Capital Requirement for Non-life Insurance Industry using D-vine Copula: An Empirical Evidence from Malaysia

Fatin Noor Najihah Abd Mutalip, Isaudin Ismail, *Member, IAENG*, Kek Sie Long

Abstract—**Generally, insurance companies face challenges in determining the Capital Requirement (CR) which is essential for business continuity. This situation could worsen if substantial number and amount of claims are requested by policyholders due to losses from catastrophic events such as floods, tsunamis, and earthquakes. In this paper, we propose a structured model to determine the CR for non-life insurance companies through loss ratios. Firstly, dependence structure among the business lines is modelled using copulas from both Elliptical and Archimedean copulas family with loss ratios from four business lines as the risk factors. The loss ratios are derived from incurred claims and earned premiums data of Malaysia's non-life insurance businesses, such as Fire insurance, Motor insurance, Marine, Aviation and Transport (MAT) insurance, and Miscellaneous insurance. Subsequently, a combination of popular risk measures such as Value-at-Risk (VaR) and Tail-Value-at-Risk (TVaR) with a Drawable Vine (D-Vine) copulas model known as hybrid model is developed to estimate insurance companies' risk capital. Finally, a simulation of Monte Carlo is commanded by calibrating data with our selected model to project the CR. This study contributes to the literature by addressing the problem of determining the appropriate CR for non-life insurance companies, proposing an empirical model based on real non-life insurance company data with a hybrid model built from risk measures and D-Vine copula.**

Index Terms—**non-life insurance, risk capital, risk measures, copula, D-vine copula.**

I. Introduction

THE model to determine the right amount of cash needed to support businesses is crucial, especially for insurance companies. It is widely known as Capital Requirement (CR) for insurance companies. Generally, life and non-life insurance are the two well-known types of insurance. However, since the main issue in determining the CR is involving high dimensional variables, therefore this paper focuses primarily on non-life insurance because its insurance nature comprises variety of business lines such as motor insurance, transport insurance and fire insurance which are suitable for modeling high dimensional data. Furthermore, this required cash determination issue could worsen if there are many claims from the policyholders due to catastrophes including floods, tsunamis, and earthquakes. This will cause the non-life insurance companies at risk of inability to paying out claims from policyholders. This study looks into how the CR for non-life insurance loss ratios in Malaysia is affected by explicit dependence models. The CR is the sum of funds an insurance company needs to achieve its goals. It refers to the amount of money companies need to start and maintain their businesses. Other than that, the CR planning is connected with all other parts within the business plan of the company. This is given that it includes all the costs that must be considered in the planning such as inventory, payroll and rent costs.

Insurance industry is a regulated industry. All insurance companies are obligated to be solvent and this means its policyholders are protected by law from biases and discriminatory treatments. In Malaysia, the insurance industry is regulated under the Ministry of Finance (MoF) which is the central bank, Bank Negara Malaysia (BNM). The Financial Service Act's (FSA's) provisions, which went into effect on June 30, 2013, govern Malaysia's non-life insurance industry. According to the BNM, the minimum CR for an insurer ranges from RM20 million to RM100 million, conditional on the type of insurance business. The BNM prescribed minimum CR is empowered by the Insurance Act 1996 (Act) to be maintained by an insurer. It is expected to build up the financial strength of insurers to assist them in making an enthusiastic approach to the country's insurance service and eventually compete with the other countries.

However, there are lack of standardized framework for calculating the minimum CR prescribed by the BNM. Therefore, this study can be served as an alternative framework to calculate the CR. Furthermore, multiple risk concerns from the asset and liability-related parts of the financial statement must be combined when assessing the insurers' overall risk and the CR. In this scenario, a particular asset or one line of insurance company is a risk factor. This risk factor is then aggregated by considering the aggregated distribution of sole risk originating within both liabilities and asset perspectives. Then, to emulate the distributional function of every risk factor, each marginal distribution must be parametrically described at this aggregation stage.

The copula methods, which are based on Sklar's theorem [1], are commonly utilized to aggregate heterogeneous margins into orderly distributions. The idea of copula was first established in the 1950s, with its discovery linked to Sklar [1], Fréchet [2], and Féron [3]. However, due to lack of proof of Sklar's theorem, Kimeldori & Sampson [4] and Whitt [5] rediscovered copula, while Scheweizer & Sklar [6] provided

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creative evidence of the two-dimensional (2D) distribution copulas in their publication. Moreover, the academic work that Schweizer & Wolff [7] authored is the first to use the idea of a copula to examine how random variables relate to one another. As such, the fundamental concept of a copula relies on a mechanism consisting the dependence structure for capturing a set of random variables. A form of multivariate distribution function that links together all individual marginal distributions is known as the copula function [8].

