# Long-Term Prediction of Oil Palm Fresh Fruit Bunch Prices in Riau Province Post-Pandemic Using a Discrete-Time Markov Chain

Sri Purwani, Rayyan Al Muddatstsir Fasa, Almeira Tsanawafa, and Sarah Sutisna

*Abstract*—**The COVID-19 pandemic has had a significant impact on many economic sectors around the world. This includes the palm oil industry, which has been especially affected in Riau Province. One of the aspects affected in this sector is the price of oil palm fresh fruit bunches (FFB), with its uncertain price due to the COVID-19 pandemic being detrimental to many parties. Therefore, decision-makers need an understanding of how to predict the price of oil palm FFB. In this study, we use a discrete-time Markov chain approach to predict the long-term prices of oil palm FFB post-pandemic in Riau Province. This approach allowed us to model the price transition from both one period and one price category to the next based on predefined transition probabilities. Historical data on oil palm FFB prices in Riau Province from January 6–12, 2021 to January 4–10, 2023 were used to build the Markov chain model. In our analysis, we considered four possible states of Riau Province's oil palm FFB prices post-pandemic: rising, extreme rising, falling, and extreme falling. Our long-term prediction results show that Riau Province's oil palm FFB prices have an opportunity for a rise of 39%, an extreme rise of 25%, a fall of 27%, and an extreme fall of 9%. This is valuable information for industry players when making decisions regarding their investment, production, and marketing strategies. By understanding the likely future price trends, they can take appropriate steps to optimize yields and mitigate risks.**

*Index Terms*— **COVID-19, fresh fruit bunch prices, Markov chain, prediction**

## I. Introduction

OIL palm is one of the major crops in Indonesia that has become very important in the last 20 years [1]. In 2018, the land area under oil palm cultivation in Indonesia reached more than 14 million hectares, which exceeded the land area used to grow rice, the country's staple food source. Palm oil production comprises approximately 2.5% of Indonesia's gross domestic product (GDP) and creates jobs for up to 8 million people in the agriculture and processing sectors [2]. Moreover, Indonesia is the world's largest producer and exporter of palm oil [1].

Manuscript received July 9, 2023; revised May 6, 2024.

This work was supported in part by the Rector and the Directorate of Research and Community Service (DRPM) Universitas Padjadjaran.

Sri Purwani is a lecturer in the Mathematics Department, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Sumedang 45363, Indonesia (corresponding author; e-mail: sri.purwani@unpad.ac.id).

Rayyan Al Muddatstsir Fasa is a graduate of Magister of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Sumedang 45363, Indonesia (e-mail: rayyan19001@mail.unpad.ac.id).

Almeira Tsanawafa is a graduate of Magister of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Sumedang 45363, Indonesia (e-mail: almeira19001@mail.unpad.ac.id).

Sarah Sutisna is a graduate of Magister of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Sumedang 45363, Indonesia (e-mail: sarah19005@mail.unpad.ac.id).

More than 40% of the total oil palm land area in Indonesia is managed by small and medium-scale family plantations rather than large oil palm companies [3]. Several studies show that smallholders benefit from oil palm cultivation as it improves living standards due to oil palm being more profitable than traditional crops such as rice or rubber [3],[4],[5]. In addition, oil palm is a labor-efficient innovation as it requires less labor per hectare compared to most traditional crops [6],[7].

The largest oil palm plantation in Indonesia is located in Riau Province, which has a plantation area of 2.9 million hectares, of which 1.8 million hectares comprise smallholder plantations [2]. Smallholder plantations are managed by the people or farmers who own small business groups of plantation crops and plantation household businesses. The plantations in Indonesia have made significant contributions to the production of oil palm yields in Indonesia in addition to producing fresh fruit bunches (FFB) [8]. As the prices of FFB occasionally change in Riau Province, plantation farmers are unable to predict the price and acquire the maximum yield when harvesting. Thus, for farmers to gain an understanding of the prediction of future prices and to develop strategies for obtaining the maximum yield required, a model for predicting the price of oil palm FFB is needed.

Several related studies have been conducted on this topic. Karia et al. [9] applied the AutoRegressive Fractionally Integrated Moving Average method to predict the price of crude palm oil (CPO), the price of which remains non-stationary in long-term data. Sugiarto [10] conducted similar research using the AutoRegressive Integrated Moving Average (ARIMA) method to forecast the price of oil palm FFB in Riau Province. They obtained the best ARIMA model (0,1,2) with the smallest mean squared error value. Arifin et al.'s [11] study centered around forecasting world palm oil prices using various time series forecasting methods, with the results showing a downward trend in CPO prices, with variations in the highest and lowest prices in different periods.

