Nonlinear Model for Aphid and Ladybugs Interaction with Pesticides as Fuzzy Parameters

Viska Noviantri, Kelvin Alexander, Dani Suandi, Said Achmad, Angel Marcelina Br Sipahutar

***Abstract*—Pesticides are the most common method used to eliminate pests, including aphids. Pesticides are the most common method used to eliminate pests, including aphids. Nonetheless, numerous farmers incorporate ladybugs into their pest management strategies as they serve as natural predators of aphids. By integrating these methods, farmers aim to achieve optimal outcomes in mitigating the detrimental effects of aphids on the agricultural sector. In this paper, the dynamics of interactions between aphids and ladybugs, including the impact of pesticides on aphid mortality, are represented using a system of nonlinear differential equations. This study treats the parameter representing aphid mortality caused by pesticides as a fuzzy number to account for variations in resistance levels. Additionally, the model incorporates four parameters that depict the interaction between aphids and ladybugs beyond considering the effect of pesticides. The parameters include the proportion of aphids consumed by ladybugs, the proportion of aphids capable of evading ladybugs, and the growth rates of both aphids and ladybugs. The triangular form is chosen to depict the fuzzy membership function because it reflects the resistance of aphids when pesticides are applied excessively. The dynamic model, incorporating a fuzzy parameter, is transformed into discrete-time models using the Non-Standard Finite Difference (NSFD) method for simulation purposes. The simulation outcomes align with the analysis findings, indicating a potential equilibrium between the populations of aphids and ladybugs. Various examinations on the impact of fuzzy pesticide parameters on the growth of aphids and ladybugs are provided. The findings demonstrate that pesticide application can substantially decrease the aphid population and can be tailored based on the interplay between aphids and ladybugs. Moreover, pesticide usage can be diminished with heightened ladybug growth and predation rates, thereby minimizing the occurrence of resistant aphids and enhancing the effectiveness of pesticide application.**

***Index Terms*—Aphid and ladybug; dynamical system; pesticide fuzzy parameters, non-standard finite difference method.**

I. INTRODUCTION

UNITED Nations take action to achieve a better life in 2030 through 17 Sustainable Development Goals (SDG), which are related to each other. Zero hunger is second among these goals, including food security and sustainable agriculture. This goal gets more attention from countries with large populations than others because it increases food demand. High population growth creates complex problems and unpredictable challenges in the food supply and brings up food insecurity [1], [2], [3]. This problem also occurs in Indonesia [4], [5], since Indonesia is the fourth country in the world with the largest population after India, China, and the United States. For these reasons, the Food and Agriculture Organization (FAO) promotes the Sustainable Food and Agriculture (SFA) program to help countries achieve SDGs, especially regarding zero hunger. It is proof that food security and the agricultural sector are closely related. Furthermore, much research has been conducted to analyze the relationship between agricultural sustainability and food security [6], [7], [8]. In improving food security, various efforts have been made to enhance the quantity and quality of agricultural production by several approaches [9], [10], [11], [12]. In Indonesia, improving the agricultural sector is also carried out to increase national income since Indonesia is an agrarian country. In 2022, Statistics Indonesia (BPS) reported that agriculture was the third-largest sector in terms of Gross Domestic Product (GDP) contribution.

However, agricultural sustainability still faces several obstacles, such as pests. FAO [13] reports that climate change is stimulating pests to become more destructive, which will be one of the biggest challenges for farmers. Moreover, climate change affects the biology, ecology, and distribution of insect pests [14]. It implies the agricultural production management strategies to minimize the significant losses [15], [16], [17]. Aphid is the most common pest distributed worldwide, which reduces plant growth by sucking sap and causing leaf deformation. Furthermore, it can be a serious pest because of its rapid evolution [18], and aphid is the primary vector to spread plant viruses [19].

Many strategies to reduce and control the aphid population include cultural, mechanical, biological, and chemical control [20], [21]. In chemical control, pesticides are a standard method of reducing the aphid population. Studies regarding the efficacy of various types of pesticides have been widely discussed for many plants. However, pesticide use may harm the environment, human health, and the plant itself [22], [23]. Moreover, some studies show that continuous and excessive use of pesticides can make aphids resistant [24], [25]. It would be better to optimize plant production if pesticide use is combined with other methods, such as companion plants and natural predators. Ladybug is the most famous predator of aphids, both commercial and home use. Framers love them since they eat aphids and insects up to 5,000 in their lifetime. Therefore, much research has been conducted to examine the relationship between aphids and ladybugs through several approaches.

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Constructing mathematical models is a method used to enhance comprehension when investigating the dynamics of interactions between aphids and ladybugs. Ge et al.[26] use predator-prey systems within thermal performance through some scenarios to analyze the climate change effect on aphids and ladybirds interaction. It shows that ladybugs predation substantially affects aphid abundance more than climate change. A mathematical model of the system parameterized by extensive experimental work [27] and a stochastic metacommunity model successfully describes ecological interaction between aphids and ladybugs in experiment systems for different spatial scales [28]. Next, Gabbriellini [29] constructs a discrete-time model for a dynamical system to analyze the effect of the presence of ants in aphid and ladybug relationship.

Generally, many researchers have carried out mathematical models related to pesticide and insecticide use. Almost every model is approached by optimization control for a nonlinear dynamical system. Herlambang et al. [30] construct a predator-prey model with pesticide as a control variable. In the next research, they modify these models by dividing the prey population into susceptible prey and infected prey [31]. Reyes et al. [32] develop a differential equation model to describe the absorption of pesticide spray droplets across the leaf. The results of the analysis of this model will help the agrochemical industry create affordable and effective pesticides. Suganya and Senthamarai [33] also analyzed awareness programs by insecticide spray for coconut trees to develop a cost-effective insecticide usage. Furthermore, several studies have also explored the application of fuzzy logic and mathematical modeling in agriculture. For example, Wang [34], and Lone et al. [35] respectively highlight the potential of fuzzy linear programming in agricultural economic management and land allocation. Mehta et al. [36] developed a fuzzy logic model for crop selection, enhancing precision and reducing ambiguity. Umadevi [37] discussed the use of fuzzy rule-based models in solving agricultural problems, while Shams et al. [38] explored the use of the modified Adomian decomposition technique to solve generalized intuitionistic fuzzy differential equations, with applications in various physical science problems. Mondal et al. [39] use a fuzzy set number to approach and examine the correct use of pesticide amounts involving five indicators in agricultural sustainable development due to pesticides. Pourjafar [40] provides a comprehensive review of fuzzy logic in agricultural systems, emphasizing its ability to handle time-varying, non-linear, and adaptive systems. Malinowski [41] expands this discussion by introducing the concept of stochastic fuzzy differential equations as a tool for modeling uncertain dynamic systems in agriculture. These studies collectively underscore the value of fuzzy logic and mathematical modeling in addressing agricultural systems' complex and uncertain nature.

Prediction of aphid and ladybug growth in the presence of fuzzy pesticides became interesting to analyze. It can be used to plan the proper treatment for plants and estimate how long the aphid can survive. In this study, the dynamical system of aphid and ladybug interaction is described by three nonlinear differential equations, completed by pesticide parameters. Pesticide use will affect the aphid death rate and change the dynamic behavior between aphids and ladybugs. This study introduces the fuzzy membership function for pesticide use, which involves the amount of pesticide and four parameters that indicate the interaction between aphids and ladybugs. The fuzzy dynamical model is solved numerically using the Non-Standard Finite Difference Method (NSFD), then compared to the 4th Runge Kutta scheme. For further analysis, the NFSD method was carried out to compile some simulations, including sensitivity analysis. The results show that the use of pesticides as a fuzzy parameter has a significant effect on the aphid population.

II. Research Methodology

Figure 1 shows that this research methodology is divided into four major stages, namely the mathematical modeling stage (blue), the analytical stage (red), the numerical stage (yellow), and the simulation stage (green).

At the mathematical modeling stage, the authors study the dynamical system that interprets the interaction between aphids and ladybugs, which has been reviewed by Dani et al. [42]. This model does not involve the fuzzy function for pesticide parameters. In this research, we modified the pesticide parameters as a fuzzy membership function. The membership function is used in the triangular form to represent the fuzzy numbers associated with interaction parameters. This triangular shape was chosen because there are conditions where aphids will be resistant to pesticides. As previously researched, the ongoing utilization of insecticides can lead to the development of insecticide resistance in the target insects [43]. Even when employing a rotation technique, the possibility of double resistance remains [44]. Finally, this stage produces the fuzzy dynamical system.

In the analytical stage, the equilibrium point can be derived from the fuzzy dynamical system to see the population condition that no longer changes over time. The eigenvalues for this dynamical system can be derived by substituting the equilibrium point into the Jacobian matrix of the model. These eigenvalues will affect the stability criteria for the analytical solution.

In the numerical stage, this study uses Python to implement the model in the numerical solution. Here, the model is solved by two numerical methods: 4th Runge Kutta and Non-Standard Finite Difference (NSFD). NFSD scheme can be derived from the discrete-time model with time step as a function of eigenvalues from the analytical stage. Unlike the Runge Kutta scheme, which applies to all equilibrium points, the NSFD scheme will differ for each equilibrium point since the time step depends on eigenvalues.

Validation is needed to check the model's validity by comparing Runge Kutta, NSFD, and analytical solutions. If these solutions are significant, the research should return to the fuzzy dynamical system in the mathematics modeling stage to check for an error. Otherwise, the research continues to the simulation stage. The simulation completed by sensitivity analysis allows further analysis of the effect of the fuzzy parameter.

