Probabilistic Operators and Hybrid Binary Logarithm Similarity Measure for MCGDM Problems Under T2SVNS Environment

Juanjuan Geng, Wanhong Ye

Abstract—**The purpose of this paper is to propose a new approach to deal with multi-criteria group decision-making (MCGDM) problems under type-2 single valued neutrosophic set (T2SVNS) environment. Firstly, we give the concepts of single valued neutrosophic set (SVNS) and T2SVNS, and then we define probabilistic operators with T2SVN information. Secondly, two type of binary logarithm similarity measures (BLSM) and weighted binary logarithm similarity measures (WBLSM) for T2SVNSs are defined. Hybrid binary logarithm similarity measure (HBLSM) and weighted hybrid binary logarithm similarity measure (WHBLSM) for T2SVNSs are also defined. In addition, a MCGDM model based on TOPSIS method is proposed under T2SVNS environment. Finally, a numerical example is given to illustrate the effectiveness and feasibility of the proposed method.**

Index Terms—**T2SVNS, probabilistic operators, binary logarithm function, MCGDM, TOPSIS method.**

I. Introduction

SMARANDACHE[1] put forward neutrosophic sets (NSs), which attracted the attention of many scholars and laid a foundation for further dealing with uncertainty and inconsistency. Wang et al. [2] developed the concept of single valued neutrosophic sets (SVNSs), which is a subclass of the NSs for solving scientific and engineering problems. SVNSs have been widely used in different fields, such as engineering problems [3], [4], medical problems [5], [6], [7], image processing problems [8], [9], [10], decision-making problems [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], social problems [24], [25], conflict problems [26].

In order to express mathematical uncertainty, Zedeh [27] proposed a fuzzy set theory and its extension, such as interval-valued IFS [28], intuitionistic fuzzy set(IFS) [29], cubic intuitionistic fuzzy set [30] and linguistic interval-valued IFS [31]. The operations between two type-2 fuzzy sets were studied in [32], [33], [34]. Mendel et al. [35] proposed a new representation of type-2 fuzzy set and defined some of its operations. Therefore, many scholars have also studied type-2 fuzzy sets, such as Yang et al. [36] introduced the similarity between type-2 fuzzy sets and discussed their properties, Hung et al. [37] proposed similarity methods between two type-2 fuzzy sets, and obtained the properties of these methods. Sing [38] introduced two type-2 fuzzy sets based on the distances between Euclidean and Hamming. Zhao et al. [39] were the first to study the type-2 intuitionistic fuzzy set(T2IFS), they gave the concept of T2IFS, and discussed the relation of T2IFS. Cuong et al. [40] introduced some operations between two T2IFSs. Sukhveer et al. [41] redefined T2IFSs as the following four functions: primary membership function (PMF), secondary membership function (SMF), primary non-membership function (PNMF), secondary non-membership function(SNMF). Many researchers have studied multi-criteria decision-making (MCDM), such as similarity measurement [42], Correlation coefficient [43], [44], Grey correlation analysis [45], TOPSIS method [46], VIKOR method [47], aggregation operator [48]. Recently, Abdel Basset et al. [49] defined the concept of type-2 neutrosophic number (T2NN). At this time, they also introduced many concepts, such as score function and precision function of T2NN, aggregation operator of T2NN. In addition, they also proposed TOPSIS method based on information of T2NN. Distance measure is an important tool for measure, especially in similarity measures and decision making. Mondal et al. [50], [51] proposed sine hyperbolic similarity measure and tangent similarity measure methods to deal with MADM problems. Lu et al. [52] proposed logarithmic similarity measure and applied it in fault diagnosis strategy under interval valued fuzzy set environment [53]. Based on the above analysis, the main objectives of this paper are as follows:To define some new probabilistic aggregation operators under SVNS and T2SVNS environment. To define binary logarithm similarity measures for T2SVNSs. To develop a MCGDM model based on proposed operators and similarity measures. To present a numerical example to illustrate the effectiveness and feasibility of the proposed method.

The structure of the rest of this paper is as follows. In section 2, the concepts of SVNSs and T2SVNSs are given. In section 3, we define BLSM, WBLSM, HBLSM and WHBLSM similarity measures between two T2SVNSs. In section 4, we propose a MCGDM method under T2SVN environment by TOPSIS method with the proposed operators and distance measure methods. In section 5, an example is given to illustrate the effectiveness and feasibility of the proposed model. In section 6, we come to the conclusion.

II. Preliminaries

A. The Single Valued Neutrosophic Sets

Definition 1[54] Let X be a universal space of points (objects), with a generic element in X denoted by x, single valued neutrosophic set (SVNS) Q $\subset X$ is characterized by truth-membership function $t_q(x)$, indeterminacy-membership

Manuscript received January 9, 2024; revised May 23, 2024.

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function $i_q(x)$ and falsity-membership function $f_q(x)$. A SVNS can be expressed as

$$Q = \{[\langle x, t_q(x), i_q(x), f_q(x)\rangle] \,|x \in X\}. \tag{1}$$

where $t_q(x)$, $i_q(x)$, $f_q(x)$ are real standard or nonstandard subsets of $[0,1]$, so that it means $t_q(x)$: $X\to [0,1]$, $i_q(x)$: $X\to [0,1]$, $f_q(x)$: $X\to [0,1]$, with the condition of $0 \le \sup t_q(x) + \sup i_q(x) + \sup f_q(x) \le 3$, for all $x \in X$.

When X is continuous, a SVNS Q can be written as

$$Q = \int_X \langle t_q(x), i_q(x), f_q(x)\rangle/x, \quad x \in X; \tag{2}$$

When X is discrete, a SVNS Q can be written as

$$Q = \sum_{i=1}^{n} \langle t_q(x_i), i_q(x_i), f_q(x_i)\rangle/x_i, \quad x_i \in X. \tag{3}$$

Definition 2[55]Let P and Q be two SVNSs, P $= \langle t_p(x), i_p(x), f_p(x)\rangle$, Q $= \langle t_q(x), i_q(x), f_q(x)\rangle$, then$\forall$ $x \in X$, operations can be defined as follows:

1) A SVNS Q is contained in the other SVNS P, denoted as Q $\subset$ P, iff, $t_p(x) \ge t_q(x), i_p(x) \le i_q(x), f_p(x) \le f_q(x)$, for all $x \in X$.

2) Two SVNSs Q and P are equal, denoted as Q = P, iff, $Q \subseteq P$ and $Q \subseteq P$, for all $x \in X$.

