A New Single-valued Neutrosophic Extended Power Average Operator based on Dombi Operations in MCDM Problem

Baolin Li, Zetao Hu, Lihua Yang*, Yanli Li

Abstract—**In real multi-criteria decision-making (MCDM), single-valued neutrosophic set (SVNS) has become a research hotspot because it can better depict complicated decision-making information. Compared to the classical power operator, extended power average (EPA) operator can handle extreme values more flexibly in practice. Dombi operations with a parameter are more flexible in integrating multiple decision values. Motivated by these merits, the extended power weighted average operator based on Dombi operations is firstly defined and expanded to SVNS. First, the Dombi operational rules of single-valued neutrosophic numbers (SVNNs) are established, as well as the Hamming distance measure and Dice similarity measure for SVNNs. Second, single-valued neutrosophic Dombi extended power weighted averaging (SVNDEPWA) operator is built, and special cases are also discussed. Eventually, an MCDM model using the operator above is constructed. Meanwhile, a case and sensitive analysis, as well as antithesis are conducted to present the effectiveness and robustness of the new model.**

Index Terms—**SVNS, EPA, Dombi, MCDM**

I. Introduction

A single-valued neutrosophic set (SVNS) [1], as prolongation of fuzzy set (FS) [2] and intuitionistic fuzzy set (IFS) [3], is more appropriate to describe the real cognitive information of indeterminate, incomplete and inconsistent. Currently, SVNS has been widely used to settle different multi-criteria decision-making (MCDM) matters [4-6], for instance, disease diagnosis, clustering, and investment strategy.

In actual MCDM issue, aggregating operator (AO) is a powerful tool for fusing several input arguments given by decision-makers (DMs) into a comprehensive value. Various operators depending on different operational rules are developed, including Einstein, Archimedean, Hamacher, Frank, Schweizer-Skla, and Dombi operations [7-12]. Dombi operations initially put forwards by Dombi [13] is more flexible with a general parameter. Subsequently, Dombi operations are utilized to handle interval neutrosophic set(INS) [14], picture fuzzy set [15], bipolar FS [16], single-valued trapezoidal NS [17], bipolar complex FS [18]. Moreover, the power averaging (PA) operator firstly defined [19], can alleviate the impact of irrational input values. Power geometric operator (PGA) is proposed by Xu [20]. A generalized power average (GPA) operator is introduced by Zhou [21]. Nevertheless, extreme data may be useful in some actual cases. Therefore, Xiong [22] defined extended PA (EPA) operator to flexibly solve inappropriate data. Afterwards, Li [23] employed EPA operator to handle multi-criteria group decision-making (MCGDM) issue with Q-RDHF information. Ning manipulated EPA operator to solve sustainable suppliers selection problem with PDHF information [24]. Kou used EPA to settle linguistic pythagorean fuzzy issue [25].

Consider the superiorities of Dombi and PA, some achievements on the combination of them are studied. Dombi power operator is applied to INS [14], 2-tuple linguistic neutrosophic set [26], SVNS [27], spherical FS [28], and bipolar complex FS [29].

Up to now, there are few literatures relating to the application of EPA operator, and it cannot deal with SVNS, especially using Dombi operations. Thus, the core aim of this thesis is to explore EPA operator using Dombi to solve MCDM issue with single-valued neutrosophic information.

Therefore, the thesis is structured as follows. In Section II, some relating preliminaries are presented. Section III describes the Dombi EPA operator for SVNS. In Section IV, a new MCDM algorithm model with single-valued neutrosophic information is constructed. Section V contains result analysis regarding an MCDM example. Eventually, Section VI outlines the conclusion.

Manuscript received July 29, 2023; revised April 27, 2024. This work was supported by the Humanities and Social Sciences Planning Fund of the Ministry of Education of China under Grant 23YJAZH060, and Doctoral Research Initiation Fund of Hubei University of Automotive Technology under Grant BK202109.

Baolin Li is a professor at the School of Economics and Management, Hubei University of Automotive Technology, Shiyan, 442000, China. (e-mail:20050002@huat.edu.cn).

Zetao Hu is a postgraduate student at the School of Economics and Management, Hubei University of Automotive Technology, Shiyan, 442000, China. (e-mail:202110038@huat.edu.cn).

Lihua Yang is a professor at the School of Economics and Management, Hubei University of Automotive Technology, Shiyan, 442000, China. (Corresponding author: +8613593745204; 20060019@huat.edu.cn).

Yanli Li is a postgraduate student at the School of Economics and Management, Hubei University of Automotive Technology, Shiyan, 442000, China. (e-mail:202210041@huat.edu.cn).

II. Preliminaries

Some notions are introduced.

A. SVNS

Definition1. [1] Assuming Y is a collection with an element in Y represented by y. An SVNS B in Y is defined as follows:

$$B=\left\{\left\langle y,T_B(y),I_B(y),F_B(y)\right\rangle \middle| y\in Y\right\} \quad (1)$$

Where $T_B(y)$, $I_B(y)$, and $F_B(y)$ represent the three memberships of the truth, the indeterminacy, and the falsity, separately. For each element y in Y, an SVNS B satisfies the following conditions:

$T_B(y),I_B(y),F_B(y)\in[0,1]$, $0\le T_B(y)+I_B(y)+F_B(y)\le 3$.

For simplification, a basic element in SVNS B is expressed as $b=\langle T_b,I_b,F_b\rangle$, which is called a single-valued neutrosophic number (SVNN).

B. Distance Measure

Definition2. Let $b_1=\langle T_1,I_1,F_1\rangle$ and $b_2=\langle T_2,I_2,F_2\rangle$ be SVNNs, the Hamming measure for b_1 and b_2 is delimited.

$$d(b_1,b_2)=\frac{1}{3}\left(|T_1-T_2|+|I_1-I_2|+|F_1-F_2|\right) \quad (2)$$

C. Dombi Operations

Definition3. Assuming $b_1=\langle T_1,I_1,F_1\rangle$ and $b_2=\langle T_2,I_2,F_2\rangle$ are SVNNs, $x\ge 1$ and $\lambda>0$, then Dombi operations of SVNNs are defined.