III. Dynamical System of Aphid and Ladybugs

Dani et al. [42] introduce the dynamical system of aphids and ladybugs as a simultaneous nonlinear differential equation. The nonlinear system consists of three populations: aphids, ladybugs, and hibernate ladybugs. Assume that

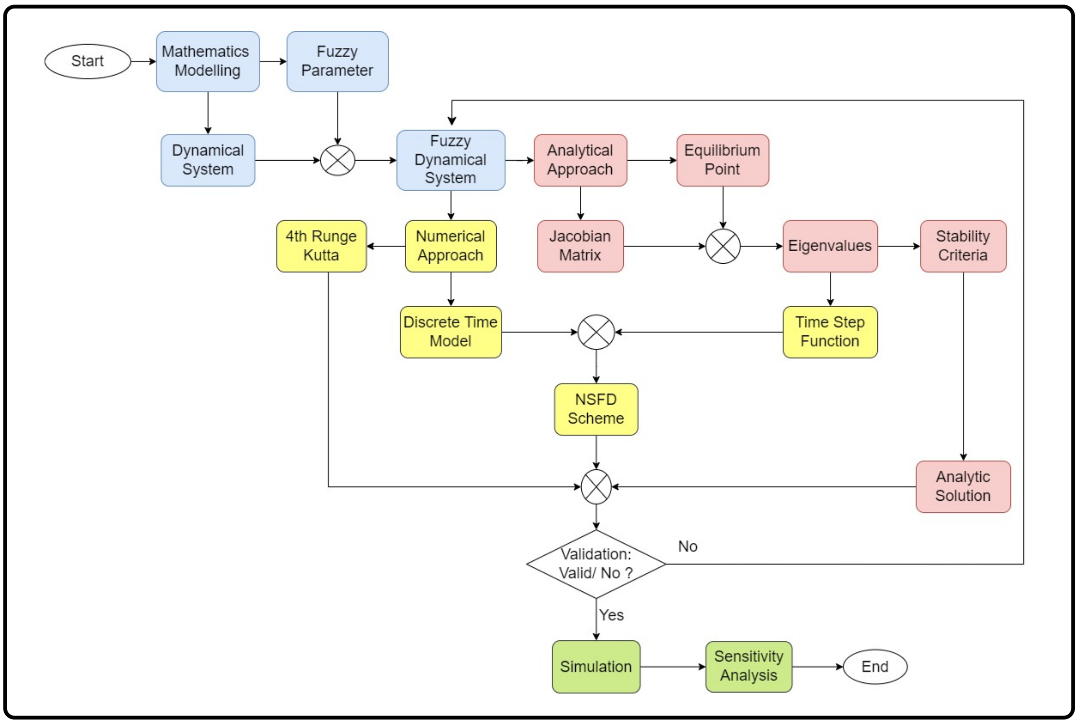

Fig. 1. Research Methodology Flowchart

aphids and ladybugs satisfy the logistic growth model with r and β as growth rates for their population, respectively. Let $x(t), y(t)$, and $z(t)$ be the proportion of the aphid, active ladybugs, and hibernate ladybugs population at time t by their carrying capacity K_1 and K_2 as follows,

$$x = \frac{X}{K_1}, y = \frac{Y}{K_2}, z = \frac{Z}{K_2}.$$

The population of aphids will decrease due to the use of pesticides, which cause the death of aphids at a rate u, and the presence of ladybugs that prey on aphids at a rate m. If some aphids escape from ladybugs with proportion ξ, then the population of aphids described by

$$\frac{dx}{dt} = rx(1-x) - ux - \frac{\mu xy}{1+cx}, \tag{1}$$

where

$$\mu = mK_2, c = \xi K_1.$$

On the other hand, the interaction between aphids and ladybugs will increase the number of ladybug populations. If the active ladybugs go to hibernate with a transition rate α and the hibernate goes to active with a transition rate γ, then the active ladybug population over time t satisfied the following differential equation.

$$\frac{dy}{dt} = \beta y(1-y) + \frac{b\mu xy}{1+cx} - \alpha y + \gamma z, \tag{2}$$

where

$$b = \frac{K_1}{K_2}\xi.$$

Last, assume that the hibernate ladybugs over time t satisfied the exponential growth so that

$$\frac{dz}{dt} = \alpha y - \gamma z. \tag{3}$$

IV. Dynamical System of Aphid and Ladybugs with Pesticide as Fuzzy Parameter

Consider the dynamical model for aphids and ladybugs in (1) - (3), let Ω be the pesticide load each day, and assume that the aphid and ladybugs dynamic will influence the effective use of pesticides. Then, the higher pesticide load leads to a higher death rate of aphids [45], but aphids may be resistant if the pesticide use is overdosed [25], [24]. Considering the pesticide load, the parameters u can be viewed as a function of the pesticide load Ω. Thus, the dynamical model (1) - (3) can be modified as the fuzzy dynamical model, represented as follows:

$$\frac{dx}{dt} = rx(1-x) - u(\Omega)x - \frac{\mu xy}{1+cx}, \tag{4}$$

$$\frac{dy}{dt} = \beta y(1-y) + \frac{b\mu xy}{1+cx} - \alpha y + \gamma z, \tag{5}$$

$$\frac{dz}{dt} = \alpha y - \gamma z, \tag{6}$$

The membership function $u(\Omega)$ is constructed by adjusting the pesticide load according to the interaction behavior between aphids and ladybugs. If the aphid's growth rate r and the proportion of the aphids that can escape from ladybugs ξ increases, then the pesticide load must be increased. Conversely, if the ladybug's growth rate β and the aphid death rate caused by ladybug predation m increases, we can reduce pesticide use. Based on this analysis, the maximum death rate of aphids because of pesticides becomes

$$u_{max} = U + u_{min}, \tag{7}$$

where

$$U = r\xi(1-\beta)(1-m).$$

The maximum death rate of aphids u_{max} occurs when $\Omega = \Omega_0$, where Ω_0 is an effective pesticide load and u_{min} as the minimum value of $u(\Omega)$.

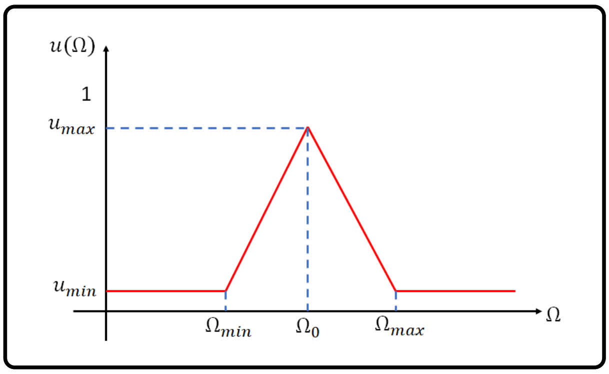

Fig. 2. Fuzzy Membership Function for Aphid Death Rate $u(\Omega)$.

Let Ω_{min} and Ω_{max} be a maximum and minimum pesticide load, respectively. If the pesticide load used does not meet the minimum requirements for effective use ($\Omega < \Omega_{min}$), then the aphid death rate becomes u_{min}. It happens because the amount of pesticide used is too small, so it cannot work perfectly in killing aphids. It also occurs when the pesticide exceeds the maximum effective limit for use ($\Omega > \Omega_{max}$); this will cause aphids to develop immunity, making them unable to kill aphids. From an agricultural perspective, the minimum and maximum aphid death rates because of pesticides also depend on their quality and efficacy [46], [47].

Furthermore, when $\Omega_{min} < \Omega < \Omega_0$, the pesticide load does not make aphid resistant yet, then the aphid death rate will increase up to u_{max}. On the other side, if the pesticide load is more than the effective pesticide load ($\Omega_0 < \Omega < \Omega_{max}$), then aphids start to become resistant so that the aphid death rate decrease. Mathematically, the death rate of aphids based on the use of pesticides $u(\Omega)$ can be expressed in a fuzzy membership function as follows:

$$u(\Omega) = \begin{cases} u_{min}, \Omega < \Omega_{min}, \Omega > \Omega_{max} \\ u_{min} + \frac{\Omega - \Omega_{min}}{\Omega_0 - \Omega_{min}} U, \Omega_{min} \le \Omega \le \Omega_0 \\ u_{min} + \frac{\Omega - \Omega_{max}}{\Omega_0 - \Omega_{max}} U, \Omega_0 \le \Omega \le \Omega_{max} \end{cases} \quad (8)$$

which can be described by Figure 2.

The alpha-cut concept from fuzzy sets can be applied to determine the pesticide load adjusted to the desired minimum death rate for aphids. Let A be the minimum expected aphid death rate, and then the pesticide load used should be

$$u_A = \{\Omega \in [\Omega_{min}, \Omega_{max}] | u(\Omega) \ge A\}, \quad (9)$$

After apply some algebraic manipulation to (8) and (9), then the pesticide load must satisfied

$$\Omega_1 \le \Omega \le \Omega_2, \quad (10)$$

where

$$\Omega_1 = \Omega_{min} + \frac{1}{U}(A - u_{min})(\Omega_0 - \Omega_{min})$$

$$\Omega_2 = \Omega_{max} + \frac{1}{U}(A - u_{min})(\Omega_0 - \Omega_{max})$$

A. Equilibrium Point

The equilibrium point for the nonlinear system can be derived by setting (4) - (6) equal to zero:

$$rx(1-x) - u(\Omega)x - \frac{\mu xy}{1+cx} = 0, \quad (11)$$

$$\beta y(1-y) + \frac{b\mu xy}{1+cx} - \alpha y + \gamma z = 0, \quad (12)$$

$$\alpha y - \gamma z = 0. \quad (13)$$

Solve (11) - (13) by some algebraic manipulation, then three equilibrium points are obtained. These are

$$E_0 = (x_0, y_0, z_0) = (0, 0, 0), \quad (14)$$

$$E_1 = (x_1, y_1, z_1) = \left(0, 1, \frac{\alpha}{\gamma}\right), \quad (15)$$

$$E_2 = (x_2, y_2, z_2) = \left(\frac{r - u(\Omega)}{r}, 0, 0\right), \quad (16)$$

where E_0 describes the extinction of all populations, whereas E_1 and E_2 describe the extinction of the aphid and ladybug populations, respectively.

B. Stability Analysis

The stability of each equilibrium point can be checked from its characteristic equation by substituting the equilibrium point into the Jacobian matrix of the nonlinear system. The Jacobian matrix (J) for the model (4) - (6) related to equilibrium point E_0 is

$$J_0 = \begin{pmatrix} r - u(\Omega) & 0 & 0 \\ 0 & \beta - \alpha & \gamma \\ 0 & \alpha & -\gamma \end{pmatrix}, \quad (17)$$

so that the characteristic equation becomes

$$[\lambda - r + u(\Omega)][\lambda^2 + (\alpha - \beta + \gamma)\lambda - \beta\gamma] = 0, \quad (18)$$

and the eigenvalues are

$$\lambda_{01} = r - u(\Omega), \quad (19)$$

$$\lambda_{02}, \lambda_{03} = \frac{-(\alpha - \beta + \gamma) \pm \sqrt{(\alpha - \beta + \gamma)^2 + 4\beta\gamma}}{2}. \quad (20)$$

Based on (20), the eigenvalues λ_{02} and λ_{03} always have different sign. It can be concluded that the equilibrium point E_0 is unstable.

Next, the Jacobian matrix related to E_1 is

$$J_1 = \begin{pmatrix} r - u(\Omega) - \mu & 0 & 0 \\ b\mu & -\beta - \alpha & \gamma \\ 0 & \alpha & -\gamma \end{pmatrix}, \quad (21)$$

and the characteristic equation related to (21) is

$$(\lambda - r + u(\Omega) + \mu)[\lambda^2 + (\alpha + \beta + \gamma)\lambda + \beta\gamma] = 0. \quad (22)$$

Solving (22) for λ gives us the following eigenvalues,

$$\lambda_{11} = r - u(\Omega) - \mu, \quad (23)$$

$$\lambda_{12}, \lambda_{13} = \frac{-(\alpha + \beta + \gamma) \pm \sqrt{(\alpha + \beta + \gamma)^2 - 4\beta\gamma}}{2}. \quad (24)$$

The equation (24) show that λ_{12} and λ_{13} always negative. Furthermore, the equilibrium point E_1 is stable if and only if λ_{11} negative, which is $r - u(\Omega) - \mu < 0$.