3) The complement of a SVNS P is denoted by P ^{c} and is defined by $t_{p^c}(x) = f_p(x)$, $i_{p^c}(x) = 1 - i_p(x)$, $f_{p^c}(x) = t_p(x)$, for all $x \in X$.

4) P$\bigcup$Q=$\langle max(t_p(x), t_q(x)), min(i_p(x), i_q(x)), min(f_p(x), f_q(x))\rangle$, for all $x \in X$.

5) P$\bigcap$Q=$\langle min(t_p(x), t_q(x)), max(i_p(x), i_q(x)), max(f_p(x), f_q(x))\rangle$, for all $x \in X$.

Definition 3[56], [57], [58] Let Q and P be two SVNSs, Q $= \langle t_q(x), i_q(x), f_q(x)\rangle$, P $= \langle t_p(x), i_p(x), f_p(x)\rangle$, then$\forall$ $x \in X$, $\forall$ $\lambda \in R$ and $\lambda > 0$, there is

1) $Q\oplus$P $= \langle t_q(x)+t_p(x)-t_q(x)\cdot t_p(x), i_q(x)\cdot i_p(x), f_q(x)\cdot f_p(x)\rangle$;

2) $Q\otimes$P $= \langle t_q(x)\cdot t_p(x), i_q(x)+i_p(x)-i_q(x)\cdot i_p(x), f_q(x) + f_p(x) - f_q(x)\cdot f_p(x)\rangle$;

3) λQ $= (1-(1-t_q(x))^\lambda, i_q(x)^\lambda, f_q(x)^\lambda)$;

4) $Q^\lambda = (t_q(x)^\lambda, 1-(1-i_q(x))^\lambda, 1-(1-f_q(x))^\lambda)$.

Definition 4[59] Let $N_j = (t_j, i_j, f_j)$ be "n" SVNNs, $\omega_j > 0$ with $\sum\limits_{j=1}^{n}\omega_j$=1 and $p_j > 0$ with $\sum\limits_{j=1}^{n} p_j$=1 be the subjective and objectively weights of N_j. A map P-SVNWA called the probabilistic single valued neurotrophic weighted average operator is defined as

$$\begin{aligned} P\!-\!SVNWA(N_1, N_2, \ldots, N_n) &= \bigoplus_{j=1}^{n} \nu_j N_j \\ &= (1-\prod_{j=1}^{n}(1-t_j)^{\nu_j}, \prod_{j=1}^{n} i_j^{\nu_j}, \prod_{j=1}^{n} f_j^{\nu_j}) \end{aligned} \tag{4}$$

where $\nu_j = (1-\beta)p_j + \beta\omega_j$ be the weight vector with $\sum\limits_{j=1}^{n}\nu_j$=1 and $\beta \in [0,1]$.

Definition 5[59] Let $N_j = (t_j, i_j, f_j)$ be "n"SVNNs, $\omega_j > 0$ with $\sum\limits_{j=1}^{n}\omega_j$=1 and $p_j > 0$ with $\sum\limits_{j=1}^{n} p_j$=1 be the subjective and objectively weights of N_j. A map P-SVNWG called probabilistic single valued neutrosophic weighted geometric operator is defined as

$$\begin{aligned} P\!-\!SVNWG(N_1, N_2, \ldots, N_n) &= \bigotimes_{j=1}^{n} N_j^{\nu_j} \\ &= (\prod_{j=1}^{n} t_j^{\nu_j}, 1-\prod_{j=1}^{n}(1-i_j)^{\nu_j}, 1-\prod_{j=1}^{n}(1-f_j)^{\nu_j}) \end{aligned} \tag{5}$$

where $\nu_j = (1-\beta)p_j + \beta\omega_j$ be the weight vector with $\sum\limits_{j=1}^{n}\nu_j$=1 and $\beta \in [0,1]$.

B. The Type-2 Single Valued Neutrosophic Sets

Definition 6[60] A T2SVNS $\tilde{N}$ is a set of pairs $\{\mu_N(a), \eta_N(a), \nu_N(a)\}$, $a \in A$, $\mu_N(a)$, $\eta_N(a)$ and $\nu_N(a)$ are respectively the degrees of the truth-membership, indeterminacy-membership and falsity membership, defined as

$$\begin{aligned} \mu_N(a) &= \int_{u_N \in j_a^T} t_a(u_N)/u_N; \\ \eta_N(a) &= \int_{n_N \in j_a^I} i_a(n_N)/n_N; \\ \nu_N(a) &= \int_{v_N \in j_a^F} f_a(v_N)/v_N \end{aligned} \tag{6}$$

where u_N,n_N and v_N are named primary truth membership function (PTMF), primary indeterminacy membership function (PIMF) and primary falsity membership function (PFMF). $t_a(u_N)$, $i_a(n_N)$ and $f_a(v_N)$ are called secondary truth-membership function (STMF), secondary indeterminacy membership function (SIMF) and secondary falsity membership function (SFMF). j_a^T, j_a^I and j_a^F are called as primary truth membership, primary indeterminant membership and primary falsity membership, respectively.

T2SVNS $\tilde{N}$ in universe of discourse A can be expressed as follows:

$$\begin{aligned} \tilde{N} =& \{\langle(a, u_N, n_N, v_N), (t_a(u_N), i_a(n_N), f_a(v_N))\rangle \\ & |a \in A, u_N \in j_a^T, n_N \in j_a^I, v_N \in j_a^F\}. \end{aligned} \tag{7}$$

For the convenience of calculation, $\tilde{N}$ can be abbreviated as $\tilde{N} = \langle(u_N, t_a(u_N), n_N, i_a(n_N), v_N, f_a(v_N))\rangle$, which is called type-2 single valued neutrosophic number (T2SVNN).

Example Let A=$(a_1, a_2, a_3, a_4, a_5)$, we give a T2SVNS $\tilde{N}$, which is shown as:

$$\begin{aligned} \tilde{N} =& \{\langle(a_1, 0.7, 0.4, 0.3), (0.4, 0.22, 0.55)\rangle, \\ & \langle(a_2, 0.2, 0.2, 0.3), (0.2, 0.1, 0.1)\rangle, \\ & \langle(a_3, 0.6, 0.5, 0.1), (0.2, 0.1, 0.1)\rangle, \\ & \langle(a_4, 0.4, 0.1, 0.7), (0.7, 0.5, 0.1)\rangle, \\ & \langle(a_5, 0.5, 0.0, 0.4), (0.5, 0.6, 0.6)\rangle\} \end{aligned}$$

