A Single Item Production Inventory Model with the Fuzzy Demand

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Abstract—**In this paper, we aim to investigate a model for a single item production system (economic production quantity) under the assumption that the demand rate is triangular fuzzy number. The other parameters, the rate production, fixed cost production and inventory cost are to be constant. Hence, the length cycle production is fuzzy number. Furthermore, the holding cost per cycle is fuzzy. Here, we need fuzzy integral to compute fuzzy holding cost. Accordingly, the annual total cost is also fuzzy. Actually, we can not formulate membership function for fuzzy holding cost and fuzzy annual total cost. The fuzzy ranking cannot be used in this model. However, we will do it numerically by alpha-cut approach to plot the fuzzy costs.**

Index Terms—**production inventory, triangular fuzzy number, fuzzy demand, fuzzy integral.**

I. Introduction

The economic production quantity (EPQ) model has been widely used in practice. An EPQ models are essential tools for optimizing production processes, minimizing costs, and maximizing efficiency in manufacturing and inventory management. In the traditional EPQ models, such as the classic Economic Order Quantity (EOQ) model, are based on deterministic assumptions and crisp mathematical formulations, which may oversimplify the complexities of real-world production environments. The parameters, such as the demand rate, production rate, setup cost, the holding cost, and the shortage cost are crisp values. However, in a real production-inventory system, these cost parameters are imprecise. Many researchers have tried to improve it with different assumptions in the parameter. Recently, the classical EPQ model has been generalized in many assumptions. The EPQ model is developed on the number of vendor and buyer, defective or perishable item, number of type item and so on. Based on properties of the parameters in the model, EPQ model divided into deterministic, probabilistic and fuzzy models. Furthermore, to meet the demand, the EPQ model was extended to the backorder model.

The basic EPQ model is given in the [6]. In the EPQ model, there are no ordering costs, but there are fixed costs of production and production costs per unit. Goods are produced at certain time intervals at a constant rate. Demand and production are assumed to be continuous.

Nadjafi [3] has been considered EPQ model where unit production and set-up costs are assumed to be continuous functions of production rate. The total cost function associated with this model is proved to be non-convex. To determine the optimal production quantity and rate of production a procedure is proposed to solve this problem.

Pandey [2] develop an economic production quantity (EPQ) inventory model for deteriorating items with the constant rate of deterioration. The rate of demand is stock dependent. Shortages are not allowed. The model is formulated by two differential equations with boundary conditions for time production and time non production.

A production inventory model with shortages are allowed and fully backlogged has been formulated by Mahapatra et al. [7]. Assumptions in the model, demand varies with the on hand inventory level and production price. The preparation time is assumed to be crisp in the first model or fuzzy in the second model. The setup cost depends on preparation time. The fuzzy preparation time is reduced to a crisp interval preparation time using nearest interval approximation and following the interval arithmetic, the reduced problem is converted to a multi-objective programming. The first model is solved by generalized reduced gradient technique and multi-objective. The second model is solved by Global Criteria Method.

Wang [10] considers the economic production quantity problem with shortage is allowed. All costs in this model, the setup cost, the holding cost and the shortage cost are fuzzy variables. This causes the annual cost to become a fuzzy variable. There are two approaches to optimize the problem. They are the expected value and chance constrained approaches. Consequently, a fuzzy expected value model and a chance constrained programming model are developed.

Biswas and Islam [1] developed an EPQ model with fuzzy cost without backorder. Here setup cost to be constant. In this model, price discount is availed for defective items. The cost parameters are assumed as triangular fuzzy numbers and to defuzzify the model signed distance method is applied.

Moghdani et al. [8] study an EPQ model with multi item. The assumptions in this model are triangular fuzzy demand and phased deliveries. The batch size in each delivery is considered since limitations in warehouse capacity and the number of order for each product. Shortage and delay in delivery are permitted. Using the a-cut approach, model is solved by meta heuristic algorithms.

Sharma et al. [6] consider production inventory system with backorder. In this model, it is assumed time dependent demand from the initial to the end of the stock level. Demand is assumed price dependent. Deterioration of item is taken time dependent and a shortage is allowed under partial backlogging. Here, all parameters are crisp value. The model is expressed by differential equations with boundary conditions.

Xie et al. [11] consider inventory control problems for demand and deteriorating items are depending on time. Defective products can be sold at a lower price due to

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a penalty fee, with the price determined according to the agreement in the contract. There are two models based on the type of inspections: the first model involves one inspection during the cycle, and the second model involves continuous monitoring throughout the cycle. In this model, the optimal time ordering and quantity ordering will be determined.