$$(1) b_1\oplus b_2=\left\langle 1-\frac{1}{1+\left\{\left(\frac{T_1}{1-T_1}\right)^x+\left(\frac{T_2}{1-T_2}\right)^x\right\}^{\frac{1}{x}}}, \frac{1}{1+\left\{\left(\frac{1-I_1}{I_1}\right)^x+\left(\frac{1-I_2}{I_2}\right)^x\right\}^{\frac{1}{x}}}, \frac{1}{1+\left\{\left(\frac{1-F_1}{F_1}\right)^x+\left(\frac{1-F_2}{F_2}\right)^x\right\}^{\frac{1}{x}}}\right\rangle$$

$$(2) b_1\otimes b_2=\left\langle \frac{1}{1+\left\{\left(\frac{1-T_1}{T_1}\right)^x+\left(\frac{1-T_2}{T_2}\right)^x\right\}^{\frac{1}{x}}}, 1-\frac{1}{1+\left\{\left(\frac{I_1}{1-I_1}\right)^x+\left(\frac{I_2}{1-I_2}\right)^x\right\}^{\frac{1}{x}}}, 1-\frac{1}{1+\left\{\left(\frac{F_1}{1-F_1}\right)^x+\left(\frac{F_2}{1-F_2}\right)^x\right\}^{\frac{1}{x}}}\right\rangle$$

$$(3) \lambda b_1=\left\langle 1-\frac{1}{1+\left\{\lambda\left(\frac{T_1}{1-T_1}\right)^x\right\}^{\frac{1}{x}}}, \frac{1}{1+\left\{\lambda\left(\frac{1-I_1}{I_1}\right)^x\right\}^{\frac{1}{x}}}, \frac{1}{1+\left\{\lambda\left(\frac{1-F_1}{F_1}\right)^x\right\}^{\frac{1}{x}}}\right\rangle$$

$$(4) b_1^{\lambda}=\left\langle \frac{1}{1+\left\{\lambda\left(\frac{1-T_1}{T_1}\right)^x\right\}^{\frac{1}{x}}}, 1-\frac{1}{1+\left\{\lambda\left(\frac{I_1}{1-I_1}\right)^x\right\}^{\frac{1}{x}}}, 1-\frac{1}{1+\left\{\lambda\left(\frac{F_1}{1-F_1}\right)^x\right\}^{\frac{1}{x}}}\right\rangle$$

D. EPA Operator

Definition4. [22] Assuming $a_i\left(i=1,2,\cdots,n\right)$ is a set of real numbers, an EPA operator is defined as follows:

$$EPA^{(\rho)}\left(a_1,a_2,\cdots,a_n\right)=\sum_{i=1}^{n}w_i^{(\rho)}a_i=\sum_{i=1}^{n}\frac{\left(\rho+T\left(a_i\right)\right)}{\sum_{j=1}^{n}\left(\rho+T\left(a_j\right)\right)}a_i \quad (3)$$

Where $\rho\in\left(-\infty,1+n\right]\cup\left[0,+\infty\right)$, $T\left(a_i\right)=\sum_{j=1,j\ne i}^{n}Sup(a_i,a_j)$, $Sup(a_i,a_j)=1-d(a_i,a_j)$. Here, $d(a_i,a_j)$ is the distance measure for a_i and a_j, and $Sup(a_i,a_j)$ denotes the support of a_i and a_j, satisfying three conditions:

(1) $0\le Sup\left(a_i,a_j\right)\le 1$

(2) $Sup\left(a_i,a_j\right)=Sup\left(a_j,a_i\right)$

(3) $Sup\left(a_i,a_j\right)\ge Sup\left(a_p,a_q\right), if\ d\left(a_i,a_j\right)\le d\left(a_p,a_q\right)$

E. Dice Similarity

Dice similarity is initially delimited by Dice [30]. Ye defined the Dice similarity [31] and the generalized Dice similarity [32] for simplified neutrosophic set. Here, the Dice similarity for SVNNs is established depending on the above definitions.

Definition5. Assuming $b_1 = \langle T_1, I_1, F_1 \rangle$ and $b_2 = \langle T_2, I_2, F_2 \rangle$ are SVNNs, the Dice similarity measure for b_1 and b_2 is defined below.

$$S_D(b_1,b_2) = \frac{2(b_1 \cdot b_2)}{|b_1|^2 + |b_2|^2} = \frac{2(T_1T_2 + I_1I_2 + F_1F_2)}{(T_1^2 + I_1^2 + F_1^2) + (T_2^2 + I_2^2 + F_2^2)} \quad (4)$$

Dice similarity for SVNNs b_1 and b_2 obeys to the conditions:

(1) $0 \le S_D(b_1,b_2) \le 1$;

(2) $S_D(b_1,b_2) = S_D(b_2,b_1)$;

(3) $S_D(b_1,b_2) = 1, if\ \ b_1 = b_2$.

III. New Operator

In this section, we will expand EPA operator using Dombi to handle SVNS. A single-valued neutrosophic Dombi extended power weighted averaging (SVNDEPWA) operator is put forward, and its special cases are also discussed.

A. SVNDEPWA Operator

Definition6. Let $b_i = \langle T_i, I_i, F_i \rangle (i = 1,2,\cdots,n)$ be a set of SVNNs, $\rho \in (-\infty, 1-n] \cup [0, +\infty)$ and $x \ge 1$ be two parameters. The weighting vector is $w = (w_1, w_2, \cdots, w_n)^T$, $\sum_{i=1}^n w_i = 1$, $w_i \in [0,1]$. Then SVNDEPWA operator is defined as below, and the aggregating result is still a SVNN.

$$SVNDEPWA(b_1,b_2,\cdots,b_n) = \mathop{\oplus}_{i=1}^{n} \frac{w_i(\rho + T(b_i))}{\sum_{j=1}^{n} w_j(\rho + T(b_j))} b_i$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ \sum_{i=1}^{n} \left(\frac{w_i(\rho + T(b_i))}{\sum_{j=1}^{n} w_j(\rho + T(b_j))} \left(\frac{T_i}{1-T_i} \right)^x \right) \right\}^{1/x}}, \frac{1}{1 + \left\{ \sum_{i=1}^{n} \left(\frac{w_i(\rho + T(b_i))}{\sum_{j=1}^{n} w_j(\rho + T(b_j))} \left(\frac{1-I_i}{I_i} \right)^x \right) \right\}^{1/x}}, \frac{1}{1 + \left\{ \sum_{i=1}^{n} \left(\frac{w_i(\rho + T(b_i))}{\sum_{j=1}^{n} w_j(\rho + T(b_j))} \left(\frac{1-F_i}{F_i} \right)^x \right) \right\}^{1/x}} \right\rangle \quad (5)$$

Where $T(b_i) = \sum_{j=1, j\ne i}^{n} w_j Sup(b_i, b_j)$ is weighted support between b_i and b_j, $Sup(b_i,b_j) = 1 - d(b_i,b_j)$, $Sup(b_i,b_j)$ is support of b_i and b_j, $d(b_i,b_j)$ is Hamming measure presented in Eq. (2).

For convenience, let $\varpi_i = \frac{w_i(\rho + T(b_i))}{\sum_{j=1}^{n} w_j(\rho + T(b_j))}$, then Eq. (5) can be recorded as:

$$SVNDEPWA(b_1,b_2,\cdots,b_n) = \mathop{\oplus}_{i=1}^{n} \varpi_i b_i$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ \sum_{i=1}^{n} \left(\varpi_i \left(\frac{T_i}{1-T_i} \right)^x \right) \right\}^{1/x}}, \frac{1}{1 + \left\{ \sum_{i=1}^{n} \left(\varpi_i \left(\frac{1-I_i}{I_i} \right)^x \right) \right\}^{1/x}}, \frac{1}{1 + \left\{ \sum_{i=1}^{n} \left(\varpi_i \left(\frac{1-F_i}{F_i} \right)^x \right) \right\}^{1/x}} \right\rangle \quad (6)$$

Proof: It is proved using mathematical induction.