Another form of $\tilde{N}$ is as follows:

$$\begin{aligned} \tilde{N} =& \langle(a_1, 0.7, 0.4, 0.4, 0.22, 0.3, 0.55)\rangle, \\ & \langle(a_2, 0.2, 0.2, 0.2, 0.1, 0.3, 0.1)\rangle, \\ & \langle(a_3, 0.6, 0.2, 0.5, 0.1, 0.1, 0.1)\rangle, \\ & \langle(a_4, 0.4, 0.7, 0.1, 0.5, 0.7, 0.1)\rangle, \\ & \langle(a_5, 0.5, 0.5, 0.0, 0.6, 0.4, 0.6)\rangle \end{aligned}$$

Definition 7[60] Let $\tilde{N}$ be a T2SVNS, x, y and z are variance margin functions of a T2SVNS $\tilde{N}$, which are defined as

$$x_N=|u_N(a_k)-t_{a_k}(u_N)|, y_N=|n_N(a_k)-i_{a_k}(n_N)|,$$
$$z_N=|v_N(a_k)-f_{a_k}(v_N)|, \forall k.$$

Definition 8 Let Γ be the collection of all T2SVNNs $\tilde{N}_j=\langle(u_N^j,t_a^j(u_N),n_N^j,i_a^j(n_N),v_N^j,f_a^j(v_N))\rangle$, $\omega_j>0$ with $\sum_{j=1}^{n}\omega_j$=1 and $p_j>0$ with $\sum_{j=1}^{n}p_j$=1 be the weight and probabilistic weight of $\tilde{N}_j$. A map P-T2SVNWA called probabilistic Type-2 single valued neutrosophic weighted average operator is defined as

$$\begin{aligned}&P{-}T2SVNWA(\tilde{N}_1,\tilde{N}_2,\ldots,\tilde{N}_n)=\bigoplus_{j=1}^{n}\delta_j\tilde{N}_j\\&=(1-\prod_{j=1}^{n}(1-u_N^j)^{\delta_j},1-\prod_{j=1}^{n}(1-t_a^j(u_N))^{\delta_j},\\&\prod_{j=1}^{n}(n_N^j)^{\delta_j},\prod_{j=1}^{n}(i_a^j(n_N))^{\delta_j},\prod_{j=1}^{n}(v_N^j)^{\delta_j},\prod_{j=1}^{n}(f_a^j(v_N))^{\delta_j})\end{aligned}\tag{8}$$

where $\delta_j=(1-\beta)p_j+\beta\omega_j$ be the weight vector with $\sum_{j=1}^{n}\delta_j$=1 and $\beta\in[0,1]$.

Definition 9 A probability Type-2 single valued neutrosophic weighted geometric (P-T2SVNWG) operator is a map P-T2SVNWG: $\Gamma^n\rightarrow\Gamma$ defined as

$$\begin{aligned}&P{-}T2SVNWG(\tilde{N}_1,\tilde{N}_2,\ldots,\tilde{N}_n)=\bigotimes_{j=1}^{n}(\tilde{N}_j)^{\delta_j}\\&=(\prod_{j=1}^{n}(u_N^j)^{\delta_j},\prod_{j=1}^{n}(t_a^j(u_N))^{\delta_j},1-\prod_{j=1}^{n}(1-n_N^j)^{\delta_j},\\&1-\prod_{j=1}^{n}(1-i_a^j(n_N))^{\delta_j},1-\prod_{j=1}^{n}(1-v_N^j)^{\delta_j},\\&1-\prod_{j=1}^{n}(1-f_a^j(v_N))^{\delta_j})\end{aligned}\tag{9}$$

where $\delta_j=(1-\beta)p_j+\beta\omega_j$ be the weight vector with $\sum_{j=1}^{n}\delta_j$=1 and $\beta\in[0,1]$.

III. Similarity Measures Between T2SVNSs

In this section, we define two type of binary logarithm similarity measures (BLSM) and weighted binary logarithm similarity measures (WBLSM), hybrid binary logarithm similarity measure (HBLSM) and weighted hybrid binary logarithm similarity measure (WHBLSM) between two T2SVNSs.

Definition 10 Let $SV_2(A)$ be the collection of all T2SVNS over the universe A, a similarity measure of T2SVNSs is a real valued function $s:SV_2(A)\times SV_2(A)\rightarrow[0,1]$, which satisfies the following four properties:

1) $0\leq s(\tilde{N}_1,\tilde{N}_2)\leq 1,\forall\tilde{N}_1,\tilde{N}_2\in SV_2(A)$;
2) $s(\tilde{N}_1,\tilde{N}_2)=1$, if and only if $\tilde{N}_1=\tilde{N}_2$;
3) $s(\tilde{N}_1,\tilde{N}_2)=s(\tilde{N}_2,\tilde{N}_1)$;
4) If $s(\tilde{N}_1,\tilde{N}_2)=1$, $s(\tilde{N}_1,\tilde{N}_3)=1$, $\tilde{N}_3\in SV_2(A)$, then $s(\tilde{N}_2,\tilde{N}_3)=1$.

Distance measures is another important measure in the T2SVNS theory. Next, based on similarity measures of the T2SVNSs, we give the concept of distance mersure of T2SVNSs:

Let d be a mapping $d:SV_2(A)\times SV_2(A)\rightarrow[0,1]$. Then the distance between $\tilde{N}_1$ and $\tilde{N}_2$ is defined as:

$$d(\tilde{N}_1,\tilde{N}_2)=1-s(\tilde{N}_1,\tilde{N}_2)\tag{10}$$

where $d(\tilde{N}_1,\tilde{N}_2)$ satisfies the following four properties:

1) $0\leq d(\tilde{N}_1,\tilde{N}_2)\leq 1,\forall\tilde{N}_1,\tilde{N}_2\in SV_2(A)$;
2) $d(\tilde{N}_1,\tilde{N}_2)=0$, if and only if $\tilde{N}_1=\tilde{N}_2$;
3) $d(\tilde{N}_1,\tilde{N}_2)=d(\tilde{N}_2,\tilde{N}_1)$;
4) If $d(\tilde{N}_1,\tilde{N}_2)=0$, $d(\tilde{N}_1,\tilde{N}_3)=0$, $\tilde{N}_3\in SV_2(A)$, then $d(\tilde{N}_2,\tilde{N}_3)=0$.

