Toward Sustainable Development Goals (SDGs) with Statistical Modeling: Recursive Bivariate Binary Probit

Vita Ratnasari, Syirrul Hadi Utama, Andrea Tri Rian Dani

Abstract— **Poverty is still a global problem that must be immediately eradicated by Sustainable Development Goals (SDGs) 1, namely ending poverty anywhere and in any form. In 2021, West Papua province will have the 2nd most significant percentage of poor people after Papua province, with 21.84% of the poor population. Poor households in West Papua province are dominated by families, with the Head of Household (KRT) working in the agricultural sector at 65.10 %. In this research, joint modelling was carried out between the level of household welfare and the employment sector of the head of the household in the West Papua province. It is suspected that these two variables have endogeneity problems, where one of the response variables becomes a predictor variable in the other equation, so a recursive bivariate binary probit regression model is used. Recursive bivariate binary probit regression parameter estimation uses Maximum Likelihood Estimation (MLE), but the results are not closed form, so it is continued using the Newton-Raphson iteration method. The results of hypothesis testing show that partially, variables that significantly influence the level of household welfare include the variable marital status, KRT formal/informal workers, health complaints, asset ownership status, migration status, number of household members, classification of area of residence (Village/City), age of head of household, and employment sector of head of household. Meanwhile, variables that significantly influence the choice of working in the agricultural sector include the director of household education, classification of area of residence (rural/city), and the age of the head of household.**

Index Terms— **Maximum Likelihood Estimation, Newton Raphson, SDGs, Recursive Bivariate Binary Probit**

I. Introduction

ON of the welfare indicators based on the expenditure approach is the percentage of poor people [1], [2]. Poverty must be eradicated immediately by the 1st goal of the Sustainable Development Goals (SDGs), namely ending poverty anywhere and in any form [1], [3].

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Vita Ratnasari is a Professor in Statistics Department and Lecturer, Faculty of Science and Data Analytics, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia (corresponding author to e-mail: vita.statistikaits@gmail.com).

Syirrul Hadi Utama is a Postgraduate Student at the Department of Statistics, Faculty of Science and Data Analytics, Sepuluh Nopember Institute of Technology, Surabaya, Indonesia (e-mail: syirrulhadiutama@gmail.com).

Andrea Tri Rian Dani is a Lecturer in Statistics Study Program, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Mulawarman University, Samarinda, Indonesia (e-mail: andreatririandani@fmipa.unmul.ac.id).

The National Socioeconomic Survey (SUSENAS) conducted by the Central Bureau of Statistics (BPS) in 2021 shows that in Indonesia, there are 10.14% of poor people. The province with the second largest percentage of poor people after Papua province is West Papua province, with 21.84% of poor people. West Papua has an Open Unemployment Rate (TPT) of 6.18%. This figure is similar to the national TPT rate of 6.26%. West Papua Province ranks 25th with the lowest TPT. But ironically, the low TPT does not lead to lower poverty in West Papua, which may be due to the low income earned from workers in the agricultural sector [4]. The agricultural sector is the primary employment sector in West Papua [5]. Based on the results of the National Labor Force Survey (SAKERNAS) conducted by BPS in February 2021, the agricultural sector is the employment sector with the highest percentage of workers, namely 32.69%, followed by the wholesale, retail, and pharmaceutical sectors, with 18.12%, administration government 13.75%, and other sectors 63.56%. Based on the March 2021, SUSENAS results, 87.74% of the poor work in the agricultural sector, and 65.10% of poor households with heads of household work there.

Another welfare indicator, according to the World Bank [6], is the disparity in spending between the population with the lowest 40% spending level compared to the highest 60% of the population [7]. Inequality is measured by calculating the proportion of expenditure of the bottom 40% of the population. Currently, in West Papua, the proportion of expenditure of the lowest 40% of the population is 22.57%. West Papua has a low level of inequality but a very high poverty rate. This shows that many people are not classified as poor but whose per capita expenditure is close to the poverty line.

Studies examining and discussing welfare issues use response variables with categorical data scales [8], [9]. One method that can be used to model some categorical response variables is probit regression[10], [11]. Probit regression is a regression method that is used to analyze the dependent variable, which is qualitative, and the independent variable, which is qualitative, quantitative, or a combination of qualitative and quantitative with the standard normal distribution [9], [12]. Cumulative Distribution Function (CDF) approach to estimate parameters so that a probit model is formed [13]. The probit model that uses two dichotomous variables as the response variable, while the independent variables can be either discrete or continuous, and qualitative variables, namely nominal or ordinal variables, is called the bivariate binary probit model[14]. To perform modeling on a bivariate model, the conditions that must be met are that the two response variables in the model must have a relationship[15], [16]. If there is an endogeneity problem between the two equations in the bivariate probit, then the bivariate probit model cannot provide accurate results. Therefore, a recursive bivariate binary probit model was developed[17].

Recursive bivariate probit regression is a method in which two probit equations whose errors are correlated, and one of the response variables becomes an endogenous factor in the other dependent variable[18]. The recursive bivariate binary probit regression model is good for modeling variables with binary response variables that have endogeneity but can also be interpreted partially [19]. The value of the correlation parameter on the recursive bivariate probit is not the same as the correlation on the bivariate probit. This is because, in recursive bivariate probit, endogeneity occurs so that it can affect the value of the correlation parameter [20], [21].

In this study, modeling was carried out jointly between the level of household welfare and the employment sector of the head of the household in West Papua province. These two variables are suspected of having an endogeneity problem, where the household head's employment sector variable influences the household welfare level. So, in modeling, the response variable of the household head's employment sector will be the predictor variable. Therefore, the analytical method used is recursive bivariate binary probit regression[22]. This study aimed to obtain parameter estimates from a recursive bivariate binary probit regression model and to model the factors that influence the level of household welfare and the employment sector of the head of the household in West Papua province. The output of this research is a theoretical and applicative study of case studies.