Although numerous bivariate copula families exist comparable to a broad area of complex dependencies, the dimensional restriction prevents them from describing the relationship between multivariate vectors. Despite the problem, an approach for high-dimensional copula models that tends to hierarchical frameworks according to building blocks was established. In numerous fields of study, high-dimensional dependence models such as the Pair Copula Construction (PCC) and the Hierarchical Archimedean Copula (HAC) have been implemented. The HAC also known as nested Archimedean, is a multivariate dependence framework generated by a multi- level hierarchical setting generators of Archimedean family. The purpose of this is to subdue the Archimedean copula basic methods versatility [9].

In addition, by associating the variable set as well as generating a configurable dependence framework, the PCC which also known as vine copula is frequently applied to lessen dimension [10], [11]. Note that the PCC is more adaptable than the HAC concerning the amount of copula functions that can be utilized for construction, but it has extra parameters to determine [12]. An algorithmic theoretical framework for the PCC has been established and then applied to a financial portfolio [13]. It also was employed by Brechmann et al. [14], Min & Czado [15], and Righi et al. [16] in calibrating problems related to interest rates. However, empirical research on combination of the insurance input with the high-dimensional asset modelling is inadequate.

A graphical model of the PCC system, known as the Regular Vine (R-Vine) was introduced [17], [11]. This structure links risk factors based on their dependency levels. Brechmann et. al [14] taken into account the characteristics of similar distribution to insurance claims which is the R-Vine coupla. The R-Vine copula is then used in modelling the dependence framework of operational risk losses. Peng et al. [18] demonstrated the relationship between market risk, credit risk and insurance risk using the utilization of R-Vine copula model in their study on integrated risk for insurance companies. Two specific examples for R-Vine models are copulas of Drawable Vine (D-Vine) and Canonical Vine (C-Vine). Both of them are frequently applied in finance and insurance. For each of the copulas, they provides a unique approach to density decomposition [19].

In tree structures, a features of a star configuration is shown by the C-Vine. It dedicates a primary risk factor is connected to all other factors. On the other hand, the D-Vine adopts a path configuration, establishing a hierarchical relationship with the primary risk factor linking the other factors [20]. Figures 1 and 2 demonstrate the stuctures of the of the C-Vine and D-Vine trees, respectively. These methods provide a straightforward graphical approach to efficiently produced a bivariate copula desities product decomposed

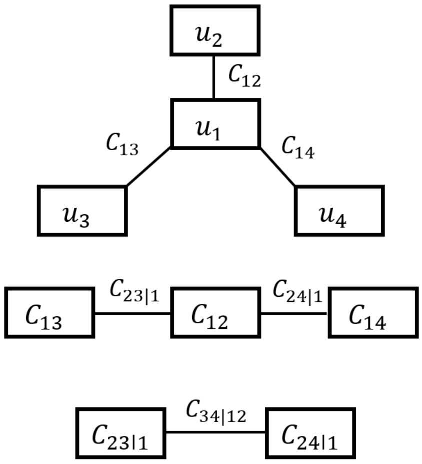

Fig. 1. The Star Structure of C-Vine Copula Tree in 4-dimensions

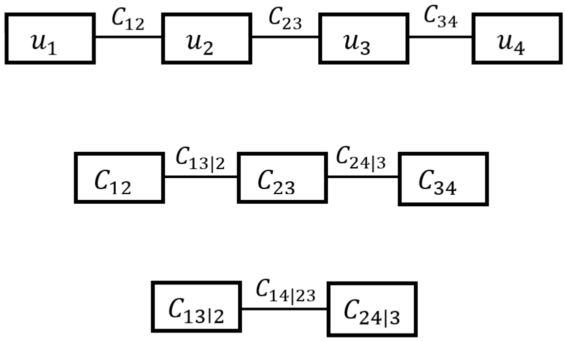

Fig. 2. The Path Structure of D-Vine Copula Tree in 4-dimensions

from the copula density functionv[21]. An enhanced predictive capabilities if offered by the C-Vine copula in comparison to the other copulas and especially the multivariate normal distribution.

Additionally, it effectively captures the interdependencies within collateral return data. Financial providers can integrate the C-Vine copula into their portfolio strategies for inventory management to lower default risks and overcome the overall profile's risk [22]. Sun & Wang [23] employed one of the R-Vine copulas which is the C-Vine copula model in performance evaluation of risk measurements. Their empirical findings indicated that the the C-Vine copula model in yielding dependencies is particularly suitable for measuring potential losses in portfolios with significant tail risks.

