Design of Ceramic Products Formed by Drawing Embryos Based on the Quasi-cubic Uniform B-spline

Juncheng Li, Chengzhi Liu, and Ge Ding

Abstract—**Due to the fact that most ceramic products formed by drawing embryos can be geometrically viewed as rotating bodies generated by the rotation of their profile curves, the design of these ceramic products can focus on the description of profile curves. A scheme for describing profile curves of ceramic products using the quasi-cubic uniform B-spline is proposed. Different shapes of C^2 profile curves can be obtained by modifying the control points or local parameters of the quasi-cubic uniform B-spline, thereby meeting the practicality and personality of ceramic product design. Furthermore, a method for determining the optimal local parameters in the quasi-cubic uniform B-spline using energy minimization is presented. The proposed method can generate smooth profile curves of ceramic products, thereby meeting the aesthetics of ceramic product design.**

Index Terms—**Cubic uniform B-spline, local parameter, ceramic product design, profile curve, smooth optimization**

I. Introduction

Daily-use ceramic products have become indispensable necessities in life. The design of daily-use ceramic products is characterized by rapidity, creativity, and repetition. With the increasing improvement of living standards, people's requirements for the daily-use ceramic product design are also increasing. Using geometric models to quickly and accurately express daily ceramic products has always challenged designers. In the daily-use ceramic product design, embryo forming is one of the common methods. Geometrically, most ceramic products formed by drawing embryos can be viewed as rotating bodies generated by the rotation of their profile curves. Thus, the design of the ceramic products formed by drawing embryos can focus on the description of profile curves.

When designing ceramic products, practicality, aesthetics, and personalization usually need to be considered [1]. In [2], the quartic ω-Bézier curves were used to describe the profile curves of ceramic products. Users could adjust shape of the profile curves by modifying the control points and shape parameters of the quartic ω-Bézier curves, which makes the obtained profile curves meet the personality of ceramic product design. However, it is necessary to reasonably set the control points of the quartic ω-Bézier curves in advance to make them smoothly connected, which brings inconvenience to the users. In addition, the profile curves only satisfy G^1 or G^2 continuity when the shape parameters of the quartic ω-Bézier curves are retained. If the profile curves are required to be C^1 or C^2, the shape parameters of the quartic ω-Bézier curves would need to be assigned. That means shape of the profile curves cannot be adjusted by the shape parameters, namely, the personalized design of the profile curve cannot be realized. The first purpose of this paper is to use the quasi-cubic uniform B-spline [3] to describe the profile curves of ceramic products. The proposed scheme only requires inputting the control points and local parameters of the quasi cubic uniform B-spline to obtain C^2 profile curves, and shape of the obtained profile curves can be freely adjusted. These advantages meet the practicality and personality of ceramic product design.

Since the quasi-cubic uniform B-spline contains local parameters, a natural idea arises: when the control points are fixed, the values of local parameters can be optimized to obtain profile curves that meet some certain specific needs. In recent years, using energy minimization to optimize the shape of curves has become a common way. Such as the shape optimization of the interpolation curve, the Bézier curve, the B-spline curve, and the Catmull-Rom spline based on energy minimization [4-9], and so on. The second purpose of this paper is to use energy minimization to optimize the values of local parameters in the quasi-cubic uniform B-spline. The proposed method can generate smooth profile curves of ceramic products, which meets the aesthetics of ceramic product design.

The rest of this paper is organized as follows. In Section II, the definition and some properties of the cubic uniform B-spline are presented. In Section III, the design scheme of ceramic products formed by drawing embryos based on the cubic uniform B-spline is given. In Section IV, the shape optimization of ceramic products formed by drawing embryos based on energy minimization is provided. Finally, a brief conclusion is given in Section V.

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II. THE CUBIC UNIFORM B-SPLINE

In curve modeling, the B-spline [10] is widely adopted as a basic component in most CAD systems due to its simplicity, flexibility, and convenience in representing curves. However, the position of the B-spline is fixed relative to its control polygon. If the shape of the B-spline needs to be adjusted, its control polygon must be changed, which to some extent limits the application of the B-spline in curve representation. To improve the shape control performance of the B-spline, some quasi B-splines with parameters were proposed [3,11-14]. These quasi B-splines allow for flexible shape adjustment by modifying the parameters contained within them without altering the control polygon, providing convenience for curve representation. To effectively and simply achieve the design of ceramic products formed by drawing embryos, a type of quasi-cubic uniform B-spline is selected here to represent the profile curves of ceramic products.

Given the control points $\boldsymbol{p}_i \in \mathbb{R}^d$ ($d = 2,3$; $i = 0,1,\cdots,n$), for $0 \le t \le 1$, the piecewise quartic polynomial curve [3] can be defined as

$$\boldsymbol{r}_i(t) = \sum_{j=0}^{3} b_{i,j}(t)\boldsymbol{p}_{i+j-1}, \quad i = 1,2,\cdots,n-2, \tag{1}$$

where $b_{i,j}(t)$ $(j = 0,1,2,3)$ are the basis functions expressed by

$$\begin{cases} b_{i,0}(t) = \dfrac{1}{24}\left((4-\lambda_i)(1-t)^4 + 4(1-\lambda_i)(1-t)^3 t\right), \\ b_{i,1}(t) = \dfrac{1}{24}\left(2(8+\lambda_i)(1-t)^4 + 8(8+\lambda_i)(1-t)^3 t + \right. \\ \qquad\qquad \left. 72(1-t)^2 t^2 + 4(7-\lambda_{i+1})(1-t)t^3 + (4-\lambda_{i+1})t^4\right), \\ b_{i,2}(t) = \dfrac{1}{24}\left((4-\lambda_i)(1-t)^4 + 4(7-\lambda_i)(1-t)^3 t + \right. \\ \qquad\qquad \left. 72(1-t)^2 t^2 + 8(8+\lambda_{i+1})(1-t)t^3 + 2(8+\lambda_{i+1})t^4\right), \\ b_{i,3}(t) = \dfrac{1}{24}\left(4(1-\lambda_{i+1})(1-t)t^3 + (4-\lambda_{i+1})t^4\right), \end{cases} \tag{2}$$

where $\lambda_i \in [-8,1]$ $(i = 1,2,\cdots,n-1)$ are the local parameters.