II. Bivariate Binary Probit Regression Model

The bivariate probit regression model is a method that describes the relationship between two response variables that are qualitative and several predictor variables that are qualitative, quantitative, or a combination of qualitative and quantitative with the normal distribution CDF (Cumulative Distribution Function) approach to estimate parameters so that a probit model is formed [23], [24]. The assumption that must be fulfilled in the bivariate probit model is that the response variables are mutually dependent. Therefore, a dependency test must be carried out between the response variables before modeling [25]. The bivariate binary probit model has two qualitative response variables, Y_1 and Y_2, which are assumed to originate from the unobserved variables y_1^* and y_2^*, each having two categories [26]. The equation of the two variables is defined as Equation (1).

$$\boldsymbol{y}_1^* = \mathbf{X}_1^T\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1 \;;\; \boldsymbol{y}_2^* = \mathbf{X}_2^T\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon}_2 \tag{1}$$

with:

$$\mathbf{y}_1^* = \begin{bmatrix} y_{11}^* \\ y_{12}^* \\ \vdots \\ y_{1n}^* \end{bmatrix}, \mathbf{X}_1^T = \begin{bmatrix} 1 & x_{111} & x_{121} & \cdots & x_{1p1} \\ 1 & x_{112} & x_{122} & \cdots & x_{1p2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{11n} & x_{12n} & \cdots & x_{1pn} \end{bmatrix}, \boldsymbol{\beta}_1 = \begin{bmatrix} \beta_{10} \\ \beta_{11} \\ \vdots \\ \beta_{1p} \end{bmatrix}, \boldsymbol{\varepsilon}_1 = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{1n} \end{bmatrix}$$

$$\mathbf{y}_2^* = \begin{bmatrix} y_{21}^* \\ y_{22}^* \\ \vdots \\ y_{2n}^* \end{bmatrix}, \mathbf{X}_2^T = \begin{bmatrix} 1 & x_{211} & x_{221} & \cdots & x_{2p1} \\ 1 & x_{212} & x_{222} & \cdots & x_{2p2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{21n} & x_{22n} & \cdots & x_{2pn} \end{bmatrix}, \boldsymbol{\beta}_1 = \begin{bmatrix} \beta_{20} \\ \beta_{21} \\ \vdots \\ \beta_{2p} \end{bmatrix}, \boldsymbol{\varepsilon}_2 = \begin{bmatrix} \varepsilon_{21} \\ \varepsilon_{22} \\ \vdots \\ \varepsilon_{2n} \end{bmatrix}$$

and p is the number of predictor variables for $\mathbf{X_1}$ of size (p+1) x n, q is the number of predictor variables for $\mathbf{X_2}$ of size (q+1) x n. $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are the regression coefficients in each equation. The error in each equation is denoted ε_1 and ε_2 which are assumed to have a standard normal distribution with mean 0 and variance 1. So y_1^* and y_2^* are denoted by y_1^*~N ($\boldsymbol{\beta}_1^{\mathrm{T}}\mathbf{X}_1$, 1) and y_2^*~N ($\boldsymbol{\beta}_2^{\mathrm{T}}\mathbf{X}_2$, 1).

Formation of categories on the response variable of the bivariate binary probit regression model is no different from the univariate probit regression model, namely by determining a certain threshold for each unobserved variable y_1^* and y_2^* for example γ and δ [27]. In the case of the bivariate binary probit regression model, the thresholds used in categorizing are assumed to be γ = 0 and δ = 0. The categories formed from the unobserved y_{1i}^* and y_{2i}^* are as in Equation (2).

$$Y_{1i} = 0 \text{ if } y_{1i}^* \leq 0 \text{ and } Y_{1i} = 1 \text{ if } y_{1i}^* > 0 \;;\; Y_{2i} = 0 \text{ if } y_{2i}^* \leq 0 \text{ and } Y_{2i} = 1 \text{ if } y_{2i}^* > 0 \tag{2}$$

In the bivariate binary probit regression model equation, there are two random variables that are normally distributed, namely y_{1i}^* and y_{2i}^*. Therefore, a bivariate normal distribution is formed. The PDF (probability density function) bivariate normal distribution is as in Equation (3).

$$f(y_{1i}^*, y_{2i}^*) = \frac{1}{2\pi\sqrt{|\Sigma|}} \exp\left(-\frac{1}{2} \begin{bmatrix} y_{1i}^* - \mathbf{x}_1^T\boldsymbol{\beta}_1 \\ y_{2i}^* - \mathbf{x}_2^T\boldsymbol{\beta}_2 \end{bmatrix}^T \Sigma^{-1} \begin{bmatrix} y_{1i}^* - \mathbf{x}_1^T\boldsymbol{\beta}_1 \\ y_{2i}^* - \mathbf{x}_2^T\boldsymbol{\beta}_2 \end{bmatrix} \right) \tag{3}$$

The function in Equation (3) can be denoted like Equation (4).

$$(y_{1i}^*, y_{2i}^*) \sim N_2\left(\begin{bmatrix} \mathbf{x}_1^T\boldsymbol{\beta}_1 \\ \mathbf{x}_2^T\boldsymbol{\beta}_2 \end{bmatrix}, \Sigma \right) \tag{4}$$

with:

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{bmatrix}$$

Where $\rho_{12} = \rho_{21} = \rho$.

Then the bivariate standard normal PDF is shown in Equation (5).

$$\phi(z_{1i}, z_{2i}) = \frac{1}{2\pi\sqrt{|\Sigma|}} \exp\left(-\frac{1}{2(1-\rho^2)}(z_{1i}^2 - 2\rho z_{1i}z_{2i} + z_{2i}^2) \right) \tag{5}$$

where $z_{1i} = \dfrac{y_{1i}^* - \mathbf{x}_{1i}^T\boldsymbol{\beta}_1}{\sigma_{11}}$, if $\sigma_{11} = 1$, then $z_{1i} = y_{1i}^* - \mathbf{x}_{1i}^{\mathrm{T}}\boldsymbol{\beta}_1$, with z_{1i}~N(0,1);

and $z_{2i} = \dfrac{y_{2i}^* - \mathbf{x}_{2i}^T\boldsymbol{\beta}_2}{\sigma_{22}}$, if $\sigma_{22} = 1$, then $z_{2i} = y_{2i}^* - \mathbf{x}_{2i}^{\mathrm{T}}\boldsymbol{\beta}_2$, with z_{2i}~N(0,1).