Now, for any two T2SVNSs $\tilde{N}_1$ and $\tilde{N}_2$, we define the some similarity measures with T2SVNSs as follows:

Definition 11 The binary logarithm similarity measures(BLSM)(type I):

$$\begin{aligned}&s_{BL_1}(\tilde{N}_1,\tilde{N}_2)=\frac{1}{m}\sum_{j=1}^{m}\log_2\{2-\frac{1}{9}(|u_{N_1}(a_j)-\\&u_{N_2}(a_j)|+|n_{N_1}(a_j)-n_{N_2}(a_j)|+|v_{N_1}(a_j)-\\&v_{N_2}(a_j)|+|t_{a_j}(u_{N_1})-t_{a_j}(u_{N_2})|+|i_{a_j}(n_{N_1})-\\&i_{a_j}(n_{N_2})|+|f_{a_j}(v_{N_1})-f_{a_j}(v_{N_2})|+|x_{N_1}(a_j)-\\&x_{N_2}(a_j)|+|y_{N_1}(a_j)-y_{N_2}(a_j)|+|z_{N_1}(a_j)-\\&z_{N_2}(a_j)|)\}\end{aligned}\tag{11}$$

Theorem 1 The defined binary logarithm similarity measure $s_{BL_1}(\tilde{N}_1,\tilde{N}_2)$ between any two T2SVNSs $\tilde{N}_1$ and $\tilde{N}_2$ satisfies the following properties:

1) $0\leq s_{BL_1}(\tilde{N}_1,\tilde{N}_2)\leq 1,\forall\tilde{N}_1,\tilde{N}_2\in SV_2(A)$;
2) $s_{BL_1}(\tilde{N}_1,\tilde{N}_2)=1$, if and only if $\tilde{N}_1=\tilde{N}_2$;
3) $s_{BL_1}(\tilde{N}_1,\tilde{N}_2)=s_{BL_1}(\tilde{N}_2,\tilde{N}_1)$;
4) If $s_{BL_1}(\tilde{N}_1,\tilde{N}_2)=1$, $s_{BL_1}(\tilde{N}_1,\tilde{N}_3)=1$, $\tilde{N}_3\in SV_2(A)$, then $s_{BL_1}(\tilde{N}_2,\tilde{N}_3)=1$.

Definition 12 The binary logarithm similarity measures(BLSM)(type II):

$$\begin{aligned}&s_{BL_2}(\tilde{N}_1,\tilde{N}_2)=\frac{1}{m}\sum_{j=1}^{m}\log_2\{2-\frac{1}{3}[max(|u_{N_1}(a_j)-\\&u_{N_2}(a_j)|,|n_{N_1}(a_j)-n_{N_2}(a_j)|,|v_{N_1}(a_j)-\\&v_{N_2}(a_j)|)+max(|t_{a_j}(u_{N_1})-t_{a_j}(u_{N_2})|,\\&|i_{a_j}(n_{N_1})-i_{a_j}(n_{N_2})|,|f_{a_j}(v_{N_1})-f_{a_j}(v_{N_2})|)+\\&max(|x_{N_1}(a_j)-x_{N_2}(a_j)|,|y_{N_1}(a_j)-y_{N_2}(a_j)|,\\&|z_{N_1}(a_j)-z_{N_2}(a_j)|)]\}\end{aligned}\tag{12}$$

Definition 13 The weighted binary logarithm similarity measures(WBLSM)(type I):

$$\begin{aligned}&s_{BL_1^W}(\tilde{N}_1,\tilde{N}_2)=\sum_{j=1}^{m}\omega_j\log_2\{2-\frac{1}{9}(|u_{N_1}(a_j)-\\&u_{N_2}(a_j)|+|n_{N_1}(a_j)-n_{N_2}(a_j)|+|v_{N_1}(a_j)-\\&v_{N_2}(a_j)|+|t_{a_j}(u_{N_1})-t_{a_j}(u_{N_2})|+|i_{a_j}(n_{N_1})-\\&i_{a_j}(n_{N_2})|+|f_{a_j}(v_{N_1})-f_{a_j}(v_{N_2})|+|x_{N_1}(a_j)-\\&x_{N_2}(a_j)|+|y_{N_1}(a_j)-y_{N_2}(a_j)|+|z_{N_1}(a_j)-\\&z_{N_2}(a_j)|)\}\end{aligned}\tag{13}$$

where $\omega_j\in[0,1]$ and $\sum_{j=1}^{m}\omega_j=1$.

Definition 14 The weighted binary logarithm similarity measures(WBLSM)(type II):

$$s_{BL_2^W}(\tilde{N}_1,\tilde{N}_2)=\sum_{j=1}^{m}\omega_j\log_2\{2-\frac{1}{3}[max(|u_{N_1}(a_j)-u_{N_2}(a_j)|,|n_{N_1}(a_j)-n_{N_2}(a_j)|,|v_{N_1}(a_j)-v_{N_2}(a_j)|)+max(|t_{a_j}(u_{N_1})-t_{a_j}(u_{N_2})|,|i_{a_j}(n_{N_1})-i_{a_j}(n_{N_2})|,|f_{a_j}(v_{N_1})-f_{a_j}(v_{N_2})|)+max(|x_{N_1}(a_j)-x_{N_2}(a_j)|,|y_{N_1}(a_j)-y_{N_2}(a_j)|,|z_{N_1}(a_j)-z_{N_2}(a_j)|)]\} \tag{14}$$

where $\omega_j\in[0,1]$ and $\sum_{j=1}^{m}\omega_j=1$.

Definition 15 The hybrid binary logarithm similarity measures(HBLSM):

$$s_{HBL}(\tilde{N}_1,\tilde{N}_2)=\frac{1}{m}\{\lambda\{\sum_{j=1}^{m}\log_2[2-\frac{1}{9}(|u_{N_1}(a_j)-u_{N_2}(a_j)|+|n_{N_1}(a_j)-n_{N_2}(a_j)|+|v_{N_1}(a_j)-v_{N_2}(a_j)|+|t_{a_j}(u_{N_1})-t_{a_j}(u_{N_2})|+|i_{a_j}(n_{N_1})-i_{a_j}(n_{N_2})|+|f_{a_j}(v_{N_1})-f_{a_j}(v_{N_2})|+|x_{N_1}(a_j)-x_{N_2}(a_j)|+|y_{N_1}(a_j)-y_{N_2}(a_j)|+|z_{N_1}(a_j)-z_{N_2}(a_j)|)]\}+(1-\lambda)\{\sum_{j=1}^{m}\log_2\{2-\frac{1}{3}[max(|u_{N_1}(a_j)-u_{N_2}(a_j)|,|n_{N_1}(a_j)-n_{N_2}(a_j)|,|v_{N_1}(a_j)-v_{N_2}(a_j)|)+max(|t_{a_j}(u_{N_1})-t_{a_j}(u_{N_2})|,|i_{a_j}(n_{N_1})-i_{a_j}(n_{N_2})|,|f_{a_j}(v_{N_1})-f_{a_j}(v_{N_2})|)+max(|x_{N_1}(a_j)-x_{N_2}(a_j)|,|y_{N_1}(a_j)-y_{N_2}(a_j)|,|z_{N_1}(a_j)-z_{N_2}(a_j)|)]\}\}\} \tag{15}$$

where $\lambda\in[0,1]$.