Because the curve expressed in Eq. (1) has a structure similar to the cubic uniform B-spline, it is called the quasi-cubic uniform B-spline here. According to Eq. (1), when $\boldsymbol{p}_i$ $(i = 0,1,\cdots,n)$ and λ_i $(i = 1,2,\cdots,n-1)$ are given, the whole quasi-cubic uniform B-spline will be automatically composed of n-2 segments. For $i = 1, \ 2,\cdots,n-2$, the following results can be obtained by calculating from Eqs. (1) and (2),

$$\begin{cases} \boldsymbol{r}_i(0) = \dfrac{1}{24}\left((4-\lambda_i)\boldsymbol{p}_{i-1} + 2(8+\lambda_i)\boldsymbol{p}_i + (4-\lambda_i)\boldsymbol{p}_{i+1}\right), \\ \boldsymbol{r}_i(1) = \dfrac{1}{24}\left((4-\lambda_{i+1})\boldsymbol{p}_i + 2(8+\lambda_{i+1})\boldsymbol{p}_{i+1} + (4-\lambda_{i+1})\boldsymbol{p}_{i+2}\right). \end{cases} \tag{3}$$

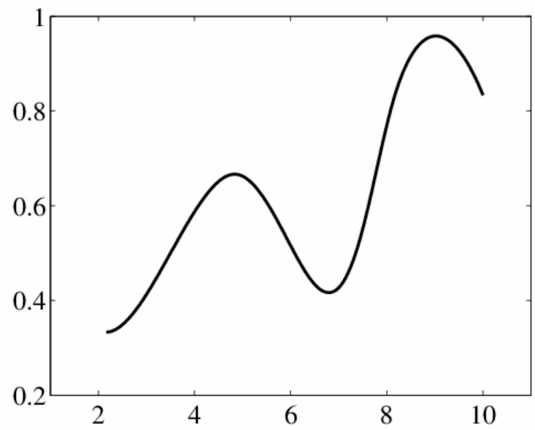
$$\begin{cases} \boldsymbol{r}_i'(0) = \dfrac{1}{2}(\boldsymbol{p}_{i+1} - \boldsymbol{p}_{i-1}), \\ \boldsymbol{r}_i'(1) = \dfrac{1}{2}(\boldsymbol{p}_{i+2} - \boldsymbol{p}_i). \end{cases} \tag{4}$$

$$\begin{cases} \boldsymbol{r}_i''(0) = \dfrac{2+\lambda_i}{2}(\boldsymbol{p}_{i-1} - 2\boldsymbol{p}_i + \boldsymbol{p}_{i+1}), \\ \boldsymbol{r}_i''(1) = \dfrac{2+\lambda_{i+1}}{2}(\boldsymbol{p}_i - 2\boldsymbol{p}_{i+1} + \boldsymbol{p}_{i+2}). \end{cases} \tag{5}$$

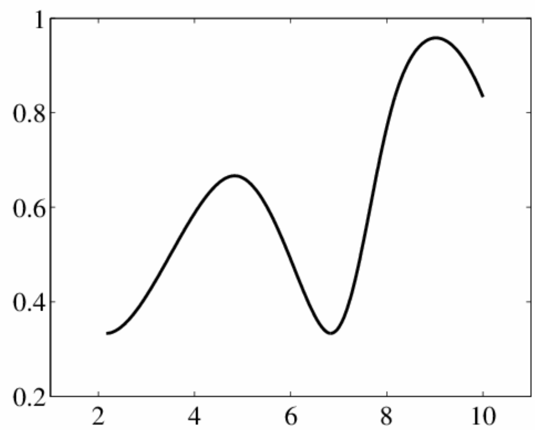
Eqs. (3), (4) and (5) yield $\boldsymbol{r}_i^{(k)}(1) = \boldsymbol{r}_{i+1}^{(k)}(0)$ $(k = 0,1,2)$, which shows that the whole quasi-cubic uniform B-spline satisfies C^2 continuity.

Due to the presence of the local parameters, shape of the quasi-cubic uniform B-spline can be adjusted locally or globally by modifying the values of the local parameters when the control points are kept unchanged.

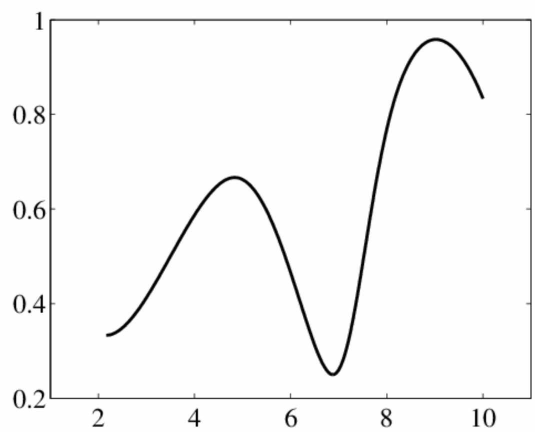
Fig. 1 shows the local adjustment of the quasi-cubic uniform B-spline composed of four segments.