So, the joint probability z_{1i} and z_{2i} shown in Equation (6).

$$(y_{1i}^* < \gamma, y_{2i}^* < \delta) = P(Z_1 < \gamma - \mathbf{x}_{1i}^{\mathrm{T}}\boldsymbol{\beta}_1, Z_2 < \delta - \mathbf{x}_{2i}^{\mathrm{T}}\boldsymbol{\beta}_2) = P(Z_1 < z_{1i}, Z_2 < z_{2i}) \tag{6}$$

$$= \int_{-\infty}^{z_{2i}} \int_{-\infty}^{z_{1i}} \phi(z_1, z_2) dz_1 dz_2$$
$$= \Phi(z_{1i}, z_{2i})$$

With $P(Y_{1i} = 0, Y_{2i} = 0)$ or $P_{00}(x)$.
$\Phi(z_{1i}, z_{1i}) = \Phi(.)$ is the CDF of the bivariate standard normal distribution.

III. RESEACRH METHODOLOGY

The data used in this study is secondary data from the March 2021 National Socioeconomic Survey (SUSENAS) conducted by the Central Statistics Agency (BPS) of West Papua Province. The data covers 5930 households spread across all Districts/Cities consisting of 12 districts and one city with a household research unit. The variables used in this study consisted of two response variables (Y) and eleven predictor variables (X) which are presented in the Table 1.

Table 1. Research Variable

Notation	Variable	Data Scale	Coding
Y_1	Household welfare level	Categorical	1 = Households with the lowest 40% spending 0 = Other
X_1	Marital status	Categorical	1 = Married 0 = Other
X_2	Employment sector status	Categorical	1 = Formal workers 0 = Informal workers
X_3	Health complaints	Categorical	1 = There are complaints 0 = No complaints
X_4	Working hours	Numeric	-
X_5	Home ownership	Categorical	1 = One's own 0 = Other
X_6	Migration	Categorical	1 = Non migrant 0 = Migrant
Y_2	KRT employment sector	Categorical	1 = Agriculture 0 = Non-agricultural
X_7	Number of household members	Numeric	-
X_8	Education completed by KRT	Categorical	1 = High School and Above 0 = Junior high school and below
X_9	Region classification	Categorical	1 = City 0 = Village
X_{10}	KRT age	Numeric	-
X_{11}	KRT Gender	Categorical	1 = Male 0 = Female

Noted: KRT is Head of household

This study aimed to obtain parameter estimates from a recursive bivariate binary probit regression model and to model the factors that influence the level of household welfare and the employment sector of the head of the household in West Papua province.

a. Estimation of Recursive Binary Bivariate Probit Regression Model

The steps in estimating the recursive bivariate binary probit regression model are as follows:

1. Form the ln likelihood function on the variable Y with the likelihood function in Equation (7)

$$Q = L(\theta) = \prod_{i=1}^{n} P\left(Y_{11i} = y_{11i}, Y_{10i} = y_{10i}, Y_{01i} = y_{01i}, Y_{00i} = y_{00i}\right)$$
$$= \prod_{i=1}^{n} P_{11i}^{y11i} P_{10i}^{y10i} P_{01i}^{y01i} P_{00i}^{y00i} \quad (7)$$

2. Get the first derivative of the likelihood function for the parameters for $\boldsymbol{\theta_1}, \boldsymbol{\theta_2}$, and $\boldsymbol{\rho}$ then equate to 0.

$$\frac{\partial \ln L(\theta)}{\partial \boldsymbol{\theta}_1} = 0 \ ; \ \frac{\partial \ln L(\theta)}{\partial \boldsymbol{\theta}_2} = 0 \ ; \ \frac{\partial \ln L(\theta)}{\partial \rho} = 0 \quad (8)$$

Based on the first derivative, it is assumed that the equation is not closed form, so that it is continued with the Newton-Raphson iteration method using the gradient vector **g** which is the first derivative component of the ln likelihood function Q with respect to $\boldsymbol{\theta_1}, \boldsymbol{\theta_2}$, and $\boldsymbol{\rho}$.

$$\mathbf{g}(\boldsymbol{\theta}) = \begin{bmatrix} \dfrac{\partial \ln L(\theta)}{\partial \boldsymbol{\theta}_1} \\ \dfrac{\partial \ln L(\theta)}{\partial \boldsymbol{\theta}_2} \\ \dfrac{\partial \ln L(\theta)}{\partial \rho} \end{bmatrix} \quad (9)$$

3. Obtain the Hessian matrix H which is the second derivative of the ln likelihood of the parameters for θ_1, θ_2, and ρ.

$$\mathbf{H}(\boldsymbol{\theta}) = \begin{bmatrix} \dfrac{\partial^2 \ln L(\theta)}{\partial \boldsymbol{\theta}_1 \partial \boldsymbol{\theta}_1} & \dfrac{\partial^2 \ln L(\theta)}{\partial \boldsymbol{\theta}_1 \partial \boldsymbol{\theta}_2} & \dfrac{\partial^2 \ln L(\theta)}{\partial \boldsymbol{\theta}_1 \partial \rho} \\ \dfrac{\partial^2 \ln L(\theta)}{\partial \boldsymbol{\theta}_2 \partial \boldsymbol{\theta}_1} & \dfrac{\partial^2 \ln L(\theta)}{\partial \boldsymbol{\theta}_2 \partial \boldsymbol{\theta}_2} & \dfrac{\partial^2 \ln L(\theta)}{\partial \boldsymbol{\theta}_2 \partial \rho} \\ \dfrac{\partial^2 \ln L(\theta)}{\partial \rho \, \partial \boldsymbol{\theta}_1} & \dfrac{\partial^2 \ln L(\theta)}{\partial \rho \, \partial \boldsymbol{\theta}_2} & \dfrac{\partial^2 \ln L(\theta)}{\partial \rho \, \partial \rho} \end{bmatrix} \quad (10)$$

