad (26)$$

Next, define a Lyapunov function as

$$V_2 = \sum_{h=1}^{H}\frac{1}{2}\xi_{h,1}^2. \quad (27)$$

Then, it produces

$$\begin{aligned}\dot{V}_2 = & \sum_{h=1}^{H}\xi_{h,1}\dot{\xi}_{h,1} \\ = & \sum_{h=1}^{H}\xi_{h,1}(-b_{h,1}\xi_{h,1}^2 + \gamma_{h,2} - \gamma_{h,1}) \\ = & -\sum_{h=1}^{H} b_{h,1}\xi_{h,1}^2 + \sum_{h=1}^{H}\xi_{h,1}(\gamma_{h,2} - \gamma_{h,1}) \\ \leq & -\sum_{h=1}^{H} b_{h,1}\xi_{h,1}^2 + \sum_{h=1}^{H}\left|\xi_{h,1}\right|\left|\gamma_{h,2} - \gamma_{h,1}\right|. \end{aligned} \quad (28)$$

According to the property of filter (10), we know that $|\gamma_{h,2} - \gamma_{h,1}| \leq \tau_h$ with τ_h being a positive constant, it yields

$$\begin{aligned}\dot{V}_2 \leq & -\sum_{h=1}^{H} b_{h,1}\xi_{h,1}^2 + \sum_{h=1}^{H}\left|\xi_{h,1}\right|\tau_h \\ \leq & -\sum_{h=1}^{H} b_{h,1}\xi_{h,1}^2 + \sum_{h=1}^{H}(0.5\xi_{h,1}^2 + 0.5\tau_h^2) \\ = & -\sum_{h=1}^{H}\left(b_{h,1} - 0.5\right)\xi_{h,1}^2 + \sum_{h=1}^{H} 0.5\tau_h^2. \end{aligned} \quad (29)$$

Define

$$\begin{cases} c_2 = 2b_{h,1} - 1, \\ d_2 = \sum\limits_{h=1}^{H} 0.5\tau_h^2. \end{cases} \quad (30)$$

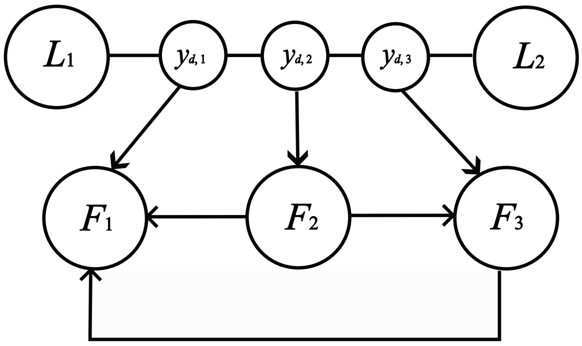

Fig. 1. Communication graph.

Thus, choosing $b_{h,1} > 0.75$ produces

$$\dot{V}_2 \leq -c_2 V_2 + d_2, \tag{31}$$

which means that $\xi_{h,1}$ are uniformly bounded. Then, one has

$$V_2(t) \leq V_2(0)e^{-ct} + \frac{d_2}{c_2}(1 - e^{-ct}), \tag{32}$$

which implies $\lim_{t\to\infty} \xi_{h,1} \leq \sqrt{2d_2/c_2}$. From (26) and $\eta_{h,1} = \varsigma_{h,1} - \xi_{h,1}$, one has

$$\lim_{t\to\infty} \varsigma_{h,1} \leq \sqrt{2d_2/c_2}. \tag{33}$$

From (33), it is concluded that the containment errors $\varsigma_{h,1}$ are bounded by $\sqrt{2d_2/c_2}$ for $t \to \infty$. Therefore, the control objective of this paper is achieved.

V. Simulation

In this section, we verify the performance of the proposed scheme by simulation. Suppose that there are three followers (F_1, F_2 and F_3) and two leaders (L_1 and L_2) under a directed communication graph shown in Figure 1. The trajectories of leaders are specified as $L_1 = \cos(t) + 2$ and $L_2 = \cos(t) - 2$, then we generate three reference signals for followers as $y_{d,1} = 0.1L_1 + 0.9L_2$, $y_{d,1} = 0.4L_1 + 0.6L_2$ and $y_{d,3} = 0.8L_1 + 0.2L_2$. The observers are defined by

$$\begin{cases} \dot{\hat{x}}_{h,1} = \hat{x}_{h,2} + \kappa_{h,1} e_{h,1}, \\ \dot{\hat{x}}_{h,2} = \frac{u_h - D_h \hat{x}_{h,2}}{M_h} + \kappa_{h,2} e_{h,1}, \end{cases} \tag{34}$$

where $\kappa_{h,1} = 0.5$ and $\kappa_{h,2} = 0.1$. The control input u_h can be determined by

$$u_h = -m_h \left(b_{h,2}\eta_{h,2} - \frac{D_h}{M_h} x_{h,2} - \dot{\gamma}_{h,2} + \eta_{h,1} \right), \tag{35}$$

where $b_{h,2} = 5$, $M_h = 1$ and $D_h = 1$. Then, $\eta_{h,1}$ is defined by

$$\eta_{h,1} = \sum_{j=1}^{H} a_{h,j}(y_h - y_j) + y_h - y_{d,h}. \tag{36}$$

Then, $\xi_{h,1}$ is generated by

$$\dot{\xi}_{h,1} = -b_{h,1}\xi_{h,1} + \gamma_{h,2} - \gamma_{h,1} \tag{37}$$

with $b_{h,1} = 1$. For command filters (10), we choose $a_{h,2} = 1$. Furthermore, ς_h can be obtained by

$$\begin{cases} \varsigma_1 = (x_{1,1} - x_{2,1}) + (x_{1,1} - x_{3,1}) + (x_{1,1} - y_{d,1}), \\ \varsigma_2 = (x_{2,1} - x_{3,1}) + (x_{2,1} - y_{d,2}), \\ \varsigma_3 = (x_{3,1} - x_{2,1}) + (x_{3,1} - y_{d,3}). \end{cases} \tag{38}$$