(a) $\lambda_i = 0$ $(i = 1,2,4,5)$, $\lambda_3 = -1$



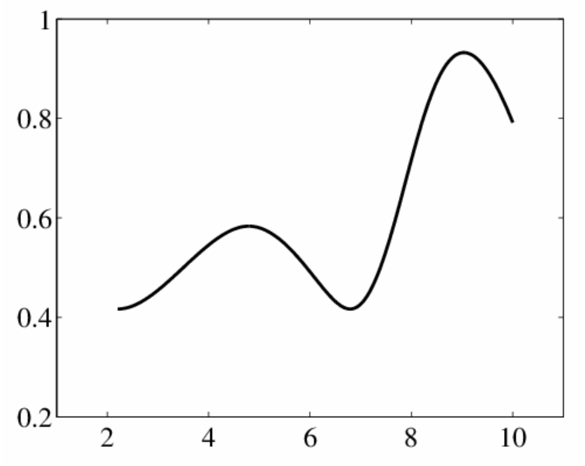
(b) $\lambda_i = 0$ $(i = 1,2,4,5)$, $\lambda_3 = 0$



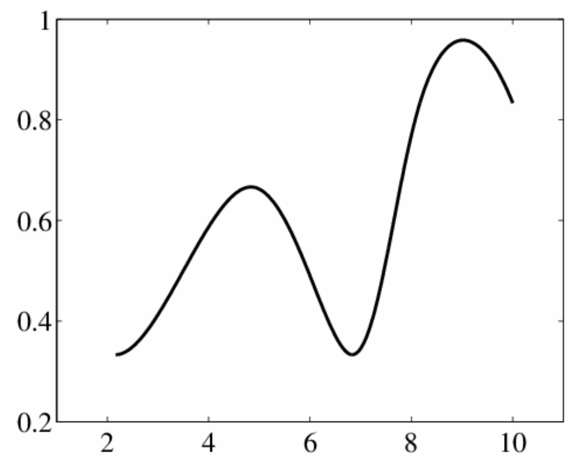
(c) $\lambda_i = 0$ $(i = 1,2,4,5)$, $\lambda_3 = 1$

Fig. 1 The local adjustment of the quasi-cubic uniform B-spline

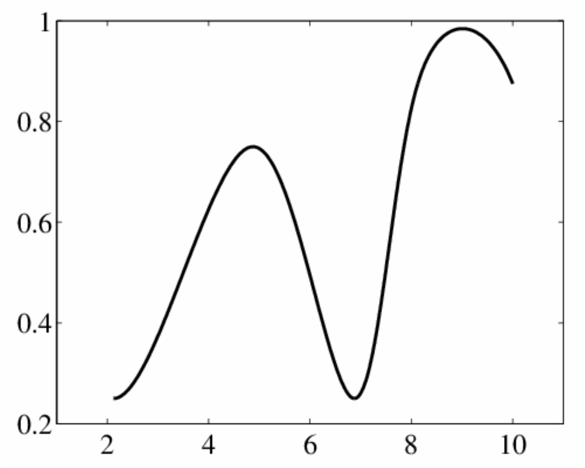
Fig. 2 shows the global adjustment of the quasi-cubic uniform B-spline composed of four segments.



(a) $\lambda_i = -1$ $(i = 1,2,34,5)$



(b) $\lambda_i = 0$ $(i = 1,2,34,5)$



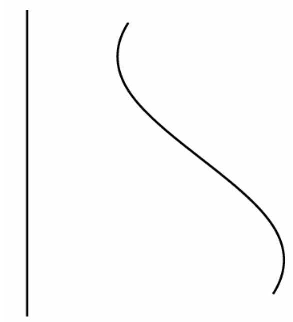
(c) $\lambda_i = 1$ $(i = 1,2,34,5)$

Fig. 2 The global adjustment of the quasi-cubic uniform B-spline

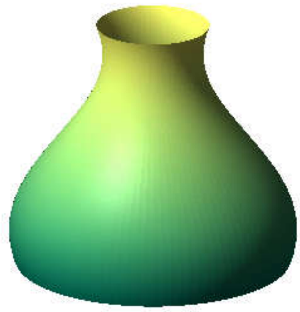
III. The Design Scheme of the Ceramic Product

A. *The mathematical model of the ceramic product*

To simplify the discussion, the ceramic product formed by drawing the embryo is viewed as a rotating body generated by its profile curve on the *yoz* plane rotating around the *z*-axis, see Fig. 3.



(a) The profile curve and rotation axis



(b) The generated ceramic product

Fig. 3 The ceramic product formed by drawing the embryo

When using the quasi-cubic uniform B-spline to describe the profile curve and setting $\boldsymbol{p}_i = (py_i, pz_i)$, Eq. (1) can be rewritten as follows,

$$\begin{cases} y_i(t) = \sum_{j=0}^{3} b_{i,j}(t)\, py_{i+j-1}, \\ z_i(t) = \sum_{j=0}^{3} b_{i,j}(t)\, pz_{i+j-1}. \end{cases} \tag{6}$$

Then, the ceramic product generated by the profile curve represented in Eq. (6) rotating around the *z*-axis can be expressed as

$$\begin{cases} x_i(t,\theta) = |y_i(t)| \cos\theta, \\ y_i(t,\theta) = |y_i(t)| \sin\theta, \\ z_i(t,\theta) = z_i(t), \end{cases} \tag{7}$$

where $0 \le t \le 1$, $0 \le \theta \le 2\pi$.