Kharvi and Pakkala [12] recently investigated the use of discrete-time Markov chains (DTMC) to determine optimal inventory for retailers when discount prices randomly appear, with the time variables being treated as discrete. In this context, the most effective inventory strategy was determined by analyzing the fluctuation in purchase prices as a discrete-time Markov chain. The profitability of the optimal solution generated by the developed model was compared to that of the economic order quantity policy, which revealed that the solution derived from the developed model yielded superior profitability.

This study is geared towards predicting oil palm FFB prices using a discrete-time Markov chain. Markov chains are typically used to carry out modeling of various conditions and determine changes that will occur. These changes can be represented in terms of dynamic variables at any given time, with each variable value denoting a predetermined time. Thus, the Markov property indicates a conditional opportunity for a future event that is not influenced by future events, only current events [13].

Areepong and Sukparungsee [14] also investigated the price of palm oil, with the extended exponentially weighted moving average (EEWMA) to monitor the MAX(q,r) process using exogenous factors. Palm oil price data was input into the EEWMA control chart in this model, thereby allowing for changes in prices to be determined and corrective action to be taken if necessary. In this study, the MAX(q,r) model with exogenous factors is used to model palm oil prices, thus demonstrating that the EEWMA control chart is more effective in monitoring palm oil price changes than other control charts.

## II. Material and Methods

This study's objective is to determine the long-term prediction of oil palm FFB prices in Riau Province. The discrete-time Markov chain is used for the forecasting method and data analysis, with the secondary data being obtained from the Riau Provincial Government. The initial data processing stage represents the serial data of oil palm FFB prices per week in tabular form. After this, the oil palm FFB price's condition is identified using the average value, maximum value, and minimum value. Furthermore, a discrete-time Markov chain is used for the analysis.

### *A. Stochastic Process*

A stochastic process $\{X_n\}$ is a collection of random variables of index *n*, defined as time, and $X_n$ is the state at time *n*. The set $n$ is called the parameter space or the set of process indices. If the values of the set *n* are integers, the stochastic process is classified as a discrete-time process. For example, $\{X_n, n = 1, 2, 3, ...\}$ is a discrete-time stochastic process with a non-negative integer index, while $\{X_n, n > 0\}$ is a continuous-time stochastic process with a non-negative real number index [15].

Markov chains comprise part of a stochastic process, in that they contain a set of random variables related to time. The stochastic process is a mathematical model in which each probability changes over time. If $X(t)$ or $X_t$ represents the number of events that occur during the interval, then this process is called the counting process of the stochastic process $X(t), t \geq 0$. Some examples of counting processes include the number of patients with COVID-19 hospitalized at time interval $t$, the number of medical personnel who come to the hospital at time *t*, or the number of COVID-19 patients who recover at time interval *t*.

The following properties must be satisfied in the counting process $X(t), t \geq 0$:

i. $X(t) \geq 0$;

ii. $X(t)$ is an integer;

iii. If $s < t$, then $X(s) < X(t)$; and

iv. For $s < t$, the number of phenomena occurring between the time intervals is determined using $X(s) - X(t)$.

### *B. Forecasting*

Forecasting entails obtaining an estimation or prediction of something that may happen in the future. For example, per capita income, population, and production are always changing. These changes are influenced by very complex factors, such as family income, labor, community culture, and land. These are difficult to determine in advance with certainty. Therefore, predictions for forecasting with the least error are necessary. Stochastic processes are important as they can be used to predict, for example, the probability of future annual rainfall using a Markov chain model [16].

### *C. Probability*

Probability denotes the determination of the probability of an uncertain event. For example, if $S$ is a sample space from a random experiment and $A$ is the event space, the probability of an event $A$ or $P(A)$ can be defined systematically as follows:

$$P(A) = \frac{n(A)}{n(S)} \tag{1}$$

where

$P(A)$ is the probability of event $A$;

$n(A)$ is the number of members of $A$; and

$n(S)$ is the number of members of the sample space $S$.

Thus, the nature of event $A$ or $P(A)$ is as follows:

1. The probability value of event $A$ is $0 \leq P(A) \leq 1$.
2. The probability value of an event that is impossible or unlikely to occur is 0.
3. The probability value of an event that must occur is 1.