Definition 16 The weighted hybrid binary logarithm similarity measures(WHBLSM):

$$s_{HBL^W}(\tilde{N}_1,\tilde{N}_2)=\lambda\{\sum_{j=1}^{m}\omega_j\log_2[2-\frac{1}{9}(|u_{N_1}(a_j)-u_{N_2}(a_j)|+|n_{N_1}(a_j)-n_{N_2}(a_j)|+|v_{N_1}(a_j)-v_{N_2}(a_j)|+|t_{a_j}(u_{N_1})-t_{a_j}(u_{N_2})|+|i_{a_j}(n_{N_1})-i_{a_j}(n_{N_2})|+|f_{a_j}(v_{N_1})-f_{a_j}(v_{N_2})|+|x_{N_1}(a_j)-x_{N_2}(a_j)|+|y_{N_1}(a_j)-y_{N_2}(a_j)|+|z_{N_1}(a_j)-z_{N_2}(a_j)|)]\}+(1-\lambda)\{\sum_{j=1}^{m}\omega_j\log_2\{2-\frac{1}{3}[max(|u_{N_1}(a_j)-u_{N_2}(a_j)|,|n_{N_1}(a_j)-n_{N_2}(a_j)|,|v_{N_1}(a_j)-v_{N_2}(a_j)|)+max(|t_{a_j}(u_{N_1})-t_{a_j}(u_{N_2})|,|i_{a_j}(n_{N_1})-i_{a_j}(n_{N_2})|,|f_{a_j}(v_{N_1})-f_{a_j}(v_{N_2})|)+max(|x_{N_1}(a_j)-x_{N_2}(a_j)|,|y_{N_1}(a_j)-y_{N_2}(a_j)|,|z_{N_1}(a_j)-z_{N_2}(a_j)|)]\}\} \tag{16}$$

where $\lambda\in[0,1]$.

IV. The MCGDM Based on TOPSIS Approach

In this section, a MCGDM based on TOPSIS approach is presented by using the probabilistic operators and above-defined similarity measures for T2SVNSs. Assume that $D=\{D_1,D_2,\ldots,D_d\}$ be a committee of decision makers, $A=\{A_1,A_2,\ldots,A_k\}$ be the alternatives, $C=\{C_1,D_2,\ldots,C_s\}$ be the attributes of each alternative. Then, the following steps are described for finding the best alternative(s).

Step 1: Determination of the T2SVN decision matrix of the decision makers (DMs)

When an expert evaluate the given alternatives A_i under different attributes C_j made by decision makers $D_m(m=1,2,\ldots,d)$ and represent their values in terms of T2SVNNs $d_{ij}^m(i=1,2,\ldots,k;j=1,2,\ldots,s)$. Hence, the T2SVN decision matrix $D_m=(d_{ij}^m)_{k\times s}$ can be written as follows:

$$D_m=(d_{ij}^m)_{k\times s}=\begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_s \end{array} \\ \begin{array}{c} A_1\\ A_2\\ \vdots\\ A_k \end{array} & \begin{pmatrix} d_{11}^m & d_{12}^m & \cdots & d_{1s}^m\\ d_{21}^m & d_{22}^m & \cdots & d_{2s}^m\\ \vdots & \vdots & \ldots & \vdots\\ d_{k1}^m & d_{k2}^m & \cdots & d_{ks}^m \end{pmatrix}\end{array} \tag{17}$$

where $d_{ij}^m=\langle u_{ij}^m,t_{A_i}^m(u_{ij}^m),n_{ij}^m,i_{A_i}^m(n_{ij}^m),v_{ij}^m,f_{A_i}^m(v_{ij}^m)\rangle$.

Step 2: Determination of the aggregating decision matrix

By using Definition 2.8, the aggregating matrix $B=(b_{ij})_{k\times s}$ is expressed as follows:

$$B=(b_{ij})_{k\times s}=\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1s}\\ b_{21} & b_{22} & \cdots & b_{2s}\\ \vdots & \vdots & \ldots & \vdots\\ b_{k1} & b_{k2} & \cdots & b_{ks} \end{pmatrix} \tag{18}$$

where $b_{ij}=\bigoplus_{m=1}^{d}(\delta_m d_{ij}^m)=(1-\prod_{m=1}^{d}(1-u_{ij}^m)^{\delta_m},1-\prod_{m=1}^{d}(1-t_{A_i}^m(u_{ij}))^{\delta_m},\prod_{m=1}^{d}(n_{ij}^m)^{\delta_m},\prod_{m=1}^{d}(i_{A_i}^m(n_N))^{\delta_m},\prod_{m=1}^{d}(v_{ij}^m)^{\delta_m},\prod_{m=1}^{d}(f_{A_i}^m(v_{ij}))^{\delta_m})$. $\delta_m=(1-\beta)p_m+\beta\omega_m$ be the weight vector with weights $\omega_m>0$, $\sum_{m=1}^{d}\omega_m=1$, a probabilities $p_m>0$, $\sum_{m=1}^{d}p_m=1$ and $\beta\in[0,1]$.

Step 3: Calculating the positive ideal solution PIS h^+ and negative ideal solution NIS h^-

The PIS $h^+=\{h_1^+,h_2^+,\ldots,h_s^+\}$ and NIS $h^-=\{h_1^-,h_2^-,\ldots,h_s^-\}$ are defined as follows:

$$h_j^+=\langle\max_i(1-\prod_{m=1}^{d}(1-u_{ij}^m)^{\delta_m},\max_i(1-\prod_{m=1}^{d}(1-t_{A_i}^m(u_{ij}))^{\delta_m}),\min_i(\prod_{m=1}^{d}(n_{ij}^m)^{\delta_m}),\min_i(\prod_{m=1}^{d}(i_{A_i}^m(n_N))^{\delta_m}),\min_i(\prod_{m=1}^{d}(v_{ij}^m)^{\delta_m}),\min_i(\prod_{m=1}^{d}(f_{A_i}^m(v_{ij}))^{\delta_m})\rangle \tag{19}$$

$$
\begin{aligned}
h_j^- = & \langle \min_i (1 - \prod_{m=1}^{d} (1 - u_{ij}^m)^{\delta_m}, \min_i (1 - \prod_{m=1}^{d} (1- \\
& t_{A_i}^m (u_{ij}))^{\delta_m}), \max_i (\prod_{m=1}^{d} (n_{ij}^m)^{\delta_m}), \\
& \max_i (\prod_{m=1}^{d} (i_{A_i}^m (n_N))^{\delta_m}), \max_i (\prod_{m=1}^{d} (v_{ij}^m)^{\delta_m}), \\
& \max_i (\prod_{m=1}^{d} (f_{A_i}^m (v_{ij}))^{\delta_m}) \rangle
\end{aligned}
\tag{20}
$$

