An Inclined MHD and Diffusive Thermo Effects on Radiative Viscoelastic Periodic Flow through Porous Medium Channel

Syed Amanullah Hussaini, Rafiuddin Mohammad, Gade Mallikarjun Reddy and Syed Mustafa

***Abstract*— In this paper, the impact of slip condition, radiation, constant heat and mass flux subjected to chemical reaction on an unsteady viscoelastic periodic flow is investigated. Further, the effect of an inclined magnetic field, heat source and diffusive thermodynamics of incompressible and electrically conductive fluid are also analyzed. The governing non-linear partial differential equations are solved by perturbation method. The final expressions for flow variables and their corresponding gradients at lower plate y = 0 are furnished. It is observed that the diffusive thermo effect Df improves the velocity and reduces skin friction for large values, an angle of inclination ψ and heat source parameter Q enhance velocity while the viscoelastic parameter decreases the same. Df and Q have opposite effects on temperature and subsides the heat transfer for enlarged values, and it is also noted that the effects of Re, Sc, and Kr increase the mass transfer as seen in the plotted graphs. Additionally, the effect of non-dimensional parameters Gr, Re, Pe, N and H on flow variables and their gradients at y = 0 are portrayed by graph's and subsequently discussed in detail.**

***Index Terms*— An angle of inclination, Diffusive thermo effect, Viscoelastic fluid, Heat source and Planer channel.**

NOMENCLATURES

T- Temperature
$\mathbf{T_0}$-Temparature at lower plate
$\mathbf{T_w}$ -Temperature at the wall
$\boldsymbol{\alpha}$- the coefficient of mean radiation absorption
C- Concentration
υ- Kinematic viscosity
ρ- Density
P- Pressure
K- Permeability
$\boldsymbol{\sigma_e}$- Electrical conductivity
J- Current density
$\mathbf{B_0}$- Magnetic field strength
$\boldsymbol{\kappa}$- Thermal conducticvity
$\mathbf{C_p}$ − Specific heat at constant pressure
$\mathbf{C_s}$ − Concentration susceptibility
$\boldsymbol{q_r}$- Radiative flux
D- Mass diffusion coefficient
U- Average velocity
a- Width of the channel
Gr- Greshoff number
Gc- Solutal Greshoff number
Re-Reynold's number
Pe-Peclet number
Kr- Chemical reaction rate parameter
N- Radiation parameter
Q- Heat source
Sc- Schmidt number
B- Coefficient of volumetric thermal expansion
$\boldsymbol{\beta_1}$- Kinematic visco-elasticity
$\boldsymbol{\beta_1^*}$- Coefficient of volumetric concentration expansion
Sf- Shape factor
U - Non-dimensional velocity
g- Acceleration due to gravity
$\boldsymbol{\theta}$ − Non-dimensional temperature
∅- Non-dimensional concentration
h- Slip coefficent
ψ − Angle of inclination
$\boldsymbol{\lambda}$- Constant
$\boldsymbol{\omega}$-Frequency

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I. Introduction

Numerous academics have focused on the slip-flow regime problem due to its extensive applications in various fields. The impact of these issues on science, technology and industrialization is significant. In actual applications, it is often observed that particles close to solid surfaces do not have surface velocities, but rather finite tangential velocities that give rise to slip effects. These effects cannot be overlooked. Dorrepaal [1] was the first to take the velocity slip impact into account. Choi [2] subsequently named the fluid that had nanoparticles suspended within it as nano fluid and described its exceptional convective heat transport abilities as well as the ability to increase thermal conductivity. When studying through nano or micro-channels, this slippage of fluid properties at solid surfaces occurs. To overcome friction, a thin coating of lubricant is applied on the surfaces of the mechanical devices while the surfaces slip over one another. Instead of a lubricant, a thin film of fragile oil can also be used. The researchers mentioned below studied periodic flow through planar channels and slip effects. In their discussion of the periodic flow of Newtonian fluid in the presence of a transverse magnetic field, Kumar *et al.* [3] discovered that the flow is reduced for high values of the magnetic parameter in a channel. Mehmood and Ali [4] came to the conclusion that Peclet number and Lorentz and buoyancy forces affect fluid slide in a channel. Das [5] discovered that as the slip parameter is increased, skin friction decreases. According to Prakash *et al.* [6], the velocity of dusty fluid and dust particles increases as the radiation parameter and Grashoff number increase.

By choosing various geometries, a typical study was carried out to investigate the impact of heat transport and chemical reaction in a steady flow by Reddy *et al.* [7], who examined the radiative effects on magnetic Newtonian fluid. It was concluded that the slip parameter and chemical reaction parameters increase velocity and concentration. Ibrahim *et al.* [8] investigated the various effects of heat and mass transfer on radiative periodic Newtonian flow via a planar channel. Hussaini *et al.* [9] studied the effects of radiation and mass transfer on MHD oscillatory Rivlin-Ericksen flow in a porous medium channel. The MHD radiative heat and mass transfer effects in vertical channel were presented by Usman *et al.* [10]. The magnetic memory flow of heat and mass transport with slip effects in horizontal channel was noted by Dev and Chaudhury [11]. By using the Laplace technique, Zulkifee *et al.* [12] obtained the correct solution of the flow variables and discovered that the Prandtl number increased the heat transport and decreased the drag. By using the perturbation technique, Reddy *et al.* [13] demonstrated how the diffusion parameter reduces the amount of dust particles and dusty fluid flow. In their study of the effects of radiation on heat and mass transmission under Newtonian heating, Zulkifee *et al.* [14] concluded that radiation parameters affect heat transfer and skin friction. Zigta [15] demonstrated that Grashoff number has a favorable impact on velocity and concentration decreases with an increase in chemical reaction number. The precise solution was found by Falade *et al.* [16], who also noted that shearing stress on the two plate's increases as injection pressure increases on the heated plate. By using a perturbation technique, Joshna *et al.* [17] deduced an analytical solution that demonstrated how the magnetic number lowers drag and the permeability number increases velocity. The present study finds applications in various fields of science. Few important applications are highlighted here.

Space exploration

In the study of space weather and the behavior of plasma in planets' magnetospheres, inclined magnetic fields are used. The aurora borealis and the aurora australis are results of the Earth's magnetic field's tilt with regard to its rotational axis.

Science of materials

Inclined magnetic fields can be used to control crystal development and to examine the characteristics of materials under strong magnetic fields. Magnetic anisotropy and magnetic domain structure are two features of magnetic materials that can be affected by magnetic field orientation.

Magnetic levitation

To lift things like trains and maglev vehicles, inclined magnetic fields are used. The magnetic field's angle can be changed to alter the levitation force and direction.