Two events are said to have conditional probability when one event needs to occur for the other to happen. In general, the conditional probability of $A$ if $B$ is defined as follows: If $A$ and $B$ are events in the sample space and the probabilities of event $B$ are not equal to zero, then the conditional probability of $A$ if $B$ had already occurred is determined as follows:

$$P(A|B)=\frac{P(A\cap B)}{P(B)}, P(B)>0 \tag{2}$$

This is only true if $P(B)>0$ because if $P(A|B)$ is undefined for the situation in which events $A$ and $B$ are independent, it can be stated that:

$$P(A\cap B)=P(B)(P(A|B)$$
$$P(A\cap B)=P(A)P(B) \tag{3}$$

**Bayes' Theorem**

If $\{B_1, B_2, B_3, \ldots B_n\}$ is the set of events that constitute the sample $S$ with $P(B_k)\neq 0$ for $k=1,2,3,...,n,$ and $A$ is an arbitrary event in $S$ with $P(A_k)\neq 0$ for $k=1,2,3,...,n,$, then:

$$P(B_k|A)=\frac{P(A|B_k)P(B_k)}{\sum_{i=1}^{n}P(B_i)P(A|B_i)} \tag{4}$$

For $A_k$, $k=1,2,3,...,n,$ the conditional probability of $A_k$, provided that $A_1, A_2, \ldots A_{n-1}$, has occurred is determined as follows:

$$P(A_k|A_1, A_2, \ldots A_{k-1})=\frac{P(A_1\cap A_2\cap\ldots\cap A_k)}{P(A_1\cap A_2\cap\ldots\cap A_n)} \tag{5}$$

*D. DTMC*

A Markov chain is a probabilistic model that describes a series of potential events, where the likelihood of each event is solely dependent on the preceding event. These chains are typically defined by their state space and time indices [17]. In essence, Markov chains use matrices and vectors to simulate and anticipate the behavior of a system as it transitions from one state to another, with the transition being solely influenced by the current state.

The term "Markov chain" is used to refer to processes that have a discrete set of parameters or time, known as DTMC. In contrast, some authors use the term "Markov process" to describe continuous-time Markov chains. However, many applications of Markov chains employ discrete time and either a finite or infinite state space, which allows for further statistical analysis. If $\{X(n), n=1,2,3,...\}$, a stochastic process with discrete parameter index and state space $i=1,2,\ldots$, satisfies:

$$P\{X(n+1)=j|X(0)=i_0, X(1)=i_1,\ldots, X(n-1)=i_{n-1}, X(n)=i\}$$
$$P\{X(n+1)=j|X(n)=i\}=p_{ij} \tag{6}$$

$\forall i_0, i_1,\ldots,i_{n-1}, i, j$ and $n,$ then the stochastic process is referred to as a DTMC and $p_{ij}$ represents the transition probability. In equation (6), the transition probability from state $i$ to state $j$ $(p_{ij})$ is solely determined by the current time. When the transition probability is not influenced by the time parameter $n$, it is considered a stationary transition probability. In these cases, the Markov chain is classified as a homogeneous Markov chain.

The one-step transition probability matrix of $\{X(n), n=0,1,2,\ldots\}$ is defined as:

$$\mathbf{P}=\left[p_{ij}\right]=\begin{bmatrix} p_{00} & p_{01} & \cdots \\ p_{10} & p_{11} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix},$$

with $p_{ij}\geq 0$ and $\sum_{j=0}^{\infty}p_{ij}=1(i, j=0,1,2,\ldots)$.

A Markov chain can be represented as a graph in which the vertex set is the state space and the transition probabilities are depicted as a directed set of edges, with the edge weights expressing the probabilities. A graph that represents a Markov chain is called a transition diagram of the Markov chain.

*E. The Chapman-Kolmogorov Equation*

The probability that a process in state $i$ will be in state $j$ after $n$ transitions is expressed by $p_{ij}^n$. It can be calculated by summing all the chances of moving from state $i$ to state $k$ in $r$ steps $(0\leq r\leq n)$ and moving from state $k$ to state $j$ in the remaining time $n-r$.

$$P_{ij}^n=P\{X(n)=j|X(0)=i\}$$
$$=\sum_{k=0}^{\infty}P\{X(n)=j|X(r)=k, X(0)=i\}P\{X(r)=k|X(0)=i\}$$
$$=\sum_{k=1}^{\infty}p_{ik}^r p_{kj}^{n-r}. \tag{7}$$

Equation (7) is called the Chapman-Kolmogorov equation. In a matrix form, it can be written as:

$$\mathbf{P}^{\mathrm{n}}=\mathbf{P}^r.\mathbf{P}^{\mathrm{n-r}} \tag{8}$$

*F. Probability Distribution*

The cumulative probability can be determined by combining the initial probability (initial distribution $\pi(0)$) and the transition probability ($P$ matrix). If $p_{ij}(n)=P\{X(n)=j\}$ and $\pi_{(n)}=[\pi_0(n), \pi_1(n),\ldots,\pi_n(n)]$ represent the distribution of $n$-step probabilities, the following relationship holds.