Depending on Dombi operations for SVNNs in Definition 3, the result is presented below.

(1) When $n = 2$,

$$\varpi_1 b_1 = \left\langle 1 - \frac{1}{1 + \left\{ \varpi_1 \left(\frac{T_1}{1-T_1} \right)^x \right\}^{1/x}}, \frac{1}{1 + \left\{ \varpi_1 \left(\frac{1-I_1}{I_1} \right)^x \right\}^{1/x}}, \frac{1}{1 + \left\{ \varpi_1 \left(\frac{1-F_1}{F_1} \right)^x \right\}^{1/x}} \right\rangle$$

And,

$$\varpi_2 b_2 = \left\langle 1 - \frac{1}{1 + \left\{ \varpi_2 \left(\frac{T_2}{1-T_2} \right)^x \right\}^{1/x}}, \frac{1}{1 + \left\{ \varpi_2 \left(\frac{1-I_2}{I_2} \right)^x \right\}^{1/x}}, \frac{1}{1 + \left\{ \varpi_2 \left(\frac{1-F_2}{F_2} \right)^x \right\}^{1/x}} \right\rangle$$

Then,

$$SVNDEPWA(b_1,b_2)=\varpi_1 b_1\oplus\varpi_2 b_2$$

$$=\left\langle 1-\frac{1}{1+\left\{\varpi_1\left(\frac{T_1}{1-T_1}\right)^x+\varpi_2\left(\frac{T_2}{1-T_2}\right)^x\right\}^{1/x}},\ \frac{1}{1+\left\{\varpi_1\left(\frac{1-I_1}{I_1}\right)^x+\varpi_2\left(\frac{1-I_2}{I_2}\right)^x\right\}^{1/x}},\ \frac{1}{1+\left\{\varpi_1\left(\frac{1-F_1}{F_1}\right)^x+\varpi_2\left(\frac{1-F_2}{F_2}\right)^x\right\}^{1/x}}\right\rangle$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{2}\left(\varpi_i\left(\frac{T_i}{1-T_i}\right)^x\right)\right\}^{1/x}},\ \frac{1}{1+\left\{\sum_{i=1}^{2}\left(\varpi_i\left(\frac{1-I_i}{I_i}\right)^x\right)\right\}^{1/x}},\ \frac{1}{1+\left\{\sum_{i=1}^{2}\left(\varpi_i\left(\frac{1-F_i}{F_i}\right)^x\right)\right\}^{1/x}}\right\rangle$$

(2) When $n=k$, the result is got depending on Eq. (6).

$$SVNDEPWA(b_1,b_2,\cdots,b_k)=\mathop{\oplus}_{i=1}^{k}\varpi_i b_i$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{k}\left(\varpi_i\left(\frac{T_i}{1-T_i}\right)^x\right)\right\}^{1/x}},\ \frac{1}{1+\left\{\sum_{i=1}^{k}\left(\varpi_i\left(\frac{1-I_i}{I_i}\right)^x\right)\right\}^{1/x}},\ \frac{1}{1+\left\{\sum_{i=1}^{k}\left(\varpi_i\left(\frac{1-F_i}{F_i}\right)^x\right)\right\}^{1/x}}\right\rangle$$

(3) Therefore, when $n=k+1$, we can get,

$$SVNDEPWA(b_1,b_2,\cdots,b_k,b_{k+1})=\mathop{\oplus}_{i=1}^{k+1}\varpi_i b_i=\mathop{\oplus}_{i=1}^{k}\varpi_i b_i\oplus\varpi_{k+1}b_{k+1}$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{k}\left(\varpi_i\left(\frac{T_i}{1-T_i}\right)^x\right)\right\}^{1/x}},\ \frac{1}{1+\left\{\sum_{i=1}^{k}\left(\varpi_i\left(\frac{1-I_i}{I_i}\right)^x\right)\right\}^{1/x}},\ \frac{1}{1+\left\{\sum_{i=1}^{k}\left(\varpi_i\left(\frac{1-F_i}{F_i}\right)^x\right)\right\}^{1/x}}\right\rangle$$

$$\oplus\left\langle 1-\frac{1}{1+\left\{\varpi_{k+1}\left(\frac{T_{k+1}}{1-T_{k+1}}\right)^x\right\}^{1/x}},\ \frac{1}{1+\left\{\varpi_{k+1}\left(\frac{1-I_{k+1}}{I_{k+1}}\right)^x\right\}^{1/x}},\ \frac{1}{1+\left\{\varpi_{k+1}\left(\frac{1-F_{k+1}}{F_{k+1}}\right)^x\right\}^{1/x}}\right\rangle$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{k}\left(\varpi_i\left(\frac{T_i}{1-T_i}\right)^x\right)+\varpi_{k+1}\left(\frac{T_{k+1}}{1-T_{k+1}}\right)^x\right\}^{1/x}},\right.$$
$$\frac{1}{1+\left\{\sum_{i=1}^{k}\left(\varpi_i\left(\frac{1-I_i}{I_i}\right)^x\right)+\varpi_{k+1}\left(\frac{1-I_{k+1}}{I_{k+1}}\right)^x\right\}^{1/x}},$$
$$\left.\frac{1}{1+\left\{\sum_{i=1}^{k}\left(\varpi_i\left(\frac{1-F_i}{F_i}\right)^x\right)+\varpi_{k+1}\left(\frac{1-F_{k+1}}{F_{k+1}}\right)^x\right\}^{1/x}}\right\rangle$$
$$=\left\langle 1-\frac{1}{1+\left\{\sum_{i=1}^{k+1}\left(\varpi_i\left(\frac{T_i}{1-T_i}\right)^x\right)\right\}^{1/x}},\right.$$
$$\left.\frac{1}{1+\left\{\sum_{i=1}^{k+1}\left(\varpi_i\left(\frac{1-I_i}{I_i}\right)^x\right)\right\}^{1/x}},\frac{1}{1+\left\{\sum_{i=1}^{k+1}\left(\varpi_i\left(\frac{1-F_i}{F_i}\right)^x\right)\right\}^{1/x}}\right\rangle$$

Thus, Eq. (6) is right for all i, namely, Eq. (5) is right for all i.

B. Special Cases

When ρ is assigned to different value, some special cases are presented as below.

(1) If $\rho=1$, then SVNDEPWA operator is simplified to single-valued neutrosophic Dombi power weighted averaging (SVNDPWA) operator, that is,

$$SVNDPWA(b_1,b_2,\cdots,b_n)=\underset{i=1}{\overset{n}{\oplus}}\frac{w_i\left(1+T(b_i)\right)}{\sum_{j=1}^{n}w_j\left(1+T(b_j)\right)}b_i$$

(2) If $\rho\to+\infty$, or $T(b_i)=m,\ \mathrm{m}\in[0,n-1]$, then SVNDEPWA operator is reduced to single-valued neutrosophic Dombi weighted average (SVNDWA) operator, that is,

$$SVNDWA(b_1,b_2,\cdots,b_n)=\underset{i=1}{\overset{n}{\oplus}}w_ib_i$$

IV. MCDM MODEL

The MCDM model with SVN information employing the above-depicted novel aggregating operator is shown in this part.