Medical imaging

MRI devices provide images of the body using inclined magnetic fields. To create the three-dimensional image of the internal structures, the magnetic field is supplied to the body at an angle.

Plasma physics

In order to comprehend how charged particles behave in a magnetic field, plasma physics makes use of inclined magnetic fields. The transport characteristics of the plasma and the emergence of instabilities can be influenced by the magnetic field's angle.

Couette [18] was the first to explain flow across parallel plates with one fixed and the other moving at a constant speed. According to Guria *et al.* [19], boundary layer thickness increases with significant values of the inclination angle with the rotational axis. According to Chauhan and Agarwal [20], the MHD flow pattern in the channel of the partially filled porous medium was dominated by the Coriolis force.

In their investigation, Seth *et al.* [21] found that the angle of inclination had an increasing impact on primary and secondary velocities and decreased the induced magnetic field. An inclined angle increases the induced magnetic field and also the flow, as suggested by Hemamalini and Shanti [22]. The Lorentz force and an inclined angle have a decreasing influence on flow, according to Sharma and Dubewar [23]. In the work of Prakash *et al.* [24], it was reported that the diffusion thermo effect increases flow while decreasing heat transmission. As stated by Dhanalakhmi *et al.* [25], the diffusion thermal effect causes the temperature to rise while the radiation parameter affects mass transfer.

As reported by Murthy *et al.* [26], mean velocity decreases with the strength of the magnetic field and also has a delaying effect on mean skin friction and heat transfer. The research of Mustafa *et al.* [27], concluded that the heat generating parameter affects the temperature profile and magnetic Reynold's number also reduces drag, and heat transmission increases as Prandtl number grows. A study by Hasanuzzaman *et al.* [28] found that the diffusion thermo-effect increases heat transmission while decreasing mass transport. Bordoloi *et al.* [29] noted that the response parameter suppresses the concentration field while the diffusion thermal effect influences the flow. According to Quader and Alam [30], cross flow is evident for prime velocity and temperature at various Dufour parameter values. Diffusion thermo-effect increases temperature profiles whereas magnetic number decreases velocity profiles, as reported by Jyothi and Selvaraj [31]. Diffusion thermo-effect has a beneficial impact on flow, heat transmission, and viscous drag, as determined by Bordoloi and Ahmed [32]. Opanuga *et al.* [33] showed the hall-effects of viscous fluid in a planar channel and examined the effects of Hall current and ion slip on time-dependent magnetic micro channels.

Agbaji *et al.* [34] studied the time independent three-dimensional free convective Newtonian boundary layer flow and large parameter spectral perturbation method is used (LSPM) to find the solution of highly non-linear coupled partial differential equations. Buzuzi *et al.* [35] presented the numerical solution of Newtonian flow over an angled stretched surface with merged magnetic field and concluded that optimal velocity is attained if angled magnetic field stretched surface required to the relatively small when the effective Prandtl number dominates radiation parameter.

Rivlin-Ericksen's [36] constitutive equations for memory fluid are

$$t = -pI + \varphi_1 A + \varphi_2 B' + \varphi_3 A^2 \qquad (i)$$

Where $t$is stress tensor, $I = \left\|\delta_{ij}\right\|$,

$$A = d_{ij} \; and \; d_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right),$$

$v_{i,j}$ is velocity gradient and $a_{i,j}$ is acceleration gradient.

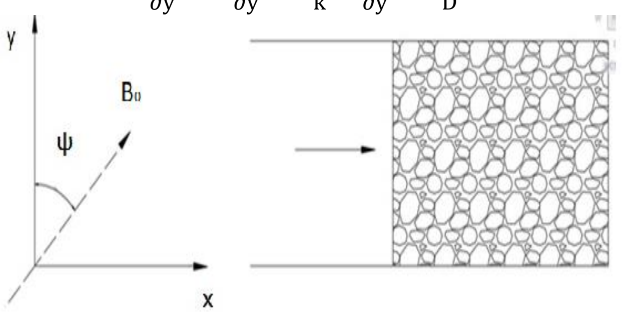
$$B' = \left\|b_{ij}\right\| \quad and \quad b_{ij} = a_{i,j} + a_{j,i} + 2v_{m,i}v_{m,j} \qquad (ii)$$

$\varphi_1, \varphi_2 \; and \; \varphi_3$ are depends on material constants. This equation is valid for the bodies obeying hook's law and many fluids like aqueous solutions of poly-C_2H_4 are well covered by the above equations.

II. PROBLEM FORMULATION

Consider the time-dependent natural convection of a two-dimensional viscoelastic fluid through a saturated porous channel subject to an angled magnetic field with the fluid being incompressible, having negligible conductivity, and optically thin. In order to test the outputs of heat and mass transfer, a magnetic field must be applied at an angle to the outward normal, or in the direction of the positive Y-axis, when first order reaction between concentration and homogeneous fluid is occurring. In order to test the outputs of heat and mass transfer as illustrated in Fig. 1, the extremely modest emf should be ignored.

$$u = h\frac{\partial u}{\partial y}, \quad \frac{\partial T}{\partial y} = -\frac{q}{k}, \quad \frac{\partial C}{\partial y} = -\frac{J}{D} \text{ on } y = a$$



$$u = 0, \quad T = T_0, \quad C = C_0 \quad \text{on } y = 0$$

Fig. 1. Geometry of the problem

To analyze the issue, a cartesian coordinate system is chosen with the real axis along the channel's midline and the imaginary axis along the X-axis normal. All flow quantities depend on y and t, and in this analysis, v is equal to zero.

Under Boussinesq's approximation and the Fig.1 assumptions, the governing equations are

Momentum equation

$$\frac{\partial u}{\partial t} = g\beta(T - T_0) + g\beta_1^*(C - C_0) + \vartheta\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\vartheta u}{K} + \beta_1\left(\frac{\partial^3 u}{\partial t\,\partial y^2}\right) - \left(\frac{\sigma_e B_0^2 sin^2\psi}{\rho}\right)u \qquad (1)$$

Equation for energy

$$\frac{\partial T}{\partial t} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \frac{D\kappa}{C_s C_p}\frac{\partial^2 C}{\partial y^2} + \frac{Q_0(T - T_0)}{\rho C_p} \qquad (2)$$

Equation for Species

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} - K_r(C - C_0) \qquad (3)$$

The boundary conditions are

$$u = h\frac{\partial u}{\partial y}, \quad \frac{\partial T}{\partial y} = -\frac{q}{k}, \quad \frac{\partial C}{\partial y} = -\frac{J}{D} \text{ on } y = a$$

$$u = 0, \quad T = T_0, \quad C = C_0 \quad \text{on } y = 0 \qquad (4)$$

The difference between the temperatures T_0 and T_w is enough to promote the radiative heat transfer, following Cogley *et al.* [37].

