Robust Optimization of Vehicle Scheduling for the Secondary Distribution of Refined Oil Products Under an Active Distribution Model

Songlin Tian, Cunjie Dai*, Xiaoquan Wang, Runyu Wu, Kexiang Bi and Tingyu Wang

***Abstract*—Considering the time-dependent characteristics and uncertainties of risk factors, distribution vehicle travel time, customer service time, and customer demand in the transportation environment, the optimization problem of scheduling vehicles for the secondary distribution of refined oil products under the active distribution mode is studied. We established a robust multi-objective vehicle scheduling optimization model with the objectives of minimum cumulative transportation risk of all vehicles, minimum total transportation cost, and the customer's satisfaction as the constraint. An improved cultural genetic algorithm (Modified Memetic Algorithm, MMA) is designed based on the model features. We used a genetic algorithm and neighborhood search to improve the algorithm's search performance. We combined the elite retention strategy and inferior solution acceptance strategy to improve search efficiency and maintain the diversity of solutions. For example, we conducted numerical experiments with a single oil product in a city PetroChina gas station. Then, the model's reasonableness and the algorithm's effectiveness are verified using different scale examples. The research results can provide a reference basis for the vehicle scheduling scheme of refined petroleum product transportation enterprises under various uncertain conditions.**

***Index Terms*—secondary distribution of refined oil, active distribution model, vehicle scheduling, robust optimization, uncertainty**

I. INTRODUCTION

IN stabilizing the world's energy landscape, refined oil products are still the primary energy source for most modern societies despite the increasing use of other alternative energy sources (natural gas, electricity, wind, solar) [1]. China has the second-highest consumption of refined oil products globally, which has become essential for its socio-economic development [2]. According to statistical data, the average Chinese annual consumption was 317 million tons throughout 2017-2022, showing a gradual increase yearly. Due to the long distance between the origin of refined oil products and refineries and the actual consumption customers, we need to distribute the primary distribution from refineries to oil depots. And the secondary distribution from oil depots to customers is usually required, resulting in the road transportation cost of all kinds of refined oil products accounting for about 65% of the logistics and transportation cost [3]. Society has widely discussed the distribution problem of refined oil products.

The problem of secondary distribution of refined oil products, also called the gas station replenishment problem, refers to the process in which oil depots distribute refined oil products according to the needs of gas stations or other customers, using their transportation systems or through third-party enterprises [2]. In the transportation process, because of the flammable and explosive properties of refined oil, once an accident occurs, it will pose a more significant threat to the public and social security. Moreover, the increase in social demand has led to an increasing demand for secondary distribution of refined oil products, and enterprises need to generate distribution plans according to different transportation needs to improve economic efficiency and avoid the occurrence of catastrophic accidents. According to Cornillier et al.[4], one of the earliest articles to study this problem was published by Brown and Graves in 1981[5], which investigated the issue of oil product distribution under full-truck-load orders. Subsequently, several articles studied this problem from different aspects.

This paper establishes a vehicle scheduling model for the secondary distribution of refined oil products under uncertain conditions through robust optimization theory, considers the functional relationship between uncertain transportation time and demand, and investigates the impact of risk sources on transportation risks, such as people, vehicles, and environment, through a population exposure risk evaluation model. Under the condition of the time window, load limit, and satisfaction threshold, we established the relevant, robust optimization model. We used a heuristic algorithm to solve the model and obtain the initial population by using the C-W algorithm in the initialization stage of the population and by the neighborhood search operation to increase the diversity of the initial population. Then, we solved the model by using the modified memetic algorithm and introduced the elite retention strategy and the inferior solution acceptance

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strategy. Finally, we analyzed the model and algorithm with some examples to verify the correctness of the model and algorithmic efficiency. It provides theoretical and technical support for developing uncertainty conditions and active distribution modes in the secondary distribution of refined oil products. It further improves the quality of distribution services, guarantees the regular operation of gas stations, and enhances customer satisfaction.

The report described in this article is structured as follows: the following section reviews the literature related to the research. Section III describes the research problem, including the active distribution model, uncertainty analysis, and the risk assessment model for transportation. Section IV develops the mathematical model and performs processing and transformation. Section V describes the algorithm solution process. Section VI shows the solution results of the solution method for different arithmetic cases and comparative analysis with other algorithms. Section VII summarizes and analyzes the researched content and gives an outlook on future research directions.

II. Literature Review

This section provides a comprehensive review of the literature related to the research content, including the problem of secondary distribution of refined petroleum products, the transportation problem under uncertainty, and the risk assessment model, so that the reader can understand the distribution of refined petroleum products, the treatment of uncertainty, and the risk metric model.

In the research on the secondary distribution problem of refined oil products, the established literature needs more research on the secondary distribution of refined oil products under the active distribution model. Bani et al. and Abderrahman et al. considered the refined oil product distribution problem with multiple depots and lead times under static conditions [1][5]. Cornillier et al. have been researching refined oil product distribution for conditions such as multi-depots, heterogeneous vehicles, and time windows to establish a distribution Route optimization model[4][6][7]. Alinaghian et al. studied and modeled the refined oil product distribution problem under multi-depot and multi-bay conditions [8]. Al-Hinai et al. solved the refined oil product distribution problem with periodicity constraints and service frequency selection conditions through a two-stage evolutionary algorithm [9]. Benantar et al. studied the problem under dynamic conditions of adjustable demand [10]. Boers et al. developed a mixed integer programming model for a multi-periodic refined oil distribution problem and solved it with a heuristic algorithm [11]. Most existing research adopts a passive distribution mode, treating the secondary distribution of refined oil products as a common VRP problem, and seldom studies the active distribution mode, with less consideration of the dynamic characteristics of the transportation environment and the changes in customer demand. The active distribution model allows suppliers to take the initiative in distribution according to customer needs, provide customized services to meet customers' individual needs, maintain close contact with customers, reduce communication and coordination costs, and has practical research significance.

In the actual distribution process, due to the complexity of the environment, it is tough to obtain accurate data on the values of the attributes of the refined oil products distribution routes, such as transportation time, customer demand, and transportation risk due to the time-varying characteristics of the road network. Therefore, studying the refined oil product distribution problem under uncertain conditions is more realistic, and scholars use different methods to deal with uncertain situations. Zandieh et al. considered the uncertainty in the hazardous materials route [12]. Mehrdad et al. considered uncertainty under hub topology on a reliable hazardous materials transportation network [13]. Mohammadi investigated demand uncertainty and dealt with it using a constrained sizing model [14]. Fatemeh et al. developed an optimization model using the fuzzy approach for hazardous material transportation risk uncertainty [15]. Zahra et al. modeled the uncertainty by considering the uncertainty in accident probability, carbon emission factor, and cost and dealt with the uncertainty using robust optimization [16]. Erfan et al. proposed a new mixed-integer linear programming model under demand uncertainty [17]. Existing researchers and scholars mainly use the methods of expected value and opportunity-constrained planning to solve the problem, and the robust optimization problem needs to be studied more. At the same time, this paper adopts robust optimization to consider the uncertainty conditions. Robust optimization entirely finds the uncertainty of the situation in the modeling process, and compared with stochastic and fuzzy planning, robust optimization does not need the distribution model of uncertain parameters and the fuzzy affiliation function of uncertain parameters, and its constraints are strictly established.

Reducing the risk arising from the transportation process requires quantifying the risk value to calculate the risk value along the distribution route. Many different risk metric models have been proposed and widely used. Alp proposed a traditional risk model for the risk along the route of a hazardous material vehicle, aiming to minimize the impact's expected value [18]. Based on this, other models have been proposed, such as the accident probability model [19], population exposure model [20], and perceived risk model [21]. The accident probability model focuses on reducing the probability of accidents, the population exposure model minimizes the impact of accidents, and the perceived risk model uses multiple criteria for weighting to balance safety and efficiency. Erkut proposed the maximization risk model, the mean-variance model, and the effects model [22]. In the maximization risk model, the minimization risk objective changed to the minimum maximization objective, the mean-variance model randomizes the Route connectivity attributes and selects the route with the smallest mean-variance, and the effects model measures the risk using the effects theoretical degree of risk. Existing studies measure risk by improving the above models and building risk assessment models according to the research.