Assume $E=\{E_1,E_2,\cdots,E_p\}$ is a limited alternative set and $A=\{A_1,A_2,\cdots,A_q\}$ is the evaluating criteria. Let $w=\{w_1,w_2,\cdots,w_q\}$ be a completely known weight set regarding on criteria, where $\sum_{i=1}^{n}w_i=1$, $w_i\in[0,1]$.

The initial decision matrix is depicted as $D=[V_{ij}]_{p\times q}=<T_{ij},I_{ij},F_{ij}>_{p\times q}$ $(i=1,2,\cdots,p;\ j=1,2,\cdots,q)$, where V_{ij} is SVNN representing the evaluation of each alternative E_i under criterion A_j. The detailed algorithm procedure of MCDM model under SVNS environment manipulating SVNDEPWA is presented below.

Algorithm: The MCDM model with SVN information

Input: Limited alternatives, criteria, and weights

Output: Ordering alternatives

Step1. Obtain the SVN decision matrix $D=[V_{ij}]_{p\times q}=<T_{ij},I_{ij},F_{ij}>_{p\times q}$ given by decision maker.

Step2. Construct the transformation matrix $D'=[U_{ij}]_{p\times q}$ if necessary. If A_j is a cost criterion, then $U_{ij}=<F_{ij},I_{ij},T_{ij}>$, otherwise, $U_{ij}=V_{ij}=<T_{ij},I_{ij},F_{ij}>$.

Step3. Obtain Hamming measure of U_{im} and $U_{in}\ (i=1,2,\cdots,p;m,n=1,2,\cdots,q;m\neq n)$.

$$d(U_{im},U_{in})=\frac{1}{3}\left(\left|T_{im}-T_{in}\right|+\left|I_{im}-I_{in}\right|+\left|F_{im}-F_{in}\right|\right) \tag{7}$$

Step4. Calculate the support for U_{im} from $U_{in}\ (i=1,2,\cdots,p;m,n=1,2,\cdots,q;m\neq n)$.

$$Sup(U_{im},U_{in})=1-d(U_{im},U_{in}) \tag{8}$$

Step5. Calculate the weighted support of $U_{im}\ (i=1,2,\cdots,p;m=1,2,\cdots,q)$.

$$T(U_{im})=\sum_{n=1,n\neq m}^{q}w_nSup(U_{im},U_{in}) \tag{9}$$

Step6. Obtain the extended power weighting (EPW) $\varpi_{im}\ (i=1,2,\cdots,p;m=1,2,\cdots,q)$.

$$\varpi_{im}=\frac{w_m\left(\rho+T(U_{im})\right)}{\sum_{n=1}^{q}w_n\left(\rho+T(U_{in})\right)} \tag{10}$$

Step7. Calculate the comprehensive value $U_i(i=1,2,\cdots,p)$.

$$SVNDEPWA(U_{i1},U_{i2},\cdots,U_{iq})=\underset{m=1}{\overset{q}{\oplus}}\varpi_{im}U_{im}$$
$$=\left\langle 1-\frac{1}{1+\left\{\sum_{m=1}^{q}\left(\varpi_{im}\left(\frac{T_{im}}{1-T_{im}}\right)^x\right)\right\}^{1/x}},\right. \tag{11}$$
$$\left.\frac{1}{1+\left\{\sum_{m=1}^{q}\left(\varpi_{im}\left(\frac{1-I_{im}}{I_{im}}\right)^x\right)\right\}^{1/x}},\frac{1}{1+\left\{\sum_{m=1}^{q}\left(\varpi_{im}\left(\frac{1-F_{im}}{F_{im}}\right)^x\right)\right\}^{1/x}}\right\rangle$$

Step8. Calculate the Dice similarity measure of each alternative $U_i = \langle T_i, I_i, F_i \rangle$ $(i = 1, 2, \cdots, p)$ and the optimal alternative $U^* = \langle 1, 0, 0 \rangle$.

$$S_D(U_i, U^*) = \frac{2T_i}{T_i^2 + I_i^2 + F_i^2 + 1} \tag{12}$$

Step9. Obtain the ranking of all alternatives.
Step10. End.

V. Example And Analysis

An actual instance originated from Jana [27] is adopted herein to verify the practicality of the above-proposed MCDM model. Meanwhile, the sensitive analysis of different parameters ρ and x on alternatives selection is performed, and the comparison analysis is conducted as well.

A. An Example

To promote economic and social development, the Indian government wants to select an optimal one from five enterprises $E = \{E_1, E_2, E_3, E_4, E_5\}$ to repair the roads. There are four criteria $A = \{A_1, A_2, A_3, A_4\}$ to assess the five enterprises, and the corresponding weighting of each attribute is $w = \{0.2, 0.1, 0.3, 0.4\}$ .The initial SVN decision matrix provided by expert is illustrated as below.

$$D = \begin{bmatrix} \langle 0.6,0.2,0.2\rangle & \langle 0.7,0.5,0.1\rangle & \langle 0.6,0.7,0.2\rangle & \langle 0.5,0.6,0.4\rangle \\ \langle 0.8,0.2,0.2\rangle & \langle 0.6,0.3,0.1\rangle & \langle 0.5,0.7,0.2\rangle & \langle 0.4,0.5,0.1\rangle \\ \langle 0.7,0.3,0.4\rangle & \langle 0.7,0.5,0.2\rangle & \langle 0.6,0.7,0.1\rangle & \langle 0.8,0.3,0.2\rangle \\ \langle 0.6,0.7,0.2\rangle & \langle 0.8,0.4,0.2\rangle & \langle 0.7,0.6,0.2\rangle & \langle 0.6,0.3,0.3\rangle \\ \langle 0.5,0.5,0.2\rangle & \langle 0.7,0.4,0.1\rangle & \langle 0.5,0.7,0.2\rangle & \langle 0.8,0.5,0.1\rangle \end{bmatrix}$$

For convenience, we assume $\rho = 3$ and $x = 1$.
The procedure of algorithm is developed below.
Step1. The Hamming distance $d(U_{im}, U_{in})$ $(i = 1, 2, 3, 4, 5; m, n = 1, 2, 3, 4; m \neq n)$ is calculated employing Eq. (7).