$$\frac{\partial q_r}{\partial y} = 4\alpha^2(T_0 - T) \qquad (5)$$

The following transformations convert the above equations into dimensionless form:

$$h' = \frac{h}{a}, \quad x' = \frac{x}{a}, \quad y' = \frac{y}{a}, \quad t' = \frac{tU}{a}, \quad P' = \frac{aP}{\rho\vartheta U},$$

$$u' = \frac{u}{U}, \quad \vartheta = \frac{\mu}{\rho}, \quad Gr = \frac{g a^2 \beta(T_w - T_0)}{\vartheta U}, \quad Sc = \frac{\vartheta}{D},$$

$$Gc = \frac{g\beta_1 a(C_w - C_0)}{\vartheta U}, Df = \frac{D\kappa}{C_s C_p}\frac{(C_w - C_0)}{(T_w - T_0)}, Q = \frac{Q_0 a^2}{\kappa}$$

$$N^2 = \frac{4\alpha^2 a^2}{\kappa}, \quad Kr' = \frac{aKr}{U}, \quad \beta_1 = \frac{\beta_1 v_0^2}{\vartheta^2},$$

$$Df = \frac{D\kappa}{C_s C_p}\frac{(C_w - C_0)}{(T_w - T_0)}, \quad Pe = \frac{Ua\rho C_p}{\kappa}, \quad Re = \frac{Ua}{\vartheta},$$

$$Da = \frac{K}{a^2}, \quad H^2 = \frac{a^2\sigma_e B_0^2}{\rho\vartheta}, \quad \theta = \frac{(T - T_0)\kappa}{aq},$$

$$\emptyset = \frac{(C-C_0)D}{\mathrm{aJ}}, \theta = \frac{(T-T_0)}{(\mathrm{T_w}-\mathrm{T_0})}, \emptyset = \frac{(C-C_0)}{(\mathrm{C_w}-\mathrm{C_0})} \quad (6)$$

Equations (1, 2, 3 and 6) together yield dimensionless equations

$$Re\frac{\partial u}{\partial t} = -\frac{\partial \mathrm{P}}{\partial \mathrm{x}} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gc\emptyset - \beta_1\left[\frac{\partial^3 u}{\partial t \partial y^2}\right] - \{(Sf)^2 + M^2\}u \quad (7)$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + (N^2+Q)\theta + Df\frac{\partial^2 \emptyset}{\partial y^2} \quad (8)$$

$$\frac{\partial \emptyset}{\partial t} = \left(\frac{1}{ScRe}\right)\frac{\partial^2 \emptyset}{\partial y^2} - (Kr)^2\emptyset \quad (9)$$

Where $M = Hsin\,\psi$

The boundary conditions in dimensionless form are

$$\mathrm{u} = \mathrm{h}\frac{\partial \mathrm{u}}{\partial \mathrm{y}}, \quad \frac{\partial \theta}{\partial \mathrm{y}} = -1, \quad \frac{\partial \emptyset}{\partial \mathrm{y}} = -1 \text{ on } \mathrm{y} = 1$$

$$u = 0, \quad \theta = 0, \quad \emptyset = 0 \quad on\ \ y = 0 \quad (10)$$

III. MATHEMATICAL SOLUTION

Consider the purely oscillatory flow

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}, \quad u(y,t) = u_0(y)e^{i\omega t},$$

$$\theta(y,t) = \theta_0(y)e^{i\omega t}, \quad \phi(y,t) = \phi_0(y)e^{i\omega t} \quad (11)$$

The equations (7, 8, and 9) together under (11), become

$$u_0'' - m_3^2 u_0 = (-\lambda - Gr\theta_0 - Gc\phi_0)\,/(1 - i\beta_1 w) \quad (12)$$

$$\theta_0'' + m_1^2\theta_0 = D_1 sinh(\mathrm{m_2 y}) \quad (13)$$

$$\phi_0'' - m_2^2\phi_0 = 0 \quad (14)$$

Where $D_1 = \frac{Dfm_2}{\cosh m_2}$

Non dimensional boundary conditions are

$$u_0 = \mathrm{h}\frac{\partial u_0}{\partial \mathrm{y}}, \quad \frac{\partial \theta_0}{\partial \mathrm{y}} = -1, \frac{\partial \phi_0}{\partial \mathrm{y}} = -1 \ \ on\ y = 1$$

$$u_0 = 0,\ \theta_0 = 0,\ \phi_0 = 0\ \ on\ \ y = 0 \quad (15)$$

$$m_1 = \sqrt{N^2 + Q - i\omega Pe}, m_2 = \sqrt{Kr^2 ScRe + i\omega ScRe},$$

$$m_3 = \sqrt{Sf^2 + M^2 + i\omega Re)/(1 - i\beta_1\omega)}$$

Solutions of differential equations of (12, 13, and 14) under boundary conditions (15) related to flow variables are

$$\theta(y,t) = [-\frac{(1 + D_2 m_2 \cosh m_2)\sin(m_1 y)}{\mathrm{m_1 cosm_1}} + D_2 \sinh(m_2 y)]\, e^{i\omega t} \quad (16)$$

$$where\ D_2 = \frac{D_1}{(m_1^2 + m_2^2)},$$

$$\phi(y,t) = -\frac{\sinh(m_2 y)}{\mathrm{m_2 coshm_2}} e^{i\omega t} \quad (17)$$