According to Eq. (7), the mathematical model of the ceramic product would be obtained once its profile curve is described using the quasi-cubic uniform B-spline. Hence, the design of ceramic products formed by drawing embryos can focus on the description of the profile curves.

B. *Description and shape adjustment of the profile curve*

According to Eq. (3), it can be obtained that $\boldsymbol{r}_0(0) = \boldsymbol{p}_0$, $\boldsymbol{r}_{n-1}(1) = \boldsymbol{p}_n$ when supplementing $\boldsymbol{p}_{-1}$, $\boldsymbol{p}_{n+1}$ to the control points and setting $\lambda_0 = \lambda_n = 4$. That means the quasi-cubic uniform B-spline passes through the first control point $\boldsymbol{p}_0$ and the last control point $\boldsymbol{p}_n$, which will bring convenience to the design of the profile curve. For convenience, the supplementary control points can be taken as $\boldsymbol{p}_{-1} = \boldsymbol{p}_0$ and

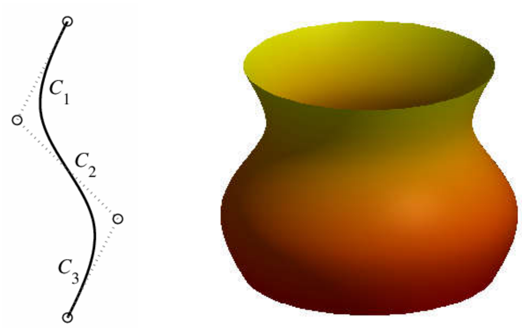
$p_{n+1} = p_n$. Then, the steps of using the quasi-cubic uniform B-spline to describe the profile curve can be described as follows,

Step 1 Input the coordinates of control points p_i $(i = -1, 0, \cdots, n+1)$, where $p_{-1} = p_0$, $p_{n+1} = p_n$.

Step 2 Input the values of local parameters λ_i $(i = 0, 1, \cdots, n)$, where $\lambda_0 = \lambda_n = 4$, $\lambda_i \in [-8, 1]$ $(i = 1, 2, \cdots, n-1)$.

Step 3 Generate the profile curve according to Eqs. (1) and (2).

Fig. 4 shows the profile curve and the corresponding ceramic product form, where the profile curve is composed of three segments C_1, C_2 and C_3 that respectively control the upper, middle, and lower parts of the ceramic product form.



(a) The profile (b) The ceramic product form

Fig. 4 The profile curve and the corresponding ceramic product form

The shape of the profile curve can be adjusted in the following three ways,

(a) Modifying the control points.

Since the ith segment of the quasi-cubic uniform B-spline $r_i(t)$ $(i = 1, 2, \cdots, n-2)$ is controlled by four control points p_{i-1}, p_i, p_{i+1} and p_{i+2}, the shape of the adjacent segments of the profile curve will change simultaneously if the position of p_i $(i = 1, 2, \cdots, n-1)$ is modified. By arbitrarily modifying the position of certain control points, the profile curve will present rich shapes.

(b) Modifying the local parameters.

Since the ith segment of the quasi-cubic uniform B-spline $r_i(t)$ $(i = 1, 2, \cdots, n-2)$ has two local parameters λ_i and λ_{i+1}, the shape of the first or the last segment of the profile curve will change when the value of λ_1 or λ_{n-1} is modified, the shape of both adjacent segments of the profile curve will change when the value of λ_i $(i = 2, 3, \cdots, n-2)$ is modified, and the profile curve will present rich shapes when arbitrarily modifying certain local parameters.

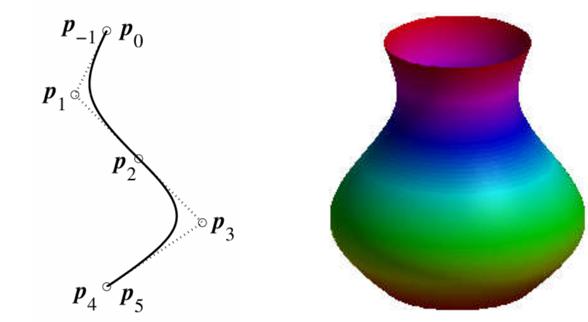
(c) Modifying the control points and local parameters simultaneously.

The positions of the control points and the values of the local parameters can be arbitrarily modified to make the profile curve present rich shapes.

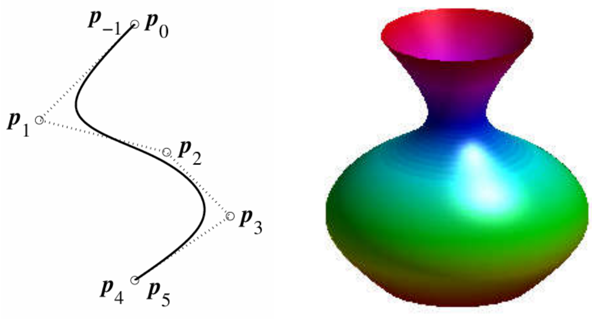
Example 1 Given the coordinates of control points p_i $(i = -1, 0, \cdots, 5)$ are