A. Preliminaries

We review theorem and definitions in fuzzy number theory [4] and fuzzy integral [5]. We use a bar notation over the letter to represent a fuzzy number. Firstly, we overview about fuzzy number.

Theorem 1:
A fuzzy set $\bar{V}$ in $\Re$ is called a fuzzy number if there exists a closed interval $[p, q] \neq \emptyset$, such that there is a function

$$\mu_{\bar{V}}(t) = \begin{cases} \ell(t) & , \quad t \in (-\infty, p) \\ 1 & , \quad t \in [p, q] \\ r(t) & , \quad t \in (q, \infty) \end{cases}$$

where $\mu_{\bar{V}}$ is the membership function for $\bar{V}$ with

i. $\ell : (-\infty, p) \to [0, 1]$ is monotonely increasing, continuous from the right and $\ell(t) = 0$ for $t \in (-\infty, w)$, for any $w \leq p$ and
ii. $r : (q, \infty) \to [0, 1]$ is monotonely decreasing, continuous from the left and $r(t) = 0$ for $t \in (u, \infty)$, for any $u \geq q$.

To abbreviate the writing, we use the notation ℓ and r to denote $\mu^L_{\bar{V}}$ and $\mu^R_{\bar{V}}$ for fuzzy set $\bar{V}$, respectively.

The definition of a triangular fuzzy number is given below

Definition 1: A fuzzy number $\bar{V}$ is called a triangular fuzzy number(TFN) if the membership function for $\bar{V}$ is

$$\mu_{\bar{V}}(t) = \begin{cases} \mu^L_{\bar{V}}(t) = \frac{t - v_l}{v - v_l} & , \quad v_l \leq t < v \\ 1 & , \quad t = v \\ \mu^R_{\bar{V}}(t) = \frac{v_u - t}{v_u - v} & , \quad v < t \leq v_u \\ 0 & , \quad t > v_u, t < v_l \end{cases}$$

The triangular fuzzy number $\bar{V}$ is denoted by the triplet $\bar{V} = (v_l, v, v_u)$, with v_l and v_u are the lower and upper limits of the support of $\bar{V}$ respectively.
The definition of α-cut and fuzzy arithmetic of fuzzy set are given below.

Definition 2: Given $\bar{V}$ be a fuzzy number in $\Re$ and $\alpha \in (0, 1]$. The α-cut of fuzzy number $\bar{V}$ is the crisp set $\bar{V}_\alpha$ given by

$$\bar{V}_\alpha = \{t \in \Re : \mu_{\bar{V}}(t) \geq \alpha\}.$$

The α-cut of fuzzy number is a closed interval. We use approach based on α-cut to develop the arithmetic of fuzzy numbers. Let $\bar{V}$ and $\bar{W}$ be two fuzzy numbers and $\bar{V}_\alpha = [v^L_\alpha, v^R_\alpha], \bar{W}_\alpha = [w^L_\alpha, w^R_\alpha]$ be α-cut, $\alpha \in (0, 1]$, of $\bar{V}$ and $\bar{W}$ respectively. The arithmetic of closed interval is defined in the following:

(i.) $\bar{V}_\alpha + \bar{W}_\alpha = [v^L_\alpha + w^L_\alpha, v^R_\alpha + w^R_\alpha]$.
(ii.) $\bar{V}_\alpha - \bar{W}_\alpha = [v^L_\alpha - w^R_\alpha, v^R_\alpha - w^L_\alpha]$.
(iii.) $\bar{V}_\alpha : \bar{W}_\alpha = [\frac{v^L_\alpha}{w^R_\alpha}, \frac{v^R_\alpha}{w^L_\alpha}], 0 \notin \bar{W}_\alpha$ for $\bar{V}$ and $\bar{W}$ in $\Re_+$.
(iv.) $\bar{V}_\alpha \cdot \bar{W}_\alpha = [\min\{v^L_\alpha \cdot w^L_\alpha,\ v^L_\alpha \cdot w^R_\alpha,\ v^R_\alpha \cdot w^L_\alpha,\ v^R_\alpha \cdot w^R_\alpha\},\ \max\{v^L_\alpha \cdot w^L_\alpha,\ v^L_\alpha \cdot w^R_\alpha,\ v^R_\alpha \cdot w^L_\alpha,\ v^R_\alpha \cdot w^R_\alpha\}]$
(v.) $(h\bar{V})_\alpha = h\bar{V}_\alpha = [hv^L_\alpha, hv^R_\alpha]$, for $h > 0$.
(vi.) $(\frac{h}{\bar{V}})_\alpha = [\frac{h}{v^R_\alpha}, \frac{h}{v^L_\alpha}]$, for $h > 0$.