The last, the Jacobian matrix related to equilibrium point E_2, written as

$$J_2 = \begin{pmatrix} -r+u(\Omega) & -\frac{\mu(r-u(\Omega))}{r+c(r-u(\Omega))} & 0 \\ 0 & \beta+\frac{b\mu(r-u(\Omega))}{r+c(r-u(\Omega))}-\alpha & \gamma \\ 0 & \alpha & -\gamma \end{pmatrix} \quad (25)$$

Then, the characteristic equation for (25) is

$$[\lambda + r - u(\Omega)]\left[\lambda^2 - (B-\alpha-\gamma)\lambda - B\gamma\right] = 0, \quad (26)$$

where

$$B = \beta + \frac{b\mu(r-u(\Omega))}{r+c(r-u(\Omega))}.$$

Solve (26) then

$$\lambda_{21} = -r + u(\Omega), \quad (27)$$

$$\lambda_{22}, \lambda_{23} = \frac{(B-\alpha-\gamma) \pm \sqrt{(B-\alpha-\gamma)^2 + 4B\gamma}}{2}. \quad (28)$$

The eigenvalues (28) has a similar form with (20), which is the eigenvalues λ_{22} and λ_{23} have different sign so that the equilibrium point E_2 always unstable.

V. Numerical Method

The Non-Standard Finite Difference (NSFD) method is a numerical scheme that can be used to solve differential equations, which is better than the classical finite difference method [48]. It shows that NSFD accuracy is the same as the 4th Runge-Kutta method, although for the large time increment. Meanwhile, the classical finite difference is only accurate for small increments. Furthermore, Yaghoubi et al. [49] stated that NSFD was more stable than the classical method.

Let an ordinary differential equation

$$\begin{cases} \frac{df}{dt} = g(f(t), t, \varepsilon), t \in [0, t_e], \\ f(0) = f_0 \end{cases} \quad (29)$$

where ε is a parameter set, t_e is the final time, and f_0 is the initial condition.

If the domain is uniformly discretized, then the time length is

$$\Delta t = t_{n+1} - t_n$$

where $n = 0, 1, 2, ...$ Based on this discretization, the approximation of $f(t_n)$ becomes f_n. According to Baleanu [50], we use the discrete approximation

$$\frac{df}{dt} \approx \frac{f_{n+1} - f_n}{\phi(\Delta t)}, \quad (30)$$

so that the NFSD scheme for (29) become

$$\frac{f_{n+1} - f_n}{\phi(\Delta t)} = G(f_{n+1}, fn, ..., t_n, \varepsilon), \quad (31)$$

where $\phi(\Delta t)$ is a modified function of time step size that must be satisfied

$$\phi(\Delta t) = h + O((\Delta t)^2)$$

and $G(f_{n+1}, fn, ..., t_n, \varepsilon)$ is a nonlinear term of (29). Note that the scheme (31) becomes the classical finite difference scheme when $\phi(\Delta t) = \Delta t$.

Gabbriellini [29] stated that the nonlinear term $g(f(t), t, \varepsilon)$ can rewrite as

$$g(f(t), t, \varepsilon) = (-1)^p f^i h(f, \varepsilon), \quad (32)$$

so that the nonlinear term $G(f_{n+1}, fn, ..., t_n, \varepsilon)$ discretization become

$$\frac{(-1)^p}{2}\left[(1-(-1)^{p+1})f_n^i + (1-(-1)^p)f_n^{i-1}f_{n+1}\right] h_n(f_n, \varepsilon). \quad (33)$$

The NSFD scheme (31) can be generalized to the system of nonlinear differential equations and applied for the nonlinear fuzzy model (4) - (6). Let $f_n = \{x_n, y_n, z_n\}$, where x_n, y_n, and z_n is the population of aphid, active, and hibernate ladybugs after discretization at time t_n, respectively. Rewrite the nonlinear term (4) - (6) one by one as (32) to get the value of p, i, and the function $h_n(f_n, \varepsilon)$, then substitute this values and function into (33) to get $G(f_{n+1}, fn, ..., t_n, \varepsilon)$. In the end, the NSFD scheme (31) for the nonlinear fuzzy system (4) - (6) becomes

$$\frac{x_{n+1} - x_n}{\phi(\Delta t)} = rx_n - rx_n^2 - u(\Omega)x_n - \frac{\mu x_n y_n}{1 + cx_n}, \quad (34)$$

$$\frac{y_{n+1} - y_n}{\phi(\Delta t)} = \beta y_n - \beta y_n^2 + \frac{b\mu x_n y_n}{1 + cx_n} - \alpha y_n + \gamma z_n, \quad (35)$$

$$\frac{z_{n+1} - z_n}{\phi(\Delta t)} = \alpha y_n - \gamma z_n. \quad (36)$$

where

$$\phi(\Delta t) = \frac{1 - e^{-q\Delta t}}{q}, \quad (37)$$

with

$$q \geq max\left\{\frac{\lambda^2}{2|Re(\lambda)|}\right\}, \quad (38)$$

as in [29]. Here λ is the eigenvalue obtained from Section III.

VI. Results and Discussion

This section shows numerical results using the NSFD scheme compared to the 4th Runge Kutta method as a validation. Some simulations are conducted here to analyze the aphid and ladybug interaction behavior, which is affected by pesticide use as a fuzzy parameter.

A. Numerical Results

The aphid, active, and ladybug populations are approximated by NSFD and the 4th Runge Kutta scheme using input parameters as in Table I. From this table, $\Omega = \Omega_0 = 5.5$ means the simulation shows effective pesticide load, so the aphid death rate $u(\Omega) = u_{max}$. Use (7), then the aphid death rate becomes $u(\Omega) = u_{max} = 0.21575$. After calculating $\mu = mK_2$, we obtain that the data satisfied $r < u(\Omega)+\mu$ and convergent to equilibrium point E_1. Substitute these parameter input into equation (23) and (24), then the eigenvalues are $\{-0.0438, -0.4562, -14.9658\}$. The equilibrium is stable since all eigenvalues are negative.

The NSFD results can be obtained by iterating (34)-(35) simultaneously for x_{n+1}, y_{n+1}, and z_{n+1}, then the results as in Figure 3. Here, choose $q = 8$, which is satisfying (38) based on the eigenvalue set. Figure 4 shows the 4th Runge Kutta approximation with the same parameter input as in Table I. Both figures show the same quantitative and qualitative results. The number of aphids over time (solid curve) grows in the opposite direction with active (dotted curve) and hibernate ladybugs (dashed curve) since the dynamical population goes to equilibrium point $E_1 =$

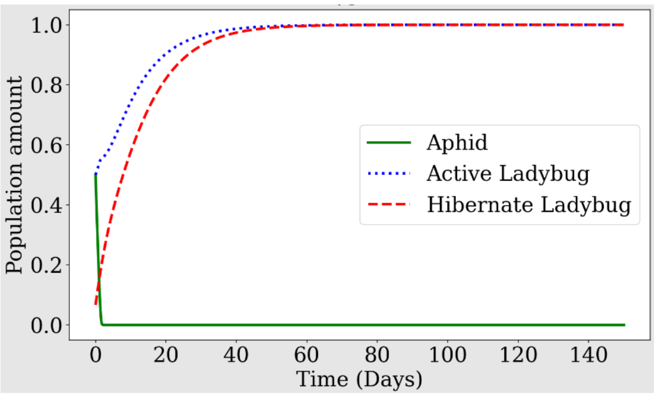

Fig. 3. Numerical Results by NSFD Scheme.

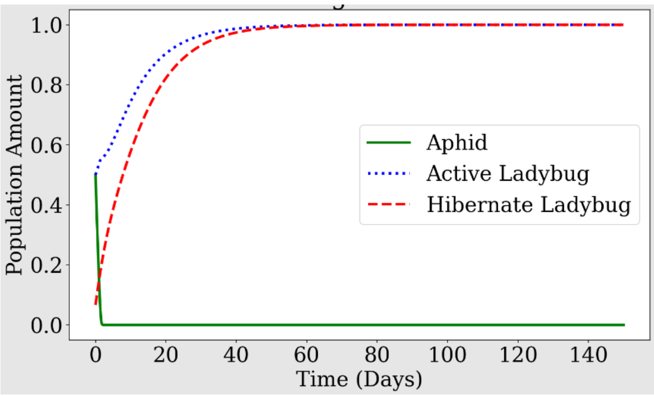

Fig. 4. Numerical Results by 4th Runge Kutta Scheme.

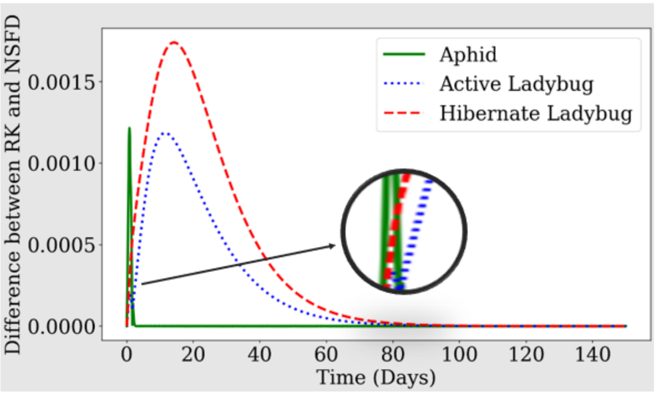

Fig. 5. NSFD and 4th Runge Kutta Difference.

$(0, 1, 1)$. These results are appropriate for stability analytical analysis. Here, the extinction of aphids occurs quickly, in less than three days. It occurs because the aphid death rate reaches the maximum value since the pesticide load satisfies the effective amount. Moreover, the existence of ladybugs as predators also accelerates aphid extinction. The active ladybugs converge to 1 faster than hibernate ladybugs since the active ladybug's growth rate α is greater than a transition rate from active to hibernate β.

To analyze the accuracy of NSFD, the difference between NSFD and the Runge Kutta scheme Δf_n is calculated by

$$\Delta f_n = |f_n(NSFD) - f_n(RK)|, \qquad (39)$$

where $f_n(NSFD)$ and $f_n(RK)$ is (x_n, y_n, z_n) by NSFD and Runge Kutta scheme.