$py_i = [1.5, 1.5, 1, 2, 3, 1.5, 1.5]$, $pz_i = [6, 6, 5, 4, 3, 2, 2]$,

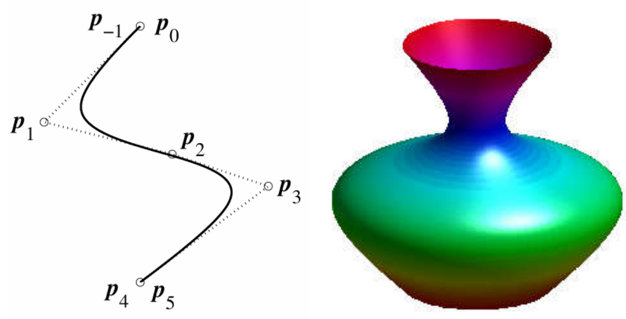
and the local parameters are $\lambda_0 = 4$, $\lambda_1 = \lambda_2 = \lambda_3 = 0$ and $\lambda_4 = 4$. The profile curves and the corresponding ceramic product forms generated by modifying some control points are shown in Fig. 5, where 5(a) and 5(b) present the original profile curve and the corresponding ceramic product form; 5(c) and 5(d) present the profile curve and the corresponding ceramic product form generated by modifying the coordinate of p_1 from $(1,5)$ to $(0,4.5)$; 5(e) and 5(f) present the profile curve and the corresponding ceramic product form generated by further modifying the coordinate of p_3 from $(3,3)$ to $(3.5,3.5)$.



(a) The original profile (b) The original form



(c) The profile by modifying p_1 (d) The form by modifying p_1



(e) The profile by modifying p_1, p_3 (f) The form by modifying p_1, p_3

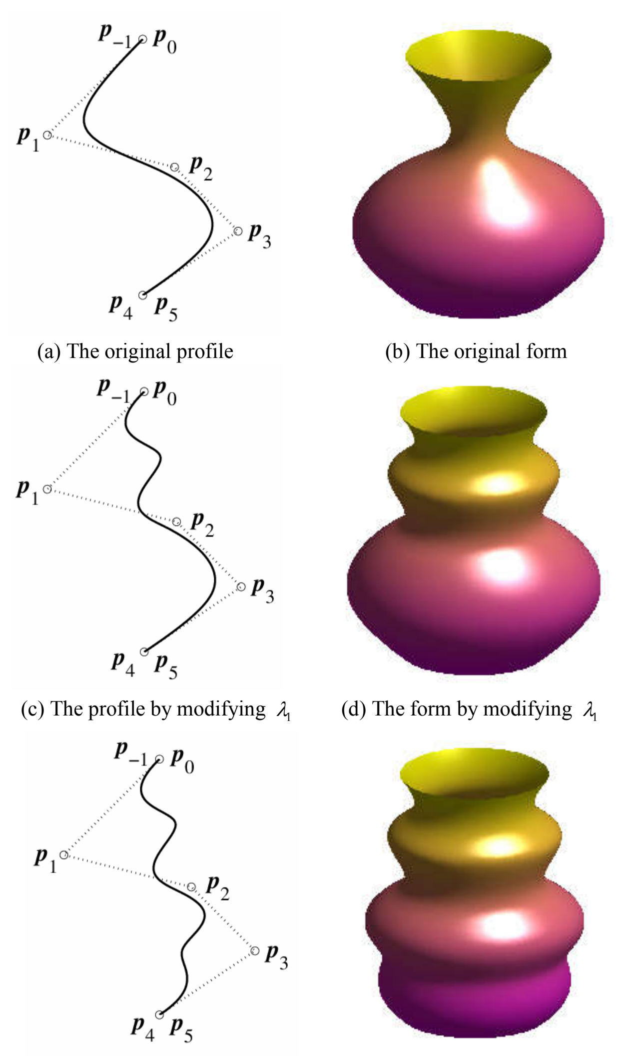
Fig. 5 The profile curves and the corresponding ceramic product forms generated by modifying the control points

Example 2 Given the coordinates of control points p_i $(i = -1, 0, \cdots, 5)$ are

$py_i = [1.5, 1.5, 0, 2, 3, 1.5, 1.5]$, $pz_i = [6, 6, 4.5, 4, 3, 2, 2]$,

and the local parameters are $\lambda_0 = 4$, $\lambda_1 = \lambda_2 = \lambda_3 = 0$ and $\lambda_4 = 4$. The profile curves and the corresponding ceramic product forms generated by modifying some local parameters are shown in Fig. 6, where 6(a) and 6(b) present the original profile curve and the corresponding ceramic product form; 6(c) and 6(d) present the profile curve and the corresponding ceramic product form generated by modifying the value of λ_1

from 0 to 8; 6(e) and 6(f) present the profile curve and the corresponding ceramic product form generated by further modifying the value of λ_3 from 0 to -2.