$$\sum_{j=0}^{\infty}\pi_j(n)=1 and\ \pi(n)=\pi(0)\mathbf{P}^{\mathrm{n}} \tag{9}$$

TABLE I
DESCRIPTIVE STATISTICS ABOUT THE STATE OF THE DATA

| Age | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| Maximum | 207.2 | 225.87 | 248.4 | 254.59 | 264.6 | 272.11 |
| Minimum | -695.44 | -762.86 | -841.85 | -863.37 | -897.39 | -923.34 |
| Average increase | 67.23 | 73.54 | 80.65 | 82.64 | 85.88 | 88.29 |
| Average decrease | -116.65 | -132.88 | -145.70 | -149.28 | -155.14 | -159.48 |

| 9 | 10-20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|
| 278.96 | 285.89 | 984.88 | 271.44 | 270.22 | 257.98 | 251.25 |
| -947.8 | -972.29 | -926 | -920.76 | -916.4 | -872.73 | -848.71 |
| 90.46 | 92.65 | 113.08 | 88.10 | 87.71 | 83.85 | 81.72 |
| -163.41 | -167.37 | -183.83 | -159.14 | -158.44 | -151.47 | -147.62 |

## III. RESULT AND DISCUSSION

### A. *Oil Palm FFB Prices*

Weekly oil palm FFB prices for the period January 6–12, 2021 to January 4–10, 2023 were used as the dataset. The price data were classified into 13 categories according to the age of the oil palm plant: 3, 4, 5, 6, 7, 8, 9, 10–20, 21, 22, 23, 24, and 25 years old.

Figure 1 shows the price of oil palm FFB by crop age. This graph demonstrates that the price of oil palm FFB from oil palms between 10 and 20 years had the highest price, which is due to the oil palm being mature and having reached peak production.

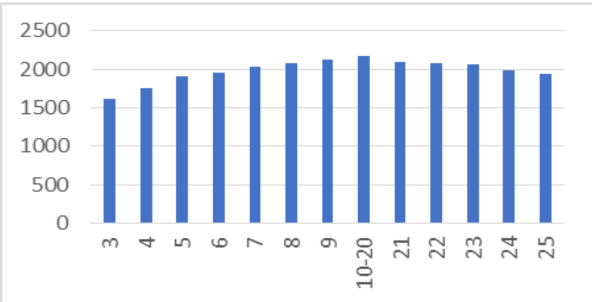


Fig. 1. Graph of Oil Palm FFB prices by Crop Age.

Figure 2 delineates the weekly oil palm FFB prices for the oil palm aged 10–20 years from January 6–12, 2021 to January 4–10, 2023. This graph demonstrates that the price of oil palm FFB fluctuated. Thus, this study estimated the future changes in the price of oil palm FFB.


Fig. 2. Graph of Oil Palm FFB prices from period January 6-12, 2021 to January 4-10, 2023.

The data represented the rising or falling conditions every week, with these conditions being categorized into 4 conditions: rising, extreme rising, falling, and extreme falling. Descriptive statistics of the data, namely the maximum value, the minimum value, the average increase, and the average decrease of each age group, were then obtained (see Table I).

After the initial data processing, the data on the price of FFB from oil palms aged 10–20 years were selected for analysis as they matched the 4 conditions. Using these data, the boundaries were established, which were used to determine each state as follows:

State 1: Rising state with the range $0 < x \le 92.65$
State 2: Extreme rising state with the range
$92.65 < x \le 285.89$
State 3: Falling state with the range $-167.37 \le x < 0$
State 4: Extreme falling state with the range
$-972.29 \le x < -167.37$

To determine the long-term opportunity, the Markov chain for oil palm FFB prices was included in an ergodic Markov chain, which was used to determine if the Markov chain is irreducible, positive recurrent, and aperiodic.

***Transition Probability Matrix***

Table II represents the changes in the state of oil palm FFB prices in Riau province according to the following conditions:

Rising state: The price of oil palm FFB is rising.
Extreme rising state: The price of oil palm FFB is rising in an extreme manner.
Falling state: The price of oil palm FFB is falling.
Extreme falling state: The price of oil palm FFB is falling at an extreme rate.