TABLE I
PARAMETER INPUT

Parameter	Value	Unit
r	0.25	day^{-1}
u_{min}	0.2	day^{-1}
m	0.1	day^{-1}
ξ	0.1	day^{-1}
K_1	200	Individuals
β	0.3	day^{-1}
K_2	150	Individuals
α	0.2	day^{-1}
γ	0.2	day^{-1}
Ω_{min}	1	day^{-1}
Ω_0	5.5	day^{-1}
Ω_{max}	10	day^{-1}
Ω	5.5	day^{-1}
x_0	100	Individuals
y_0	75	Individuals
z_0	10	Individuals
T	150	day
Δt	0.01	day

TABLE II
PARAMETER INPUT FOR PESTICIDE LOAD SIMULATION

Parameter	Value	Unit
r	0.7	day^{-1}
u_{min}	0.2	day^{-1}
m	0.1	day^{-1}
ξ	0.6	day^{-1}
K_1	200	Individuals
β	0.1	day^{-1}
K_2	150	Individuals
α	0.2	day^{-1}
γ	0.2	day^{-1}

TABLE III
PESTICIDE LOAD AND FUZZY PARAMETER FUNCTION $u(\Omega)$

Simulation	Ω_{min}	Ω_0	Ω_{max}	Ω	u_{max}	$u(\Omega)$
Simulation A (Different Ω)	5	15	25	8	0.5402	0.3021
	5	15	25	12	0.5402	0.4381
	5	15	25	15	0.5402	0.5402
	5	15	25	18	0.5402	0.4381
	5	15	25	22	0.5402	0.3021
Simulation B (Different Ω_{min})	10	15	25	11	0.5402	0.2680
	5	15	25	11	0.5402	0.4041
	1	15	25	11	0.5402	0.4430
Simulation C (Different Ω_0)	5	20	25	11	0.5402	0.3361
	5	15	25	11	0.5402	0.4041
	5	10	25	11	0.5402	0.6082
	5	20	25	19	0.5402	0.6082
	5	15	25	19	0.5402	0.4041
	5	10	25	19	0.5402	0.3361

The result (39) is plotted by Figure 5. It can be seen that the difference for each population increases quickly at the initial observation. The difference for aphid, active, and hibernate ladybugs rich the maximum value $\Delta x \approx 0.00125$ at time $t = 1.5$ days, $\Delta y \approx 0.00110$ at time $t \approx 12$ days, and $\Delta z \approx 0.0018$ at time $t = 18$ days, then decrease afterward before convergent to 0. Furthermore, the time

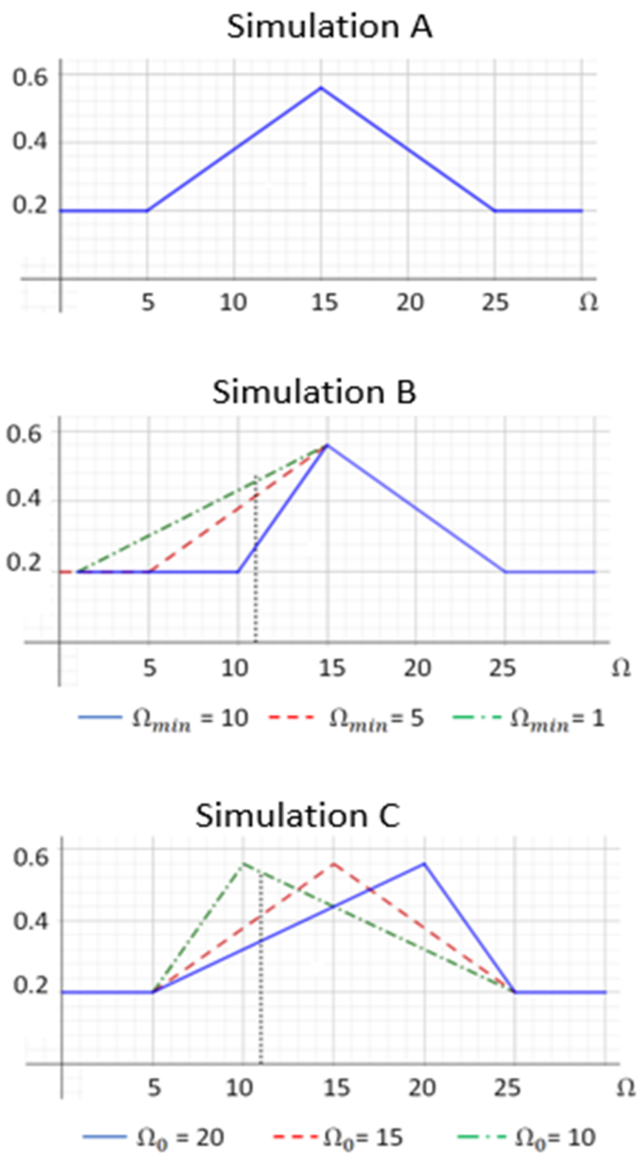

Fig. 6. Fuzzy Membership Function $u(\Omega)$ for Simulation A, B, and C.

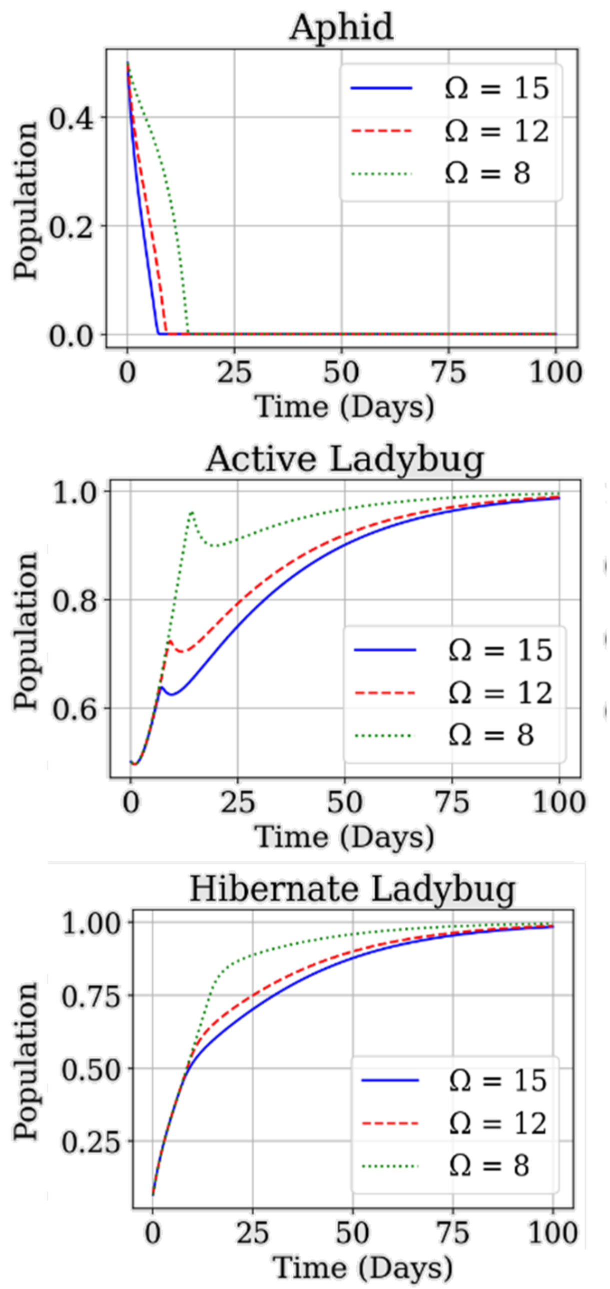

Fig. 7. Aphid, Active, and Hibernate Ladybugs Population: Simulation A.

required for the difference Δf_n to converge to 0 is the same as the time needed for each population to converge to the equilibrium point. It is why the hibernate ladybug has the most significant difference between NSFD and Runge Kutta, followed by active ladybugs and aphid populations. Since the difference between NSFD and the 4th Runge Kutta is less than $0.0018 = 0.18\%$, then the NSFD scheme has a high accuracy in solving model (4) - (6).

B. Pesticide Load Simulation

This section gives the simulation results to analyze the effect of pesticide load on the dynamical system. The simulation uses the data in Table II. Use these data to conduct three simulations, named Simulation A (different value Ω), B (different value Ω_{min}), and C (different value Ω_0). Each simulation gives some variation of fuzzy parameter function $u(\Omega)$ as shown in Table III. From an agricultural perspective, the minimum and effective pesticide load can be derived based on pesticide quality. Then, the fuzzy membership function for each simulation is described in Figure 6. Based on Table III, u_{max} always has the same value for all simulations since u_{max} depend on r, ξ, β, and m as in Table II so that pesticide load Ω does not change u_{max}. The fuzzy parameter $u(\Omega)$ will change by pesticide load. Changing the pesticide load also changes the aphid, active, and hibernate ladybugs as in Figure 7 - 9.

In simulation A, $u(8) = u(22) = 0.3021, u(12) = u(18) = 0.4381$, and $u(15) = 0.5$. It is related to Figure 7, which shows that the aphid population decreases when Ω gets closer to $\Omega_0 = 15$. When effective pesticide loads are used $\Omega_0 = 15$, the aphid extinction occurs for about eight days. When pesticide loads 12 and 18, the aphid extinction occurs for about ten days. For pesticide loads 8 and 22, the aphid extinction occurs about 15 days. Therefore, if the pesticide load Ω is closer to an effective pesticide load Ω_0, the aphid death rate u becomes bigger so that the aphid population gets smaller, and the aphid extinction becomes faster.

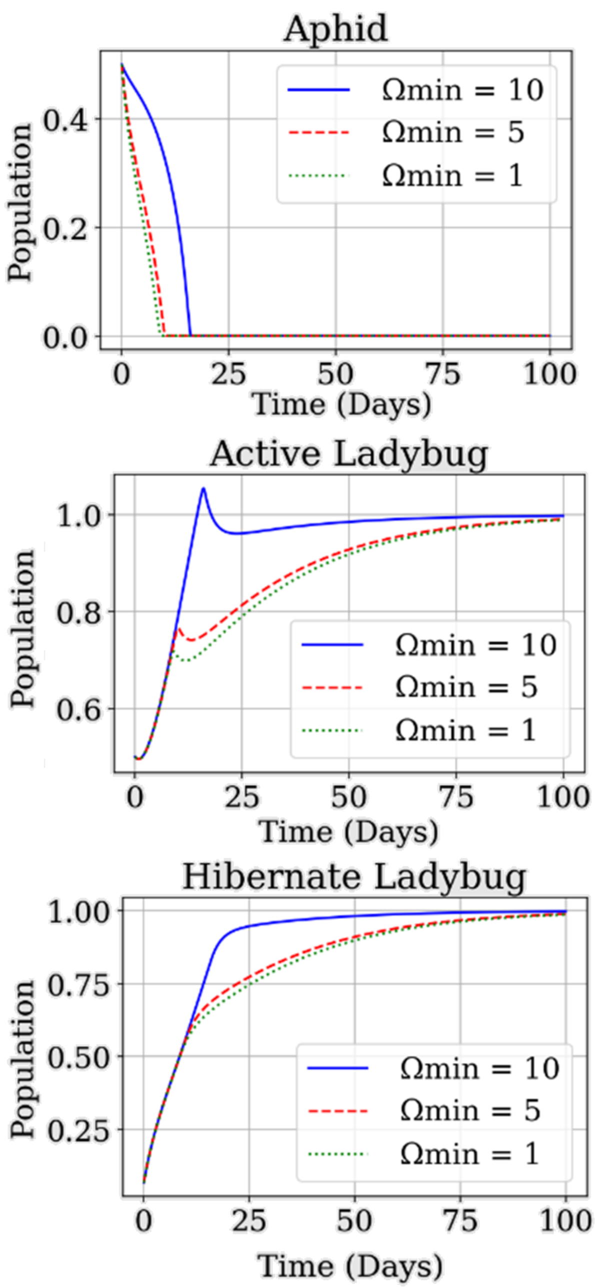

Fig. 8. Aphis, Active, and Hibernate Ladybugs for Simulation B.