III. Problem Definition

In this article, we can describe the vehicle scheduling problem of secondary distribution of refined oil products under active distribution mode as follows: in the refined oil products transportation network, based on the active distribution mode, vehicles are scheduled by the distribution center according to the inventory information of gas stations, and the cars depart from the distribution center to distribute oil products to multiple demand points and return to the distribution center after the distribution is completed and returned to the distribution center. Due to the influence of natural conditions, how can a reasonable vehicle scheduling

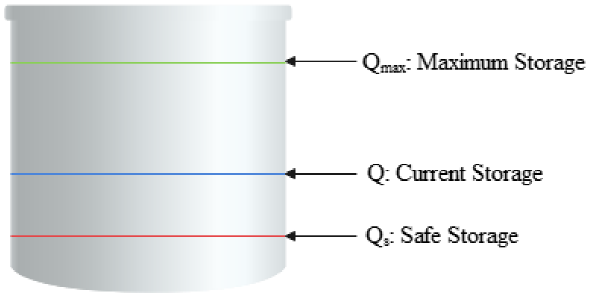

Fig 1 Schematic diagram of tank inventory

plan be found to get the minimum transportation cost and transportation risk under the uncertainty of customer demand and transportation time, time window constraints, and load limitations?

In this study, we make the following assumptions:

(1) Distribution center inventory meets all customer demands;

(2) The exposed population density between nodes is known;

(3) The fixed costs of the vehicles are known;

(4) The transportation oil is a single species, and the vehicle is a homogeneous, single-compartment vehicle;

(5) Customer requirements are not separable;

(6) The daily working hours of petrol stations and the average unit consumption rate of oil products are fixed.

A. Active distribution model

The active distribution model performs a judgmental analysis of the existing inventory at the gas station, predicts how much oil we need to distribute and when we distribute the oil, and then optimizes the scheduling of the vehicles. Let's simplify this problem to solve, and the focus is on the robust optimization of vehicle scheduling in active distribution mode.

(1) Active distribution judgment mechanism

Before establishing the judgment mechanism of active distribution, we need to clarify the relevant storage capacity of the tank, as shown in Figure 1, including the maximum storage capacity of Q_{max}, the current storage capacity of Q, the safe storage capacity of Q_s, through the changes in the storage capacity of the tank of oil to establish the corresponding judgment mechanism and the initial prediction of the demand for gas stations.

We build the judgment mechanism in two steps. First, we judge the current storage capacity of the gas station, assuming that the sales volume of the gas station obeys a normal distribution. According to the service level, we determine the safe storage capacity, which we calculate by the average unit time consumption rate of oil as V_{oil} and the daily working time of the gas station as T_{work}. Then, we construct the satisfaction function related to the current and safe storage volumes to improve the oil depot's transportation efficiency and the gas station's satisfaction and establish a double judgment distribution mechanism considering the satisfaction threshold. The specific judgment mechanism is as follows:

①**Storage capacity judgment:** When the current storage capacity cannot meet the day's fuel consumption on a secure depot basis, we need to arrange distribution services for that station, and if the current storage capacity can meet the demand, distribution is unnecessary.

$$\begin{cases} if \quad Q - V_{oil} \times T_{work} \geq Q_s, & refuse \\ else &, \quad agree \end{cases} \tag{1}$$

②**Satisfaction judgment**: Since we don't consider the gas station's inventory cost when we distribute, the greater the customer's demand when arriving there, the higher the customer's satisfaction. When $Q \leq Q_{max}$-Q_s, the satisfaction $\varphi \leq$ 1; When $Q \geq Q_{max}$-Q_s, the satisfaction $\varphi \geq 0$, and the specific functional relationship as shown in Eq. 2:

$$\varphi = \frac{Q_p}{Q - Q_s} + \frac{Q - Q_{\max}}{Q - Q_s} \tag{2}$$

We need to set the satisfaction threshold "θ" by considering the transportation time and practical factors, which we can adjust according to different situations. When we choose a more significant satisfaction threshold, the current storage quantity Q is close to the safety stock quantity Q_s; it needs to be delivered quickly to avoid the risk of running out of oil, and transportation is difficult. Usually, scheduling the vehicle with a satisfaction threshold of 1 is impossible, but arriving at the gas station with a customer satisfaction level of 1 is possible. Choosing the right threshold conditions can improve transportation efficiency and average customer satisfaction.

(2) Determination of demand

In real life, we find that the consumption rate of refined oil products at gas stations shows different consumption patterns over time, and the average consumption rate of oil products remains generally stable. Based on the current storage volume and the oil consumption rate, we guarantee that the storage volume Q is above the safe storage volume Q_s when the vehicle is delivered. Since we don't consider the inventory cost, we maximize the revenue of the gas station, and customer satisfaction is highest when the oil inventory reaches just the safe storage Q_s. Based on the satisfaction thresholds, the initial and maximum delivery volumes can be derived, and the actual demand of the gas station should be located within this range.

The formula for calculating demand Q_d based on the satisfaction threshold is Eq 3:

$$Q_d = \theta \times (Q - Q_s) + Q_{\max} - Q \tag{3}$$

The formula for calculating the maximum demand Q_u is Eq 4:

$$Q_u = Q_{\max} - Q_s \tag{4}$$

We show the theoretical maximum distribution volume in Eq 4. Still, we should consider the distribution vehicle's actual capacity in reality, which is the maximum acceptable demand of the gas station. The demand for the gas station should be within the range [Q_d, Q_u]. We determine the service time window from the gas station's demand and the oil product's average unit time consumption rate.

B. Uncertainty analysis

The uncertainty factors in the secondary distribution of refined oil products mainly include the uncertainty of transportation time and the uncertainty of demand for gas stations. The uncertainty of transportation time is primarily due to the existence of a large number of uncertain factors in the transportation process of the natural environment, such as the road being congested, road surface temporary maintenance, and other unforeseen circumstances; we can't guarantee the weather changes, the emergence of inclement weather will have an impact on the transportation time, the driver's choice of routes, the speed of the vehicle control and

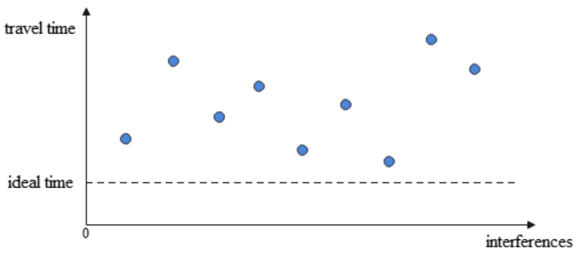

Fig 2 Travel time under interference

other factors, so that these extrinsic factors and intrinsic factors together determine the uncertainty of the transportation time, as shown in Figure 2, under the interference of external and internal factors, the transportation time will be fluctuating so that the vehicle can't arrive at the ideal time.

In the passive distribution mode, the staff gives the expected demand quantity of gas stations based on historical data and their experience perception. However, in the whole system operation process, the staff's subjective behavior, holidays, weather, and other changes in the objective environment may lead to demand deviation. Therefore, we can't determine the demand for some types of oil when the next moment arrives. In the active distribution model, we assume that there is a functional relationship between customer demand and transportation time. Since the transportation time is an uncertain variable, according to the relevant definition of uncertain variables in the uncertainty theory [23], it is known:

Definition 1: Let ξ_1, ξ_2, ξ_3, …, ξ_n be uncertain variables, and let f be a real-valued measurable function. Then is an uncertain variable defined by:

$$\xi(\gamma) = f(\xi_1(\gamma), \xi_2(\gamma), \xi_3(\gamma), ..., \xi_n(\gamma)), \forall \gamma \in \Gamma)$$

With the above definition, we find that an uncertain variable remains uncertain after it has been varied through a function, so customer demand is also uncertain. We processed both uncertain variables through the robust optimization discrete model proposed by Bertsimas and Sim [24] and gave the transformation results in Section IV C.

C. *Risk assessment model*

In research, we usually express transportation risk as the product of the consequences of an accident and the probability of an accident, and the risk assessment of refined oil transportation requires an understanding of the characteristics of the transported goods and the risk characteristics. In the transportation process, people, vehicles, roads, and the environment are the essential elements of transportation and the primary sources of transportation risk, which we need to consider for the impact on the transportation risk. Erukt et al. proposed in their study that the consequences of accidents should be evaluated directly as risks to avoid neglecting the prevention and control of high-loss, low-probability accidents [25], such as the population exposure model [20]. We combined the above research to set the risk index of hazardous sources, and the risk evaluation model was established by considering the explosion impact area of oil products:

$$R = C_{ij} \tag{5}$$

$$C_{ij} = S_{ij}\rho_{ij}P_{ij}V_{ij}E_{ij} \tag{6}$$

$$S_{ij} = \pi \times \lambda^2 \tag{7}$$

Where R is the total risk within the sphere of influence of the accident, C_{ij} is the consequences of an accident, usually expressed as the number of people in the area of the accident, S_{ij} is the size of the area affected at the time of the accident, λ is the radius of the area affected, the density of the exposed population in the area of the accident, ρ_{ij} is the density of the exposed population in the area of the accident, P_{ij} is the risk index of violations by drivers, V_{ij} is the hazard index of the state of the vehicle itself and its attachments and the state of its operation, E_{ij} is the hazard index of the adverse natural environment and the driving environment.