Table II shows that the weekly oil palm FFB price remained in the rising state for 19 weeks, transitioned from the rising to extreme rising states over 7 weeks, went from the rising to falling states over 10 weeks, and from the rising

TABLE II
CHANGES IN THE PRICE OF OIL PALM FFB IN RIAU PROVINCE

| Price of oil palm FFB in Riau Province | | State | | | |
|---|---|---|---|---|---|
| | | Rising | Extreme Rising | Falling | Extreme Falling |
| State | Rising | 19 | 7 | 10 | 4 |
| | Extreme Rising | 9 | 11 | 3 | 2 |
| | Falling | 9 | 5 | 11 | 2 |
| | Extreme Falling | 3 | 2 | 3 | 1 |

to extreme falling states over 4 weeks, and so on. The frequency of transition in the state of oil palm FFB prices is represented in the following matrix:

$$F_P = \begin{pmatrix} 19 & 7 & 10 & 4 \\ 9 & 11 & 3 & 2 \\ 9 & 5 & 11 & 2 \\ 3 & 2 & 3 & 1 \end{pmatrix}$$

This matrix was used to calculate the probability of the value of each condition.

$$P = \begin{pmatrix} 0.48 & 0.17 & 0.25 & 0.1 \\ 0.36 & 0.44 & 0.12 & 0.08 \\ 0.33 & 0.19 & 0.41 & 0.07 \\ 0.33 & 0.23 & 0.33 & 0.11 \end{pmatrix}$$

The transition probability diagram for weekly oil palm FFB prices in Riau Province is shown in Figure 3.

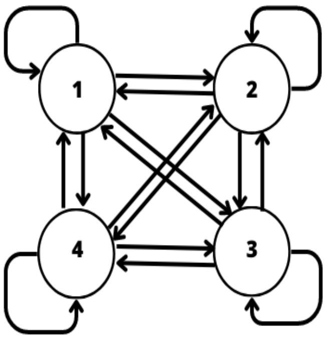


Fig. 3. Transition Probability Diagram of oil palm FFB price in Riau Province.

This figure demonstrates that states all states only had one equivalent class {1, 2, 3, 4}. Therefore, the Markov Chain of oil palm FFB prices in Riau Province is irreducible.

### *The n-Step Transition Probability Matrix*

Using equation (8), the one-step transition probability matrix of oil palm FFB price was determined as follows:

$$P = \begin{pmatrix} 0.48 & 0.17 & 0.25 & 0.1 \\ 0.36 & 0.44 & 0.12 & 0.08 \\ 0.33 & 0.19 & 0.41 & 0.07 \\ 0.33 & 0.23 & 0.33 & 0.11 \end{pmatrix}$$

Accordingly, the *n*th-transition probability matrix of $P^n$ was determined as follows:

$$P^2 = P \times P$$

$$P^2 = \begin{pmatrix} 0.40529167 & 0.22864352 & 0.27493519 & 0.09112963 \\ 0.39606667 & 0.2966 & 0.21835556 & 0.08897778 \\ 0.38549383 & 0.23172154 & 0.29622771 & 0.08655693 \\ 0.38648148 & 0.24253086 & 0.28283951 & 0.08814815 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.39684681 & 0.24269414 & 0.27114731 & 0.08931173 \\ 0.39735211 & 0.26002472 & 0.2532276 & 0.08939557 \\ 0.3941242 & 0.24351075 & 0.27371771 & 0.08864734 \\ 0.39455237 & 0.246314 & 0.2703377 & 0.08879593 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.39602514 & 0.24629313 & 0.268573 & 0.08910872 \\ 0.39655887 & 0.25070718 & 0.26350632 & 0.08910963 \\ 0.39566122 & 0.24650434 & 0.26881608 & 0.08910866 \\ 0.39579663 & 0.24721979 & 0.267932 & 0.08910882 \end{pmatrix}$$

Based on this *n*-step transition probability matrix, the $P^n$ for state 1 had a value greater than 0, with the greatest common divisor (GCD) of *n* for $n = 2, 3, 4, \ldots$ equaling 1. Thus, state 1 was aperiodic. Accordingly, as all the states are dependent on each other, all of them were determined to be aperiodic.

State 1:

$$\sum_{n=1}^{\infty} p_{11}^n = p_{11}^1 + p_{11}^2 + p_{11}^3 + p_{11}^4 + \ldots = 0.48 + 0.41 + 0.40 + 0.40 + \ldots = \infty$$

Based on the necessary and sufficient conditions of recurrent and transient states, state 1 was recurrent. As such, due to all the states relying on one another, all of the states 1 were considered recurrent.

By continuing the *n*-step transition probability, the following was determined:

$$P^{14} = P^{15} = \begin{pmatrix} 0.39 & 0.25 & 0.27 & 0.09 \\ 0.39 & 0.25 & 0.27 & 0.09 \\ 0.39 & 0.25 & 0.27 & 0.09 \\ 0.39 & 0.25 & 0.27 & 0.09 \end{pmatrix}$$

Since $P^{14}$ and $P^{15}$, and due to states 1, 2, 3, and 4 being recurrent and aperiodic, $P_{jj}^{n} \to \frac{1}{\mu_j}, j = 1,2,3,4$ for $n \to \infty$, thus $\mu_j = [2.56 \;\; 4.00 \;\; 3.70 \;\; 11.11]$. As $\mu_j < \infty$, the Markov chain of oil palm FFB price is positive recurrent.