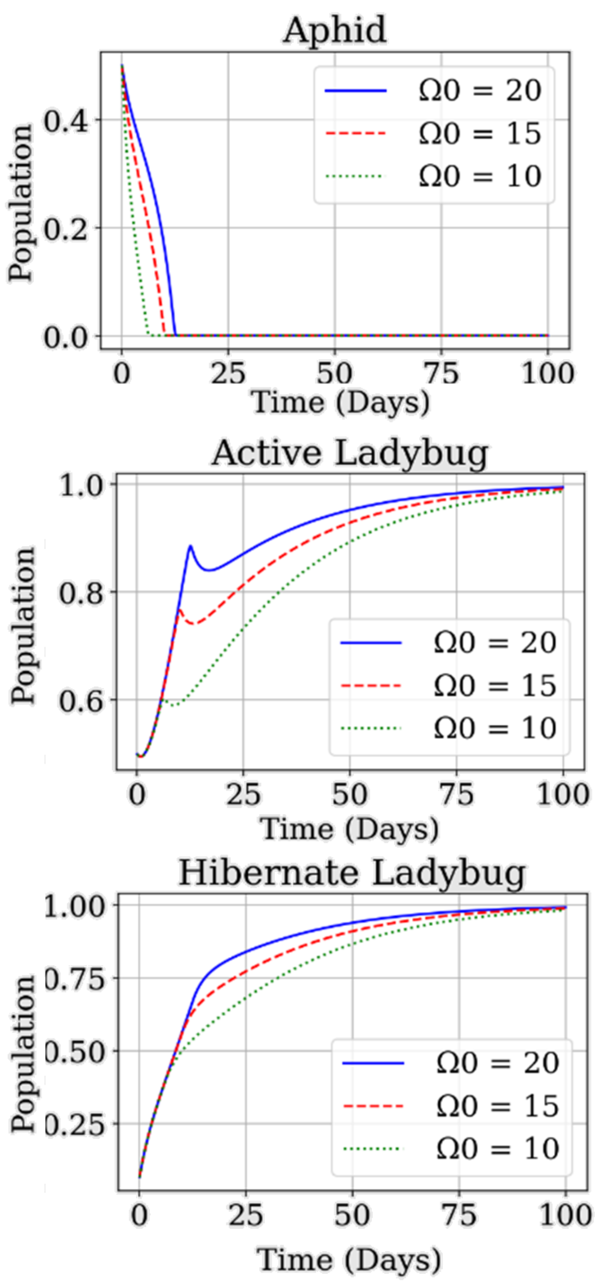

Fig. 9. Aphis, Active, and Hibernate Ladybugs for Simulation C.

TABLE IV
PESTICIDE LOAD VS THE DESIRED MINIMUM DEATH RATE AND APHIDS EXTINCTION TIME

The Desired Minimum Death Rate for Aphids (A)	Aphid Extinction Time (Days)	Pesticide Load Ω
0.2680	18	$7 \leq \Omega \leq 23$
0.3361	13	$9 \leq \Omega \leq 21$
0.4041	10	$11 \leq \Omega \leq 19$
0.4721	8	$13 \leq \Omega \leq 17$
0.5402	7	$\Omega = \Omega_0 = 15$

In simulation B, changing the minimum pesticide load will change the fuzzy parameter $u(\Omega)$ so that the line gradient in domain $[\Omega_{min}, \Omega_0]$ will be different as in the Figure 6 (middle). In this domain, a bigger Ω_{min} leads to a bigger line gradient so that $u(\Omega)$ becomes smaller. For example, $\Omega = 11$, which is described by a dotted vertical line in the Figure 6 (middle), will intersect $u(\Omega)$ in the different point, which gives us $u(11)$ as in Table III. When $u(\Omega)$ gets smaller, the aphid population gets bigger, as in Figure 8.

In simulation C, changing the effective pesticide load Ω_0 will change the fuzzy membership function $u(\Omega)$ as in Figure 6 (right). Without changing the value of Ω_{min} and Ω_{max}, a bigger Ω_0 leads a smaller line gradient in domain $[\Omega_{min}, \Omega_0]$ and a bigger line gradient in domain $[\Omega_0, \Omega_{max}]$. That is why the value $u(11)$ become smaller when Ω_0 getting bigger since $11 \in [\Omega_{min}, \Omega_0]$. It can be concluded that for $\Omega \in [\Omega_{min}, \Omega_0]$, the aphid, active, and hibernate ladybug population will increase over Ω_0 as in Figure 6. In contrast, $u(19)$ will be proportional to the Ω_0 and inversely proportional to the number of aphids, active, and hibernate ladybugs since $19 \in [\Omega_0, \Omega_{max}]$.

Overall, the number of active and hibernated ladybugs is proportional to the number of aphids since more aphids give

TABLE V
SENSITIVITY PARAMETER INPUT TO FUZZY PARAMETER $u(\Omega)$ AND u_{max}

Simulation	r	ξ	β	m	Ω_{min}	Ω_0	Ω_{max}	Ω	u_{min}	u_{max}	$u(\Omega)$
1^{st} Simulation: Different Value of r	0.7	0.6	0.1	0.05	5	15	25	13	0.2	0.5591	0.4873
	0.4	0.6	0.1	0.05	5	15	25	13	0.2	0.4052	0.3642
	0.1	0.6	0.1	0.05	5	15	25	13	0.2	0.2513	0.2411
2^{nd} Simulation: Different Value of ξ	0.7	0.6	0.1	0.05	5	15	25	13	0.2	0.5591	0.4873
	0.7	0.35	0.1	0.05	5	15	25	13	0.2	0.4095	0.3676
	0.7	0.1	0.1	0.05	5	15	25	13	0.2	0.2599	0.2479
3^{rd} Simulation: Different Value of β	0.7	0.6	0.7	0.05	5	15	25	13	0.2	0.3197	0.2958
	0.7	0.6	0.4	0.05	5	15	25	13	0.2	0.4394	0.3915
	0.7	0.6	0.1	0.05	5	15	25	13	0.2	0.5591	0.4873
4^{th} Simulation: Different Value of m	0.7	0.6	0.1	0.75	5	15	25	13	0.2	0.2945	0.2756
	0.7	0.6	0.1	0.4	5	15	25	13	0.2	0.4268	0.3814
	0.7	0.6	0.1	0.05	5	15	25	13	0.2	0.5591	0.4873

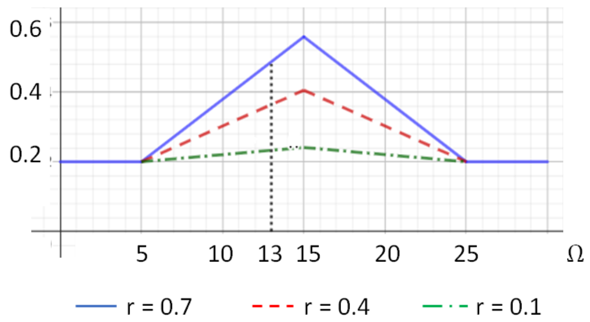

Fig. 10. Fuzzy Membership Function $u(\Omega)$ for Different r.

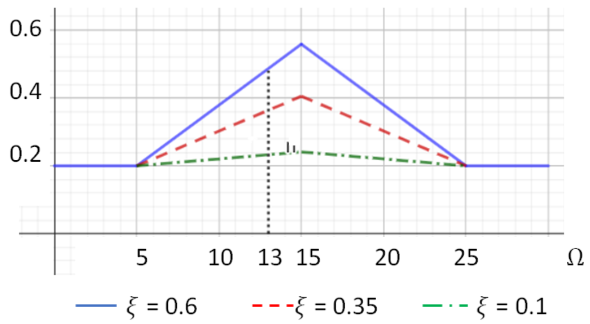

Fig. 11. Fuzzy Membership Function $u(\Omega)$ for Different ξ.

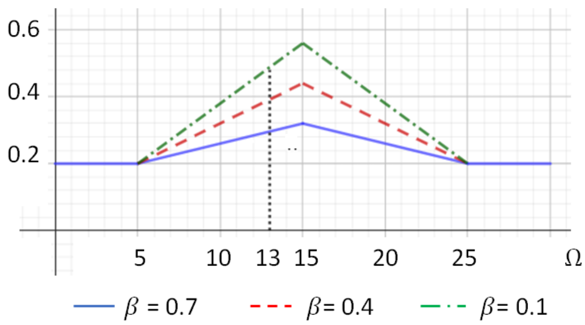

Fig. 12. Fuzzy Membership Function $u(\Omega)$ for Different β.

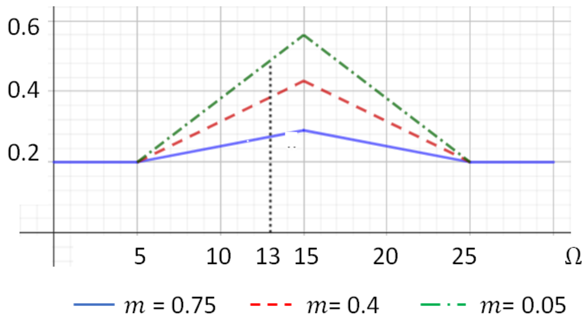

Fig. 13. Fuzzy Membership Function $u(\Omega)$ for Different m.

more food for ladybugs. The active ladybug population fluctuates in domain $t \in [0, 25]$, and the maximum point occurs when aphids are extinct. All populations converge toward the equilibrium point (0,1,1) according to the analytical solution described previously.

Furthermore, simulation results can be used to determine the load of pesticide (Ω) required that will produce the desired minimum death rate of aphid or the expected time to achieve aphid extinction through the alpha-cut concept in equation (9) - (10). Table IV gives some examples for pesticide load selection to get the desired $u(\Omega)$ based on parameter input in Simulation A. It can be seen that the length of time aphids are extinct is inversely proportional to the death rate of aphids. Other scenarios can be considered when making decisions about controlling aphids with pesticides.

C. Sensitivity Analysis

Consider the definition of fuzzy membership function (8), changing parameter r, ξ, β, and m will affect $u(\Omega)$. For this reason, four simulations were conducted here as in Table V where the membership function $u(\Omega)$ for each simulation is described in Figure 10 - 13. The simulation results align with the analytical analysis, which stated that u_{max} is proportional to aphid growth rate r and the proportion of aphids that can escape from ladybugs ξ. Moreover, it is inversely proportional to ladybugs' growth rate β and the proportion of aphids eaten by ladybugs m.

Figure 14 - 17 describes the aphid population for Simulation 1 - 4. Each figure consists of three subfigures corresponding to three different parameter values (which produce three different u). Each subfigure shows aphid populations over time with pesticides (red solid line) and without pesticides (green dashed line) to see the effect of pesticide use. Almost all subfigures show that aphid populations dropped drastically after pesticide use.