D. *Transportation cost model*

In the process of refined oil distribution, transportation costs are incurred, mainly including three parts: (1) fixed costs of vehicles C_1: fixed costs incurred by vehicle distribution, such as vehicle dispatch costs and vehicle wear and tear costs; (2) vehicle driving costs C_2: vehicle driving costs include the cost of drivers in the distribution process, fuel consumption costs and vehicle maintenance costs, which are related to the transportation distance; (3) penalty costs C_3: penalty costs Refers to the penalty cost when the oil storage volume reaches under the safe storage after the vehicle arrives later than the service time. We calculate the three transportation costs as follows:

$$\begin{aligned} C_1 &= \sum_{i\in M}\sum_{k\in K} x_{0ik}C_g \\ C_2 &= \sum_{i\in M}\sum_{j\in M}\sum_{k\in K} x_{ijk}d_{ij}C_f \\ C_3 &= \sum_{i\in M} a\times \max\{(t_i - t_{LT}^i, 0)\} \end{aligned} \tag{8}$$

In the above equation, C_g is a fixed cost factor for dispatch, C_f is a unit cost of travel factor for vehicles, and α is a cost penalty factor.

IV. Mathematical Model

A. *Relevant parameters*

Table 1 Set Parameter Definition

Symbol	Meaning [a]
G	transportation networks, undirected graphs
V	Set of nodes $V = \{0,1,2,3,...,N\}$, 0 is depot, $M \subseteq N$
L	Customer set $M = \{1,2,3,...,M\}$
M	Route set $L = \{(i,j) \mid i,j \in V, i \neq j\}$
K	Customer set $K = \{1,2,3,...,K\}$

B. *Robust optimization model*

Based on the above analysis, we construct a robust optimization model for vehicle scheduling for the secondary distribution of refined oil products using the discrete robust optimization theory of Bertsimas and Sim [24].

$$\min C = C_1 + C_2 + C_3 \tag{9}$$

$$\min R = \sum_{i\in N}\sum_{j\in N} R_{ij}x_{ijk} \tag{10}$$

$$\sum_{i\in M}\sum_{j\in V} x_{ijk} \le 1 \quad \forall i \in M, j \in V, k \in K \tag{11}$$

$$\sum_{i\in V}\sum_{j\in M}\sum_{k\in K} x_{ijk} = \sum_{i\in M}\sum_{j\in V}\sum_{k\in K} x_{ijk} = 1 \quad \forall k \in K, i \neq j \tag{12}$$

Table 2 Symbol Description

Symbol	Meaning
[ET, LT]	Time window for customer acceptance of service
C	Transportation cost
R	Transportation risk
C_q	Vehicle fixed cost
C_f	Vehicle running costs
W	Vehicle weight limits
k	service to customers through vehicle k , $k \in K$
t_{ij}	Nominal transportation time for distribution vehicles from gas station i to gas station j
Δt_{ij}	the deviation of the variable transportation time of a distribution vehicle from gas station i to gas station j , relative to its nominal value, $\Delta t_{ij} > 0$
$\tilde{t}_{ij}$	Variable transportation time of a distribution vehicle from gas station point i to gas station point j , $\tilde{t}_{ij} \in [t_{ij} - \Delta t_{ij}, t_{ij} + \Delta t_{ij}]$
q_i	Nominal value of demand at gas station i
Δq_i	Variable demand at gas station i , deviation from its nominal value, $\Delta q_i > 0$
q_i	Variable demand at gas station i , $q_i \in [q_i, q_i + \Delta q_i]$
J_i^t	The set consisting of the column subscripts j of all uncertain data $\tilde{t}_{ij}$ in row i of the variable time matrix $\tilde{t}_{ij}$, $\lvert J_i^t \rvert \leq n$
Γ_i^t	The parameter Γ_i^t is used to regulate the robustness as well as the degree of conservatism of the transport time of the robustly optimized discrete model, which can be a float, $\Gamma_i^t \in [0, \lvert J_i^t \rvert]$
$\lfloor \Gamma_i^t \rfloor$	Largest integer less than or equal to Γ_i^t
S_i^t	The set of column subscripts j in row i of the variable time matrix $\tilde{t}_{ij}$ that have changed as a result of the uncertainty, $S_i^t \in J_i^t$
J_i^q	The set consisting of all column subscripts j of uncertain data q_i in row i of the variable demand matrix q_i , $\lvert J_i^q \rvert \leq n$
Γ_i^q	The parameter Γ_i^q is used to regulate the robustness as well as the degree of conservatism of the transport time of the robustly optimized discrete model, and can be a float, $\Gamma_i^q \in [\lvert 0, \lvert J_i^q]$
$\lfloor \Gamma_i^q \rfloor$	Largest integer less than or equal t $\lfloor \Gamma_i^q \rfloor$
S_i^q	The set consisting of the column subscripts j of the columns of q_i in row i of the variable demand matrix q_i that change as a result of uncertainty, $S_i^q \subseteq J_i^q$
T_{jk}	Specific time of arrival of vehicle k at gas station j
T_i	Service hours at gas stations i
x_{ijk}	vehicle k from gas station i to gas station j is 1, otherwise 0, $i \neq j$

$$\sum_{i \in V} x_{ijk} - \sum_{j \in V} x_{jlk} = 0 \quad \forall l \in V, k \in K \tag{13}$$

$$\sum_{i \in V} \sum_{j \in V} q_i x_{ijk} + \max_{\{S_k^q \cup \{t_k^q\} \mid S_k^q \subseteq J_k^q, \lvert S_k^q \rvert = \lfloor \Gamma_k^q \rfloor, t_k^q \in J_k^q \setminus S_k^q\}} \{\sum_{i \in V} \sum_{j \in S_k^q} \Delta q_i x_{ijk} + (\Gamma_k^q - \lfloor \Gamma_k^q \rfloor) \sum_{i \in V} \Delta q_{it_k^q} x_{it_k^q k}\} \leq W \quad \forall k \in K \tag{14}$$

$$T_{jk} = T_{ik} + T_i + \sum_{i,j \in V} t_{ij} x_{ijk} + \max_{\{S_k^t \cup \{t_k^t\} \mid S_k^t \subseteq J_k^t, \lvert S_k^t \rvert = \lfloor \Gamma_k^t \rfloor, t_k^t \in J_k^q \setminus S_k^q\}} \{\sum_{i \in V} \sum_{j \in S_k^t} \Delta t_{ij} x_{ijk} + (\Gamma_k^t - \lfloor \Gamma_k^t \rfloor) \sum_{i \in V} \Delta t_{it_k^t} x_{it_k^t k}\} \quad \forall k \in K \tag{15}$$

$$ET \leq T_{jk} \leq LT \tag{16}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \leq \lvert S \rvert - 1, \quad \forall S \in V, 1 < \lvert S \rvert < n \tag{17}$$

Eq. (9)-(10) denotes the transportation cost and transportation risk objective function. Eq. (11) states that only one vehicle serviced each gas station and only made one delivery. Eq. (12)-(14) represent vehicle-related constraints, respectively, the delivery vehicle departs from the distribution center and returns to the distribution center after completing the delivery task; the car arrives at a particular gas station, and departs from the gas station after completing the service; customer demand to satisfy the vehicle load constraint. Eq. (15)-(16) are time dependent constraints, denote the specific time when the vehicle arrives at the gas station and the time window constraint. Eq. (17) is the elimination of sub-loop constraint, S denotes the proper subset of the set V (set S ≠ V), and |S| denotes the number of vertices in the set S.

C. Robust peer-to-peer model

To solve the robust optimization problem, we must transform the original model to a certain degree to transform the uncertain variables into deterministic conditions. In robust optimization theory, we generally assume that the uncertain variables obey some specific convex set. In this paper, we think that the uncertain variables follow the budget uncertainty set proposed by Bertsimas and Sim [24] and define the uncertain transportation time and demand uncertainty sets. We transform the robust discrete optimization model.