(a) The original profile (b) The original form

(c) The profile by modifying λ_1 (d) The form by modifying λ_1

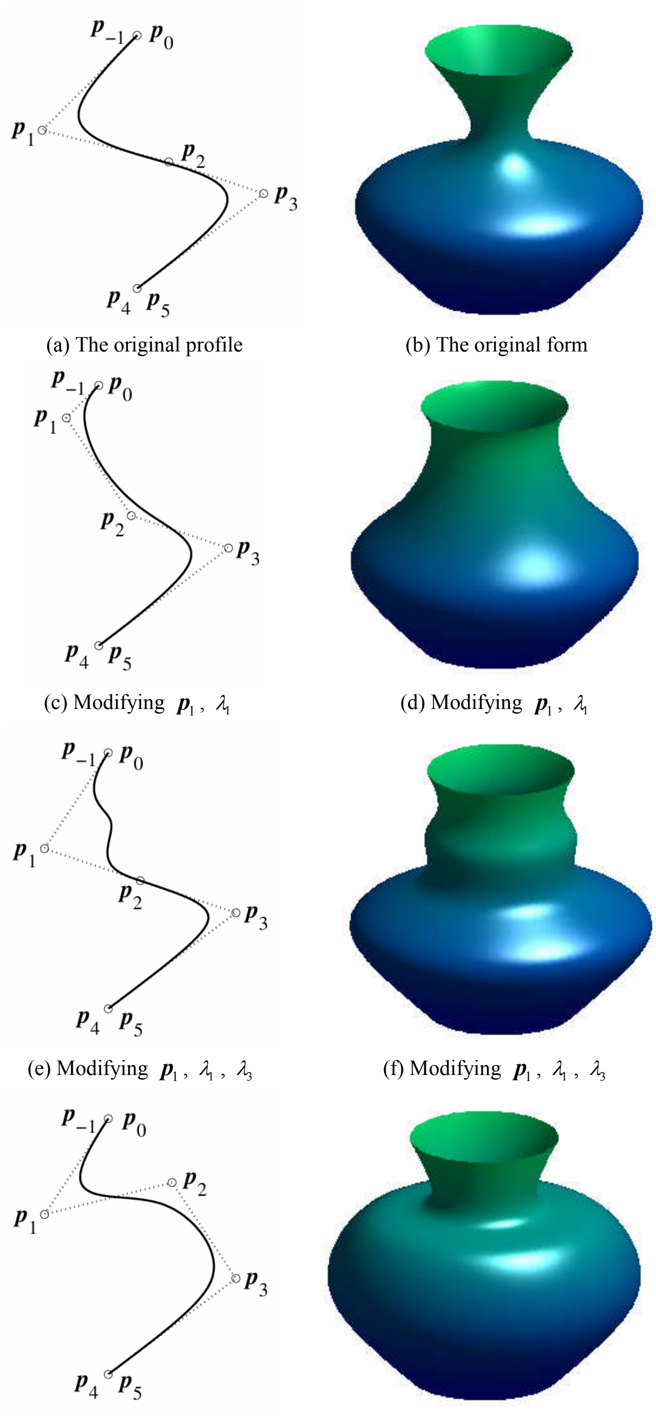
(e) The profile by modifying λ_1, λ_3 (f) The form by modifying λ_1, λ_3

Fig. 6 The profile curves and the corresponding ceramic product forms generated by modifying the local parameters

Example 3 Given the coordinates of control points $\boldsymbol{p}_i$ $(i=-1,0,\cdots,5)$ are

$py_i=[1.5,1.5,0,2,3.5,1.5,1.5]$, $pz_i=[6,6,4.5,4,3.5,2,2]$,

and the local parameters are $\lambda_0=4$, $\lambda_1=\lambda_2=\lambda_3=0$ and $\lambda_4=4$. The profile curves and the corresponding ceramic product forms generated by modifying some control points and local parameters simultaneously are shown in Fig. 7, where 7(a) and 7(b) present the original profile curve and the corresponding ceramic product form; 7(c) and 7(d) present the profile curve and the corresponding ceramic product form generated by modifying the coordinate of $\boldsymbol{p}_1$ from $(0,4.5)$ to $(1,5.5)$ and the value of λ_1 from 0 to 1; 7(e) and 7(f) present the profile curve and the corresponding ceramic product form generated by modifying the coordinate of $\boldsymbol{p}_1$ from $(0,4.5)$ to $(0.5,4.5)$, the value of λ_1 from 0 to -6, and the value of λ_3 from 0 to 1; 7(g) and 7(h) present the profile curve and the corresponding ceramic product form generated by modifying the coordinate of $\boldsymbol{p}_1$ from $(0,4.5)$ to $(0.5,4.5)$, the coordinate of $\boldsymbol{p}_2$ from $(2,4)$ to $(2.5,5)$, the value of λ_1 from 0 to -1, and the value of λ_3 from 0 to 1.



(a) The original profile (b) The original form

(c) Modifying $\boldsymbol{p}_1$, λ_1 (d) Modifying $\boldsymbol{p}_1$, λ_1

(e) Modifying $\boldsymbol{p}_1$, λ_1, λ_3 (f) Modifying $\boldsymbol{p}_1$, λ_1, λ_3

(g) Modifying $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, λ_1, λ_3 (h) Modifying $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, λ_1, λ_3

Fig. 7 The profile curves and the corresponding ceramic product forms generated by modifying the control points and local parameters simultaneously

Overall, using the quasi-cubic uniform B-spline to describe the profile curve of ceramic product formed by drawing embryo can meet the practical and personalized requirements, which are reflected as follows,

(a) Users only need to input the coordinates of control points and the values of local parameters to automatically generate the profile curve. There is no need to smoothly connect multiple curves, and the profile curve reaches C^2

continuity. These advantages meet the practicality of the ceramic product design.

(b) Users can adjust shape of the profile curve locally or globally by modifying the control points or local parameters, which meets the personality of the ceramic product design.

IV. THE SMOOTH OPTIMIZATION OF THE CERAMIC PRODUCT

The shape of the profile curve can be adjusted by modifying the control points or local parameters of the quasi-cubic uniform B-spline, which brings convenience to the design of ceramic products formed by drawing embryos. But the control points of the quasi-cubic uniform B-spline may sometimes be unable to be modified. For example, the control points are taken from the mould of the ceramic product. Then the adjustment of the profile curve can only be achieved by modifying the local parameters. However, the generated profile curve might not be smooth enough if the values of the local parameters are inappropriate, as shown in Fig. 4(c). To generate the profile curve as smoothly as possible, it can be considered to optimize the local parameters in the quasi-cubic uniform B-spline while keeping the control points fixed.

Although there is currently no recognized standard in mathematics to describe the smoothness of curves, using strain energy minimization or curvature variation energy minimization to construct curves as smooth as possible has become a common method [4-9]. Here, both the strain energy minimization and the curvature variation energy minimization are adopted to generate the profile curve as smoothly as possible.