$d(U_{11}, U_{12}) = d(U_{12}, U_{11}) = 0.1667;$
$d(U_{11}, U_{13}) = d(U_{13}, U_{11}) = 0.1667;$
$d(U_{11}, U_{14}) = d(U_{14}, U_{11}) = 0.2333;$
$d(U_{12}, U_{13}) = d(U_{13}, U_{12}) = 0.1333;$
$d(U_{12}, U_{14}) = d(U_{14}, U_{12}) = 0.2000;$
$d(U_{13}, U_{14}) = d(U_{14}, U_{13}) = 0.1333;$
$d(U_{21}, U_{22}) = d(U_{22}, U_{21}) = 0.1333;$
$d(U_{21}, U_{23}) = d(U_{23}, U_{21}) = 0.2667;$
$d(U_{21}, U_{24}) = d(U_{24}, U_{21}) = 0.2667;$
$d(U_{22}, U_{23}) = d(U_{23}, U_{22}) = 0.2000;$
$d(U_{22}, U_{24}) = d(U_{24}, U_{22}) = 0.1333;$
$d(U_{23}, U_{24}) = d(U_{24}, U_{23}) = 0.1333;$
$d(U_{31}, U_{32}) = d(U_{32}, U_{31}) = 0.1333;$
$d(U_{31}, U_{33}) = d(U_{33}, U_{31}) = 0.2667;$
$d(U_{31}, U_{34}) = d(U_{34}, U_{31}) = 0.1000;$
$d(U_{32}, U_{33}) = d(U_{33}, U_{32}) = 0.1333;$
$d(U_{32}, U_{34}) = d(U_{34}, U_{32}) = 0.1000;$
$d(U_{33}, U_{34}) = d(U_{34}, U_{33}) = 0.2333;$
$d(U_{41}, U_{42}) = d(U_{42}, U_{41}) = 0.1667;$
$d(U_{41}, U_{43}) = d(U_{43}, U_{41}) = 0.0667;$
$d(U_{41}, U_{44}) = d(U_{44}, U_{41}) = 0.1667;$
$d(U_{42}, U_{43}) = d(U_{43}, U_{42}) = 0.1000;$
$d(U_{42}, U_{44}) = d(U_{44}, U_{42}) = 0.1333;$
$d(U_{43}, U_{44}) = d(U_{44}, U_{43}) = 0.1667;$
$d(U_{51}, U_{52}) = d(U_{52}, U_{51}) = 0.1333;$
$d(U_{51}, U_{53}) = d(U_{53}, U_{51}) = 0.0667;$
$d(U_{51}, U_{54}) = d(U_{54}, U_{51}) = 0.1333;$
$d(U_{52}, U_{53}) = d(U_{53}, U_{52}) = 0.2000;$
$d(U_{52}, U_{54}) = d(U_{54}, U_{52}) = 0.0667;$
$d(U_{53}, U_{54}) = d(U_{54}, U_{53}) = 0.2000;$

Step2. The support $Sup(U_{im}, U_{in})$ $(i = 1, 2, 3, 4, 5; m, n = 1, 2, 3, 4; m \neq n)$ is calculated employing Eq. (8).

$Sup(U_{11}, U_{12}) = Sup(U_{12}, U_{11}) = 0.8333;$
$Sup(U_{11}, U_{13}) = Sup(U_{13}, U_{11}) = 0.8333;$
$Sup(U_{11}, U_{14}) = Sup(U_{14}, U_{11}) = 0.7667;$
$Sup(U_{12}, U_{13}) = Sup(U_{13}, U_{12}) = 0.8667;$
$Sup(U_{12}, U_{14}) = Sup(U_{14}, U_{12}) = 0.8000;$
$Sup(U_{13}, U_{14}) = Sup(U_{14}, U_{13}) = 0.8667;$
$Sup(U_{21}, U_{22}) = Sup(U_{22}, U_{21}) = 0.8667;$
$Sup(U_{21}, U_{23}) = Sup(U_{23}, U_{21}) = 0.7333;$
$Sup(U_{21}, U_{24}) = Sup(U_{24}, U_{21}) = 0.7333;$
$Sup(U_{22}, U_{23}) = Sup(U_{23}, U_{22}) = 0.8000;$
$Sup(U_{22}, U_{24}) = Sup(U_{24}, U_{22}) = 0.8667;$
$Sup(U_{23}, U_{24}) = Sup(U_{24}, U_{23}) = 0.8667;$
$Sup(U_{31}, U_{32}) = Sup(U_{32}, U_{31}) = 0.8667;$
$Sup(U_{31}, U_{33}) = Sup(U_{33}, U_{31}) = 0.7333;$
$Sup(U_{31}, U_{34}) = Sup(U_{34}, U_{31}) = 0.9000;$
$Sup(U_{32}, U_{33}) = Sup(U_{33}, U_{32}) = 0.8667;$
$Sup(U_{32}, U_{34}) = Sup(U_{34}, U_{32}) = 0.9000;$
$Sup(U_{33}, U_{34}) = Sup(U_{34}, U_{33}) = 0.7667;$
$Sup(U_{41}, U_{42}) = Sup(U_{42}, U_{41}) = 0.8333;$
$Sup(U_{41}, U_{43}) = Sup(U_{43}, U_{41}) = 0.9333;$
$Sup(U_{41}, U_{44}) = Sup(U_{44}, U_{41}) = 0.8333;$
$Sup(U_{42}, U_{43}) = Sup(U_{43}, U_{42}) = 0.9000;$
$Sup(U_{42}, U_{44}) = Sup(U_{44}, U_{42}) = 0.8667;$
$Sup(U_{43}, U_{44}) = Sup(U_{44}, U_{43}) = 0.8333;$

$$Sup(U_{51},U_{52}) = Sup(U_{52},U_{51}) = 0.8667;$$
$$Sup(U_{51},U_{53}) = Sup(U_{53},U_{51}) = 0.9333;$$
$$Sup(U_{51},U_{54}) = Sup(U_{54},U_{51}) = 0.8667;$$
$$Sup(U_{52},U_{53}) = Sup(U_{53},U_{52}) = 0.8000;$$
$$Sup(U_{52},U_{54}) = Sup(U_{54},U_{52}) = 0.9333;$$
$$Sup(U_{53},U_{54}) = Sup(U_{54},U_{53}) = 0.8000;$$

Step3. The weighted support $T(U_{ij})$ $(i=1,2,3,4,5;\ j=1,2,3,4)$ is calculated employing Eq. (9). Then, the matrix $[T(U_{ij})]_{5\times4}$ is gained below.

$$\left[T\left(U_{ij}\right)\right]_{5\times4} = \begin{bmatrix} 0.6400 & 0.7467 & 0.6000 & 0.4933 \\ 0.6000 & 0.7600 & 0.5733 & 0.4933 \\ 0.6667 & 0.7933 & 0.5400 & 0.5000 \\ 0.6967 & 0.7833 & 0.6100 & 0.5033 \\ 0.7133 & 0.7867 & 0.5867 & 0.5067 \end{bmatrix}$$

Step4. The EPW ϖ_{ij} $(i=1,2,3,4,5;\ \ j=1,2,3,4)$ is calculated employing Eq. (10). Then, the matrix $[\varpi_{ij}]_{5\times4}$ is gained below.