For any real number c, it is used (c, c, c) as fuzzy number and its $\alpha-$cut is $[c, c]$. We called $\bar{V} < \bar{W}$ if $v^R_\alpha < w^L_\alpha$ for $\alpha \in (0, 1]$. Here, we denoted $\bar{\mathcal{F}}$ is the set of all fuzzy. numbers

The definition of the fuzzy integral from [5] is used in this research. To determine inventory level in any time, we use this fuzzy integral.

Definition 3: Let a fuzzy function $\bar{g} : \bar{\mathcal{F}} \to \bar{\mathcal{F}}$. Given two fuzzy numbers $\bar{V}$, $\bar{W}$ and $\bar{V} < \bar{W}$, the fuzzy integral $\int_{\bar{V}}^{\bar{W}} \bar{g}tdt = \bar{Z}$ is defined through α-level,

$$(\mu^\ell_{\bar{Z}})^{-1}(\alpha) = \int_{(\mu^\ell_{\bar{V}})^{-1}(\alpha)}^{(\mu^\ell_{\bar{W}})^{-1}(\alpha)} (\mu^\ell_{\bar{g}})^{-1}(\alpha)tdt$$

and

$$(\mu^r_{\bar{Z}})^{-1}(\alpha) = \int_{(\mu^r_{\bar{V}})^{-1}(\alpha)}^{(\mu^r_{\bar{W}})^{-1}(\alpha)} (\mu^r_{\bar{g}})^{-1}(\alpha)tdt \ .$$

The fuzzy integral $\int_{\bar{V}}^{\bar{W}} \bar{g}tdt$ is exist, if $\bar{Z}$ is a fuzzy number. The index "-1" above represents the inverse of the function.

II. Mathematical Models

In the production process, products are usually produced in batches of a certain measurable size, known as the lot size in one production period. In the deterministic model, all parameters are crisp, then all output variables such as lot size, length of time production and costs are crisp. However, if there is an imprecision parameter then all decision variables are not precise. For example, if the demand is an imprecise, then the length of the production time will also be imprecise. In this current research, we formulate EPQ model with the following assumptions:

(i.) There is a single product.
(ii.) Production rate is assumed to be constant. Production process is occur continuously.
(iii.) There is a setup cost production in each early production process.
(iv.) The demand rate is imprecise and is assumed to be a triangular fuzzy number.
(v.) There is no backorders.

We use the notations of parameters and variables in the following:

- F= setup cost production
- h= the cost per unit-year of holding inventory (inventory cost)
- $\bar{D}$= fuzzy demand rate
- P= production rate (unit per year)
- $\bar{T}$= fuzzy time interval per cycle
- $\bar{Q}$= fuzzy lot size.

It is very difficult to draw system with fuzzy parameter. So, we illustrate the system, but there is no fuzzy parameter in Figure 1.

In the crisp EPQ model, we define the utilization coefficient $\rho = \frac{D}{P}$, where D is demand rate. The optimal time interval per cycle T is

$$T^* = \sqrt{\frac{2F}{hD(1 - \rho)}}.$$

In this fuzzy EPQ model, we let the fuzzy demand rate $\bar{D}$. So the length of production time in each cycle is fuzzy, we will denote by $\bar{t_p}$. Then to meet demand during one period(cycle),

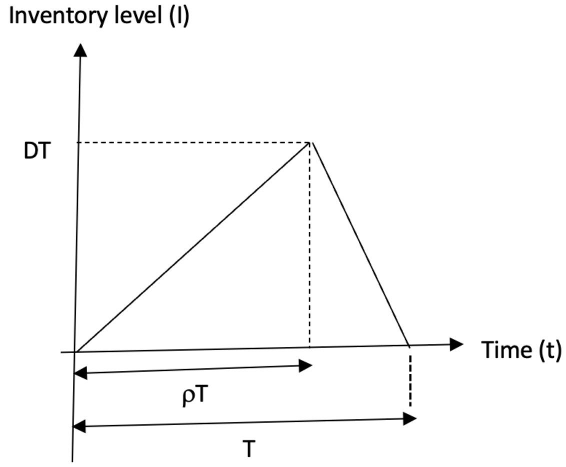

Fig. 1. The graphic inventory level versus time in the EPQ model

vendor must produce item amount $\bar{D}\bar{T}$. In this EPQ model, there is no backorders, so the number of products is equal to the number of demand. Consider the equations

$$P\bar{t_p} = \bar{D}\bar{T}$$

$$\bar{t_p} = \frac{\bar{D}\bar{T}}{P}. \quad (1)$$

Let $\bar{D}$ is a triangular fuzzy number.