Due to the Markov chain being irreducible, aperiodic, and positive recurrent, the Markov chain of oil palm FFB is ergodic. Thus, the stationary distribution can be determined as follows:

$$\lim_{n \to \infty} P_{ij}^{n} = \pi_j > 0 \; (i, j = 0,1,2,\ldots) \tag{10}$$

By using equation (10), it becomes clear that, starting from $P^{14}$, the $n$-step distribution matrix is of the same value. Therefore, the stationary distribution of the Markov chain of weekly oil palm FFB prices in Riau Province was obtained as follows:

$$\pi_j = [0.39 \;\; 0.25 \;\; 0.27 \;\; 0.09].$$

Based on these observations, it can be concluded that Riau Province's oil palm FFB prices have an opportunity for a rise of 39%, an extreme rise of 25%, a fall of 27%, and an extreme fall of 9%. Hence, we can conclude that the oil palm FFB price will rise in the future because the cumulative probability of rising and extreme rising is 64%.

### B. *CPO Prices*

The daily CPO price data for the period January 3, 2022 to December 30, 2022 were used herein. Figure 4 details the CPO prices in that period.


Fig. 4. Graph of CPO prices from period January 3, 2022 to December 30, 2022.

This graph indicates that the price increased from the beginning of the year to the middle of the year but fell in the middle of the year to the end of the year, with the price stabilizing at the end of the year.

The data represented the rising or falling conditions every week. It was hence categorized into the same 4 conditions: rising, extreme rising, falling, and extreme falling. The descriptive statistics of the data, namely the maximum value, the minimum value, the average increase, and the average decrease of each age group, were then obtained (see Table III).

TABLE III
DESCRIPTIVE STATISTICS ABOUT THE STATE OF THE DATA

| Descriptive Statistics | |
|---|---|
| Maximum | 9632.39 |
| Minimum | -5443.95 |
| Average increase | 660.14 |
| Average decrease | -727.34 |

After the data were processed, the boundaries were obtained, which were then used to determine each state as follows:

State 1: Rising state with the range $0 \le x \le 660.14$
State 2: Extreme rising state with the range $660.14 < x \le 9632.39$
State 3: Falling state with the range $-727.34 \le x < 0$
State 4: Extreme falling state with the range $-5443.95 \le x < -727.34$

To determine the long-term opportunity, the Markov chain of CPO prices was included in an ergodic Markov chain if the Markov chain was determined to be irreducible, positive recurrent, and aperiodic.

#### *Transition Probability Matrix*

Table IV delineates the changes in the state of CPO prices in Riau province according to the following conditions:

Rising state: The price of oil palm FFB is rising.
Extreme rising state: The price of oil palm FFB is rising at an extreme rate.
Falling state: The price of oil palm FFB is falling.
Extreme falling state: The price of oil palm FFB is falling at an extreme rate.

Table IV shows that the daily CPO price remained in the rising state for 38 weeks, transitioned from the rising to extreme rising states over 8 weeks, moved from the rising to falling states over 29 weeks, shifted from the rising to extreme falling states over 7 weeks, and so forth. The frequency of transitions in oil palm FFB prices was represented in the following matrix:

$$F_P = \begin{pmatrix} 38 & 8 & 29 & 7 \\ 5 & 8 & 12 & 15 \\ 35 & 11 & 22 & 9 \\ 4 & 13 & 14 & 8 \end{pmatrix}$$

From this, the probability of the value of each condition was calculated

TABLE IV
CHANGES IN THE PRICE OF CPO IN INDONESIA

| Price of CPO in Indonesia | | State | | | |
|---|---|---|---|---|---|
| | | Rising | Extreme Rising | Falling | Extreme Falling |
| State | Rising | 38 | 8 | 29 | 7 |
| | Extreme Rising | 5 | 8 | 12 | 15 |
| | Falling | 35 | 11 | 22 | 9 |
| | Extreme Falling | 4 | 13 | 14 | 8 |

$$P=\begin{pmatrix}0.46 & 0.10 & 0.35 & 0.09\\ 0.13 & 0.20 & 0.30 & 0.38\\ 0.45 & 0.14 & 0.29 & 0.12\\ 0.10 & 0.33 & 0.36 & 0.21\end{pmatrix}$$

The transition probability diagram for daily CPO prices in Riau Province is shown in Figure 5, which demonstrates that all the states had only one equivalent class {1, 2, 3, 4}. Therefore, the Markov Chain of CPO prices in Riau Province was irreducible.