For further analysis, each simulation is completed by Figure 18 - 21. These figures show how big the difference in aphid population over time between pesticide use and non-pesticide for different parameter values based on the

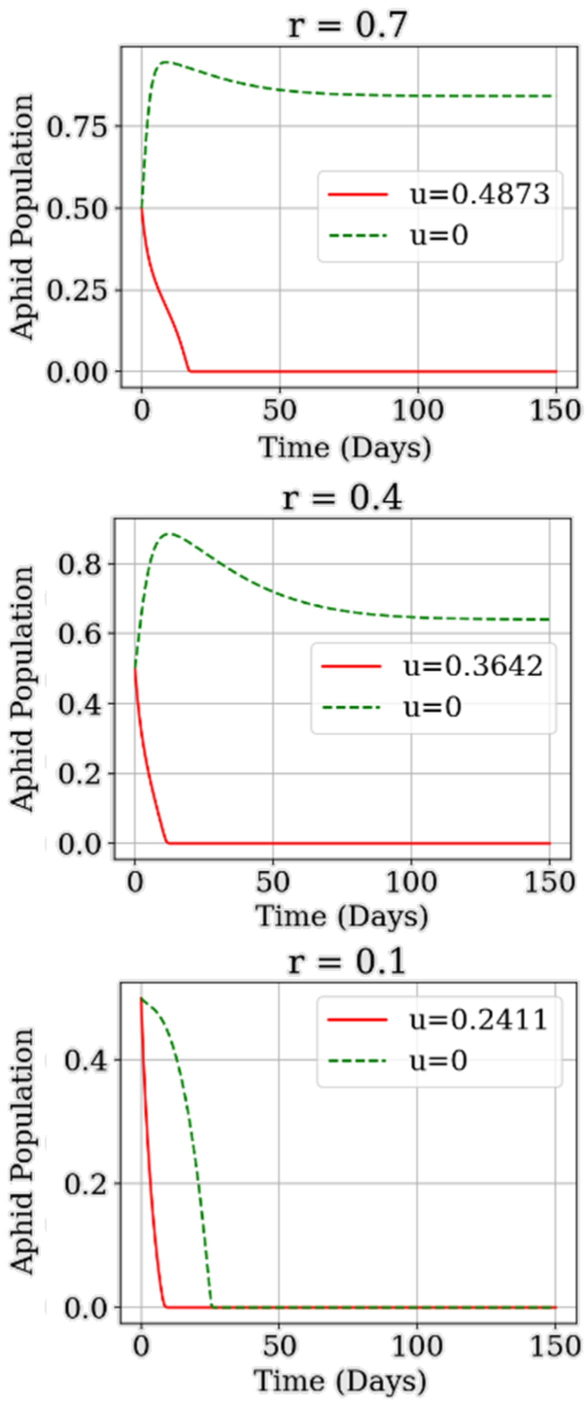

Fig. 14. Aphid Population Between Pesticide Use ($u \neq 0$) and Non Pesticide ($u = 0$) for Simulation 1.

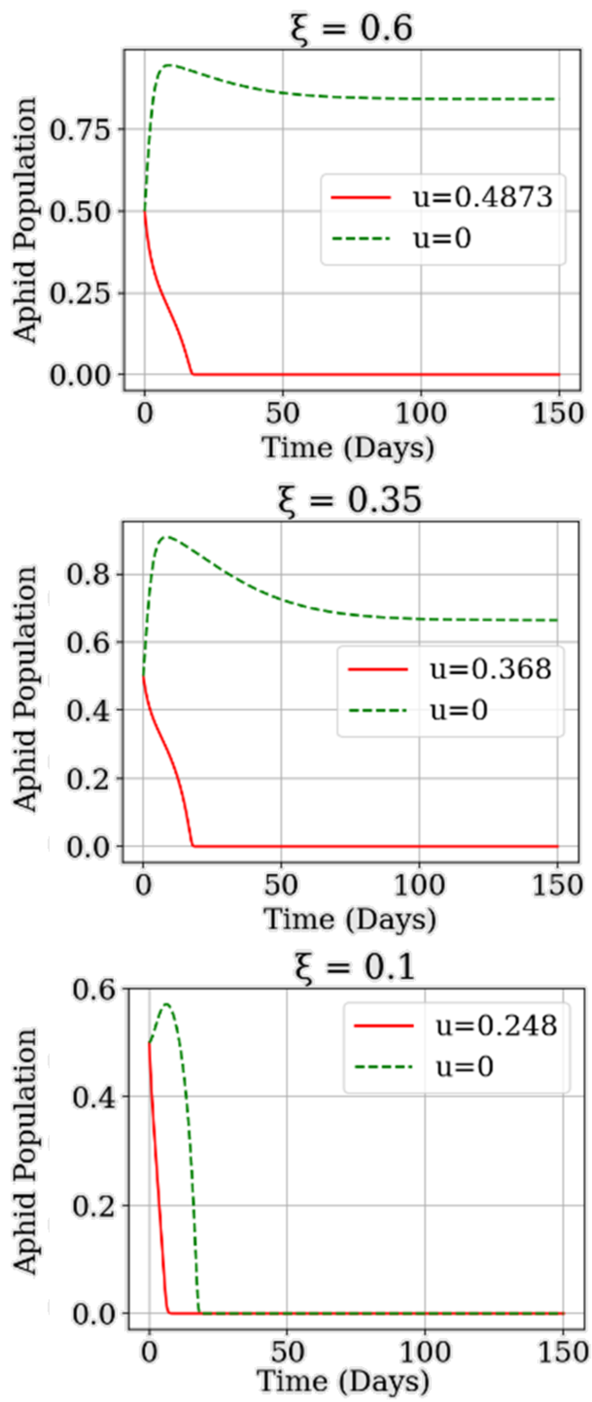

Fig. 15. Aphid Population Between Pesticide Use ($u \neq 0$) and Non Pesticide ($u = 0$) for Simulation 2.

following formula,

$$\Delta x_n = x_n|_{u(\Omega)\neq 0} - x_n|_{u=0}. \tag{40}$$

In Figure 14, when $r = 0.7$ and $u = 0$, the aphid population will increase sharply up to $x \approx 0.95$ within ten days, then decrease slowly afterward, convergent to $x \approx 0.83$. The use of pesticides changes u from 0 to 0.4873, so the aphid population will drop quickly and become extinct within 15 days. This pattern also occurs for $r = 0.4$, but the distance between the green dotted line ($u \neq 0$) and the red solid line ($u = 0$) becomes smaller. The extreme condition occurs when the aphid growth rate is too small. Without pesticide use, the aphid population will be extinct within 25 days for $r = 0.1$. This extinction becomes faster (within eight days) when the pesticide is used. In detail, Figure 18 shows that a bigger r leads to a more significant difference in the aphid population between pesticide and non-pesticide. This means that the use of pesticides will kill more aphids when the aphid growth rate r is higher. The same explanation applies to Simulation 2, described by Figure 15 and 19. The proportion of aphids that can escape from ladybugs ξ is proportional to the number of aphids killed by pesticides.

Figure 16 related to Simulation 3, aphid population based on different ladybugs growth rate β. This parameter is inversely proportional to u, so the parameter u becomes smaller when β gets bigger. That is why aphid extinction

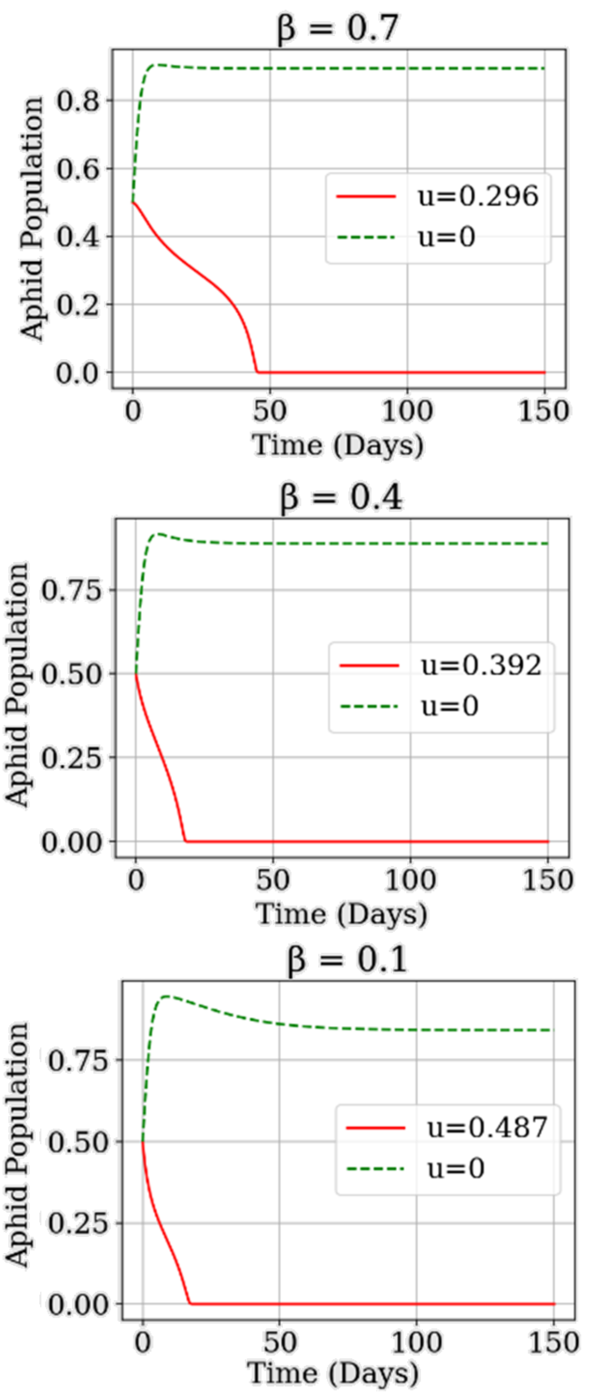

Fig. 16. Aphid Population Between Pesticide Use ($u \neq 0$) and Non Pesticide ($u = 0$) for Simulation 3.

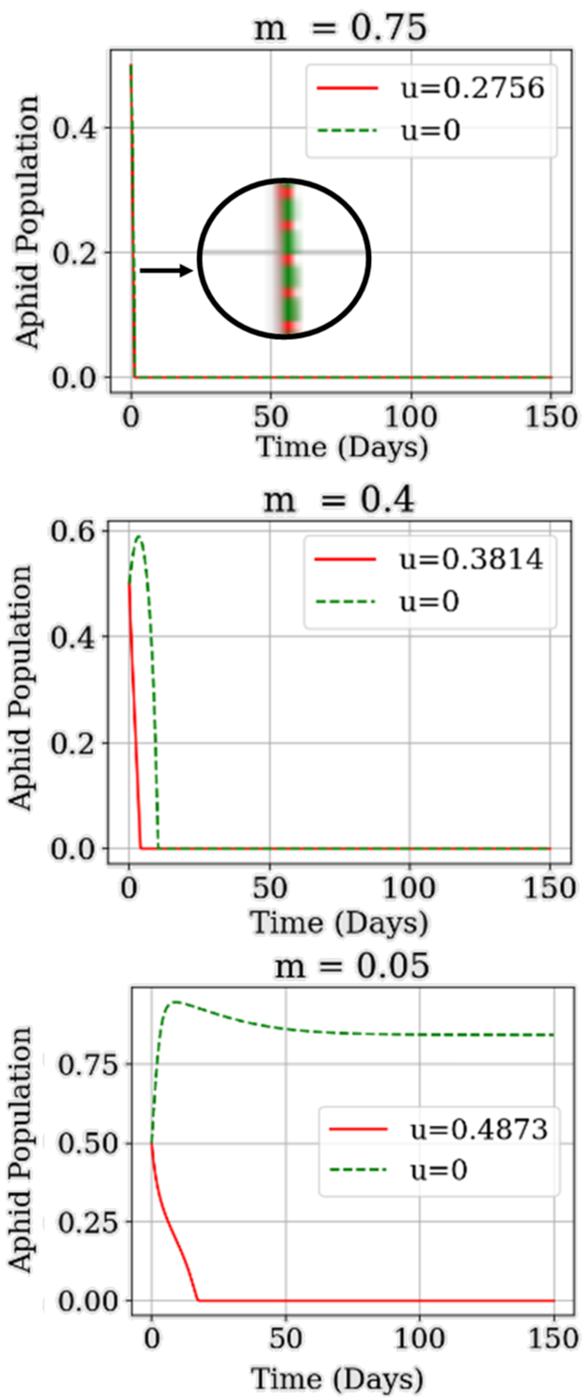

Fig. 17. Aphid Population Between Pesticide Use ($u \neq 0$) and Non Pesticide ($u = 0$) for Simulation 4.

becomes slower when β gets bigger. It is in line with the Figure 20. However, this figure does not follow the pattern in Figure 18 and 19. Here, a bigger β leads to a smaller difference between pesticide use and non-pesticide within 50 days (before aphid extinction because of pesticide use). After 50 days, this order will be reversed.