$$u(y,t) = u_0(y,t)e^{i\omega t} \quad (18)$$

$$\begin{aligned}
u_0(y,t) &= \frac{e^{m_3 y}}{2(sinhm_3 - hm_3 coshm_3)}\Bigg[\left\{\frac{\lambda}{m_3^2(1 - i\beta_1\omega)}\right\}\{e^{-m_3}(hm_3 + 1) - 1\} \\
&+ \frac{Gr}{(1 - i\beta_1\omega)}\Bigg[\left\{\frac{(1 + D_2 m_2 \cosh m_2)}{\mathrm{m_1}(m_1^2 + m_3^2)}\right\} tan\mathrm{m_1} \\
&- h\left\{\frac{(1 + D_2 m_2 \cosh m_2)}{(m_1^2 + m_3^2)}\right\} + \frac{D_2}{(m_2^2 - m_3^2)}\sinh \mathrm{m_2} \\
&- \frac{D_2}{(m_2^2 - m_3^2)}\mathrm{hm_2}\cosh \mathrm{m_2}\Bigg] \\
&- \frac{Gc}{(1 - i\beta_1\omega)}\left[\left\{\frac{1}{\mathrm{m_2}(m_2^2 - m_3^2)}\right\} tan\mathrm{hm_2} - \frac{\mathrm{h}}{(m_2^2 - m_3^2)}\right]\Bigg] \\
&- \frac{e^{-m_3 y}}{2(sinhm_3 - hm_3 coshm_3)}\Bigg[\left\{\frac{\lambda}{m_3^2(1 - i\beta_1\omega)}\right\}\{e^{-m_3}(hm_3 + 1) - 1\} \\
&+ \frac{Gr}{(1 - i\beta_1\omega)}\Bigg[\left\{\frac{(1 + D_2 m_2 \cosh m_2)}{\mathrm{m_1}(m_1^2 + m_3^2)}\right\} tan\mathrm{m_1} \\
&- h\left\{\frac{(1 + D_2 m_2 \cosh m_2)}{(m_1^2 + m_3^2)}\right\} + \frac{D_2}{(m_2^2 - m_3^2)}\sinh \mathrm{m_2} \\
&- \frac{D_2}{(m_2^2 - m_3^2)}\mathrm{hm_2}\cosh \mathrm{m_2}\Bigg] \\
&- \frac{Gc}{(1 - i\beta_1\omega)}\left[\left\{\frac{1}{\mathrm{m_2}(m_2^2 - m_3^2)}\right\} tan\mathrm{hm_2} - \frac{\mathrm{h}}{(m_2^2 - m_3^2)}\right] \\
&+ \frac{\lambda}{m_3^2(1 - i\beta_1\omega)}\Bigg] + \frac{\lambda}{m_3^2(1 - i\beta_1\omega)} \\
&- \frac{Gr}{(1 - i\beta_1\omega)}\Bigg[\left\{\frac{(1 + D_2 m_2 \cosh m_2)}{\mathrm{m_1 cosm_1}(m_1^2 + m_3^2)}\right\} sin(\mathrm{m_1 y}) \\
&+ \frac{D_2}{(m_2^2 - m_3^2)} sinh(\mathrm{m_2 y})\Bigg] \\
&+ \frac{Gc}{(1 - i\beta_1\omega)}\left\{\frac{1}{\mathrm{m_2}\cosh \mathrm{m_2}\,(m_2^2 - m_3^2)} sinh(\mathrm{m_2 y})\right\}
\end{aligned}$$

Sherwood number, shearing stress, and Nusselt's number are crucial physical characteristics for flows. Known values of the velocity field are used to determine the dimensionless skin-friction values for the channel plates.

$$\tau = -\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= \left[\frac{m_3 e^{m_3 y}}{2(sinhm_3 - hm_3 coshm_3)}\left[\left\{\frac{\lambda}{m_3^2(1-i\beta_1\omega)}\right\}\{e^{-m_3}(hm_3 + 1) - 1\}\right.\right.$$
$$+ \frac{Gr}{(1-i\beta_1\omega)}\left[\left\{\frac{(1+D_2 m_2 \cosh m_2)}{m_1(m_1^2+m_3^2)}\right\} tanm_1 - h\left\{\frac{(1+D_2 m_2 \cosh m_2)}{(m_1^2+m_3^2)}\right\} + \frac{D_2}{(m_2^2-m_3^2)}\sinh m_2 - \frac{D_2}{(m_2^2-m_3^2)} hm_2 \cosh m_2\right]$$
$$\left.- \frac{Gc}{(1-i\beta_1\omega)}\left[\left\{\frac{1}{m_2(m_2^2-m_3^2)}\right\} tanhm_2 - \frac{h}{(m_2^2-m_3^2)}\right]\right]$$
$$+ \frac{m_3 e^{-m_3 y}}{2(sinhm_3 - hm_3 coshm_3)}\left[\left\{\frac{\lambda}{m_3^2(1-i\beta_1\omega)}\right\}\{e^{-m_3}(hm_3 + 1) - 1\}\right.$$
$$+ \frac{Gr}{(1-i\beta_1\omega)}\left[\left\{\frac{(1+D_2 m_2 \cosh m_2)}{m_1(m_1^2+m_3^2)}\right\} tanm_1 - h\left\{\frac{(1+D_2 m_2 \cosh m_2)}{(m_1^2+m_3^2)}\right\} + \frac{D_2}{(m_2^2-m_3^2)}\sinh m_2 - \frac{D_2}{(m_2^2-m_3^2)} hm_2 \cosh m_2\right]$$
$$\left.- \frac{Gc}{(1-i\beta_1\omega)}\left[\left\{\frac{1}{m_2(m_2^2-m_3^2)}\right\} tanhm_2 - \frac{h}{(m_2^2-m_3^2)}\right] + \frac{\lambda}{m_3^2(1-i\beta_1\omega)}\right]$$
$$- \frac{Gr}{(1-i\beta_1\omega)}\left[\left\{\frac{(1+D_2 m_2 \cosh m_2)}{\cos m_1(m_1^2+m_3^2)}\right\} cos(m_1 y) + \frac{D_2 m_2}{(m_2^2-m_3^2)}\cosh(m_2 y)\right]$$
$$\left.+ \frac{Gc}{(1-i\beta_1\omega)}\left\{\frac{1}{\cosh m_2\,(m_2^2-m_3^2)}\cosh(m_2 y)\right\}\right] e^{i\omega t} \quad (19)$$

Nusselt's number in terms of temperature gradients is

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \frac{1}{\cos m_1}\{1 - D_2 m_2(\cos m_1 - \cosh m_2\} e^{i\omega t} \quad (20)$$

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=1} = e^{i\omega t} \quad (21)$$

Sherwood's number is

$$Sh = -\left(\frac{\partial\phi}{\partial y}\right)_{y=0} = \frac{e^{i\omega t}}{\cosh m_2} \quad (22)$$

$$Sh = -\left(\frac{\partial\phi}{\partial y}\right)_{y=1} = e^{i\omega t} \quad (23)$$

IV. OUTCOMES AND DISCUSSIONS

The problem's final expressions are arrived at numerically and the results are represented graphically. To investigate the effects of various parameters on velocity, temperature, and concentration, a comprehensive analysis is presented. The fictitious portion of the final result is disregarded in favour of confirming the analytical findings. By altering the non-dimensional numbers Gr, Pe, H, N, Re, Sf, Sc, Kr and Df in the governing equations, it is possible to calculate the trend of the flow variables and their gradients. Gr = 1, Pe = 0.71, H = 1, N = 1, Q = 0, Re = 3, Sc = 2, Sf = 1, Kr = 1.5, Df = 0.3, t = 0.02, ω = 1, λ = 1, h = 0.1, Gc = 1, β_1 = - 0.01 and $\psi = \frac{\pi}{3}$ are the invariant values of the parameters utilized. Unless otherwise stated, all graphs are portrayed with these specific values.