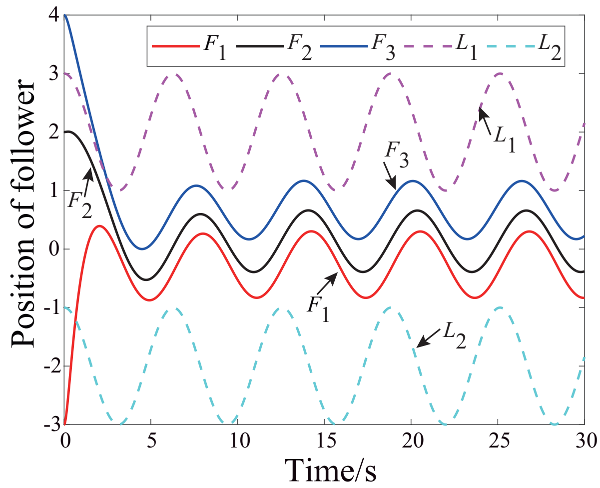

Fig. 2. Tracking performance.

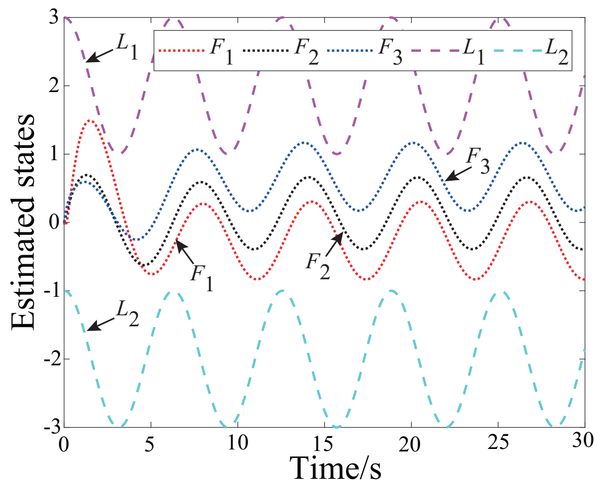

Fig. 3. Estimation performance.

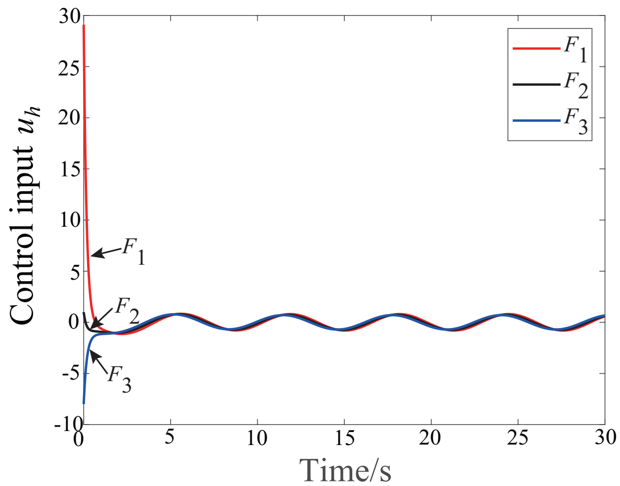

Fig. 4. Control laws.

For the initial conditions, we set $x_{1,1}(0) = -3$, $x_{2,1}(0) = 2$, $x_{3,1}(0) = 4$ and the rest initial values are set to 0. Figures 2–4 show the simulation results. Figure 2 is the tracking performance of the three followers, it can be seen that the three followers start from their own initial state, then F_1 and F_3 enter the convex hull shortly. During the process of moving, there is no collision. By setting the adjustable reference signal $y_{d,h}$, the positions of the three followers are specified arbitrarily within the convex hull. Figure 3 is the observation performance of the observers, the initial values of the observers are 0, it can be seen that the estimated values fit well with the follower movement trajectories in Figure 2. Figure 4 shows the actual control input signals of the three followers, and it shows that the control signals are large at the beginning, and after the followers enter the convex hull, the control signal gradually decreases and oscillates in the neighborhood around zero. Therefore, it can be concluded that the proposed control scheme is effective. The containment control of MASs is realized by arbitrarily setting the reference signals in the convex hull, and the probability of collision among agents is reduced.

VI. Conclusion

In this paper, we propose an novel containment control scheme, which takes into account the case where the states of MASs are unmeasurable. In order to reduce the probability of collision after the follower agents enter the convex hull, we design a mechanism with adjustable reference signals, and combine it with backstepping to design the distributed controllers. The observers are proved to be asymptotically stable and the containment errors are bounded through Lyapunov theory. Finally, the proposed method is verified to be effective by simulation. In the next step, the transient and steady-state performance of MASs can be further improved by adopting prescribed performance control and finite−time control method.

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