$$\left[\varpi_{ij}\right]_{5\times4} = \begin{bmatrix} 0.2034 & 0.1047 & 0.3017 & 0.3903 \\ 0.2019 & 0.1055 & 0.3007 & 0.3919 \\ 0.2051 & 0.1061 & 0.2971 & 0.3916 \\ 0.2053 & 0.1050 & 0.3007 & 0.3890 \\ 0.2063 & 0.1052 & 0.2989 & 0.3896 \end{bmatrix}$$

Step5. The comprehensive value U_i $\left(i=1,2,3,4,5\right)$ is got employing Eq. (11).

$$U_1 = \langle 0.5819, 0.4334, 0.2199 \rangle;$$
$$U_2 = \langle 0.6044, 0.3884, 0.1336 \rangle;$$
$$U_3 = \langle 0.7325, 0.3808, 0.1674 \rangle;$$
$$U_4 = \langle 0.6681, 0.4249, 0.2298 \rangle;$$
$$U_5 = \langle 0.6978, 0.5314, 0.1338 \rangle.$$

Step6. The Dice similarity measure $S_D(U_i, U^*)$ $\left(i=1,2,3,4,5\right)$ is got employing Eq. (12).

$$S_D(U_1,U^*) = 0.7391;$$
$$S_D(U_2,U^*) = 0.7880;$$
$$S_D(U_3,U^*) = 0.8569;$$
$$S_D(U_4,U^*) = 0.7955;$$
$$S_D(U_5,U^*) = 0.7809.$$

Step7. The final ranking outcome is $E_3 \succ E_4 \succ E_2 \succ E_5 \succ E_1$, the optimal enterprise is E_3.

B. Sensitive Analysis

To demonstrate the influence of parameter, the detailed ranking results with parameters ρ and x are shown in Figs. 1 and 6.

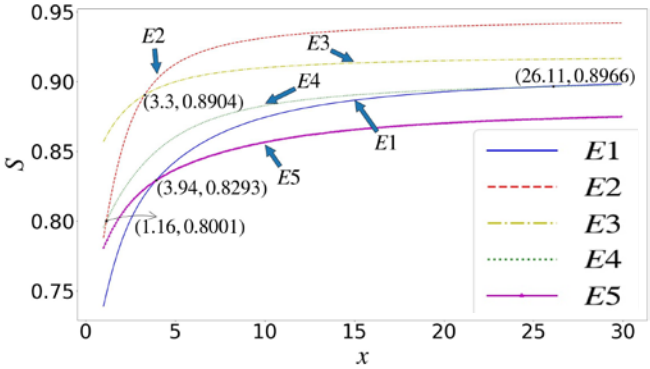

Fig. 1. Dice similarity with $\rho = 3$ and $x \geq 1$.

In Fig. 1, parameter ρ is assigned to a fixed positive value, and parameter x is greater than or equal to 1. As we can see from Fig. 1, parameter ρ is set to 3. It is clearly that the Dice value regarding alternative $E_p\,(p=1,2,3,4,5)$ becomes larger and larger as x grows. Meanwhile, when x is different value, the alternative ranking is also slightly different. When $x \in [1, 1.16]$, the ranking is $E_3 \succ E_4 \succ E_2 \succ E_5 \succ E_1$. When $x \in (1.16, 3.3]$, the ranking is $E_3 \succ E_2 \succ E_4 \succ E_5 \succ E_1$. When $x \in (3.30, 3.94]$, the ranking is $E_2 \succ E_3 \succ E_4 \succ E_5 \succ E_1$. When $x \in (3.94, 26.11]$, the ranking is $E_2 \succ E_3 \succ E_4 \succ E_1 \succ E_5$. When $x \geq 26.11$, the ranking is $E_2 \succ E_3 \succ E_1 \succ E_4 \succ E_5$. Generally, DMs can delimit parameter value using their preferences. Therefore, the variation of ranking outcome illustrates the adaptability of the proposed model with the general parameter x.

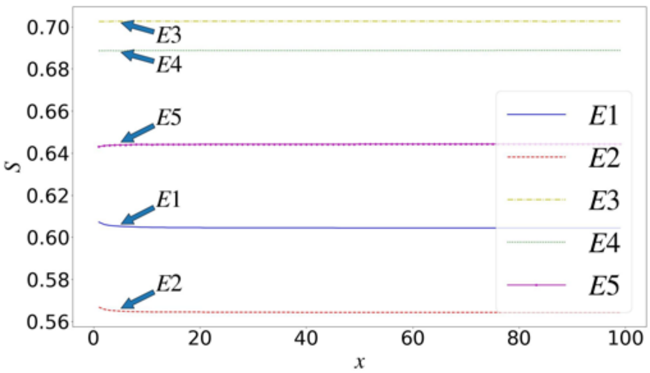

Fig. 2. Dice similarity with $\rho = -3$ and $x \geq 1$.

In Fig. 2, ρ is assigned to a fixed negative value, and parameter x is greater than or equal to 1. As we can see from Fig. 2, parameter ρ is set to -3. It is evident that the ranking orders remain unchanged with the increasing of parameter x, and the ranking outcome is always $E_3 \succ E_4 \succ E_5 \succ E_1 \succ E_2$. Therefore, the ranking indicates the robustness of the proposed model in this study.

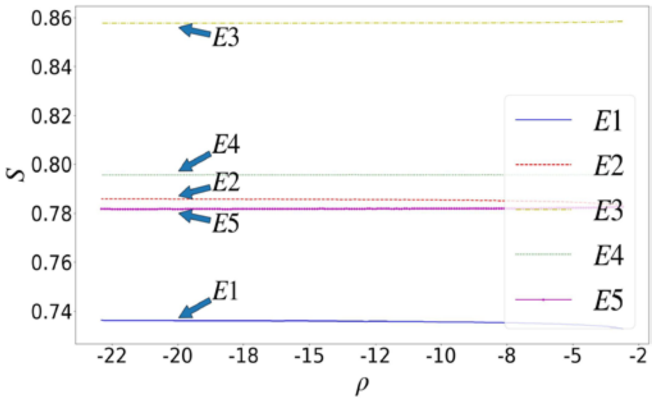

Fig. 3. Dice similarity with $\rho \in (-\infty, -3]$ and $x = 1$.

In Fig. 3, parameter x is assigned to a fixed positive value, and parameter ρ varies within a reasonable range. As we can see from Fig. 3, parameter ρ is smaller than or equal to 1. It is evident that the ranking orders remain unchanged as ρ grows. The ranking is $E_3 \succ E_4 \succ E_2 \succ E_5 \succ E_1$ regardless of parameter ρ. Similarly, the ranking verifies the robustness of proposed model in this study.