The velocity field is largest at the upper plate of the channel, while it is least at the lower plate, as seen in the plots of the current article.

The lower plate with y = 0 has the highest temperature field θ. Following that, gradually, it decreases until it reaches its lowest point at the upper plate of the channel. The results for the provided parameter values indicate that the temperature gradually decreases from its peak on the lower plate of the channel, where y = 0, to its lowest point on the upper plate, where y = 1.

At the corresponding sites shown in Figs. 2–6, the effect of the parameters Df, ψ, Kr, Sc, Gr, Sf, N, Q and Re is to upgrade the velocity and flow recedes for large values of H and β_1. A departure from past findings is noted when it is discovered that the maximum and minimum values in the depicted graphs are attained on the channel plates rather than in the channel's middle.

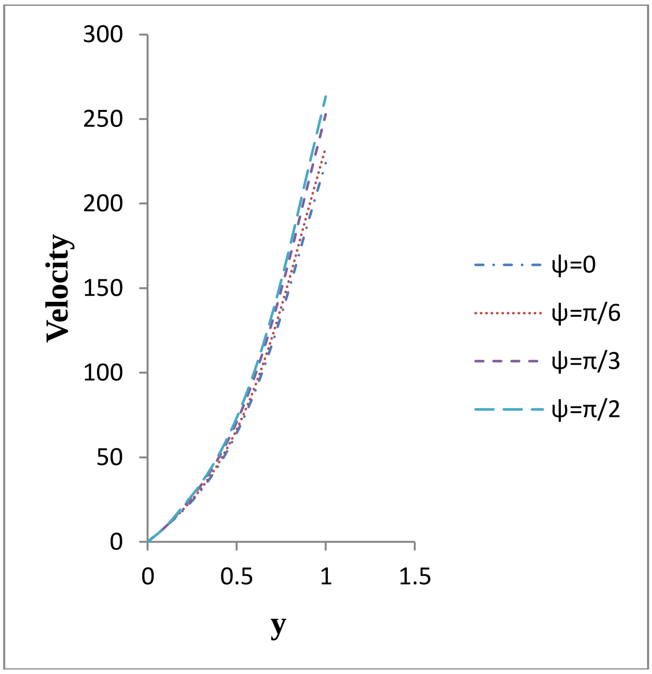


Fig. 2. Velocity profiles for different angle of inclinations ψ with fixed values of all other parameters.

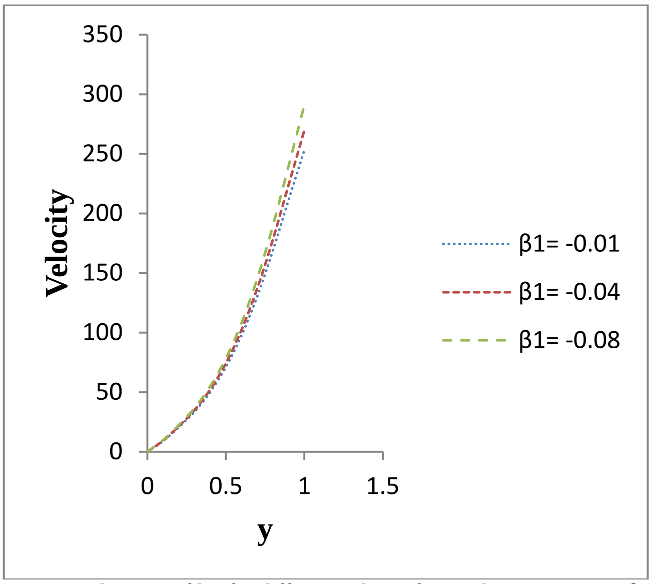


Fig. 3. Velocity profiles for different values of viscoelastic parameter β_1 with fixed values of all other parameters.

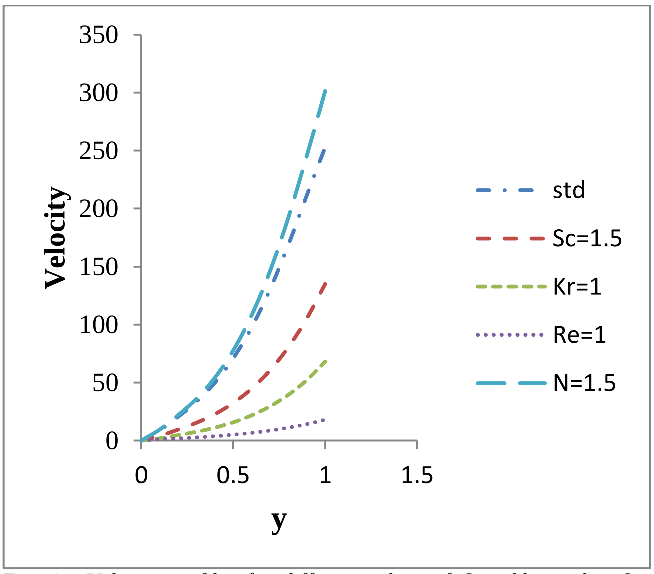


Fig. 4. Velocity profiles for different values of Scmidth number Sc, Chemical reaction parameter Kr, Renold's number Re and Radiation parameter N with fixed values of all other parameters.

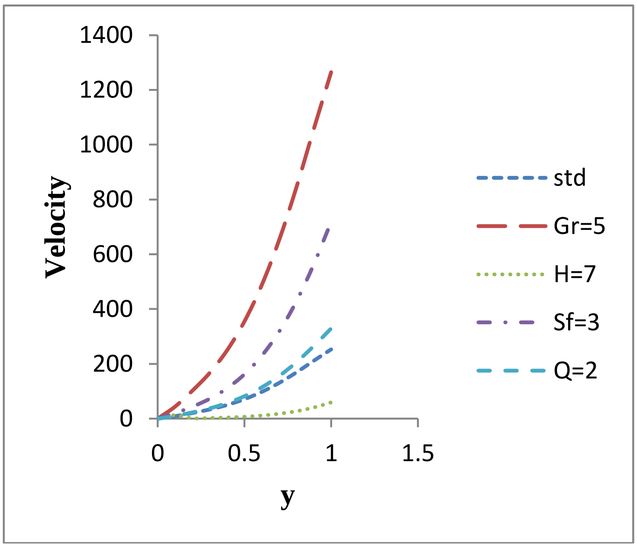


Fig. 5. Velocity profiles for different values of Grashoff number Gr, Shape factor Sf, Heat source parameter Q and Magnetic parameter H with fixed values of all other parameters.