After the control points $\boldsymbol{p}_{-1} = \boldsymbol{p}_0$ and $\boldsymbol{p}_{n+1} = \boldsymbol{p}_n$ are supplemented, the strain energy and curvature variation energy can be approximately expressed as

$$E_k = \sum_{i=0}^{n-1} \int_0^1 \left\| \boldsymbol{r}_i^{(k)}(t) \right\|^2 \mathrm{d}t \, , \tag{8}$$

where $\boldsymbol{r}_i(t)$ is the quasi-cubic uniform B-spline represented according to Eq. (1). Eq. (8) represents the strain energy when $k = 2$, and it represents the curvature variation energy when $k = 3$.

According to Eq. (2), Eq. (1) can be rewritten as

$$\boldsymbol{r}_i(t) = \boldsymbol{A}_i(t)\lambda_i + \boldsymbol{B}_i(t)\lambda_{i+1} + \boldsymbol{C}_i(t) \, , \tag{9}$$

where

$$\boldsymbol{A}_i(t) = -\frac{1}{24}\left((1-t)^4 + 4(1-t)^3 t\right)(\boldsymbol{p}_{i-1} - 2\boldsymbol{p}_i + \boldsymbol{p}_{i+1}) \, ,$$

$$\boldsymbol{B}_i(t) = -\frac{1}{24}\left(t^4 + 4(1-t)t^3\right)(\boldsymbol{p}_i - 2\boldsymbol{p}_{i+1} + \boldsymbol{p}_{i+2}) \, ,$$

$$\begin{aligned}\boldsymbol{C}_i(t) = \frac{1}{6}\Big(& ((1-t)^4 + (1-t)^3 t)\boldsymbol{p}_{i-1} + \\ & (4(1-t)^4 + 16(1-t)^3 t + 18(1-t)^2 t^2 + 7(1-t)t^3 + t^4)\boldsymbol{p}_i + \\ & (4t^4 + 16(1-t)t^3 + 18(1-t)^2 t^2 + 7(1-t)^3 t + (1-t)^4)\boldsymbol{p}_{i+1} + \\ & (t^4 + (1-t)t^3)\,\boldsymbol{p}_{i+2}\Big) \, .\end{aligned}$$

By submitting Eq. (9) into Eq. (8), then

$$E_k = \sum_{i=0}^{n-1}\left(a_{k,i}\lambda_i^2 + b_{k,i}\lambda_{i+1}^2 + c_{k,i} + 2d_{k,i}\lambda_i\lambda_{i+1} + 2e_{k,i}\lambda_i + 2f_{k,i}\lambda_{i+1}\right) \, , \tag{10}$$

where

$a_{k,i} = \int_0^1 \left\| \boldsymbol{A}_i^{(k)}(t) \right\|^2 \mathrm{d}t$, $b_{k,i} = \int_0^1 \left\| \boldsymbol{B}_i^{(k)}(t) \right\|^2 \mathrm{d}t$,

$c_{k,i} = \int_0^1 \left\| \boldsymbol{C}_i^{(k)}(t) \right\|^2 \mathrm{d}t$, $d_{k,i} = \int_0^1 \left(\boldsymbol{A}_i^{(k)}(t) \cdot \boldsymbol{B}_i^{(k)}(t) \right) \mathrm{d}t$,

$e_{k,i} = \int_0^1 \left(\boldsymbol{A}_i^{(k)}(t) \cdot \boldsymbol{C}_i^{(k)}(t) \right) \mathrm{d}t$,

$f_{k,i} = \int_0^1 \left(\boldsymbol{B}_i^{(k)}(t) \cdot \boldsymbol{C}_i^{(k)}(t) \right) \mathrm{d}t$, $k = 2,3$.

When the control points $\boldsymbol{p}_i$ $(i = -1, 0, \cdots, n+1)$ and k are given, $a_{k,i}$, $b_{k,i}$, $c_{k,i}$, $d_{k,i}$, $e_{k,i}$, $f_{k,i}$ are all constants. Recall that $\lambda_0 = \lambda_n = 4$. Hence, to make the profile curve described by the cubic uniform B-spline as smooth as possible, the following optimization model can be obtained,

$$\min_{\lambda_i \in [-8,1]} \quad E_k(\lambda_1, \lambda_2, \cdots, \lambda_{n-1}) \, , \tag{11}$$

where $k = 2,3$. Since Eq. (10) is a quadratic function about λ_i $(i = 1, 2, \cdots, n-1)$, it is easy to get the solution of Eq. (11) with the help of some mathematical software.

The steps of using the strain energy minimization or the curvature variation energy minimization to generate the smooth profile curve can be described as follows,

Step1 Input the coordinates of control points $\boldsymbol{p}_i$ $(i = -1, 0, \cdots, n+1)$, where $\boldsymbol{p}_{-1} = \boldsymbol{p}_0$, $\boldsymbol{p}_{n+1} = \boldsymbol{p}_n$.

Step2 Let $\lambda_0 = \lambda_n = 4$, and select k to determine what type of the energy used.

Step 3 Solve Eq. (11) to obtain the optimal values of local parameters λ_i $(i = 1, 2, \cdots, n-1)$.

Step 4 Generate the smooth profile curve according to Eqs. (1) and (2).

Example 4 Given the coordinates of control points $\boldsymbol{p}_i$ $(i = -1, 0, \cdots, 5)$ are

$$py_i = [1.5, 1.5, 0, 2, 3, 1.5, 1.5] \, , \; pz_i = [6, 6, 4.5, 4, 3, 2, 2] \, ,$$

and let $\lambda_0 = \lambda_4 = 4$, the following results can be gotten,

(a) When $k = 2$, the optimal values of local parameters are

$$\lambda_1 = -1.9492 \, , \; \lambda_2 = -1.6598 \, , \; \lambda_3 = -1.7580 \, .$$

The generated profile curve with minimal strain energy and the corresponding ceramic product form are shown in Fig. 8.