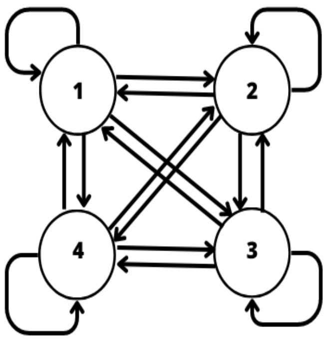


Fig. 5. Transition Probability Diagram of CPO price in Indonesia.

***The n-Step Transition Probability Matrix***

Using equation (8) the one-step transition probability matrix of CPO prices was obtained as follows:

$$P=\begin{pmatrix}0.46 & 0.10 & 0.35 & 0.09\\ 0.13 & 0.20 & 0.30 & 0.38\\ 0.45 & 0.14 & 0.29 & 0.12\\ 0.10 & 0.33 & 0.36 & 0.21\end{pmatrix}$$

Accordingly the $n$th-transition probability matrix of $P^n$ was determined as follows:

$$P^2 = P \times P$$

$$P^2=\begin{pmatrix}0.39646 & 0.14370 & 0.32485 & 0.13499\\ 0.25775 & 0.22005 & 0.32454 & 0.19766\\ 0.37036 & 0.15269 & 0.32720 & 0.14975\\ 0.27341 & 0.196331 & 0.31247 & 0.21779\end{pmatrix}$$

$$P^3=\begin{pmatrix}0.36319 & 0.15882 & 0.32459 & 0.15339\\ 0.31474 & 0.18141 & 0.32085 & 0.18300\\ 0.35480 & 0.16333 & 0.32403 & 0.15784\\ 0.31561 & 0.18318 & 0.32305 & 0.17816\end{pmatrix}$$

$$P^4=\begin{pmatrix}0.35144 & 0.16499 & 0.32390 & 0.15997\\ 0.33314 & 0.17382 & 0.32310 & 0.16994\\ 0.34831 & 0.16618 & 0.32372 & 0.16179\\ 0.33427 & 0.17296 & 0.32283 & 0.16994\end{pmatrix}$$

Based on this $n$-step transition probability matrix, the $P^n$ for state 1 had a value greater than 0, with the GCD of $n$ for $n=2,3,4,\ ...$ equaling 1. Thus, the first state was determined to be aperiodic. Due to this and all the states being linked to each other, all the states were determined to be aperiodic.

State 1

$$\sum_{n=1}^{\infty} p_{11}^{n} = p_{11}^{1} + p_{11}^{2} + p_{11}^{3} + p_{11}^{4} + \ldots = 0.46 + 0.40 + 0.36 + 0.35 + \ldots = \infty$$

Based on the necessary and sufficient conditions of recurrent and transient states, state 1 was recurrent. Due to this as well as all the states being dependent on each other, states 1, 2, 3, and 4 were considered recurrent.

By continuing the $n$-step transition probability, the following matrix was obtained.

$$P^{16}=P^{17}=\begin{pmatrix}0.35 & 0.17 & 0.32 & 0.16\\ 0.35 & 0.17 & 0.32 & 0.16\\ 0.35 & 0.17 & 0.32 & 0.16\\ 0.35 & 0.17 & 0.32 & 0.16\end{pmatrix}$$

Since $P^{16}$ and $P^{17}$ in addition to states 1, 2, 3, and 4 being recurrent and aperiodic, $P_{jj}^{n} \to \frac{1}{\mu_j}, j=1,2,3,4$ for $n \to \infty$, thus $\mu_j = \begin{bmatrix}2.86 & 5.88 & 3.125 & 6.25\end{bmatrix}$. As $\mu_j < \infty$, the Markov chain of CPO price was categorized as positive recurrent.

As it was irreducible, aperiodic, and positive recurrent, the Markov chain of CPO was determined to be an ergodic Markov chain. Thus, the stationary distribution was determined as follows:

$$\lim_{n\to\infty} P_{ij}^{n} = \pi_j > 0 \ (i, j = 0,1,2,\dots) \quad (11)$$

According to equation (10), starting from $P^{16}$, the $n$-step distribution matrix is of the same value. Therefore, the stationary distribution of the Markov chain of CPO prices in Riau Province was obtained as follows:

$$\pi_j = \begin{bmatrix} 0.35 & 0.17 & 0.32 & 0.16 \end{bmatrix}.$$

These findings indicate that Indonesia's CPO price has an opportunity for a rise of 35%, an extreme rise of 17%, a fall of 32%, and an extreme fall of 16%. Hence, we conclude that the CPO price will rise in the future because the cumulative probability of rising and extreme rising is 52%.