The budget uncertainty set is shown in Eq. (18):

$$u=\{\phi:\sum|\phi|\le\Gamma,|\phi|\le 1 \quad \forall i\in N\} \tag{18}$$

Where N denotes the set, for transportation time $t_{ij}\in u$ and customer demand $q_i\in u_q$, the uncertainty constraints are time window and load weight constraints, the deterministic transformation of the above limitations, we transformed the uncertainty constraints of customer demand (14) as shown in Eq (19) and transform the uncertainty constraint (15) on the transportation time into what showing in Eq (20). After changing the model, the analysis indicates that the model is a mixed-integer planning model, which can be solved using commercial solvers or other means, and we give the details of the solution in Section V.

$$\begin{array}{ll}
\sum_{j\in[N]} q_i x_{ijk}+\phi_i^q\Gamma_i^q+\sum_{j\in J_i^q} p_{ij}^q\le W & \forall i\in M, \\
\phi_i^q+p_{ij}^q\ge q_i y_i & \forall j\in J_i^q, i\in M, \\
-y_j^q\le x_{ijk}\le y_j^q & \forall j\in N, \\
l_j^q\le x_{ijk}\le u_j^q & \forall j\in N, \\
p_{ij}^q\ge 0 & \forall j\in J_i^q, i\in M, \\
y^q\ge 0 & \forall j\in N, \\
\phi_i^q\ge 0 & \forall i\in M\cup\{0\}, \\
x_{ijk}\in\{0,1\} & \forall i\in M.
\end{array} \tag{19}$$

$$\begin{array}{ll}
ET\le T_{ik}+T_i+\sum_{j\in[N]} t_{ij}x_{ijk}+\phi_i^t\Gamma_i^t+\sum_{j\in J_i^t} p_{ij}^t\le LT & \forall i\in M, \\
\phi_i^t+p_{ij}^t\ge \hat{t}_{ij}y_j^t & \forall j\in J_i^t, i\in M, \\
-y_j^t\le x_{ijk}\le y_j^t & \forall j\in N, \\
l_j^t\le x_{ijk}\le u_j^t & \forall j\in N, \\
p_{ij}^t\ge 0 & \forall j\in J_i^t, i\in M, \\
y^t\ge 0 & \forall j\in N, \\
\phi_i^t\ge 0 & \forall i\in M\cup\{0\}, \\
x_{ijk}\in\{0,1\} & \forall i\in M.
\end{array} \tag{20}$$

V. SOLUTION METHOD

The refined oil secondary distribution vehicle scheduling problem is a specific application of the VSP problem, a typical NP-hard problem [26], and we often solve it by heuristic algorithms [1]-[5], [7]-[14], [16]-[17], [27]-[28]. In this paper, we first use the improved C-W conservation algorithm to obtain the initial solution and then get the optimal scheduling plan by the Memetic Algorithm (MA) [29]. The Memetic Algorithm is an optimization algorithm based on simulated cultural evolution proposed by Pablo in 1989 [30], which is essentially a combination of population-based global search and individual-based local heuristic search. This combination mechanism makes its search efficiency several orders of magnitude faster than traditional genetic algorithms in some problem domains, which we can apply to various disciplines and obtain satisfactory results. We use a genetic algorithm for the global optimization strategy and a neighborhood search for the local optimization strategy. We then introduce an elite retention strategy and an inferior solution acceptance strategy called the Modified Memetic Algorithm (MMA).

A. Chromosome coding and decoding

We use natural number coding, assuming there are M customers, one distribution center, and the chromosome length is the number of customers M. Their arrangement is the order in which we serve customers. We must ensure we service all the customers and examine the vehicle distribution routes using vehicle load constraints.

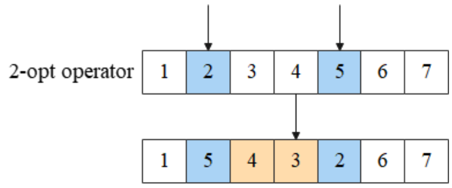

Fig 3 2-opt execution process

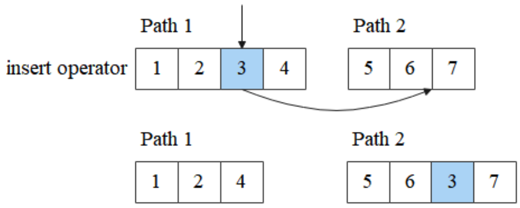

Fig 4 insert execution process

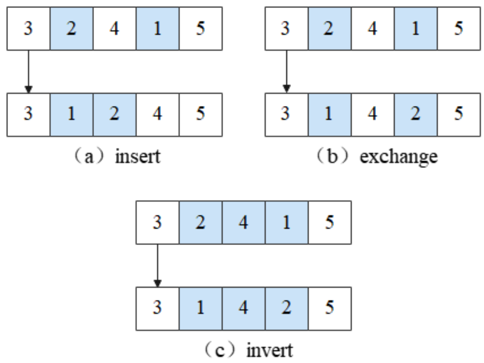

Fig 5 mutation execution process

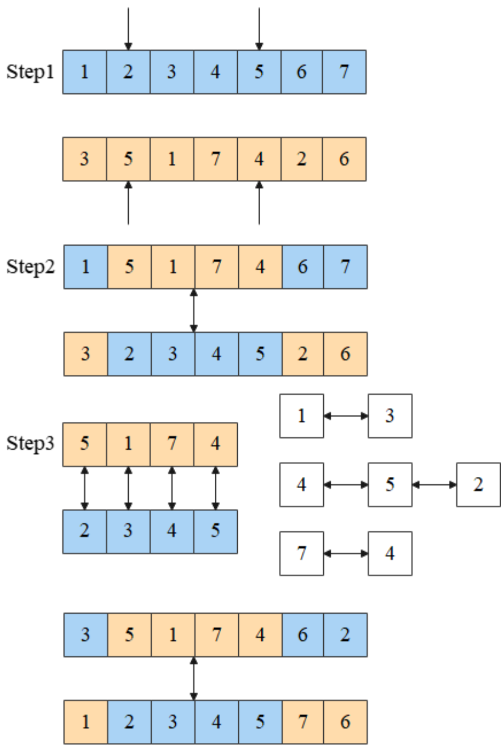

Fig 6 PMX crossover execution process

B. Population initialization

The C-W algorithm is a method proposed by Charke and Wright to solve the CVRP [29]. Still, it can't obtain the optimal solution to the vehicle route problem, but it can be a better feasible solution. This paper uses the improved C-W algorithm to bring the initial population based on considering load constraints. The initial population is diversified using two neighborhood operations: 2-opt operator and insert operator to get a better initial solution. 2-opt operator, Figure 3: randomly selects two nodes in a route to exchange positions and flips the sequence of routes between them. Insert operator, Figure 4: removes a node from the current route and inserts it into another position on a different route. Calculate the saving value; if it is greater than the original route, keep it; otherwise, preserve the original route.

C. Evolutionary operations

We make the population generation proceed using the genetic algorithm of crossover and mutation. The purpose of crossover is to pass the superior gene structure in the parent chromosome to the offspring and thus obtain a better solution. The goal of mutation is to increase the diversity of the population through the perturbation of uncertainty. This paper uses PMX (Partial Matching Crossover) crossover and three types of mutation; when the population is performing the crossover, we need to conflict detection and correction of crossover chromosomes, and the mutation methods are insertion, exchange, and inversion mutation. The specific mechanism of action is shown in Figure 6.

D. Local search

Cultural and genetic algorithms evolve populations through crossover and mutation generation of chromosomes; the choice of crossover and mutation parameters affects the quality of the solution; we use =local search to improve the efficiency of the algorithm's solution. In this paper, we conduct the local search process through neighborhood search, which operates within and between routes. We work on routes through the partial neighborhood operation proposed by Bezerra [31], which uses customer satisfaction thresholds and load capacity constraints to eliminate non-feasible solutions. We study only feasible solutions at each action, and if a different operation produces a non-feasible solution, we execute the operator again until we find a possible solution.

In the process of the in-route operator, when we select gene position 1 of the chromosome for the operation, the departure time of the vehicle changes according to the service time window of gene position 1. At this time, we update the vehicle's departure time and compute the function's objective value, which we keep if it is better than the original scheme and not retained if it is better than the original scheme. The departure time of the vehicle remains unchanged when we select the rest of the gene positions. We give the following three operators in a route, and the specific operation steps are as follows, Figure 7-9:

Reinsert operator: removes a client from the current route and inserts it into another location in that route; Or-opt operator: removes two consecutive clients and inserts them into another position in the current route; 3-opt operator: selects the routes between the three nodes in the route that are not adjacent to each other to delete, and then test all the possibilities of swapping between them to generate a new route.