4. Iterate with Newton-Raphson until it converges with the iteration formula.

$$\hat{\boldsymbol{\theta}}^{(m)} = \hat{\boldsymbol{\theta}}^{(m-1)} - \left[\boldsymbol{H}\left(\hat{\boldsymbol{\theta}}^{(m-1)}\right)\right]^{-1} \mathbf{g}\left(\hat{\boldsymbol{\theta}}^{(m-1)}\right) \quad (11)$$

b. Modelling with Recursive Binary Bivariate Probit Regression

The steps for modeling a case with a recursive bivariate probit regression model are as follows:

1. Identify the relationship (dependency) between the response variables Y_1 and Y_2 by using the Chi-square test.
2. Perform the response variable endogenity test
3. Modeling the response variable and predictor variable with a recursive bivariate binary probit regression model.
4. Modeling the response variable and predictor variable with a recursive bivariate binary probit regression model.
5. Testing the significance of model parameters simultaneously and partially.

IV. RESULTS AND DISCUSSION

a. Estimation of Recursive Binary Bivariate Probit Regression Model Parameters

To obtain parameter estimates in the bivariate recursive binaryprobit model, the maximum likelihood estimation (MLE) method is used, it is known that the variable $\mathbf{Y} = [Y_{11} \ Y_{10} \ Y_{01i} \ Y_{00i}]^T$ multinomial distribution so that it can be denoted as $\mathbf{Y} \sim M(1; P_{11i}, P_{10i}, P_{01i}, P_{00i})$. The recursive bivariate binary probit model equation can be written as in Equation (12).

$$\begin{aligned} \boldsymbol{y_1^*} &= \mathbf{X}_1^T \boldsymbol{\beta}_1 + \upsilon \boldsymbol{y_2^*} + \boldsymbol{\varepsilon_1} \\ \boldsymbol{y_2^*} &= \mathbf{X}_2^T \boldsymbol{\beta}_2 + \boldsymbol{\varepsilon_2} \end{aligned} \quad (12)$$

Where:
$Y_1 = 1$ if $y_1^* > 0$ and $Y_1 = 0$ if $y_1^* \leq 0$
$Y_2 = 1$ if $y_2^* > 0$ and $Y_2 = 0$ if $y_2^* \leq 0$
To facilitate parameter estimation, the recursive bivariate binary probit model can be simplified by assuming $\boldsymbol{\eta_1} = \mathbf{W}^T\boldsymbol{\theta_1}$ and $\boldsymbol{\eta_2} = \mathbf{X}^T\boldsymbol{\theta_2}$ where $\mathbf{W}$ is the predictor variable matrix of the first equation consisting of $\mathbf{X_1}$ and $\boldsymbol{y_2^*}$. Then $\mathbf{X}$ is the predictor variable of the second equation consisting of $\mathbf{X_2}$. For each observation $\mathbf{w_{1i}} = (1, x_{1i}, x_{2i}, x_{3i}, \dots, x_{pi}, y_{2i}^*)^T$ and $\mathbf{x_i} = (1, x_{1i}, x_{2i}, x_{3i}, \dots, x_{qi})^T$. $\boldsymbol{\theta_1}$contains components $\boldsymbol{\beta}_1^T$ and δ, then $\boldsymbol{\theta_2}$ contains components $\boldsymbol{\beta}_2^T$ as in the Equation (13).

$$\begin{aligned} y_{1i}^* &= \mathbf{w_i^T}\boldsymbol{\theta_1} + \varepsilon_{1i} \\ y_{2i}^* &= \mathbf{x_i^T}\boldsymbol{\theta_2} + \varepsilon_{2i} \end{aligned} \tag{13}$$

With:
$\mathbf{Y_{1i}} = 1$ if $\boldsymbol{y_{1i}^*} > 0$ and $\mathbf{Y_{1i}} = 0$ if $\boldsymbol{y_{1i}^*} \leq 0$
$\mathbf{Y_{2i}} = 1$ if $\boldsymbol{y_{2i}^*} > 0$ and $\mathbf{Y_{2i}} = 0$ if $\boldsymbol{y_{2i}^*} \leq 0$

Opportunities for a respondent to be categorized into one of the categories are as follows:

$$\begin{aligned} P_{11} &= P(Y_{1i}=1, Y_{2i}=1) \\ &= P(y_{1i}^* > 0, y_{2i}^* > 0) \\ &= P(\varepsilon_{1i} > -\mathbf{w}_i^T\boldsymbol{\theta}_1, \varepsilon_{2i} > -\mathbf{x}_i^T\boldsymbol{\theta}_2) \\ &= P(\varepsilon_{1i} \leq \mathbf{w}_i^T\boldsymbol{\theta}_1, \varepsilon_{2i} \leq \mathbf{x}_i^T\boldsymbol{\theta}_2) \\ &= \int_{-\infty}^{\eta_{2i}}\int_{-\infty}^{\eta_{1i}} \phi(\eta_{1i}, \eta_{2i}) d\eta_{1i} d\eta_{2i} \\ &= \Phi(\eta_{1i}, \eta_{2i}) \end{aligned} \tag{14}$$