Step 4: Determination of the distance between each alternative and PIS h^+ / NIS h^-

$$
\begin{cases}
d_i^+ = \sum\limits_{j=1}^{s} d_\triangle (b_{ij}, h_j^+) \\
d_i^- = \sum\limits_{j=1}^{s} d_\triangle (b_{ij}, h_j^-)
\end{cases}
for \quad i = 1, 2, \ldots, k. \tag{21}
$$

where $d_\triangle = 1 - s_\triangle$, $s_\triangle (\triangle \in \{BL_1, BL_2, BL_1^W, BL_2^W, HBL, HBL^W\})$ are defined in section 3.

Step 5: Calculating the closeness coefficients of alternatives

$$
CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad for \quad i = 1, 2, \ldots, k. \tag{22}
$$

Step 6: Ranking the alternatives

The highest value of closeness coefficients CC_i, the best alternative A_i is.

V. Numerical Example

In this section, we applies the proposed MCGDM method for the low carbon logistics service provider selection. Due to the increasingly serious problems caused by carbon emissions, the concept of low carbon economy has gradually attracted the attention of the international community. The logistics industry is the basis and artery industry of national economic development, and it is also an industry with large energy consumption and carbon emission. With the advocacy and implementation of the concept of low carbon economy, the transformation and development of low carbon logistics industry will be an inevitable trend. However, the real market competition is not the competition between enterprises, but the competition between supply chains. How to choose a suitable low carbon logistics supplier is of great significance to reduce the carbon emissions of the whole supply chain and enhance the market competitiveness of the supply chain. We will use a numerical example of the low carbon logistics service provider selection problem provided by Chen et al.[61]. There are three DMs (D_1, D_2, D_3) to evaluate, with four alternatives $A_i (i = 1, 2, 3, 4)$ and four attributes: C_1:low-carbon technology, C_2: cost, C_3: risk factor, C_4: capacity. The weight of criteria is $\tau = (0.30, 0.35, 0.15, 0.20)$. Assume that the importance to the subjective (probability) and objective (weightage) information is taken as 40% and 60%, respectively. Probabilistic data is $p = (0.45, 0.35, 0.20)$ and the importance of each T2SVNN is $\omega = (0.30, 0.25, 0.45)$, so $\delta_m = (1 - \beta) p_m + \beta \omega_m$ is calculated as $\delta = (0.36, 0.29, 0.35)$. Then the complete MCGDM model based on TOPSIS method is summarized by the following steps:

Step 1: DMs evaluate alternatives for each criteria by linguistic grade represented in Table 1. Tables 2-4 show their evaluations.

$$
D_1 = \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \\ d_{14} \\ d_{21} \\ d_{22} \\ d_{23} \\ d_{24} \\ d_{31} \\ d_{32} \\ d_{33} \\ d_{34} \\ d_{41} \\ d_{42} \\ d_{43} \\ d_{44} \end{bmatrix} = \begin{bmatrix}
\langle 0.762, 0.500, 0.400, 0.238, 0.762, 0.500 \rangle \\
\langle 1.000, 0.858, 0.400, 0.400, 1.000, 0.868 \rangle \\
\langle 0.858, 0.248, 0.000, 0.238, 0.868, 0.238 \rangle \\
\langle 0.858, 0.763, 0.762, 0.238, 0.868, 0.762 \rangle \\
\langle 1.000, 0.762, 0.762, 0.762, 1.000, 0.762 \rangle \\
\langle 1.000, 0.858, 0.762, 1.000, 1.000, 0.868 \rangle \\
\langle 1.000, 0.762, 0.238, 0.400, 1.000, 0.762 \rangle \\
\langle 1.000, 0.858, 0.238, 0.600, 1.000, 0.868 \rangle \\
\langle 0.762, 0.500, 0.238, 0.000, 0.762, 0.500 \rangle \\
\langle 0.858, 0.762, 0.500, 1.000, 0.868, 0.762 \rangle \\
\langle 1.000, 0.858, 0.600, 0.500, 1.000, 0.868 \rangle \\
\langle 0.762, 0.500, 0.400, 0.238, 0.762, 0.500 \rangle \\
\langle 0.762, 0.858, 0.238, 0.400, 0.868, 0.868 \rangle \\
\langle 0.762, 0.762, 1.000, 0.762, 0.500, 0.500 \rangle \\
\langle 0.762, 0.400, 0.238, 1.000, 0.762, 1.000 \rangle \\
\langle 1.000, 0.858, 0.500, 0.500, 1.000, 0.868 \rangle
\end{bmatrix}
$$

$$
D_2 = \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \\ d_{14} \\ d_{21} \\ d_{22} \\ d_{23} \\ d_{24} \\ d_{31} \\ d_{32} \\ d_{33} \\ d_{34} \\ d_{41} \\ d_{42} \\ d_{43} \\ d_{44} \end{bmatrix} = \begin{bmatrix}
\langle 0.762, 0.248, 1.000, 0.000, 0.762, 0.238 \rangle \\
\langle 0.500, 0.500, 0.400, 0.600, 0.500, 0.500 \rangle \\
\langle 0.762, 0.248, 0.238, 0.400, 0.762, 0.238 \rangle \\
\langle 0.762, 0.248, 0.000, 0.500, 0.762, 0.238 \rangle \\
\langle 0.858, 1.000, 0.000, 0.238, 0.868, 1.000 \rangle \\
\langle 0.858, 0.248, 0.762, 0.400, 0.868, 0.238 \rangle \\
\langle 0.858, 1.000, 0.000, 0.238, 0.868, 1.000 \rangle \\
\langle 0.858, 1.000, 1.000, 0.000, 0.868, 1.000 \rangle \\
\langle 0.858, 0.500, 0.500, 1.000, 0.868, 0.500 \rangle \\
\langle 0.762, 0.248, 0.400, 0.600, 0.762, 0.238 \rangle \\
\langle 0.858, 0.500, 0.762, 0.600, 0.868, 0.500 \rangle \\
\langle 0.858, 0.500, 0.238, 0.238, 0.868, 0.500 \rangle \\
\langle 0.858, 0.762, 0.400, 0.500, 1.000, 0.868 \rangle \\
\langle 0.858, 0.762, 0.400, 0.500, 0.868, 0.762 \rangle \\
\langle 0.858, 0.762, 0.400, 0.762, 0.868, 1.000 \rangle \\
\langle 0.858, 1.000, 0.400, 1.000, 0.868, 1.000 \rangle
\end{bmatrix}
$$