When we operate in the active distribution mode for customers who need to make distributions. Firstly, for departure time, we treat it in the same way as the in-route operator. Then, when the vehicle's departure time and arrival time violate the customer's time window constraints, we eliminate the solutions in this category directly. In the end, for the solutions that do not violate the time window constraints, we compute the satisfaction level of each customer and the load factor of the route. After that, we eliminate the solutions that do not reach the threshold and solutions that violate the load factor constraint. The inter-route operator includes the following three operations, Figure 10-12.

Swap operator: exchanges a customer in one route with a customer in another route; Shift operator: moves a client from one route to another; Ex-opt2 operator: removes two consecutive clients from one route and inserts them randomly into another route.

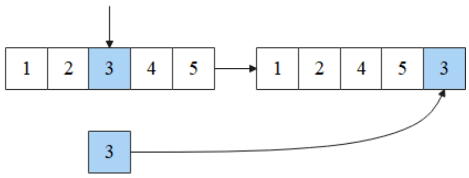

Fig 7 Reinsert execution process

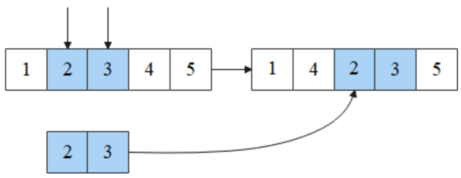

Fig 8 Or-opt2 operator execution process

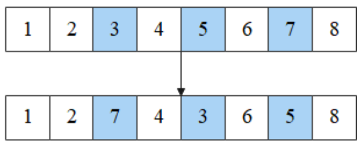

Fig 9 3-opt execution process

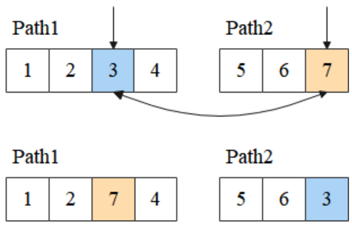

Fig 10 Swap execution process

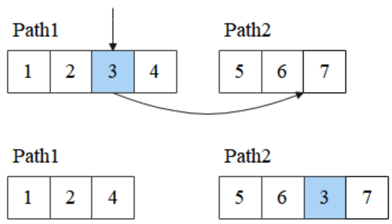

Fig 11 Shift execution process

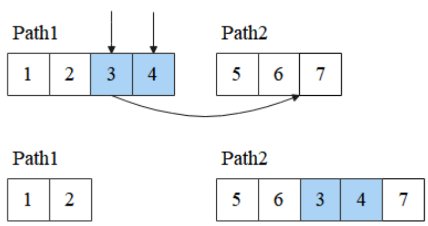

Fig 12 Ex-opt2 execution procesS

E. Elite retention strategies

We classify populations with fast, non-dominated sorting [32], where each individual is compared to all individuals in the population to decide whether it is dominated or not and determine and decide whether the new population can accommodate that domination level. If it can, populate the new population with all individuals of that dominance class and continue to determine whether it can fully accommodate the next dominance class until it cannot accommodate all individuals Fi, assuming class i+1.

Calculate the crowding distance, which generally refers to the distance between an individual in the Pareto frontier or other dominance hierarchy and other individuals after the population is sorted non-dominantly according to the dominance relationship [32], indicating the degree of crowding (denseness) among individuals. Define the crowding comparison operator, assuming that *i*, *j* denotes the individuals in the population. If any condition of Eq (21) is satisfied, individual *i* is considered to be superior to individual *j*, and individual *N-Fi* is selected to populate the new population.

$$\begin{cases} i_{rank} < j_{rank} \\ i_{rank} = j_{rank},\ and\ J_i > J_j \end{cases} \tag{21}$$

F. Inferior solution acceptance strategy

For the vehicle scheduling problem, the departure time of the vehicle and the delivery route are the key to the problem. The elite retention strategy obtains a large number of non-inferior solutions. Still, there may be routes with too much similarity in the solutions [33], and we need to eliminate the related routes to expand the search space further and maintain the diversity of solutions. Considering the effect of different departure times on the solutions, combined with related research [33][34], we calculate the similarity of the solutions and eliminate the solutions with high likeness. Then, the solutions with lower similarity at the next level are sequentially selected to join the population. We established an adaptive mechanism: the probability of choosing an inferior solution should gradually reduce with the increase in the number of iterations. The similarity calculation method of adding time difference and adaptive mechanisms are as follows:

$$\lambda(s_i,s_j)=\frac{2c_a(s_i,s_j)+2c_b(s_i,s_j)}{M(s_i)+M(s_j)+N(s_i)+N(s_j)}+\sum\frac{1}{\Delta t(s_i,s_j)} \tag{22}$$

Where $c_a(s_i, s_j)$ refers to the number of identical sections within the route, $c_b(s_i, s_j)$ means the number of identical nodes within the route, $M(s_i)$ denotes the number of road segments within the route, $N(s_i)$ denotes the number of routes to the solution, $\Delta(s_i, s_j)$ indicates the difference in time to reach the same section within the route.

$$P_x=\begin{cases} 1\ ,\ \lambda(s_i,s_j)\le\lambda(s_i,s_j)^{'} \\ exp(\dfrac{-(\lambda(s_i,s_j)-\lambda(s_i,s_j)^{'})}{Max\_Gen\text{-}Gen})\ ,\ \lambda(s_i,s_j)>\lambda(s_i,s_j)^{'} \end{cases} \tag{23}$$

Where: P_x denotes the probability of accepting the solution, $\lambda(s_i,s_j)$ denotes the similarity for the solution, $\lambda(s_i,s_j)^{'}$ denotes the similarity of the solution after iteration, *Max_Gen* denotes the maximum number of iterations, which is a pre-given value, *Gen* is the number of iterations, and *exp* denotes the natural index.

VI. Experimental Results

Due to the unique nature of the transport of refined oil products, the specific practical use of case data is challenging to obtain. As an example, a city within the inner ring of PetroChina part of the gas station point of a single oil product, endowed with the characteristics of its commodity to carry out numerical experiments, including a distribution center, 61 customer points. The coordinates of the points by the topology of the map showing in Table 3, any two nodes in this undirected network graph have a route connection, independent of the direction of vehicular travel. According to the law, hazardous materials vehicles traveling speed within the city road is not higher than 60km/h. This paper assumes that we generate the rate based on the interval [30-50] in different sections of the road randomly and the vehicle is allowed to carry a weight of 15t.

For the designed MMA algorithm, we carried out programming simulation experiments on Visual Studio 2022. The model is solved and analyzed based on the examples under nominal values. The relevant parameters of the algorithm are as follows: the satisfaction threshold is 0.5; in the cost parameter, the fixed cost is 300 RMB/vehicle; the comprehensive cost of fuel consumption is 58 RMB/km; the penalty coefficient is 2, the crossover probability is 0.5, and the mutation probability is 0.05.

A. Analysis of results

The algorithm runs under Intel(R) Core(TM) i5-8250U CPU, Windows11 system; the algorithm obtains the first three groups of optimal results for each objective value as shown in Table 4, the optimal results for each objective and the distribution path as shown in Table 5, and the number of vehicles used under optimal conditions and the delay time as shown in Table 6.

From the data in Table 4, we can find out that: (1) The various optimization objectives of the secondary distribution routes of refined oil products are in apparent conflict, and the case of satisfying the optimum at the same time does not exist; (2) The transportation enterprises and the government can choose the corresponding distribution routes according to their respective preferences, and the enterprises are inclined to the optimization of the transportation cost, while the government is inclined to the optimization of the transportation risk, and the selection of the routes is appropriately compromised concerning the different optimization objectives, to avoid the taking of the worst value under the part of the objectives; (3) The number of vehicles in use is relatively low when the transportation cost is low, and the number of vehicles in use is relatively large when the transportation risk is low.

Comparative analysis of the optimal results of each objective, from Table 5-6: (1) When the transportation cost C optimal, the transportation risk R relative to the optimal transportation risk increased by 21.98%, but its transportation cost reduced by 31.43%, the number of vehicles used by 5 vehicles, accounting for 45.45%. For the transport enterprises, can effectively reduce transportation costs, improve economic efficiency. The delay time reduced by 54.33%, for gas stations, it can effectively improve customer satisfaction and minimize the risk of fuel shortage at gas stations, improve the service efficiency of gas stations. And the reduction in the use of vehicles can effectively reduce carbon emissions and reduce environmental pollution; (2) When the transportation risk R is optimal, the transportation cost C increases by 45.83% relative to the optimal transportation cost, but its transportation risk is reduced by 18.02%. The program for the transport enterprise, the economic benefits are not considerable, the number of vehicles use and delay time are increased more, especially the latter more than 100%, making the customer satisfaction decreased. But from the government level, social level, effectively reduce the transportation risk so as to protect the people's safety and increase the happiness of the people.