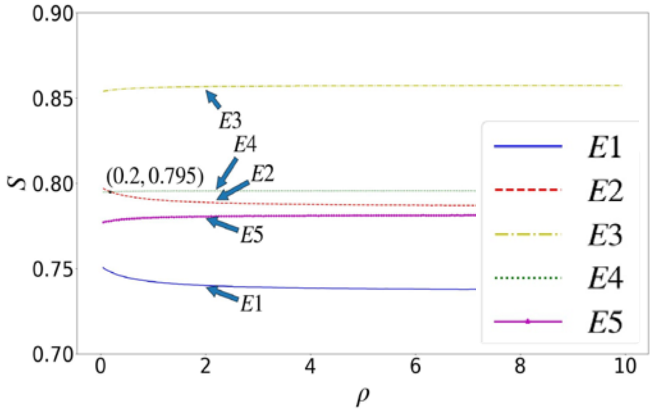

Fig. 4. Dice similarity with $\rho \in [0, +\infty)$ and $x = 1$.

In Fig. 4, parameter x is assigned to a fixed positive value, and parameter ρ varies within a reasonable range. As we can see from Fig. 4, parameter $\rho \geq 0$. When $\rho \in [0.0.2]$, the ranking is $E_3 \succ E_2 \succ E_4 \succ E_5 \succ E_1$, and when $\rho > 0.2$, the ranking outcome is $E_3 \succ E_4 \succ E_2 \succ E_5 \succ E_1$. The best choice is always E_3, whereas the worst choice is always E_1. From Fig. 4, the stability of ranking outcome illustrates the reliability of the model with parameter ρ.

In Figs. 5 and 6, parameters ρ and x vary within a reasonable range. As we can see from Figs. 5 and 6, it is obvious that the changes of parameter ρ in EPA operator and parameter x in Dombi operations may affect the ranking orders of five alternatives. The main reason is that parameters ρ and x can be assigned to different values according to DMs' preferences, so diverse Dice similarity measures are gained. The detailed results are demonstrated in Figs. 5 and 6.

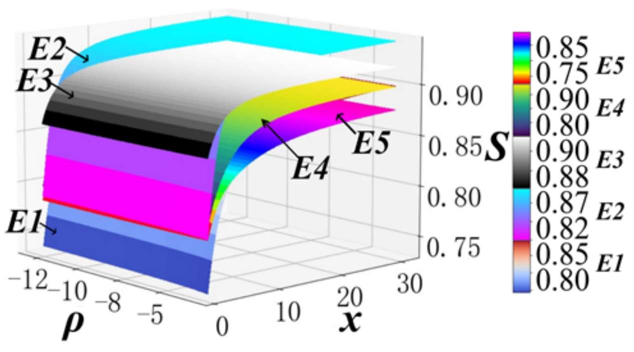

Fig.5. Dice similarity with $\rho \in (-\infty, -3]$ and $x \geq 1$.

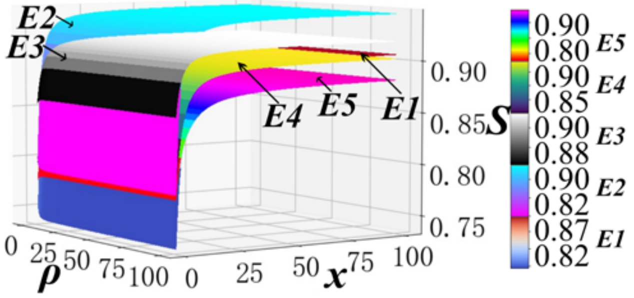

Fig.6. Dice similarity with $\rho \in [0, +\infty)$ and $x \geq 1$.

C. *Comparative Analysis*

To reveal more advantages of the model, the contrast is performed in this subsection. The comparison of outcomes of the new model with the other models are illustrated in TABLE I.

TABLE I
THE COMPARISON OF THREE METHODS

Models	Handle flexibly irrational data	Parameter numbers	Decision matrix
Xiong [22]	yes	ρ	numerical value
Jana [27]	no	x	SVN
Proposed model	yes	ρ, x	SVN

Some conclusions are obtained:

(1) The proposed model in this paper can solve not only single-valued information but also numerical values of unit interval [22] in MCDM problem.

(2) The proposed model in this study can handle flexibly irrational data by assigning different weights rather than only reduce the impact of extreme value [27].

(3) The new model integrates both the merits of EPA and Dombi. The SVNDPWA and SVNDWA operators are special instance of the developed operator.

Hence, the proposed model can be regarded as an extension of those introduced by Xiong [22] and Jana [27], it can handle not only MCDM issue with SVN information but also flexibly irrational values in complex environment. The comparative analysis confirms the developed model provided is more flexible comparing with the existing models.

VI. CONCLUSION

SVNS providing more information, as a generalization of IFS, is important since it can better express incomplete and indeterminate cognitive information, thus, a MCDM algorithm model managing SVNS is established in this paper. The developed model for SVNS is considered as a further extension of the model for IFS. The existing model for IFS is special instance of the proposed model in this study. The new single-valued neutrosophic algorithm not only solve MCDM problem with SVNS information but also the MCDM issue under IFS environment. An example of the presented model verifies its superiority and flexibility when facing extreme input values in SVN.

The central contributions are presented: (1) The proposed MCDM model under SVNS environment is more suitable for tackling sophisticated inconsistent decision information. (2) The EPA operator is initially expanded to accommodate SVN environment, which can dynamically assign weights to input values and cope with irrational data. (3) The SVNDEPWA aggregating operator is firstly proposed, which can provide more flexible and general parameter given by DMs according to realistic environment. (4) Dice similarity of SVNNs is delimited, and special instance for SVNDEPWA operator is studied. (5) To verify the powerfulness of the new model, an MCDM case is conducted, the sensitivity and contrast are performed as well. The analyses manifest the adaptability and robustness for the proposed model.

In future, the applications of the proposed SVNDEPWA operator and the above-mentioned MCDM model need to be explored to diverse fields with uncertain information, for instance, clustering, garbage treatment, supply demand matching.

REFERENCES

[1] H. Wang, F. Smarandache, Y. Zhang and R. Sunderraman, "Single Valued Neutrosophic Sets," *Proceedings of the 8th Joint Conference on Information Sciences in USA*, vols.1-3, pp. 94-97, 2005.

[2] L. A. Zadeh, "Fuzzy Sets," *Information and Control*, vol. 8, no. 4, pp. 338-353, 1965.

[3] K. T. Atanassov, "Intuitionistic Fuzzy-Sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87-96, 1986.

[4] Q. Y. Li, Y. C. Ma, F. Smarandache and S. G. Zhu, "Single-Valued Neutrosophic Clustering Algorithm Based on Tsallis Entropy Maximization," *Axioms*, vol. 7 no. 3, pp. 57, 2018.

[5] K. Mohana and M. Mohanasundari, "On Some Similarity Measures of Single Valued Neutrosophic Rough Sets," *Neutrosophic Sets and Systems*, vol. 24, pp. 10-22, 2019.

[6] L. H. Yang and B. L. Li, "An Extended Single-valued Neutrosophic Normalized Weighted Bonferroni Mean Einstein Aggregation Operator," *IAENG International Journal of Applied Mathematics*, vol. 48, no. 4, pp. 373-380, 2018.