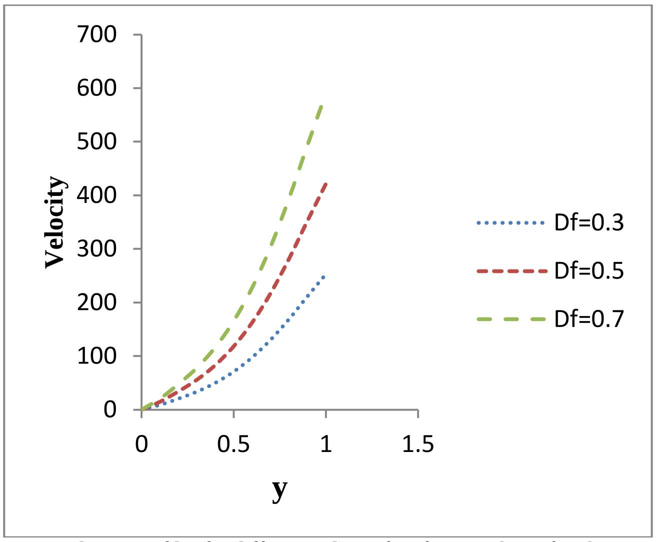


Fig. 6. Velocity profiles for different values of Dufour number Df with fixed values of all other parameters.

The diffusive thermal effect, a greater inclination angle, buoyancy forces, and improved permeability all contribute to the velocity rise. Viscoelasticity and the Lorentz force oppose the velocity flow. The temperature field is lowered as a result of the parameters Df, Sc, Kr, Re N as well as Q have an enhancing impact. The Figs. 7- 9 can be referred for details on how viscous forces and diffusion caused the decline.

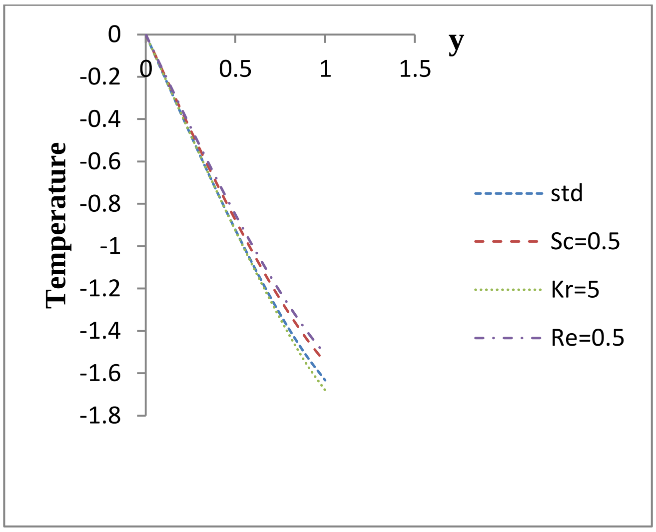


Fig. 7. Temperature profiles for different values of Scmidth number Sc, Chemical reaction parameter Kr and Reynold's number Re with fixed values of all other parameters.

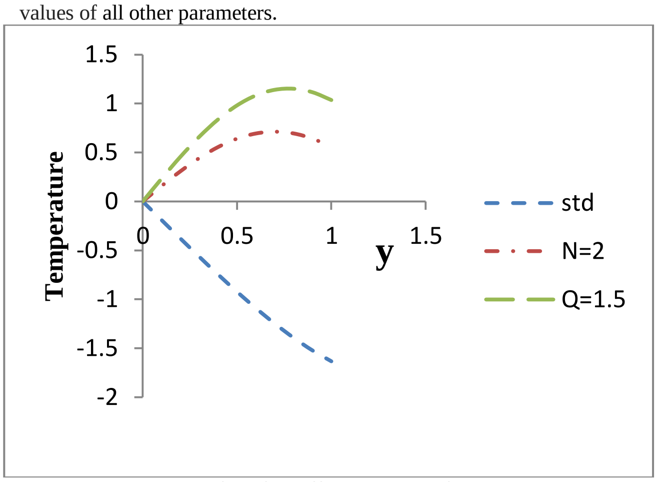


Fig. 8. Temperature profiles for different values of Radiation parameter N and Heat source parameter Q with fixed values of all other parameters.

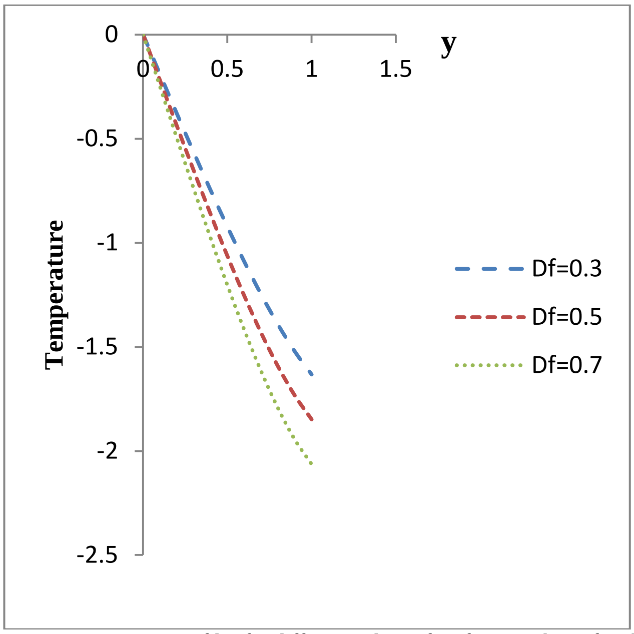


Fig. 9. Temperature profiles for different values of Dufour number Df with fixed values of other parameters.

Fluid is optically thin and having low-density, the dimensionless concentration φ is always non-positive, reaches its maximum at the bottom plate of the channel, and its minimum at the upper plate. The dimensionless concentration may be above real-axis, positive, or below it, negative in the flow field. It is also discovered that the concentration is consistently negative for varied values of the different non dimensional numbers. The top plate of the channel has a lower concentration than that of the lower plate. The concentration field gradually decreases for the tested values as they move from their high value on the bottom plate to their low value on the upper plate of the channel.

The concentration is negative for low-density and grey liquids. In the viscoelastic example, as seen from Fig. 10, the concentration rises at similar places for large values of Sc, Kr and Re.