In summary, reasonable vehicle scheduling arrangements can reduce transportation costs and risks. Decision makers start from different levels of consideration to choose reasonable scheduling and transportation solutions. However, there is always a contradiction between transportation cost and transportation risk when one side meets the optimal; the other side is a non-inferior solution. We can set a threshold to prevent the optimization level of a particular aspect of the bias from being high to choose a compromise solution, to improve the acceptance of the decision makers and implementers.

Table 3 Node Attributes

Node	X	Y	Service time	Demand	Node	X	Y	Service time	Demand
0	40	40	0	0	31	45	60	[14.7,15.4]	[1,1.05]
1	19	54	[11.2,11.8]	[1,1.05]	32	45	55	[10.1,10.6]	[0.9,0.945]
2	19	51	[12.8,13.4]	[1.5,1.575]	33	45	51.5	[10.9,11.4]	[1.1,1.155]
3	19	47	[10.5,11]	[1.2,1.26]	34	45	46	[11.1,11.7]	[1,1.05]
4	16	41	[13.2,13.9]	[1.3,1.365]	35	45	40	[10.2,10.7]	[0.7,0.735]
5	13	36	[14.5,15.2]	[1.1,1.155]	36	45	35	[11.4,12]	[0.7,0.735]
6	10	33	[10.5,11]	[1.5,1.575]	37	45	31	[11.9,12.5]	[0.7,0.735]
7	24	58	[10.7,11.2]	[1.3,1.365	38	45	24	[10.6,11.1]	[0.5,0.525]
8	24	55	[13.7,14.4]	[1.2,1.26]	39	49	55	[14.6,15.3]	[0.6,0.63]
9	24	51	[14.6,15.3]	[1.5,1.575]	40	49.5	51.5	[14.1,14.8]	[0.7,0.735]
10	24	47	[10.6,11.1]	[1.2,1.26]	41	49.5	46	[11.8,12.4]	[1,1.05]
11	23	41	[11.8,12.4]	[1.3,1.365]	42	49.5	40	[12.9,13.5]	[0.6,0.63]
12	21.5	35	[10,10.5]	[1.4,1.47]	43	49.5	35	[12.2,12.8]	[0.8,0.84]
13	17.5	31	[12,12.6]	[1.5,1.575]	44	49.5	31.5	[12.5,13.1]	[0.5,0.525]
14	30	55	[13.4,14.1]	[1.5,1.575]	45	49.5	24	[12.3,12.9]	[1,1.05]
15	30	51	[10.6,11.1]	[1.3,1.365]	46	52.5	60	[10.7,11.2]	[1,1.05]
16	30	46	[11.3,11.9]	[1.5,1.575]	47	56	55	[11.8,12.4]	[0.5,0.525]
17	30	38	[14.1,14.8]	[1.5,1.575]	48	56	51.5	[14.5,15.2]	[0.8,0.84]
18	28	33	[15,15.8]	[1.5,1.575]	49	56	46	[10.7,11.2]	[0.6,0.63
19	26	30.5	[14.3,15]	[1.5,1.575]	50	56	37.5	[10.1,10.6]	[0.6,0.63]
20	35	55	[11.2,11.8]	[1.1,1.155]	51	56	31.5	[12.3,12.9]	[0.6,0.63]
21	35	51	[10.8,11.3]	[0.6,0.63]	52	56	24	[13.9,14.6]	[0.5,0.525]
22	35	46	[12.5,13.1]	[1,1.05]	53	60	62	[15,15.8]	[0.7,0.735]
23	35	40	[10.7,11.2]	[0.6,0.63]	54	64	55	[14.7,15.4]	[1.1,1.155]
24	35	34	[11,11.6]	[0.8,0.84]	55	61	51.5	[13.5,14.2]	[0.7,0.735]
25	35	31	[12.9,13.5]	[0.8,0.84]	56	66	51.5	[14.7,15.4]	[0.6,0.63]
26	35	27.5	[11.3,11.9]	[0.6,0.63]	57	66	46	[14.9,15.6]	[0.5,0.525]
27	41	51	[10.6,11.1]	[1,1.05]	58	62	40	[15,15.8]	[0.6,0.63]
28	41	46	[14.7,15.4]	[0.5,0.525]	59	61	35	[12.3,12.9]	[1.1,1.155]
29	40	34	[11,11.6]	[0.8,0.84]	60	61.5	31.5	[10.1,10.6]	[0.9,0.945]
30	40	27.5	[13.9,14.6]	[0.6,0.63]	61	61	24	[11.6,12.2]	[0.9,0.945]

Table 4 Optimal distribution routes for the first 3 groups of each objective

Sequences	Total cost	Total risk	Departure time	Num	Routes set
C-1	69184.93	3708.6	00:02	1	0-30-53-2-19-26-11-54-3-8-61-46-49-5-0
			00:24	2	0-24-32-16-59-39-50-37-22-10-12-0
			00:50	3	0-21-15-18-6-57-43-42-56-4-13-58-0
			01:37	4	0-35-52-7-28-47-55-38-44-14-25-20-1-0
			02:49	5	0-17-51-41-29-31-45-48-36-0
			09:03	6	0-33-40-9-60-27-23-34-0
			00:02	1	0-30-53-2-19-26-11-54-3-23-45-48-36-0
			00:24	2	0-24-32-16-59-39-50-37-22-40-10-12-0

C-2	69793.34	3625.5	01:37	3	0-35-27-52-7-28-34-9-55-38-44-14-49-5-60-0
			02:49	4	0-17-51-41-33-29-31-8-61-46-0
			08:52	5	0-25-20-1-0
			23:48	6	0-47-21-15-18-6-57-43-42-56-4-13-58-0
C-3	70038.54	3582.4	00:02	1	0-30-53-2-19-26-11-54-3-8-61-31-46-0
			00:24	2	0-24-17-32-16-59-39-50-37-22-10-12-0
			00:50	3	0-21-41-45-15-18-6-57-43-48-42-56-4-13-58-0
			01:37	4	0-35-52-7-28-47-55-38-44-14-49-5-0
			06:01	5	0-29-33-51-40-9-60-0
			08:52	6	0-25-20-1-27-36-23-34-0
R-1	100891.9	3040.4	00:02	1	0-30-53-40-42-20-1-0
			00:50	2	0-21-47-9-59-39-17-56-55-41-50-38-0
			01:37	3	0-35-51-8-34-0
			02:00	4	0-16-3-7-2-19-26-33-60-27-13-4-0
			05:14	5	0-28-34-24-5-0
			06:01	6	0-29-15-61-18-57-32-52-0
			06:48	7	0-37-6-36-45-48-0
			07:48	8	0-22-10-12-0
			08:04	9	0-14-49-46-0
			08:52	10	0-25-11-54-43-44-0
			10:45	11	0-23-58-0
R-2	96523.82	3043.3	00:02	1	0-30-53-40-42-20-1-6-36-45-48-0
			00:50	2	0-21-57-47-9--59-39-17-56-55-46-0
			01:37	3	0-35-51-8-62-31-23-58-0
			02:00	4	0-16-3-7-2-19-26-18-44-14-49-10-12-0
			05:14	5	0-28-34-24-5-0
			06:01	6	0-29-15-61-33-60-27-13-4-0
			07:48	7	0-22-41-50-37-38-0
			08:52	8	0-25-11-54-43-32-52-0
R-3	89435.52	3059.5	00:06	1	0-33-24-5-0
			00:50	2	0-21-47-9-39-17-11-54-43-45-48-60-0
			02:00	3	0-16-3-4-2-19-26-25-56-55-41-50-37-38-0
			02:52	4	0-32-30-53-40-42-20-7-1-6-36-52-0
			05:14	5	0-28-34-15-57-35-27-51-8-31-23-58-0
			06:01	6	0-29-59-61-18-44-1-49-46-0
			07:48	7	0-22-10-13-12-0