Conversely, the C-Vine structure is less versatile in comparison to the D-Vine structure because the D-Vine structure exhibits a superior flexibility. It demonstrates greater effectiveness when sequentially analyzing all the observed random variables mutual intercorrelations, which are the focus of this study [24]. In a separate application related to credit risk, utilization of D-Vine copula to model in equity dynamics is established by Dalla Valle et al. [25]. Therefore, the D-Vine copula adeptly captures various asymmetries and tail dependencies for different variable pairs. Additionally, a two-part D-Vine copula model was established by Yang & Czado [26]. The aim of the model is to analyze longitudinal mixed insurance claim data. The time dependence of binary outcomes is one of the part for this model. It is indicates the condition of the claim either it has been made or not.

The other part examines the condition of the occurrence in the claim size of the dependence. This model facilitates straightforward predictions for the quantiles of severity and the probability of claims once a claim has occurred.

Moreover, D-Vine copula model is chosen due to its simple path structure. The structure shows that in any tree, there is no node is linked to more than two edges. The order of the sequence in the first tree defined the entire structure. It is solely by the order of the sequence in the first tree, simplifying the calculations and enhancing flexibility. In contrast, the C-Vine copula resembles factor models, with a specific variable acting as a pivot (or factor) in each tree [27]. As this study does not aim to investigate any factor structures within the data sets, the model of D-Vine copula was deemed more appropriate.

The purpose of multivariate modeling in this study is to estimate risk capital for the non-life insurance company effectively. The risk capital is the financial security needed to ensure a company's survival in the worst-case scenario [28]. According to Bandt & Overton [28], they highlighted the significance of effective risk management by revealing that, during the crisis of financial in 2007-2008, 48% of occurrances derived from questionnaire surveys of 31 national supervisory authorities represented instances of failed insurance companies attributed to inadequate risk management practices.

As stated by Gayareta et al. [29], inadequate risk capital management results in compromised corporate governance and incurs penalties in the form of elevated the CR. Insufficient understanding and incompetence in addressing solvency within the CR framework contribute to the downfall of insurance companies, posing a potential threat to the financial industry as a whole. This situation increases systemic risk and has unfortunate consequences on the overall economy. Other than that, the calculation of risk capital can be obtained from the process of risk aggregation under different copula constraints.

The risk capital model presented in this study focuses on a risk aggregation-specific methodology which is dependent to the D-Vine copula. This aim of this study is to regulate the CR of non-life insurance companies and the appropriate risk measures for risk capital. Furthermore, this study also aims to analyze if the difference in types and values of risk measures as the independent variable affects the CR as the dependent variable. It is helpful for the non-life insurance companies and regulators to determine better risk management and solvency decision-making.

In this study, the data used in the determination of the CR is obtained from the non-life insurance company in Malaysia which includes the total of 17 non-life insurance companies. The data are extracted from four business lines' loss ratios which are Fire insurance, Motor insurance, Marine, Aviation and Transport (MAT) insurance and Miscellaneous insurance. Next, we established the D-Vine copula model to resolve the CR problem of the non-life insurance companies. Throughout the process, we explored the appropriate risk measures for the insurance risk capital. Finally, we conducted Monte Carlo simulation analysis to achieve the minimum CR of this study. Therefore, in Section II, methodology of our selected method is discussed. Results and discussion of our analysis is discussed in Section III and lastly, Section IV presents the conclusion.

II. METHODOLOGY

Our empirical analysis refers to the multiple business lines focusing on the second type insurance which is the non-life. This study utilized two types of data: incurred claims and earned premiums. All data analyzed in this study were issued by the BNM and extracted from an online data platform, CEIC. The CEIC is a data company provider and helps in navigate the world of macroeconomic data. Note that the data frequency are semi-annually collected from June 2009 to June 2022. For simplicity, this study taken into account four business lines of the non-life insurance industry. These four lines are sufficient for our analysis without involves the unwanted complications along the process. In this context, we focuses on the following four non-life business lines: the Fire insurance (Fire), the Motor insurance (Motor), the Marine, Aviation and Transport insurance (MAT), and the Miscellaneous insurance (Miscellaneous). These four business lines are the top four highest claims by the earned premium. Therefore, this section consists of related concepts and theories used for our model which is the D-Vine copula.