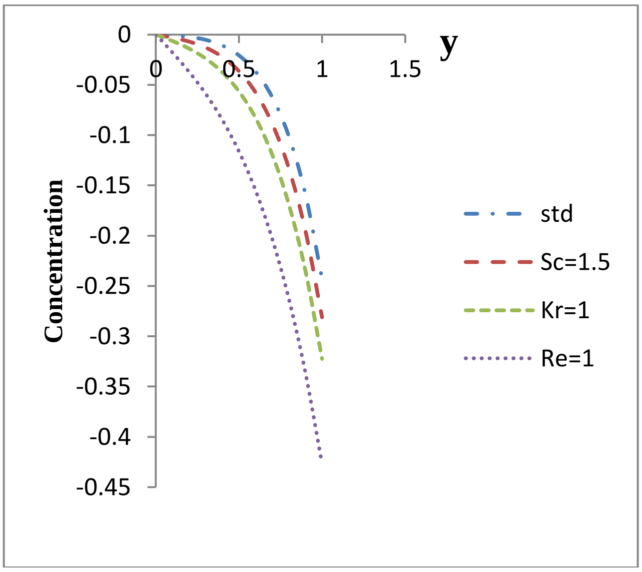


Fig. 10. Concentration profiles for different values of Scmidth number Sc, Chemical reaction parameter Kr and Reynold's number Re with fixed values of all other parameters.

With higher values of Sc, Kr, and Re, Sherhood's number predominates. As time passes, mass transfer decreases and transport increases are seen in Fig. 11.

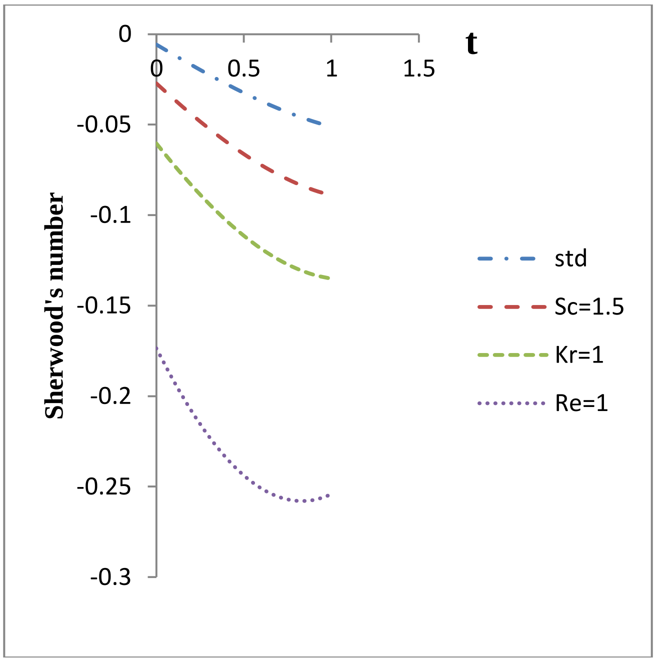


Fig. 11. Sherhood's number for different values of Scmidth number Sc, Chemical reaction parameter Kr and Reynold's number Re with fixed values of all other parameters.

Kr and Sc lessen the quantity of Nusselt's which can be seen from the graphs, their dominance results in less heat transfer. With high Peclet number Pe and heat source parameter Q, heat transmission is predominant. Heat transfer decreases over time for Df, Pe, Q and Kr, whereas it reverses for Sc (see Figs. 12–14). At first, a rapid reduction is observed for Sc and Kr.

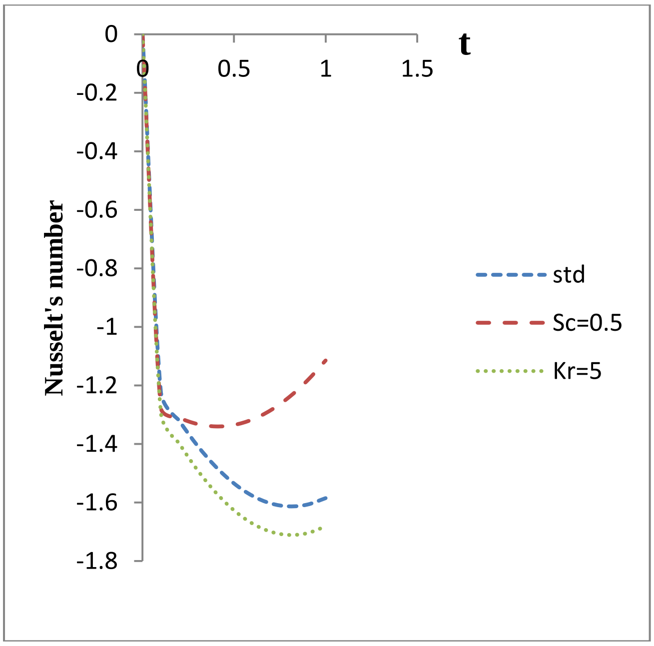


Fig. 12. Nusselt's number for different values of Scmidth number Sc and Chemical Reaction parameter Kr with fixed values of all other parameters.

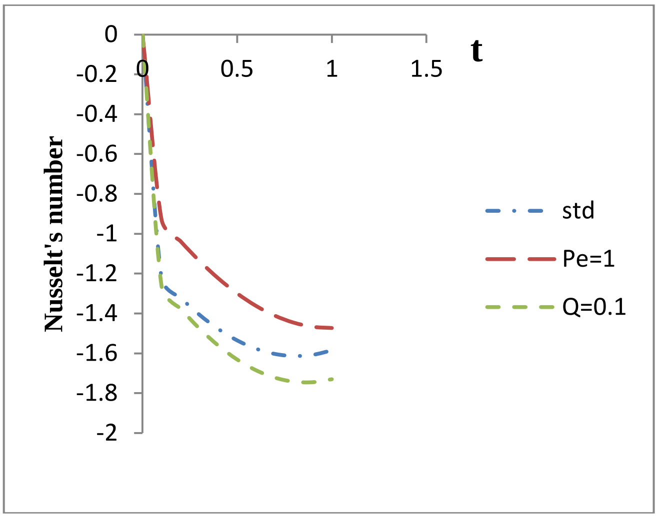


Fig. 13. Nussult's number for different values of Peclet number Pe and Heat source parameter Q with fixed values of all other parameters.