Table 5 Optimization of objectives and their distribution routes

Total cost	Total risk	Departure time	Num	Routes set
69184.93	3708.6	00:02	1	0-30-53-2-19-26-11-54-3-8-61-46-49-5-0
		00:24	2	0-24-32-16-59-39-50-37-22-10-12-0
		00:50	3	0-21-15-18-6-57-43-42-56-4-13-58-0
		01:37	4	0-35-52-7-28-47-55-38-44-14-25-20-1-0
		02:49	5	0-17-51-41-29-31-45-48-36-0
		09:03	6	0-33-40-9-60-27-23-34-0
100891.9	3040.4	00:02	1	0-30-53-40-42-20-1-0
		00:50	2	0-21-47-9-59-39-17-56-55-41-50-38-0
		01:37	3	0-35-51-8-34-0
		02:00	4	0-16-3-7-2-19-26-33-60-27-13-4-0
		05:14	5	0-28-34-24-5-0
		06:01	6	0-29-15-61-18-57-32-52-0
		06:48	7	0-37-6-36-45-48-0
		07:48	8	0-22-10-12-0

08:04	9	0-14-49-46-0
08:52	10	0-25-11-54-43-44-0
10:45	11	0-23-58-0

Table 6 Optimal Number of Vehicles Used and Delays for Each Objective

Total cost	Total risk	Number of vehicles	Delay time
69184.93	3708.6	6	2420.67
100891.9	30404	11	5300.46

Table 7 Load Ratio and Gap Values under Demand Fluctuations

Objective optimality	Route set	loading rate	1% fluctuating load rate	3% fluctuating load rate	5% fluctuating load rate	Gap1	Gap3	Gap5
cost-optimized	1	72%	72.72%	74.16%	75.60%	0.72%	2.16%	3.60%
	2	45.33%	45.77%	46.69%	47.60%	0.44%	1.36%	2.27%
	3	49.33%	49.83%	50.81%	51.80%	0.50%	1.48%	2.47%
	4	64%	64.64%	65.92%	67.20%	0.64%	1.92%	3.20%
	5	88.67%	89.55%	91.32%	93.10%	0.88%	2.65%	4.43%
	6	65.33%	65.99%	67.79%	68.60%	0.66%	2.46%	3.27%
risk-optimized	1	61.33%	61.94%	63.17%	64.40%	0.61%	1.84%	3.07%
	2	42.67%	43.09%	43.95%	44.80%	0.42%	1.28%	2.13%
	3	22.67%	22.89%	23.34%	23.80%	0.22%	0.67%	1.13%
	4	89.33%	90.22%	92.01%	93.80%	0.89%	2.68%	4.47%
	5	24.00%	24.24%	24.72%	25.20%	0.24%	0.72%	1.20%
	6	31%	31.65%	32.27%	32.90%	0.32%	0.94%	1.57%
	7	31.33%	31.65%	32.27%	32.90%	0.32%	0.94%	1.57%
	8	30%	30.30%	30.90%	31.50%	0.30%	0.90%	1.50%
	9	20.67%	20.87%	21.29%	21.70%	0.20%	0.62%	1.03%
	10	23.33%	23.57%	24.03%	24.50%	0.24%	0.70%	1.17%
	11	8%	8.08%	8.24%	8.40%	0.08%	0.24%	0.40%
Average load rate	-	45%	45.71%	46.64%	47.52%	-	-	-

B. Robustness analysis

The robustness of the model is reflected in the external interference and maintains the stability of its solution. The uncertainty of transportation time leads to uncertainty in demand for gas stations, and the demand for gas stations has changed with the change in transportation time and present positive correlation. Since the demand has a certain degree of volatility, respectively, we analyze the results of the solution for the 1%, 3%, and 5% fluctuations in demand. They include the cost of the optimal, with the optimal conditions for the transportation risk of the vehicle's average load factor, the stability of the vehicle route analysis.

As can be seen from Table 7, under different demand fluctuations, the depot carries out the distribution service under the robust optimization model solution results. And the stability of the route can still be maintained under the optimal situation of each objective, indicating that the solution results can accept a certain degree of anti-disturbance ability. In the case of the optimal position of each goal, in which the risk is optimal, the loading rate of route 4 is 89.33%. The tanker truck must consider the thermal expansion and contraction, and its full-load coefficient is between 0.9-0.95. The limit of demand fluctuation that it can withstand is 6%, and after exceeding 6%, the loading rate of route 4 is more than 95%, the route is infeasible. It is necessary to re-arrange the vehicle for distribution. So we can see that the model has a certain degree of robustness.

Except for exceptional cases, under realistic conditions, the fluctuation of fuel consumption demand at gas stations has a certain degree of regularity. The maximum fluctuation of 6% efficiently satisfies the demand for guaranteeing the replenishment of gas stations under optimal route conditions. Comparison with the solution results shows that after the fluctuation of demand, the average load factor of the vehicle is improved by 0.71%, 1.64%, and 2.52%. Respectively, it can also effectively enhance transportation efficiency and reduce transportation costs under the resistance to external interference. Still, it will sacrifice a certain amount of transportation cost and risk. Among them, the Gap value refers to the difference between the load factor after fluctuation and the solved value load factor after fluctuation in demand occurs. The Gap value of some routes is significantly higher than the average load factor enhancement under the current fluctuation in demand. It indicates that the routes can better resist disturbances and significantly enhance transportation efficiency when the demand fluctuates.

C. Algorithm performance analysis

The NSGA-II algorithm is widely used in solving multi-objective problems. To verify the performance of the MMA algorithm, we analyzed it in comparison with the

NSGA-II algorithm. We give the Pareto solution sets and Pareto frontiers generated by the two different algorithms in Figure 13-16. A comparison of the optimal total transportation cost and total transportation risk, the average delay time, the number of solutions, and the number of Pareto fronts in Table 8 reveals that the MMA algorithm outperforms the NSGA- II algorithm. The MMA algorithm manifesting itself in the number of solutions. It suggests that constructing the initial by the C-W algorithm solution, the introduction of the neighborhood search operator can accelerate the solution efficiency. And the opening of the inferior solution acceptance strategy, which searches more fully in the solution space and increases the diversity of solutions. The transportation cost is reduced by 0.27% in the risk-optimal case, the transportation risk is reduced by 5.98% in the cost-optimal case. The average delay time is reduced by 15.91%, which can effectively the risk of running out of fuel by gas by stations. The transportation cost is reduced by 0.27% in the risk-optimal case, the transportation risk is reduced by 5.98% in the cost-optimal case. The average delay time is reduced by 15.91%, which can effectively the risk of running out of fuel by gas by stations.

In order to avoid the chance of the arithmetic cases, two algorithms are used to solve the classical arithmetic issues, including 100, 200 and 400 nodes, and the optimization objective, the number of solutions and the solution time are compared and analyzed. The specific results are shown in Tables 9-10, and Figures 17-18, which show that: In solving the small-scale arithmetic cases and large-scale arithmetic cases, MMA algorithm compares with the remaining three algorithms, and in the transportation cost by 0.3%-6.1%, with an average reduction rate of 3.44%. In terms of transportation risk, it reduces by 1.3%-15.42%, with an average reduction rate of 4.4%. It indicates that the MMA algorithm has a higher optimization ability in objective solving than the remaining three heuristic algorithms. The algorithm's solution efficiency is embodied in the set of Pareto solutions