$$
D_3 = \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \\ d_{14} \\ d_{21} \\ d_{22} \\ d_{23} \\ d_{24} \\ d_{31} \\ d_{32} \\ d_{33} \\ d_{34} \\ d_{41} \\ d_{42} \\ d_{43} \\ d_{44} \end{bmatrix} = \begin{bmatrix}
\langle 0.858, 0.762, 0.000, 0.000, 1.000, 0.868 \rangle \\
\langle 0.248, 0.762, 1.000, 0.000, 0.238, 0.762 \rangle \\
\langle 0.248, 0.762, 0.000, 0.238, 0.238, 0.762 \rangle \\
\langle 0.500, 0.762, 0.000, 1.000, 0.500, 0.762 \rangle \\
\langle 0.858, 0.248, 0.238, 0.400, 0.868, 0.238 \rangle \\
\langle 0.858, 0.762, 0.600, 0.600, 0.868, 0.762 \rangle \\
\langle 0.858, 0.858, 0.400, 0.600, 0.868, 0.868 \rangle \\
\langle 0.858, 0.500, 0.000, 0.000, 0.868, 0.500 \rangle \\
\langle 0.142, 0.000, 0.500, 0.500, 0.132, 0.000 \rangle \\
\langle 0.248, 0.500, 0.500, 0.500, 0.238, 0.500 \rangle \\
\langle 0.248, 0.762, 0.238, 0.238, 0.238, 0.762 \rangle \\
\langle 0.500, 0.762, 0.400, 0.400, 0.500, 0.762 \rangle \\
\langle 0.142, 0.500, 0.400, 0.000, 0.132, 0.500 \rangle \\
\langle 0.142, 0.500, 0.600, 0.400, 0.132, 0.500 \rangle \\
\langle 1.000, 0.858, 1.000, 1.000, 1.000, 0.868 \rangle \\
\langle 1.000, 0.762, 0.238, 0.500, 1.000, 0.762 \rangle
\end{bmatrix}
$$

Step 2: Determination of the aggregating decision matrix $B = (b_{ij})_{k \times s}$.

According to the weight $\delta = (0.36, 0.29, 0.35)$ and E-

TABLE I
EVALUATIONS OF THE ALTERNATIVES BY THE LINGUISTIC VARIABLES.

Grades	PTMF	STMF	PIMF	SIMF	PFMF	SFMF
Very good (VG)	1.000	1.000	0.000	0.000	0.000	0.000
Good (G)	0.858	0.858	0.238	0.238	0.132	0.132
Medium good (MG)	0.762	0.762	0.400	0.400	0.238	0.238
Fairly (F)	0.500	0.500	0.500	0.500	0.500	0.500
Medium poor (MP)	0.248	0.248	0.600	0.600	0.762	0.762
Poor (P)	0.142	0.142	0.762	0.762	0.868	0.868
Very poor (VP)	0.000	0.000	1.000	1.000	1.000	1.000

TABLE II
LINGUISTIC DECISION MATRIX BY DECISION MAKER D_1.

Alternatives	C_1	C_2	C_3	C_4
A_1	(MG, F, MG, G, MP, F)	(VG, G, MG, MG, VP, P)	(G, MP, VG, G, P, MG)	(G, MG, P, G, P, MP)
A_2	(VG, MG, P, P, VP, MP)	(VG, G, P, VP, VP, P)	(VG, MG, G, MG, VP, MP)	(VG, G, G, MP, VP, P)
A_3	(MG, F, G, VG, MP, F)	(G, MG, F, VP, P, MP)	(VG, G, MP, F, VP, P)	(MG, F, MG, G, MP, F)
A_4	(MG, G, G, MG, P, P)	(MG, MG, VP, P, F, F)	(VG, MG, MG, G, VP, MP)	(VG, G, F, F, VP, P)

TABLE III
LINGUISTIC DECISION MATRIX BY DECISION MAKER D_2.

Alternatives	C_1	C_2	C_3	C_4
A_1	(MG, MP, VP, VG, MP, MG)	(F, F, MG, MP, F, F)	(MG, MP, G, MG, MP, MG)	(MG, MP, VG, F, MP, MG)
A_2	(G, VG, VG, G, P, VP)	(G, MP, P, MG, P, MG)	(G, VG, VG, G, P, VP)	(G, VG, VP, VG, P, VP)
A_3	(G, F, F, VP, P, F)	(MG, MP, MG, MP, MP, MG)	(G, F, P, MP, P, F)	(G, F, G, G, P, F)
A_4	(G, MG, MG, F, VP, P)	(G, MG, MG, F, P, MP)	(G, MG, MG, P, P, VP)	(G, VG, MG, VP, P, VP)

TABLE IV
LINGUISTIC DECISION MATRIX BY DECISION MAKER D_3.

Alternatives	C_1	C_2	C_3	C_4
A_1	(G, MG, VG, VG, VP, P)	(MP, MG, VP, VG, MG, MP)	(MP, MG, VG, G, MG, MP)	(F, MG, VG, VP, F, MP)
A_2	(G, MP, G, MG, P, MG)	(G, MG, MP, MP, P, MP)	(G, G, MG, MP, P, P)	(G, F, VG, VG, P, F)
A_3	(P, VP, F, F, G, VG)	(MP, F, F, F, MG, F)	(MP, MG, G, G, MG, MP)	(F, MG, MG, MG, F, MP)
A_4	(P, F, MG, VG, G, F)	(P, F, MP, MG, G, F)	(VG, G, VP, VP, VP, P)	(VG, MG, G, F, VP, MP)

q.(2.8), B matrix can be constructed. For example

$$\begin{aligned} b_{11} =&\langle 1-(1-0.762)^{0.36}\cdot(1-0.762)^{0.29}\cdot(1-0.858)^{0.35},\\ &1-(1-0.500)^{0.36}\cdot(1-0.248)^{0.29}\cdot(1-0.762)^{0.35},\\ &0.400^{0.36}\cdot 1.000^{0.29}\cdot 0.000^{0.35},\\ &0.238^{0.36}\cdot 0.000^{0.29}\cdot 0.000^{0.35},\\ &0.762^{0.36}\cdot 0.762^{0.29}\cdot 1.000^{0.35},\\ &0.500^{0.36}\cdot 0.238^{0.29}\cdot 0.868^{0.35}\rangle\\ =&\langle 0.801, 0.566, 0.000, 0.000, 0.838, 0.489\rangle \end{aligned}$$