$$\begin{aligned} P_{10} &= P(Y_{1i}=1, Y_{2i}=0) \\ &= P(y_{1i}^* > 0, y_{2i}^* \leq 0) \\ &= P(\varepsilon_{1i} > -\mathbf{w}_i^T\boldsymbol{\theta}_1, \varepsilon_{2i} \leq -\mathbf{x}_i^T\boldsymbol{\theta}_2) \\ &= P(\varepsilon_{1i} \leq \mathbf{w}_i^T\boldsymbol{\theta}_1, \varepsilon_{2i} > \mathbf{x}_i^T\boldsymbol{\theta}_2) \\ &= \int_{\eta_{2i}}^{\infty}\int_{-\infty}^{\eta_{1i}} \phi(\eta_{1i}, \eta_{2i}) d\eta_{1i} d\eta_{2i} \\ &= \Phi(\eta_{1i}) - \Phi(\eta_{1i}, \eta_{2i}) \end{aligned} \tag{15}$$

$$\begin{aligned} P_{01} &= P(Y_{1i}=0, Y_{2i}=1) \\ &= P(y_{1i}^* \leq 0, y_{2i}^* > 0) \\ &= P(\varepsilon_{1i} \leq -\mathbf{w}_i^T\boldsymbol{\theta}_1, \varepsilon_{2i} > -\mathbf{x}_i^T\boldsymbol{\theta}_2) \\ &= P(\varepsilon_{1i} > \mathbf{w}_i^T\boldsymbol{\theta}_1, \varepsilon_{2i} \leq \mathbf{x}_i^T\boldsymbol{\theta}_2) \\ &= \int_{-\infty}^{\eta_{2i}}\int_{\eta_{1i}}^{\infty} \phi(\eta_{1i}, \eta_{2i}) d\eta_{1i} d\eta_{2i} \\ &= \Phi(\eta_{2i}) - \Phi(\eta_{1i}, \eta_{2i}) \end{aligned} \tag{16}$$

$$\begin{aligned} P_{00} &= P(Y_{1i}=0, Y_{2i}=0) \\ &= P(y_{1i}^* \leq 0, y_{2i}^* \leq 0) \\ &= P(\varepsilon_{1i} \leq -\mathbf{w}_i^T\boldsymbol{\theta}_1, \varepsilon_{2i} \leq -\mathbf{x}_i^T\boldsymbol{\theta}_2) \\ &= P(\varepsilon_{1i} > \mathbf{w}_i^T\boldsymbol{\theta}_1, \varepsilon_{2i} > \mathbf{x}_i^T\boldsymbol{\theta}_2) \\ &= \int_{\eta_{2i}}^{\infty}\int_{\eta_{1i}}^{\infty} \phi(\eta_{1i}, \eta_{2i}) d\eta_{1i} d\eta_{2i} \\ &= 1 - \Phi(\eta_{1i}) - \Phi(\eta_{2i}) + \Phi(\eta_{1i}, \eta_{2i}) \end{aligned} \tag{17}$$

The likelihood function of the bivariate random variable can be formed based on the probability in Equations (14), (15), (16), and (17) so that Equation (18) is obtained.

$$\begin{aligned} Q = L(\theta) &= \prod_{i=1}^{n} P(Y_{11i} = y_{11i}, Y_{10i} = y_{10i}, Y_{01i} = y_{01i}, Y_{00i} = y_{00i}) \\ &= \prod_{i=1}^{n} P_{11i}^{y_{11i}} P_{10i}^{y_{10i}} P_{01i}^{y_{01i}} P_{00i}^{y_{00i}} \end{aligned} \tag{18}$$

Where $\boldsymbol{\theta}= (\boldsymbol{\theta_1}, \boldsymbol{\theta_2}, \rho)$.
After obtaining the recursive bivariate binary probit likelihood function, then the natural logarithm of the likelihood function is formed as in Equation (19).

$$\begin{aligned} \ln L(\boldsymbol{\theta}) &= \ln \prod_{i=1}^{n} \left(P_{11i}^{y_{11i}} P_{10i}^{y_{10i}} P_{01i}^{y_{01i}} P_{00i}^{y_{00i}} \right) \\ &= \sum_{i=1}^{n} \left(y_{11i} \ln p_{11i} + y_{10i} \ln p_{10i} + y_{01i} \ln p_{01i} + y_{00i} \ln p_{00i} \right) \end{aligned} \tag{19}$$

To get parameter estimates, the next step is to maximize the function $\ln L(\boldsymbol{\theta})$ to parameters $\boldsymbol{\theta_1}, \boldsymbol{\theta_2}$, and ρ then equal to zero. As for the first derivative $\ln L(\boldsymbol{\theta})$ to $\boldsymbol{\theta_1}, \boldsymbol{\theta_2}$, and ρ is as follows:

$$\frac{\partial \ln L(\boldsymbol{\theta})}{\partial \theta_1} = \sum_{i=1}^{n} \boldsymbol{w_i}\phi(\eta_{1i}) \left[\left(\frac{y_{11i}}{p_{11i}} - \frac{y_{01i}}{p_{01i}} \right) \Phi\left(\frac{\eta_{2i} - \rho\eta_{1i}}{\sqrt{(1-\rho^2)}} \right) + \left(\frac{y_{10i}}{p_{10i}} - \frac{y_{00i}}{p_{00i}} \right) \left(1 - \Phi\left(\frac{\eta_{2i} - \rho\eta_{1i}}{\sqrt{(1-\rho^2)}} \right) \right) \right] \tag{20}$$

$$\frac{\partial \ln L(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_2} = \sum_{i=1}^{n} \boldsymbol{x_i}\phi(\eta_{2i}) \left[\left(\frac{y_{11i}}{p_{11i}} - \frac{y_{10i}}{p_{10i}} \right) \Phi\left(\frac{\eta_{1i} - \rho\eta_{2i}}{\sqrt{(1-\rho^2)}} \right) + \left(\frac{y_{01i}}{p_{01i}} - \frac{y_{00i}}{p_{00i}} \right) \left(1 - \Phi\left(\frac{\eta_{1i} - \rho\eta_{2i}}{\sqrt{(1-\rho^2)}} \right) \right) \right] \tag{21}$$

$$\frac{\partial \ln L(\boldsymbol{\theta})}{\partial \rho} = \sum_{i=1}^{n} \left(\frac{y_{11i}}{p_{11i}} - \frac{y_{10i}}{p_{10i}} - \frac{y_{01i}}{p_{01i}} + \frac{y_{00i}}{p_{00i}} \right) \phi(\eta_{1i}, \eta_{2i}) \frac{4\exp(2\rho^*)}{(\exp(2\rho^*)+1)^2} \tag{22}$$