In Simulation 4, when $m = 0.75$ and $m = 0.4$, the aphid population will drastically decline at the initial time, as shown in Figure 17. Pesticides did not cause it, but the proportion of aphids eaten by ladybugs was too big. In this case, we do not need pesticides. In contrast, when $m = 0.05$, the proportion of aphids eaten by ladybugs is too small, so the pesticide use needs more here to get a more considerable aphid death rate u. It is described clearly in Figure 21 that m is inversely proportional to the effect of pesticide use. In this case, the use of pesticides will appear significant at very small m.

Not only the aphid population but also the active ladybug population over time is described in Figure 22 - 25 for Simulation 1-4 as in Table V. Based on Figure 22 - 25, we can derive the difference in active ladybug population over time between pesticide use and non-pesticide for different parameter values, as in equation (40), based on the following formula,

$$\Delta y_n = y_n|_{u(\Omega)\neq 0} - y_n|_{u=0}, \quad (41)$$

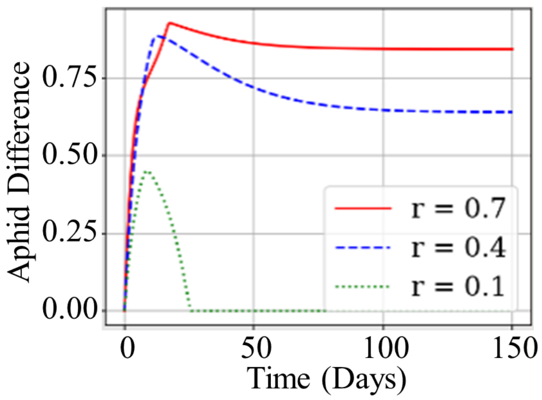

Fig. 18. Aphid Difference Between $u \neq 0$ and $u = 0$ for Simulation 1.

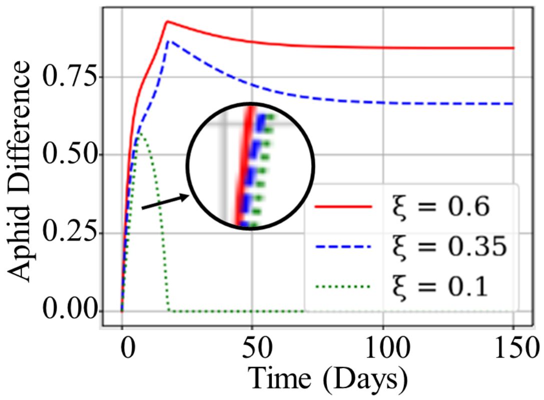

Fig. 19. Aphid Difference Between $u \neq 0$ and $u = 0$ for Simulation 2.

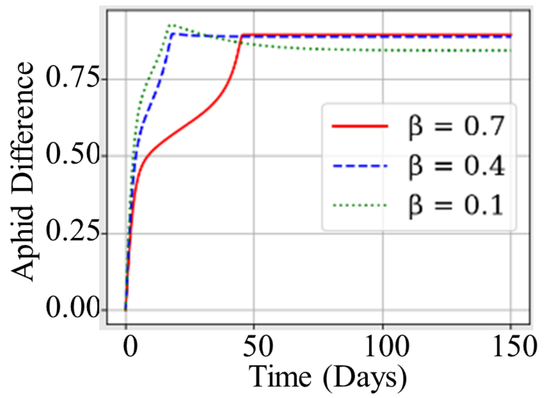

Fig. 20. Aphid Difference Between $u \neq 0$ and $u = 0$ for Simulation 3.

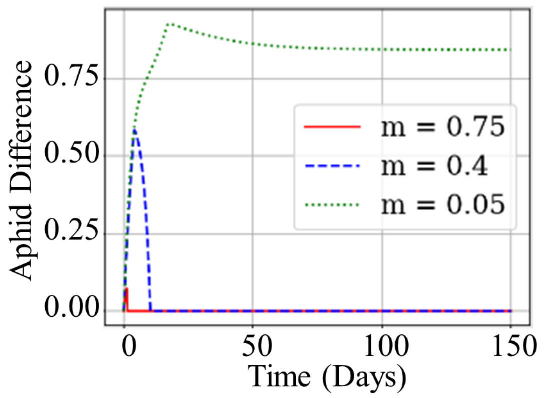

Fig. 21. Aphid Difference Between $u \neq 0$ and $u = 0$ for Simulation 4.

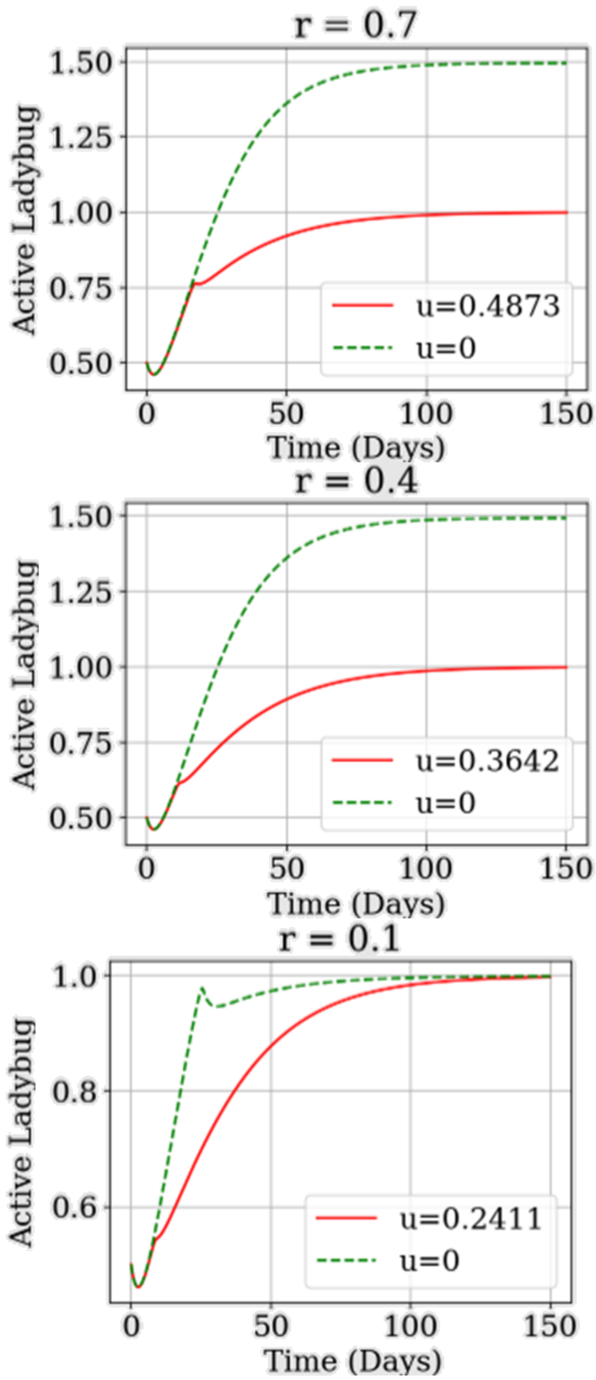

Fig. 22. Active Ladybug Population Between Pesticide Use ($u \neq 0$) and Non Pesticide ($u = 0$) for Simulation 1.

then Figure 26 - 29 can be plotted.

Figure 22 - 25 shows the active ladybug population when $u = 0$ is always greater than $u \neq 0$. Moreover, the active ladybug's population always goes to 1 as its equilibrium condition E_1 for $u \neq 0$, but not for $u = 0$. It means the equilibrium points E_1 will not be achieved without pesticide use.

From Figure 22 and 23, a bigger value of r has significant results: the red and green curves dispersed to different points. This condition does not occur when $r = 0.1$ and $\xi = 0.1$. These are why Figure 26 and 27 show a unique pattern for $r = 0.1$ and $\xi = 0.1$. The same explanation for Figure 25 and 29. In varying parameter, β, Figure 24 implies that a smaller value leads the red and green curve to more disperse

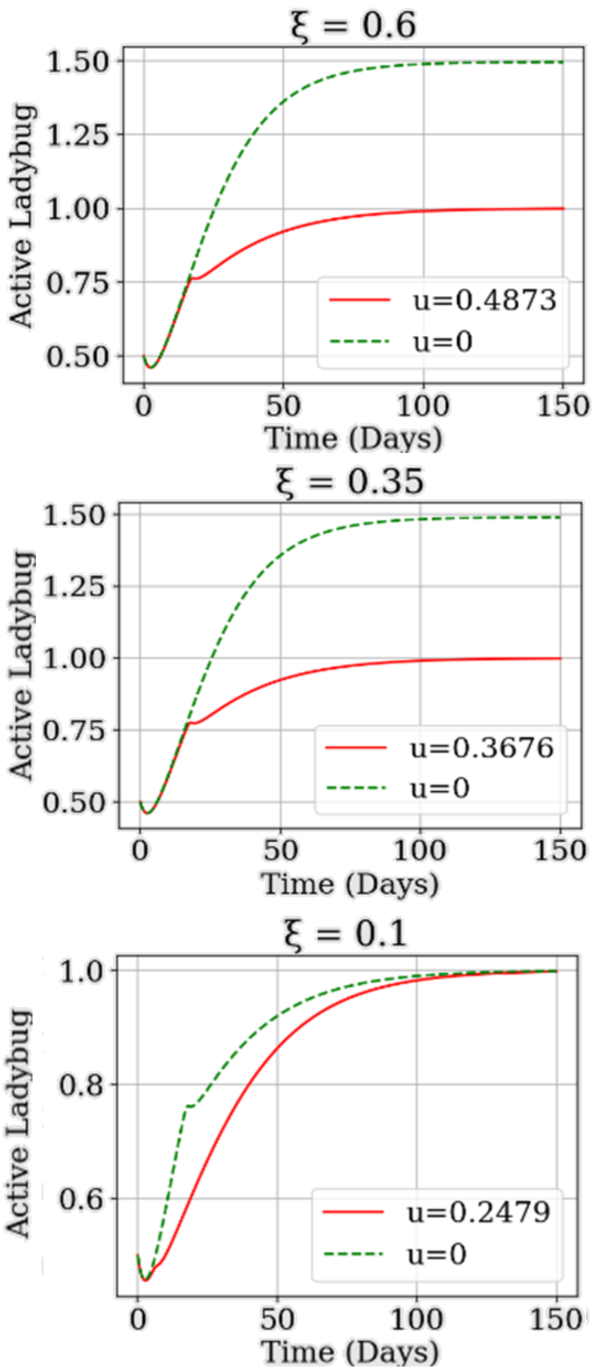

Fig. 23. Active Ladybug Population Between Pesticide Use ($u \neq 0$) and Non Pesticide ($u = 0$) for Simulation 2.