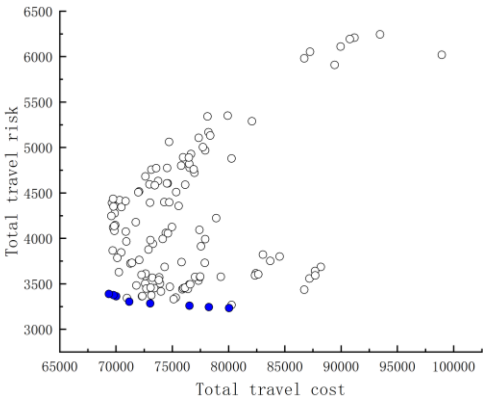

Fig 13 NSGA-II algorithm Pareto solution Set

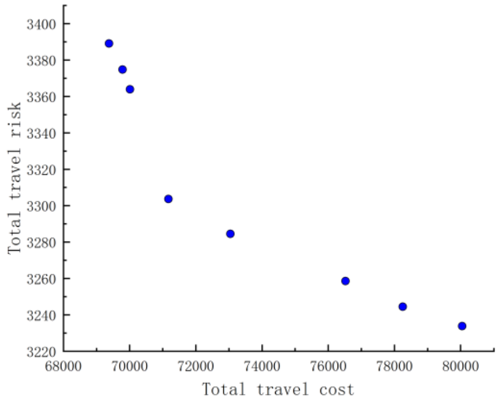

Fig 14 NSGA-II Algorithm Pareto frontier

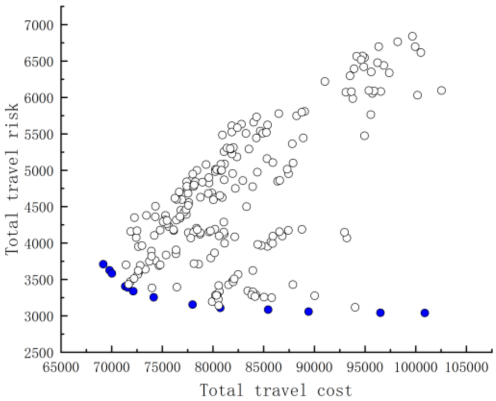

Fig 15 MMA algorithm Pareto solution set

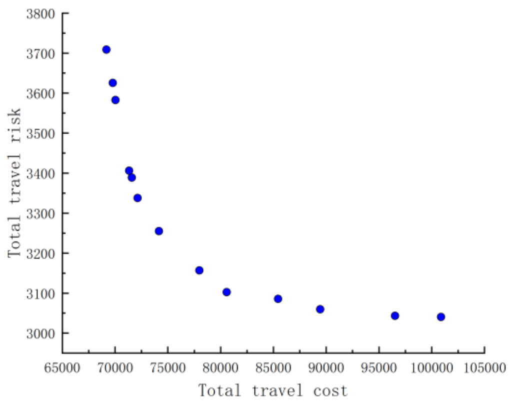

Fig 16 MMA Algorithm Pareto frontier

Table 8 Solution results

Algorithm	Cost-optimized	Risk-optimized	Delay time	Average number of vehicles	Number of solutions	Number of Pareto fronts
NSGA-II	(69376.23,3389.1)	(80048.61,3233.8)	1910.11	6.87	130	8
MMA	(69184.93,3708.6)	(100891.9,3040.4)	1606.18	6.68	221	13

Table 9 Optimal values of Risk R under different arithmetic examples

Algorithm \ Instance	RC101	RC201	C121	C122	C141	C142
VNS	2497.25	12496.35	32588.41	32049.59	1732583	1704524
GA	2579.51	13854.39	32588.41	32615.24	1711825	1695125
NSGA-II	2579.51	12087.27	32693.17	31079.63	1701024	1695125
MMA	2410.84	11717.92	31095.28	30179.62	1695827	1672890

Table 10 Optimal values of cost C under different arithmetic examples

Algorithm \ Instance	RC101	RC201	C121	C122	C141	C142
VNS	35024.68	30825.42	51583.74	50643.58	262453.2	232843.7
GA	36248.43	31543.25	54249.93	51249.28	264371.3	240385.2
NSGA-II	35024.68	30849.93	54249.93	50149.57	259682.5	232581.5
MMA	34205.47	29551.06	51583.74	49827.35	246485.3	231879.5

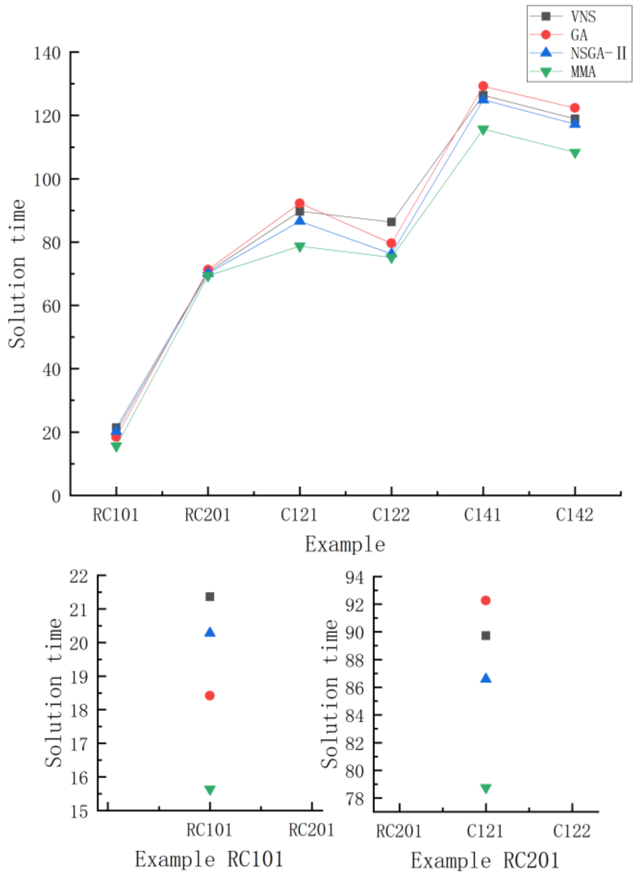

Fig 17 Solving time

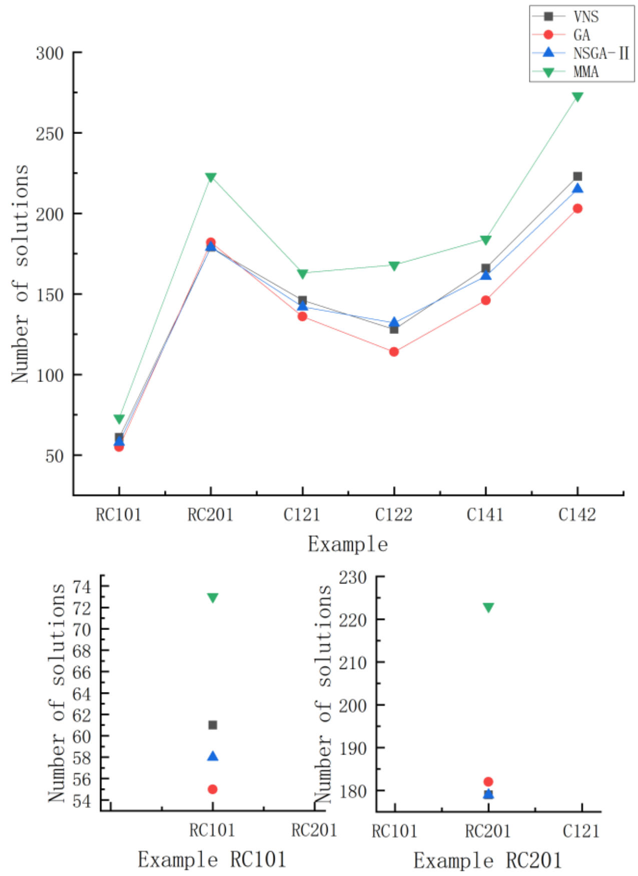

Fig 18 Number of Pareto solutions

and the speed of the solving process. The MMA algorithm has an average reduction rate of 4.4% in terms of the number of Pareto solutions, and the average reduction rate of 4.4% in terms of the number of heuristics. In terms of the number of Pareto solutions, the MMA algorithm improves by 24.29% on average, and the solution speed improves by 10.02% on average, which is more evident for the example RC101, where the solution speed improves by 21.58% on average. The above data show that the MMA algorithm has a better solving effect for both small-scale and large-scale arithmetic cases and improves the searching ability of the hybrid algorithm for superior and inferior solutions as well as the solving efficiency through the neighborhood search strategy and the inferior solution acceptance strategy.

VII. Conclusion

In this paper, we established a multi-objective robust optimization model for the vehicle scheduling problem of secondary distribution of refined oil products with a time window in active distribution mode. We designed a modified memetic algorithm with an elite retention strategy and an inferior solution acceptance strategy to solve the problem. We have the following conclusions: (1) Compared with the conventional model, the model has a certain degree of robustness, can resist small-amplitude external interference, and is suitable for the natural environment; (2) The test cases show that the hybrid algorithm has better optimization results compared with the other algorithms, obtains more Pareto solution sets and better Pareto frontier distribution and reduces the average delay time of the vehicle, which improves the efficiency of the service.

The robust optimization model of vehicle scheduling for refined oil distribution in this paper can ensure the stability of the paths when each objective is optimal under demand fluctuations, improve the vehicle load factor, and be suitable for refined oil distribution networks with a small range of demand fluctuations. However, there are still numerous possibilities for future research. First, the robust optimization model mainly considers the worst case of transportation time, which leads to a similar demand situation, and the solution obtained is conservative. Therefore, we can appropriately increase the degree of fluctuation of uncertain variables and comprehensively consider the inventory cost for modeling and optimization. Then, considering the efficiency of other algorithms for solving this problem, we can simplify the model by some mathematical method and use an exact algorithm to solve the problem. So, the next step will be to consider more complex cases and find better optimization algorithms.

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