Other values in B matrix can be made by similar way as follows:

$$B=\begin{bmatrix} \langle 0.801,0.566,0.000,0.000,0.838,0.489\rangle\\ \langle 1.000,0.755,0.551,0.000,0.495,0.707\rangle\\ \langle 0.704,0.497,0.000,0.277,0.531,0.358\rangle\\ \langle 0.744,0.668,0.000,0.488,0.689,0.544\rangle\\ \langle 1.000,1.000,0.000,0.434,0.913,0.549\rangle\\ \langle 1.000,0.724,0.701,0.641,0.913,0.570\rangle\\ \langle 1.000,1.000,0.000,0.340,0.913,0.863\rangle\\ \langle 1.000,1.000,0.000,0.000,0.913,0.746\rangle\\ \langle 0.679,0.363,0.383,0.000,0.428,0.000\rangle\\ \langle 0.704,0.569,0.469,0.677,0.531,0.469\rangle\\ \langle 1.000,0.755,0.465,0.407,0.581,0.707\rangle\\ \langle 0.734,0.614,0.344,0.285,0.683,0.579\rangle\\ \langle 0.679,0.744,0.332,0.000,0.468,0.716\rangle\\ \langle 0.679,0.691,0.641,0.538,0.368,0.564\rangle\\ \langle 1.000,0.723,0.457,0.924,0.870,0.952\rangle\\ \langle 1.000,1.000,0.361,0.611,0.960,0.864\rangle \end{bmatrix}$$

Step 3: Determination of the PIS and NIS.

By using Eqs.(4.3) and (4.4), we can get the PIS h^+ and NIS h^- as following:

$$h^+=\begin{bmatrix} \langle 1.000,0.755,0.000,0.000,0.495,0.358\rangle\\ \langle 1.000,1.000,0.000,0.000,0.913,0.549\rangle\\ \langle 1.000,0.755,0.344,0.000,0.428,0.000\rangle\\ \langle 1.000,1.000,0.332,0.000,0.368,0.564\rangle \end{bmatrix}$$

$$h^-=\begin{bmatrix} \langle 0.704,0.497,0.551,0.488,0.838,0.707\rangle\\ \langle 1.000,0.724,0.701,0.641,0.913,0.863\rangle\\ \langle 0.679,0.363,0.469,0.677,0.683,0.707\rangle\\ \langle 0.679,0.691,0.641,0.924,0.960,0.952\rangle \end{bmatrix}$$

Step 4: Determination of the distance between each alternative and the PIS h^+ / the NIS h^-.

By using distance measure and similarity measures formulas given between T2SVNSs in section 3. the separation measures d_i^+ and d_i^- are shown in Table 5. The value of λ does not affect the result of the operation, but the precision of the operation can be adjusted. For the convenience of calculation, the special case $\lambda = 0.55$ be chosen.

Step 5: Calculating the closeness coefficients CC_i of alternatives.

By using Eq.(4.6), the closeness coefficients can be obtained and shown in Table 6.

Step 6: Ranking the alternatives.

According to Table 6, the resulting ranking order is $A_1 \succ A_4 \succ A_3 \succ A_2$, alternative which has maximum closeness coefficient is A_1, according to all of distance measures. Hence, the best supplier is A_1.

Compared with the approach proposed by Karaaslan [60], the difference is that this paper proposes some similarity

TABLE V
SEPARATION MEASURES d_i^+ AND d_i^- ACCORDING TO DISTANCE MEASURES.

	d_1^+	d_2^+	d_3^+	d_4^+	d_1^-	d_2^-	d_3^-	d_4^-
d_{BL_1}	0.0774	0.1621	0.1498	0.1321	0.1840	0.1186	0.1310	0.1439
$d_{BL_1}^W$	0.0780	0.1566	0.1363	0.1167	0.1840	0.1179	0.1342	0.1522
d_{BL_2}	0.1953	0.3765	0.2965	0.2739	0.3745	0.2535	0.2640	0.2912
$d_{BL_2}^W$	0.1991	0.3909	0.2697	0.2513	0.3719	0.2537	0.2749	0.3262
d_{HBL}	0.1304	0.2586	0.2158	0.1959	0.2697	0.1793	0.1908	0.2102
d_{HBL}^W	0.1325	0.2620	0.1963	0.1773	0.2686	0.1790	0.1975	0.2305

TABLE VI
THE CLOSENESS COEFFICIENTS OF ALTERNATIVES.

	CC_1	CC_2	CC_3	CC_4	Ranking order
d_{BL_1}	0.7039	0.4225	0.4665	0.6502	$A_1 \succ A_4 \succ A_3 \succ A_2$
$d_{BL_1}^W$	0.7023	0.4295	0.4961	0.5660	$A_1 \succ A_4 \succ A_3 \succ A_2$
d_{BL_2}	0.6572	0.4024	0.4710	0.5153	$A_1 \succ A_4 \succ A_3 \succ A_2$
$d_{BL_2}^W$	0.6513	0.3935	0.5048	0.5648	$A_1 \succ A_4 \succ A_3 \succ A_2$
d_{HBL}	0.6741	0.4094	0.4693	0.5176	$A_1 \succ A_4 \succ A_3 \succ A_2$
d_{HBL}^W	0.6696	0.4059	0.5015	0.5652	$A_1 \succ A_4 \succ A_3 \succ A_2$

measures and new distance measures between T2SVNSs, but our ranking results and optimal supplier have the same values to calculate the same decision problem as that of Karaaslan [60], which is able to show our approach is practical and effective.

VI. CONCLUSION

In this study, we presented a MCGDM method for the low carbon logistics service provider selection under a type-2 single valued neutrosophic set environment. The main contributions of this paper can be summarized as follows: (1) The concept of type-2 single valued neutrosophic sets is defined. Using the existing theory of T2SVNSs, we define the probabilistic operators with T2SVN information. (2) In order to obtain the best alternative(s), we propose some similarity measures and new distance measures between two T2SVNSs. (3) A new TOPSIS based approach for MCGDM under type-2 single valued neutrosophic environment is developed by integrating proposed similarity measures and distance measures. (4) To illustrate validity and process of the proposed MCGDM, a numerical example is given.

In the future, we hope that more information measures and techniques are developed for decision-making problems under type-2 single valued neutrosophic environment. The proposed approach can be used for dealing with decision-making problems, such as personal selection in academia, project evaluation, manufacturing systems and many other areas of management systems.

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