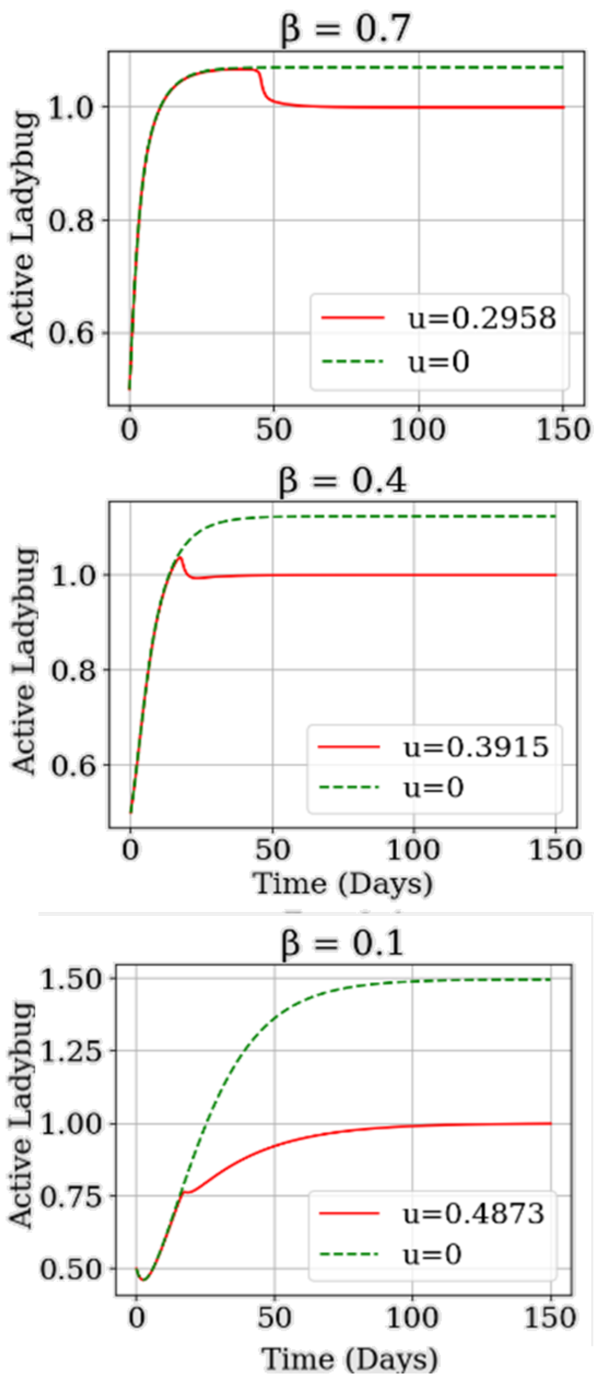

Fig. 24. Active Ladybug Population Between Pesticide Use ($u \neq 0$) and Non Pesticide ($u = 0$) for Simulation 3.

so that Δy_n become bigger as in Figure 28.

This subsection implies that pesticides are used based on the interaction between aphids and ladybugs. We may measure the effectiveness of pesticide use by combining parameters through the fuzzy parameter. Furthermore, based on Figure 18 - 21, the simulation results show that the parameters that influence the effectiveness of pesticide use, from largest to smallest, are: β, ξ, r, m. In addition, pesticide use kills aphids and can control the number of ladybugs, so they do not have more than their carrying capacity.

VII. Conclusion

The fuzzy membership function has successfully modeled the aphid death rate because pesticides depend on pesticide load. It becomes a fuzzy parameter for the dynamical system of aphid and ladybug interaction. The membership function is depicted triangular to represent the possibility of aphids being resistant to pesticides when the pesticide load exceeds the effective pesticide load. The pesticide load must be proportional to the aphid's growth rate and the proportion of the aphids that can escape from ladybugs. Conversely, if the ladybug's growth rate and the aphid death rate caused by ladybug predation increase, we can reduce pesticide use to minimize the cost of controlling aphids. Based on the pesticide fuzzy parameter, we may examine the required pesticide load to produce the desired minimum death rate of aphids or the expected time to achieve aphid extinction.

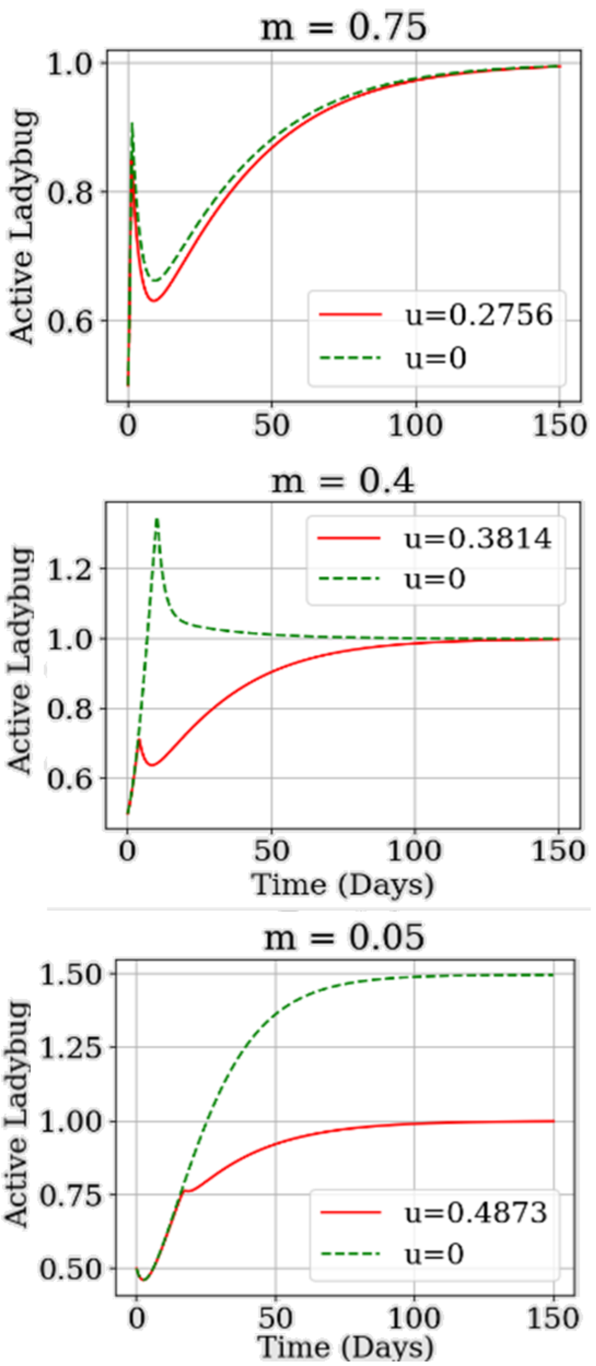

Fig. 25. Active Ladybug Population Between Pesticide Use ($u \neq 0$) and Non Pesticide ($u = 0$) for Simulation 4.

The simulation results show that the usage of pesticides can reduce the number of aphids and speed up aphid extinction. It can also control the number of ladybugs so they do not have more than their carrying capacity. Furthermore, the aphid and ladybug interaction parameters that influence the effectiveness of pesticide use, from largest to smallest, are ladybug growth rate, the proportion of aphids that can escape from ladybugs, aphid growth rate, and the proportion of aphids eaten by ladybugs.

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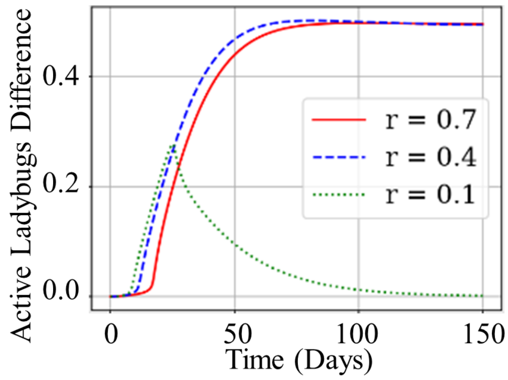

Fig. 26. Active Ladybug Difference Between $u \neq 0$ and $u = 0$ for Simulation 1.

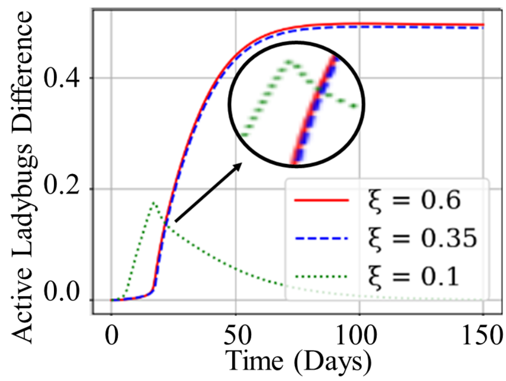

Fig. 27. Active Ladybug Difference Between $u \neq 0$ and $u = 0$ for Simulation 2.

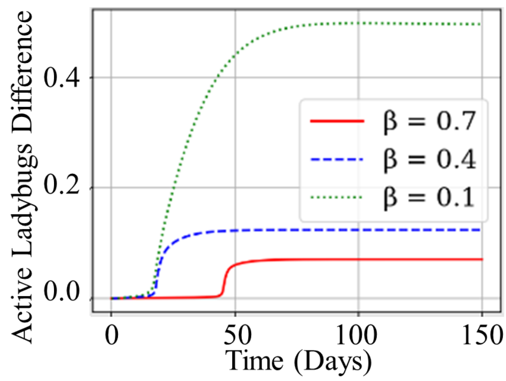

Fig. 28. Active Ladybug Difference Between $u \neq 0$ and $u = 0$ for Simulation 3.

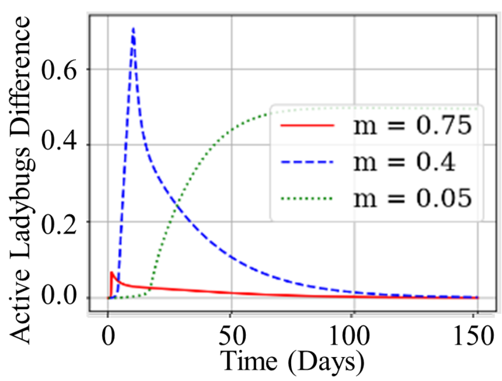

Fig. 29. Active Ladybug Difference Between $u \neq 0$ and $u = 0$ for Simulation 4.

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