Denoting $\overline{I(t)}$ the fuzzy inventory level at time t that is formulated as:

$$\overline{I(t)} = \begin{cases} (P-\bar{D})t, & \bar{0} \leq t < \bar{t_p}, \\ \bar{D}(\bar{T}-t), & \bar{t_p} \leq t \leq \bar{T}. \end{cases} \quad (2)$$

The first, compute the fuzzy holding cost per cycle $\bar{H}$ that can be formulated by

$$\bar{H} = h(\int_0^{\bar{t_p}} (P-\bar{D})tdt + \int_{\bar{t_p}}^{\bar{T}} \bar{D}(\bar{T}-t)dt). \quad (3)$$

In one year, there is $\frac{1}{\bar{T}}$ fuzzy cycle. The fuzzy holding cost per year is

$$\frac{\bar{H}}{\bar{T}}. \quad (4)$$

Whereas the setup cost per year is obviously

$$\bar{S} = \frac{F}{\bar{T}}. \quad (5)$$

The fuzzy annual total cost is sum of the fuzzy holding cost per year and the fuzzy setup cost per year. Here, the fuzzy annual total cost is a function of the cycle length $\bar{T}$ becomes

$$\overline{TC} = \frac{F}{\bar{T}} + \frac{\bar{H}}{\bar{T}}. \quad (6)$$

$$= \frac{F+\bar{H}}{\bar{T}} \quad (7)$$

We have $\overline{TC}$ is a function over $\bar{T}$. Generally, we can minimize the total cost by fuzzy ranking to convert the fuzzy model to the crisp model. However, in this case, it is very difficult to get membership function for fuzzy holding cost and fuzzy annual total cost, because the costs are functions over $\bar{T}$. Here, we propose a different approach. Since, for $\alpha = 1$, all output variables in the fuzzy EPQ will be the same as in the deterministic EPQ. Then the initial step, decision makers must find T, where T is the cycle length for deterministic EPQ. It is understood that for $\alpha = 1$, $\mu_{\bar{T}}(T) = 1$. The decision maker can take a symmetry triangular fuzzy number. Here $\bar{T} = (T-s, T, T+s)$ for positive number s. The $\bar{T}$ will minimize annual total cost that dependent to a value s. We call s as violation to the cycle length. Then, we solve them numerically to find fuzzy holding cost and fuzzy annual total cost. The fuzzy lot size $\bar{Q}$ can be computed by

$$\bar{Q} = P\bar{T}.$$

III. Numerical Example

Given an EPQ model with a fuzzy demand rate as assumed in this model with the following parameters :

$F = \$50$
$h = \$1$ per unit per year
$\bar{D} = (192, 240, 288)$ unit per year in fuzzy
$P = 600$ unit per year

Using Equation (1), we compute fuzzy time length of production

$$\bar{t_p} = \frac{\bar{D}\bar{T}}{P} = \frac{1}{600}(192,240,288)\bar{T} = \frac{1}{25}(8,10,12)\bar{T}.$$

Refer to Equation (3), we compute holding cost per cycle $\bar{H}$,

$$\bar{H} = \int_0^{\frac{1}{25}(8,10,12)\bar{T}} (600-(192,240,288))tdt+$$
$$+\int_{\frac{1}{25}(8,10,12)\bar{T}}^{\bar{T}} (192,240,288)(\bar{T}-t)dt$$

For degree of membership function of fuzzy number $\alpha = 1$, the fuzzy model is the same the crisp model. For fuzzy $\bar{D} = (192, 240, 288)$, in the deterministic case, that means $D = 240$ unit per year. Furthermore, we get the optimal T is close to 0.8. Here, we let $T = 0.8$. Here, we assume that $\bar{T}$ is triangular fuzzy number, $\bar{T} = (T-s, T, T+s)$. The α-cut for $\bar{T}$,

$$\bar{T}_\alpha = [s\alpha + T - s, -s\alpha + T + s].$$

We have

$$\bar{T}_\alpha = [s\alpha + 0.8 - s, -s\alpha + 0.8 + s].$$

For fuzzy number $\bar{D}$, we have $\bar{D}_\alpha = [48\alpha + 192, -48\alpha + 288]$. The decision maker need to take s, here we take $s = 0.04$ year. The fuzzy lot size $\bar{Q}$ is