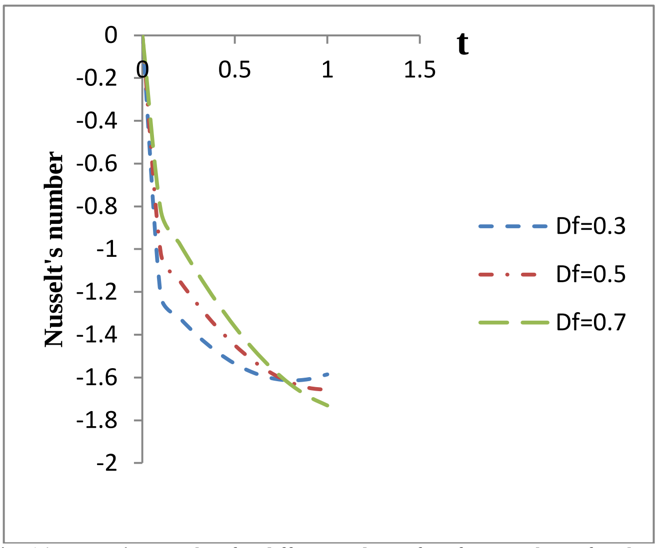


Fig. 14. Nussult's number for different values of Dufour number Df with fixed values of other parameters.

A stronger Lorentz force and a viscoelastic parameter reduce skin friction, while other parameters like Df, Gr, Sf, Re, Q and N cause the shear stress to rise quickly at first before gradually falling off as seen in Fig. 15-18.

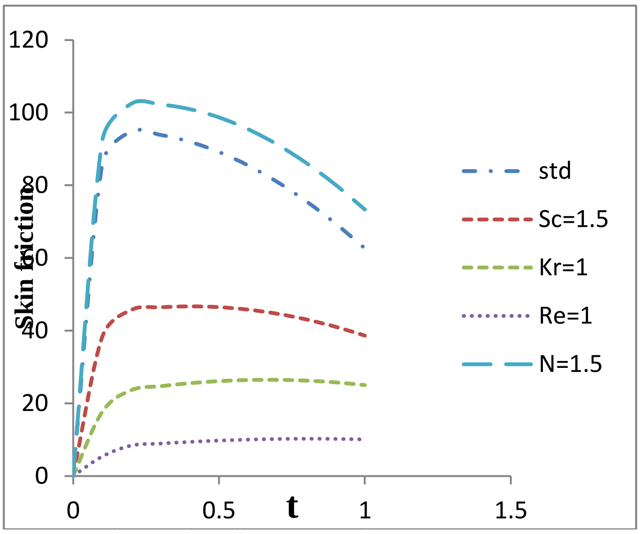


Fig. 15. Skin friction for different values of Scmidth number Sc, Chemical parameter Kr, Reynold's number Re and Radiation parameter N with fixed values of all other parameters.

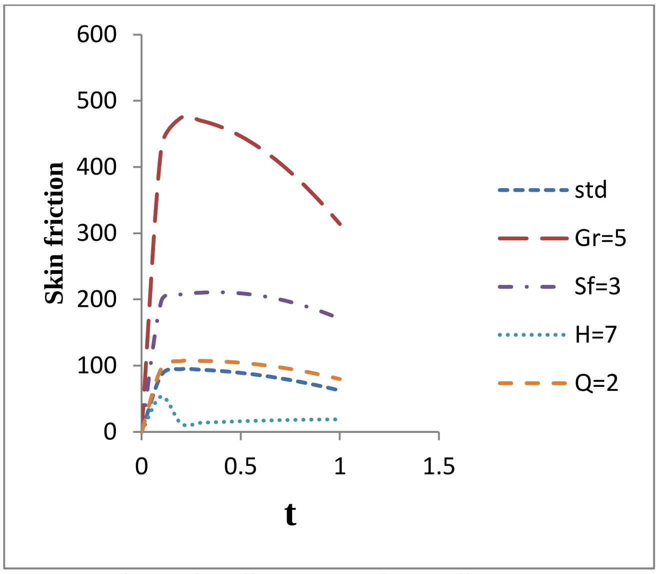


Fig. 16. Skin friction for different values of Grashoff number Gr, Shape factor Sf, Magnetic parameter H and Heat source parameter Q with fixed values of all other parameters.

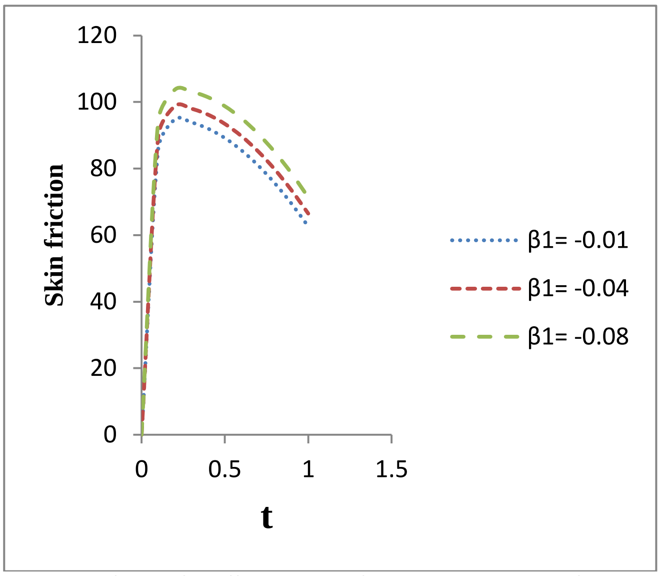


Fig. 17. Skin friction for different values of visco-elastic parameter β_1 with fixed values of all other parameters.

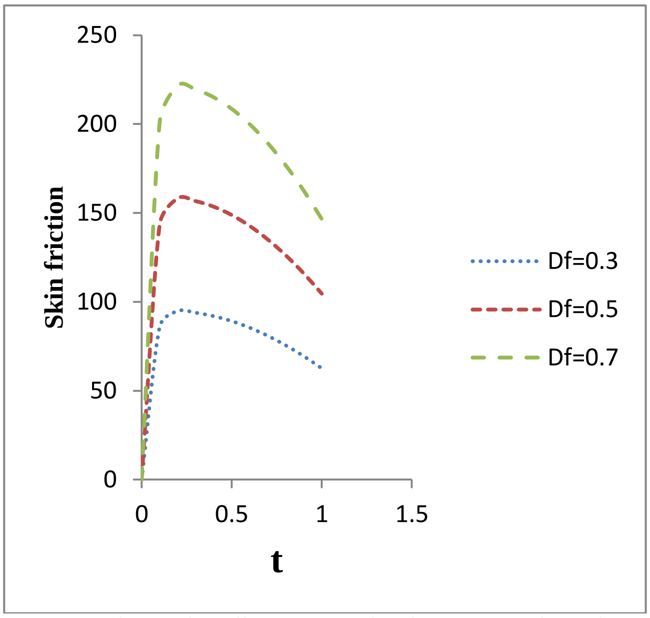


Fig. 18. Skin friction for different values of Dufour number Df with fixed values of all other parameters.

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