From the first derivative of the ln likelihood function, it is equated to 0. However, closed form results were not obtained, so a parameter estimation approach was carried out using the Newton-Raphson method. This method uses a vector **g(θ)** whose elements are the first derivative of the ln likelihood function with respect to the parameters and the Hessian matrix **H**. To obtain the components of the matrix **H**, we perform derivatives on each element of the vector **g(θ)** against θ_1, θ_2, and ρ as follows:

$$\frac{\partial \ln L(\boldsymbol{\theta})}{\partial\theta_1\,\partial\theta_1} = \sum_{i=1}^{n}\phi(\eta_{1i})\boldsymbol{w_i^2}\left\{\left(\frac{y_{11i}}{p_{11i}} - \frac{y_{10i}}{p_{10i}} - \frac{y_{01i}}{p_{01i}} + \frac{y_{00i}}{p_{00i}}\right)\left(-\eta_{1i}\Phi\left(\frac{\eta_{2i}-\rho\eta_{1i}}{\sqrt{(1-\rho^2)}}\right) - \frac{\rho}{\sqrt{(1-\rho^2)}}\phi\left(\frac{\eta_{2i}-\rho\eta_{1i}}{\sqrt{(1-\rho^2)}}\right)\right) + \left(\left(\frac{y_{00i}}{p_{00i}} - \frac{y_{10i}}{p_{10i}}\right)\eta_{1i}\right)\right\} \quad (23)$$

$$\frac{\partial \ln L(\boldsymbol{\theta})}{\partial\theta_2\,\partial\theta_2} = \sum_{i=1}^{n}\boldsymbol{x_i^2}\phi(\eta_{2i})\left(\left(\frac{y_{11i}}{p_{11i}} - \frac{y_{10i}}{p_{10i}} + \frac{y_{01i}}{p_{01i}} + \frac{y_{00i}}{p_{00i}}\right)\left[-\eta_{2i}\Phi\left(\frac{\eta_{1i}-\rho\eta_{2i}}{\sqrt{(1-\rho^2)}}\right) + \frac{\rho}{\sqrt{(1-\rho^2)}}\phi\left(\frac{\eta_{1i}-\rho\eta_{2i}}{\sqrt{(1-\rho^2)}}\right)\right] + \left(\frac{y_{00i}}{p_{00i}} - \frac{y_{01i}}{p_{01i}}\right)\eta_{2i}\right) \quad (24)$$

$$\frac{\partial \ln L(\boldsymbol{\theta})}{\partial\theta_1\,\partial\rho} = \sum_{i=1}^{n}\boldsymbol{w_i}\phi(\eta_{1i})\left[\left(\frac{y_{11i}}{p_{11i}} - \frac{y_{01i}}{p_{01i}} - \frac{y_{10i}}{p_{10i}} + \frac{y_{00i}}{p_{00i}}\right)\phi\left(\frac{\eta_{2i}-\rho\eta_{1i}}{\sqrt{(1-\rho^2)}}\right)\right] \frac{-\eta_{1i}(1-\rho^2)^{\frac{1}{2}} + (\eta_{2i}-\rho\eta_{1i}) - \rho(1-\rho^2)^{-\frac{1}{2}}}{(1-\rho^2)} \quad (25)$$

$$\frac{\partial \ln L(\boldsymbol{\theta})}{\partial\theta_2\,\partial\rho} = \sum_{i=1}^{n}\boldsymbol{x_i}\phi(\eta_{2i})\left[\left(\frac{y_{11i}}{p_{11i}} - \frac{y_{01i}}{p_{01i}} - \frac{y_{10i}}{p_{10i}} + \frac{y_{00i}}{p_{00i}}\right)\left(\frac{\phi\left(\frac{\eta_{1i}-\rho\eta_{2i}}{\sqrt{(1-\rho^2)}}\right)}{\frac{-\eta_{2i}(1-\rho^2)^{\frac{1}{2}} + (\eta_{1i}-\rho\eta_{2i}) - \rho(1-\rho^2)^{-\frac{1}{2}}}{(1-\rho^2)}}\right)\right] \quad (26)$$

$$\frac{\partial^2 \ln L(\boldsymbol{\theta})}{\partial\rho\,\partial\rho} = \sum_{i=1}^{n}\left(\frac{y_{11i}}{p_{11i}} - \frac{y_{10i}}{p_{10i}} - \frac{y_{01i}}{p_{01i}} + \frac{y_{00i}}{p_{00i}}\right)\left[\left(\phi(\eta_{1i},\eta_{2i})\frac{(-\eta_{1i}\eta_{2i}) + 2\rho^2\eta_{1i}\eta_{2i})}{(1-\rho^2)}\frac{4\exp(2\rho^*)}{(\exp(2\rho^*)+1)^2}\right) + \phi(\eta_{1i},\eta_{2i})\frac{8\exp(2\rho^*)(\exp(2\rho^*)+1) - 16\exp(2\rho^*)}{(\exp(2\rho^*)+1)^3}\right] \quad (27)$$

After obtaining the Hessian matrix **H**, it is iterated with the Newton-Raphson method until it converges with the iteration formula as in Equation (28).