## IV. Conclusion

The price of oil palm FFB can rise by 39%, there is a 25% chance that oil palm FFB prices will rise at an extreme rate, the price of oil palm FFB can fall by 27%, and there is a 9% chance that the price of oil palm FFB will fall at an extreme rate. Moreover, the price of CPO can rise by 35%, there is a 17% chance that the price of CPO will rise at an extreme rate, the price of CPO can fall by 32%, and there is a 16% chance that the price of CPO will fall at an extreme rate. Thus, the probability that the CPO price has a chance of increasing in the future is 35%, which is greater than that of other conditions. Therefore, CPO prices being predicted to rise reinforces the conclusion that the price of oil palm FFB in Riau Province will increase.

## References


[1] M. Qaim, K. T. Sibhatu, H. Siregar, and I. Grass, "Environmental, economic, and social consequences of the oil palm boom," in *Annu. Rev. Resour. Econ.*, vol. 12, no. 1, pp. 321–344, 2020.

[2] Ministry of Agriculture of the Republic of Indonesia, *MOA Number 18 of 2016*. Jakarta: Ministry of Agriculture of the Republic of Indonesia, 2016.

[3] M. Euler, S. Schwarze, H. Siregar, and M. Qaim, "Oil palm expansion among smallholder farmers in Sumatra, Indonesia," *J. Agric. Econ.*, vol. 67, no. 3, pp. 658–676, 2016.

[4] V. Krishna, M. Euler, H. Siregar, and M. Qaim, "Differential livelihood impacts of oil palm expansion in Indonesia," *Agric. Econ.*, vol. 48, no. 5, pp. 639–653, 2017.

[5] C. Kubitza, V. V. Krishna, Z. Alamsyah, and M. Qaim, "The economics behind an ecological crisis: Livelihood effects of oil palm expansion in Sumatra, Indonesia," *Hum. Ecol.*, vol. 46, pp. 107–116, 2018.

[6] L. Feintrenie, W. K. Chong, and P. Levang, "Why do farmers prefer oil palm? Lessons learnt from Bungo district, Indonesia," *Small-scale For.*, vol. 9, pp. 379–396, 2010.

[7] D. Chrisendo, H. Siregar, and M. Qaim, "Oil palm and structural transformation of agriculture in Indonesia," *Agric. Econ.*, vol. 52, no. 5, pp. 849–862, 2021.

[8] Central Bureau of Statistics, *Indonesia Palm Oil Statistics 2020*. Jakarta: Central Bureau of Statistics, 2021.

[9] A. A. Karia, I. Bujang, and I. Ahmad, "Forecasting on crude palm oil prices using artificial intelligence approaches," *Am. J. Oper. Res.*, vol. 3, no. 2, pp. 259–267, 2013.

[10] S. Sugiarto, "Estimator of the ARIMA model for forecasting the price of oil palm fresh fruit bunches in Riau Province" ["Penduga Model Arima Untuk Peramalan Harga Tbs Kelapa Sawit Di Propinsi Riau"], *J. Sci., Tech. Ind.* [*Jurnal Sains, Teknologi dan Industri (SITEKIN)*], vol. 15, no. 1, pp. 35–40, 2017.

[11] B. Arifin, I. G. L. P. Tanaya, and A. Usman, "Forecasting world palm oil prices in 2020–2024" ["Peramalan Harga Kelapa Sawit Dunia Pada Tahun 2020–2024"], *SAINTEK Proceedings* [*Prosiding SAINTEK*], vol. 3, pp. 349–368, 2021.

[12] S. Kharvi and T. P. M Pakkala, "An optimal ordering inventory policy when the purchase price follows a discrete-time Markov chain," *Commun. Stat-Simul. C.*, pp. 1–18, 2022.

[13] D. Suhartono, "Markov Chain." Retrieved from Binus University website: http://socs.binus.ac.id/2013/06/30/markov-chain

[14] Y. Areepong and S. Sukparungsee, "Capability process on a two-sided extended EWMA control chart for moving average with exogenous factors model," *IAENG International Journal of Applied Mathematics,* vol. 53, no. 3, pp. 201–213, 2023.

[15] S. M. Ross, *Stochastic Processes*, 2nd ed. New York, N. Y.: John Willey & Sons, 1996.

[16] A. O. Ibeje, J. Osuagwu, and R. Onosakponome, "A Markov model for prediction of annual rainfall," *J. Eng. Appl. Sci.*, vol. 3, no. 11, pp. 1–5, 2018.

[17] P. A. Gagniuc, *Markov chains: From theory to implementation and experimentation*. Hoboken, N. J.: John Wiley & Sons, 2017.