(b) When $k = 3$, the optimal values of local parameters are

$$\lambda_1 = -0.2765 \, , \; \lambda_2 = -1.1013 \, , \; \lambda_3 = 0.1601 \, .$$

The generated profile curve with minimal curvature variation energy and the corresponding ceramic product form are shown in Fig. 9.

Example 5 Given the coordinates of control points $\boldsymbol{p}_i$ $(i = -1, 0, \cdots, 5)$ are

$$py_i = [1.5, 1.5, 0.5, 2.5, 3.5, 1.5, 1.5] \, , \; pz_i = [6, 6, 4.5, 5, 3.5, 2, 2] \, ,$$

and let $\lambda_0 = \lambda_4 = 4$, the following results can be gotten,

(a) When $k = 2$, the optimal values of local parameters are

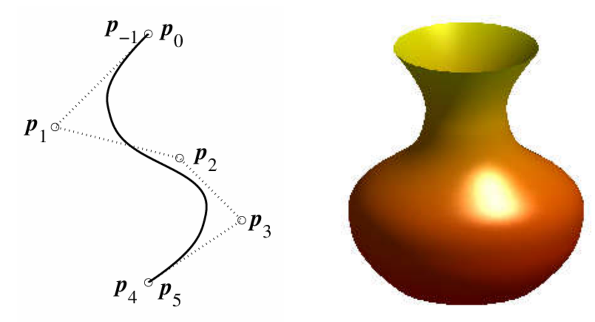
$$\lambda_1 = -2.0022 \, , \; \lambda_2 = -1.5388 \, , \; \lambda_3 = -1.8558 \, .$$

The generated profile curve with minimal strain energy and the corresponding ceramic product form are shown in Fig. 10.

(b) When $k=3$, the optimal values of local parameters are

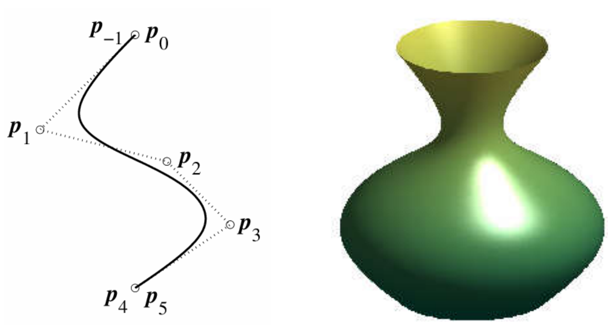
$$\lambda_1=-0.4049,\ \lambda_2=-0.8650,\ \lambda_3=0.1554.$$

The generated profile curve with minimal curvature variation energy and the corresponding ceramic product form are shown in Fig. 11.



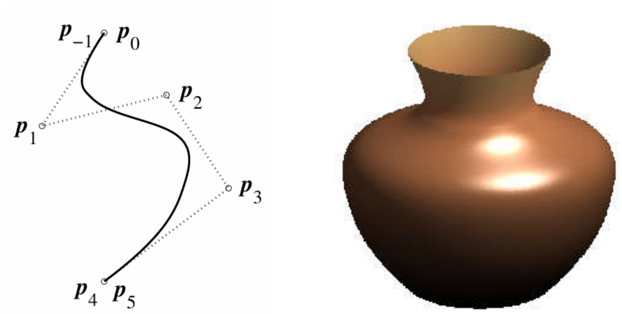
(a) The profile (b) The ceramic product form

Fig. 8 The profile curve generated by the strain energy minimization and the corresponding ceramic product form in Example 4



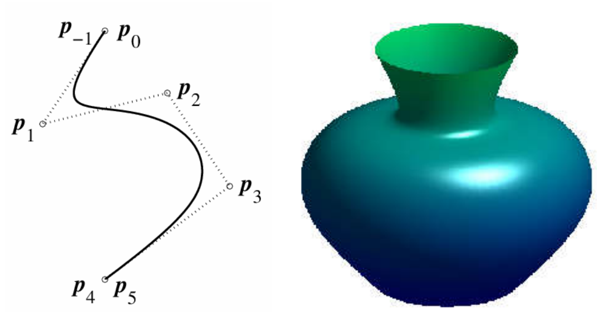
(a) The profile (b) The ceramic product form

Fig. 9 The profile curve generated by the curvature variation energy minimization and the corresponding ceramic product form in Example 4



(a) The profile (b) The ceramic product form

Fig. 10 The profile curve generated by the strain energy minimization and the corresponding ceramic product form in Example 5



(a) The profile (b) The ceramic product form

Fig. 11 The profile curve generated by the curvature variation energy minimization and the corresponding ceramic product form in Example 5

V. Conclusion

Aiming at the design of ceramic products formed by drawing embryos, the cubic uniform B-spline is adopted to describe the profile curves. The profile curves with rich shapes can be obtained by adjusting the control points or local parameters of the cubic uniform B-spline, which meets the practicality and personality of ceramic product design. Furthermore, the optimal values of local parameters in the quasi-cubic uniform B-spline can be obtained by the strain energy minimization or variation energy minimization while the control points remain fixed. The energy minimization methods can generate smooth profile curves, which meets the aesthetics of ceramic product design. The proposed design methods for ceramic products formed by drawing embryos would have a relatively broad application prospect due to their small computational complexity and simple operation.

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