[7] L. H. Yang and B. L. Li, "Picture Hesitant Fuzzy Normalized Weighted Bonferroni Mean Operator based on Einstein Operations and Its Application," *Engineering Letters*, vol. 29, no. 1, pp. 35-42, 2021.

[8] A. Keikha, "Archimedean t-Norm and t-Conorm-Based Aggregation Operators of HFNs, with the Approach of Improving Education," *International Journal of Fuzzy Systems*, vol. 24, no. 1, pp. 310-321, 2021.

[9] T. Mahmood, U. U. Rehman, J Ahmmad and Santos-García, "Bipolar Complex Fuzzy Hamacher Aggregation Operators and Their Applications in Multi-Attribute Decision Making," *Mathematics*, vol. 10, no. 1, pp. 23, 2021.

[10] H. Garg, "Novel Single-Valued Neutrosophic Aggregated Operators Under Frank Norm Operation and Its Application to Decision-Making Process," *International Journal for Uncertainty Quantification*, vol. 6, no. 4, pp. 361-375, 2016.

[11] P. D. Liu, Q. Khan and T. Mahmood, "Multiple-Attribute Decision Making Based on Single-Valued Neutrosophic Schweizer-Sklar Prioritized Aggregation Operator," *Cognitive Systems Research*, vol. 57, pp. 175-196, 2019.

[12] L. H. Yang and B. L. Li, "Multiple-Valued Neutrosophic Uncertain Linguistic Sets With Dombi Normalized Weighted Bonferroni Mean Operator and Their Applications in Multiple Attribute Decision Making Problem," *IEEE Access*, vol. 8, pp. 5906-5927, 2020.

[13] J. Dombi, "A General-Class of Fuzzy Operators, the Demorgan Class of Fuzzy Operators and Fuzziness Measures Induced by Fuzzy Operators," *Fuzzy Sets and Systems*, vol. 8, no. 2, pp. 149-163, 1982.

[14] Q. Khan, P. Liu, T. Mahmood, F. Smarandache and K. Ullah, "Some Interval Neutrosophic Dombi Power Bonferroni Mean Operators and Their Application in Multi–Attribute Decision–Making," *Symmetry*, vol. 10, no. 10, pp. 459, 2018.

[15] C. Jana, T. Senapati, M. Pal and R. R. Yager, "Picture Fuzzy Dombi Aggregation Operators: Application to MADM Process," *Applied Soft Computing, vol*. 74, pp. 99-109, 2019.

[16] C. Jana, M. Pal and J. Q. Wang, "Bipolar Fuzzy Dombi Prioritized Aggregation Operators in Multiple Attribute Decision Making," Soft Computing, vol. 24, no. 5, pp. 3631-3646, 2020.

[17] C. Jana, G. Muhiuddin and M. Pal, "Multi-Criteria Decision Making Approach Based on Svtrn Dombi Aggregation Functions," *Artificial Intelligence Review*, vol. 54, no. 5, pp. 3685-3723, 2020.

[18] T. Mahmood and U. U. Rehman, "A Method to Multi-Attribute Decision Making Technique Based on Dombi Aggregation Operators Under Bipolar Complex Fuzzy Information," *Computational & Applied Mathematics*, vol. 41, no. 1, pp. 23, 2022.

[19] R. R. Yager, "The Power Average Operator," *Ieee Transactions on Systems Man and Cybernetics Part a-Systems and Humans*, vol. 31, no. 6, pp. 724-731, 2001.

[20] Z. S. Xu and R. R. Yager, "Power-Geometric Operators and Their Use in Group Decision Making,". *Ieee Transactions on Fuzzy Systems*, vol. 18, no. 1, pp. 94-105, 2010.

[21] L. G. Zhou, H. Y. Chen and J. P. Liu, "Generalized Power Aggregation Operators and Their Applications in Group Decision Making," *Computers & Industrial Engineering*, vol. 62, no. 4, pp. 989-999, 2012.

[22] S. H. Xiong, Z. S. Chen, J. P. Chang and K. S. Chin, "On Extended Power Average Operators for Decision-Making: A Case Study in Emergency Response Plan Selection of Civil Aviation," *Computers & Industrial Engineering*, vol. 130, pp. 258-271, 2019.

[23] L. Li, C. Ji and J. Wang, "A Novel Multi-attribute Group Decision-Making Method Based on q-Rung Dual Hesitant Fuzzy Information and Extended Power Average Operators," *Cognitive Computation*, vol. 13, no. 5, pp. 1345-1362, 2019.

[24] B. Ning, G. Wei, R. Lin and Y. Guo, "A Novel MADM Technique Based on Extended Power Generalized Maclaurin Symmetric Mean Operators Under Probabilistic Dual Hesitant Fuzzy Setting and Its Application To Sustainable Suppliers Selection," *Expert Systems with Applications*, vol. 204, pp. 1345-1362, 2022.

[25] Y. Kou, J. Wang, W. Xu and Y. Xu, "Multi-Attribute Group Decision-Making Based on Linguistic Pythagorean Fuzzy Copula Extended Power Average Operator," *Expert Systems*, vol. 40, no. 7, pp. , 2023.

[26] P. D. Liu, Q. Khan, T. Mahmood ,F. Smarandache and Y. Li, "Multiple Attribute Group Decision Making Based on 2-Tuple Linguistic Neutrosophic Dombi Power Heronian Mean Operators," *Ieee Access*, vol. 7, pp. 100205-100230, 2019.

[27] C. Jana and M. Pal, "Multi-Criteria Decision Making Process Based on Some Single-Valued Neutrosophic Dombi Power Aggregation Operators," *Soft Computing*, vol. 25, no. 7, pp. 5055-5072, 2021.

[28] H. Y. Zhang, G. W. Wei and X. D. Chen, "Spherical Fuzzy Dombi Power Heronian Mean Aggregation Operators for Multiple Attribute Group Decision-Making," *Computational & Applied Mathematics*, vol. 41, no. 3, pp. 54, 2022.

[29] T. Mahmood, U. U. Rehman and G. Santos-Garcia, "The Prioritization of Solutions for Reducing the Influence of Climate Change on the Environment by Using the Conception of Bipolar Complex Fuzzy Power Dombi Aggregation Operators," *Frontiers in Environmental Science*, vol. 11, no. 24, 2023.

[30] L. R. Dice, "Measures Of The Amount Of Ecologic Association Between Species," *Ecology*, vol. 26, no. 3, pp. 297-302, 1945.

[31] J. Ye, "Vector Similarity Measures of Simplified Neutrosophic Sets and Their Application in Multicriteria Decision Making," *International Journal of Fuzzy Systems*, vol. 16, no. 2, pp. 204-211, 2014.

[32] J. Ye, "The Generalized Dice Measures for Multiple Attribute Decision Making under Simplified Neutrosophic Environments," *Journal of intelligent & fuzzy systems: Applications in Engineering and Technology*, vol. 31, no. 1, pp. 663-671, 2016.