$$\widehat{\boldsymbol{\theta}}^{(m)} = \widehat{\boldsymbol{\theta}}^{(m-1)} - \left[\boldsymbol{H}\left(\widehat{\boldsymbol{\theta}}^{(m-1)}\right)\right]^{-1}\mathbf{g}\left(\widehat{\boldsymbol{\theta}}^{(m-1)}\right) \quad (28)$$

The iteration stages use the Newton-Raphson method as follows.

a. Specifies the starting value or determines the initial value of $\hat{\theta}$ when m=0.
b. Then do the first iteration starting from m = 1 by counting

$$\widehat{\boldsymbol{\theta}}^{(m)} = \widehat{\boldsymbol{\theta}}^{(m-1)} - \left[\boldsymbol{H}\left(\widehat{\boldsymbol{\theta}}^{(m-1)}\right)\right]^{-1}\widehat{\boldsymbol{\theta}}\left(\widehat{\boldsymbol{\theta}}^{(m-1)}\right) \quad (29)$$

If $\left\|\widehat{\boldsymbol{\theta}}^{(m)} - \widehat{\boldsymbol{\theta}}^{(m-1)}\right\| \leq \Theta$, where Θ is a very small number close to 0, then the iteration stops and the parameter estimation results are obtained.

b. Estimation of Recursive Binary Bivariate Probit Regression Model Parameters

1. Response Variable Independence Test

To carry out bivariate modeling, the first assumption that must be fulfilled is the existence of a relationship (dependency) between the response variables. The hypothesis used in testing the independence of the response variable uses the Chi-square test as follows:

H_0 : There is no relationship between household welfare and the employment sector of the head of the household

H_1 : There is a relationship between household welfare and the employment sector of the head of the household

Based on the results with the Chi-Square test, a p-value of 0.00 was obtained and the H_0 decision was rejected. It can be concluded if there is a relationship between household welfare and the employment sector of the head of the household.

2. Response Variable Endogenity Test

One novelty why one should use recursive bivariate binary probit regression modeling is when there is an endogeneity problem. If there is no endogeneity, then bivariate binary probit regression is used. Hypothesis testing to test endogeneity was carried out using the Lagrange Multiplier Test with the following hypothesis:

H_0 : $\rho = 0$ (there is no endogeneity problem)
H_1 : $\rho \neq 0$ (there is endogeneity problem)

The test results show that the p-value is 0.00 and the decision H_0 is rejected. It can be concluded if there is a residual relationship between the two models. This shows that there is an endogeneity problem in the model so that the suggested modeling is bivariate recursive binary probit regression.

3. Modelling with Recursive Binary Probit

To get the best model, in the modeling process, the predictor variables are eliminated which do not have a significant effect on the partial test. Elimination of variables is carried out using the backward method, which removes the variables one by one starting from the variables that have no effect. The final modeling results using significant variables are shown in Table 2.

Table 2. Testing the significance of parameters in the model

Variable	Recursive Bivariate Binary Probit	
	Estimate	P-value
Household welfare level		
Constant	-2.5512	0.0000
X_1	0.4804	0.0004
X_2	0.1469	0.0053
X_3	-0.2189	0.0011
X_5	-0.1235	0.0355
X_6	0.3422	0.0000
X_7	0.2986	0.0000
X_9	-0.4154	0.0000
X_{10}	-0.0080	0.0000
y_2^*	1.3608	0.0000
Household Head Employment Sector		
Constant	0.3540	0.0000
X_8	-0.5923	0.0000
X_9	-1.5590	0.0000
X_{10}	0.0075	0.0000

Based on the parameter estimation results, a recursive bivariate binary probit model is:

$$y_{1i}^{*} = -2.5512 + 0.4804X_1 + 0.1469X_2 - 0.2189X_3 - 0.1235X_5 + 0.3422X_6 + 0.2986X_7 - 0.4154X_9 - 0.0080X_{10} + 1.3608y_2^{*}$$

$$y_{2i}^{*} = 0.3540 - 0.5923X_8 - 1.5590X_9 + 0.0075X_{10}$$

The recursive bivariate binary probit model has an AIC value of 12539.77. Based on the selection of the best model, then the value $\eta_{1i} = y_{1i}^{*}$ dan $\eta_{2i} = y_{2i}^{*}$ obtained using the recursive bivariate binary probit model. Furthermore, to interpret the recursive bivariate binary probit model, for example a household with a married $X_1 = 1$, the head of household works in the formal sector $X_2 = 1$, the head of household does not experience health complaints (X_3=0), the household live in their own house (X_5=1), head of household is a non-migrant (X_6=1), the number of household members is 11 people (X_7=11), the highest education completed by the head of household is junior high school (X_8=0), classification of area where they live in rural areas (X_9=0), head of household is 55 years old (X_{10}=55). From the equation above, the following probabilities are obtained.

$$P_{11i} = \phi(\eta_{1i,}\eta_{2i}) = \phi(0.7681\ ;\ 2.1821) = 0.7643$$

$$P_{10i} = \phi(\eta_{1i}) - \phi(\eta_{1i,}\eta_{2i}) = \phi(0.7681) - \phi(0.7681\ ;\ 2.1821) = 0.2211$$

$$P_{01i} = \phi(\eta_{2i}) - \phi(\eta_{1i,}\eta_{2i}) = \phi(2.1821) - \phi(0.7681\ ;\ 2.1821) = 0.0144$$

$$P_{00i} = 1 - \phi(\eta_{1i}) - \phi(\eta_{2i}) + \phi(\eta_{1i,}\eta_{2i}) = 1 - \phi(0.7681) - \phi(2.1821) + \phi(0.7681; 2.1821) = 0.0001$$

Based on the calculation above, the greatest opportunity value is obtained at ρ_{11}, namely the probability $Y_1 = 1$ and $Y_2 = 1$ with a value of 0.7643. This shows that these households have a greater chance of being categorized into households with a low level of welfare and the head of household works in the agricultural sector.