$$\bar{Q} = P\bar{T} = 600(0.76, 0.8, 0.84) = (456, 480, 504).$$

Using Definition 3, we compute $\bar{H}$ through α-cut. We have

$$(\mu^L_{\bar{H}})^{-1}(\alpha) = \int_0^{\frac{1}{25}(2\alpha+8)(s\alpha+0.8-s)} (600-48\alpha-192)tdt+$$
$$\int_{\frac{1}{25}(2\alpha+8)(s\alpha+0.8-s)}^{(s\alpha+0.8-s)} (48\alpha+192)(s\alpha+0.8-s-t)dt$$

and

TABLE I
THE DATA TO PLOT FUZZY HOLDING COST PER CYCLE

α	$(\mu^L_{\bar{H}})^{-1}(\alpha)$	$(\mu^R_{\bar{H}})^{-1}(\alpha)$
1	46.08	46.08
0.95	45.694	46.4619
0.9	45.3041	46.8398
0.85	44.9103	47.209
0.8	44.5127	47.5829
0.75	44.1114	47.9478
0.7	43.7065	48.3085
0.65	43.2981	48.6642
0.6	42.8862	48.8664
0.55	42.4711	49.3621
0.5	42.0526	49.7038
0.45	41.6309	50.0406
0.4	41.2062	50.3724
0.35	40.7784	50.6991
0.3	40.3477	51.0206
0.25	39.9141	51.3368
0.2	39.4776	51.6476
0.15	39.0385	51.953
0.1	38.5967	52.2528
0.05	38.1525	52.5469
0	37.7057	52.8354

TABLE II
THE DATA TO PLOT FUZZY ANNUAL TOTAL COST

α	$(\mu^L_{\overline{TC}})^{-1}(\alpha)$	$(\mu^R_{\overline{TC}})^{-1}(\alpha)$
1	120.1	120.1
0.95	119.3192	120.8796
0.9	118.5374	121.6580
0.85	117.7547	122.4295
0.8	116.9712	123.2107
0.75	116.1869	123.9846
0.7	115.4021	124.7570
0.65	114.6168	125.5270
0.6	113.8311	126.1051
0.55	113.0454	127.0615
0.5	112.2593	127.8254
0.45	111.4731	128.5869
0.4	110.6871	129.3459
0.35	109.9012	130.1022
0.3	109.1156	130.8557
0.25	108.3302	131.6062
0.2	107.5452	132.3536
0.15	06.7608	133.0979
0.1	105.9769	113.8387
0.05	105.1939	134.5760
0	104.4115	135.3097

$$(\mu^R_{\bar{H}})^{-1}(\alpha) = \int_0^{\frac{1}{25}(-2\alpha+12)(-s\alpha+0.8+s)}(600-288+48\alpha)tdt + \int_{\frac{1}{25}(-2\alpha+12)(-s\alpha+0.8+s)}^{(s-s\alpha+0.8)}(-48\alpha+288)(s-s\alpha+0.8-t)dt$$

After that, the fuzzy annual cost can be computed using the arithmetic of fuzzy number. Numerically, to plot fuzzy holding cost $\bar{H}$, we take some values α to compute $\alpha - cut$ as Table I. Using the same method, we list data as Table II to plot fuzzy annual cost $\overline{TC}$. Then we plot fuzzy holding cost per cycle in Figure 2 and fuzzy annual total cost in Figure 3.

IV. CONCLUSION

In this research, the demand rate is a triangular fuzzy number. We need fuzzy integral to formulate fuzzy holding cost. For triangular fuzzy cycle length, the time length of production will be non linear membership function. However, it is difficult to formulate the membership function. Furthermore, the fuzzy annual total cost has non linear membership function. Hence, we work numerically. By data of alpha-cut for some alpha, we plot fuzzy holding cost and fuzzy annual cost by Phyton.

Fig. 2. Graphic of Fuzzy Holding Cost per Cycle

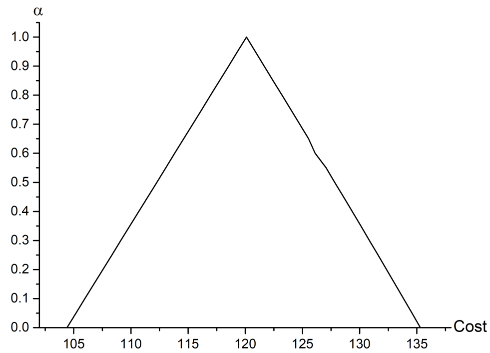

Fig. 3. Graphic of Fuzzy Annual Total Cost

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