4. Bivariate Marginal Effect

The bivariate marginal effect is calculated on the variables used in the equations y_{1i}^{*} and y_{2i}^{*} together, namely the variable of residential area classification (X_9) and the variable age of the household head (X_{10}). The marginal effect for the variable area of residence can be calculated by the following equation.

$$\frac{\partial p_{11i}}{\partial x_9} = \frac{\partial \phi(\eta_{1i,}\eta_{2i})}{\partial x_9} = \beta_{1.9}\phi(\eta_{1i})\phi\left(\frac{\partial\phi(\eta_{2i}-\rho\eta_{1i})}{\sqrt{1-\rho^2}}\right) + \beta_{2.9}\phi(\eta_{2i})\phi\left(\frac{\partial\phi(\eta_{1i}-\rho\eta_{2i})}{\sqrt{1-\rho^2}}\right) = -0.41541\phi(\eta_{1i})\phi\left(\frac{\partial\phi(\eta_{2i}-\rho\eta_{1i})}{\sqrt{1-\rho^2}}\right) - 1.55901\phi(\eta_{2i})\phi\left(\frac{\partial\phi(\eta_{1i}-\rho\eta_{2i})}{\sqrt{1-\rho^2}}\right) = -0.4777$$

$$\frac{\partial p_{10i}}{\partial x_9} = \frac{\partial\phi(\eta_{1i}) - \partial\phi(\eta_{1i,}\eta_{2i})}{\partial x_9} = \beta_{19}\phi(\eta_{1i}) - \beta_{19}\phi(\eta_{1i})\phi\left(\frac{\partial\phi(\eta_{2i}-\rho\eta_{1i})}{\sqrt{1-\rho^2}}\right) - \beta_{29}(\eta_{2i})\phi\left(\frac{\partial\phi(\eta_{1i}-\rho\eta_{2i})}{\sqrt{1-\rho^2}}\right) = -0{,}4154\phi(\eta_{1i}) + 0{,}4154\phi(\eta_{1i})\phi\left(\frac{\partial\phi(\eta_{2i}-\rho\eta_{1i})}{\sqrt{1-\rho^2}}\right) + 1{,}5590\phi(\eta_{2i})\phi\left(\frac{\partial\phi(\eta_{1i}-\rho\eta_{2i})}{\sqrt{1-\rho^2}}\right) = 0.4624$$

The highest marginal effect value is -0.4777 at probability ρ_{11}, so that it can be concluded that the variable area of rural residence reduces the contribution for households categorized into groups of households with low levels of welfare and household heads work in the agricultural sector by 47.77%. However, it has a marginal effect with a value of 0.4624 at ρ_{10} which means that the rural area variable increases the contribution for households categorized into groups of households with low levels of welfare and household heads work in the non-agricultural sector by 46.24%.

The marginal effect for the age variable of the household head can be calculated by the following equation.

$$\frac{\partial p_{10i}}{\partial x_{10}} = \frac{\partial\phi(\eta_{1i}) - \partial\phi(\eta_{1i,}\eta_{2i})}{\partial x_{10}} = \beta_{1.10}\phi(\eta_{1i}) - \beta_{1.10}\phi(\eta_{1i})\phi\left(\frac{\partial\phi(\eta_{2i}-\rho\eta_{1i})}{\sqrt{1-\rho^2}}\right) - \beta_{210}\phi(\eta_{2i})\phi\left(\frac{\partial\phi(\eta_{1i}-\rho\eta_{2i})}{\sqrt{1-\rho^2}}\right) = -0.0080\phi(\eta_{1i}) + 0.0080\phi(\eta_{1i})\phi\left(\frac{\partial\phi(\eta_{2i}-\rho\eta_{1i})}{\sqrt{1-\rho^2}}\right) - 0.0075\phi(\eta_{2i})\phi\left(\frac{\partial\phi(\eta_{1i}-\rho\eta_{2i})}{\sqrt{1-\rho^2}}\right) = -\ 0.0022$$

The highest marginal effect value is obtained at the probability of p_{10} with a value of -0.0022 so that it can be concluded that the age of the head of household 55 years reduces the contribution for households categorized into groups of households with low levels of welfare and household heads working in the non-agricultural sector is -0.0022.

V. CONCLUSIONS

Based on the analysis and discussion, modeling using the recursive binary bivariate probit model shows that variables that significantly affect the level of household welfare (Y_1) include marital status (X_1), employment sector status (X_2), health complaints (X_3), home ownership (X_5), migration status (X_6), number of household members (X_7), region classification (X_9), age of head of household (X_{10}), and sector of work of head of household (Y_2). Meanwhile, the variables that significantly influence the choice of the KRT's employment sector (Y_2) include the KRT education (X_8), region classification (X_9), and the age of the head of the household (X_{10}). In modelling poverty problems, endogeneity problems inevitably occur, so the recursive bivariate binary probit model is an alternative that can be used to solve similar cases.

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Vita Ratnasari born in Jember on September 10, 1970. She is a Professor in the Statistics Department, Faculty of Science and Data Analytics, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia. She received Ph.D Degree at Sepuluh Nopember Institute of Technology, Surabaya, Indonesia. Her research interest is Categorical Data Analysis, Nonparametric-Semiparametric Regression and Its Applications. The results of his research are mostly published in National Journals, International Proceedings, and International Journals Indexed by Scopus.

Syirrul Hadi Utama born on July 24, 1989. He is a Postgraduate student in the Statistics Department, Faculty of Science and Data Analytics, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia. He is currently a research team member with Vita Ratnasari in Social and Population Statistics Laboratorium.

Andrea Tri Rian Dani born in Samarinda on January 16, 1998. He is a Lecturer at the Statistics Study Program, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Mulawarman University, Samarinda, Indonesia. He received M. Stat. Degree at Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia. His research interest is Statistical Modeling with the Nonparametric-Semiparametric Regression approach and Its Applications. The results of his research have been published in several National Journals, International Proceedings, and International Journals Indexed by Scopus.