A. Copula Functions

For the copula utilization process, the marginal distribution functions and their joint distributions must be separated when modeling the dependence structure among insurance loss ratios. The copula is essential in explaining asymmetric dependence structures without making assumptions about the marginal distributions' parametric nature [30]. Sklar [1] was the first scholar to introduced the concept of copula to the literature, while McNeil et al [31] were among the first to introduce its application in finance. A copula, in Mathematical definition, is a multivariate distribution function with uniformly distributed margins, defined on the unit cube $[0,1]^n$. For instance, we take into account $(x_1, ..., x_n)^T$ as a random vector and F is the joint distribution function.

Sklar [1] stated that a particular copula function C occurs when:

$$F(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)), \tag{1}$$

which n-dimensional distribution function with margins $(F_1, \ldots, F_n)$ is denoted by F. Consequently, from Eq. 1, the expression of C is as written below:

$$C(u_1, ..., u_n) = F\{F_1^{-1}(u_1), ..., F_n^{-1}(u_n)\}, \tag{2}$$

in which the inverse distribution functions is represents by $F_1^{-1}(u_i)$ with respect to the marginal. The joint probability density function f of a copula C concerning an absolutely continuous F having rigorously rising continuous marginal $F_1, \ldots, F_n$ is expressed as below:

$$f(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)) \times [\prod_{i=1}^{n} f_i(x_i)], \tag{3}$$

in which the product with respect to the copula density and the univariate marginal densities is conveyed by f, whereas $c(.)$ represents the density of copula given by

$$c(u_1, ..., u_n) = \frac{\vartheta^n c(u_1, ..., u_n)}{\vartheta u_1 \cdot \vartheta u_2 ... \vartheta u_n}.$$

There are several different types of copula families. We focus especially on Archimedean and Elliptical copulas in this particular study. Student-t and Gaussian copulas are the two types of Elliptical copulas, originate from Elliptical multivariate distributions. They take the linear dependence into consideration. On the other hand, the Archimedean copulas thoroughly describe various dependence structures, including asymmetric dependencies where the coefficients values of the upper and lower tails are contrary. In this study, four distinct families of Archimedean copulas are utilized. A positive dependence with an upper tail is shows by the Gumbel copula, while positive dependence but with a lower tail is demonstrates by the Clayton copula. Both positive and negative dependence and tail independence is characterized by the Frank copula. Lastly, positive dependence with an upper bound is exhibits by the Joe copula.

Furthermore, the concept of a rotating copula function is also taken into account. A rotating copula function only applies for copula functions with an asymmetric dependence structure. From a practical standpoint, if two variables u and v are modeled using a Gumbel copula, it can be observed that the variables $1-u$ and $1-v$ exhibit a rotated Gumbel copula. This rotated copula signifies a shift in the nature of dependency, with a high dependence in the lower tail instead of the upper tail. The rotated copulas are also known as the respective family's survival copulas. Note that an Archimedean copula's survival copula is not an Archimedean copula. Mixtures of two or more copulas, which simply represent a convex combination of the copula functions under consideration, can also be taken into account, in addition to the aforementioned copula families. This allows for the creation of any desired dependent structure [32].

B. Pair Copula Construction (PCC) Model

In high-dimensional copula modeling, hierarchies based on the PCC model, popularly referred to as vine copulas, have recently gained popularity. Aas et al. [13] introduced the D-Vine copula and the C-Vine copula which are the two distinct R-Vine copulas. A n-dimensional vine is depicts as below:

1) Trees (T) of $(n-1)$;
2) The tree j has $(n-j)$ edges and $(n-1+j)$ nodes;
3) A bivariate copula density corresponds with each edge;
4) The nodes of the tree $j+1$ is the borders of the tree j;
5) The concept of complete decomposition is characterised by the presence of $n(n-1)/2$ edges and the inclusion of marginal densities for each variable. Specifically, this entails the consideration of $n(n-1)/2$ bivariate copula densities and N marginal.

The PCC as a product of several bivariate pair-copulas can be use to articulates the joint copula density c. Consider a collection of n independent random variable $X=(X_1,...,X_n)$; the joint density function is represent by f,

$$f(X_1,\dots,X_n) = f(X_n)\cdot f(X_{n-1}\mid X_n) \cdot f(X_{n-2}\mid X_{n-1},X_n) f(X_1\mid X_1,...,X_n), \tag{4}$$

where conditional density is denotes by $f(.|.)$. Furthermore, since the database does not consists any pivot variables, the D-Vine copula model is relevant in this study. There is $n!$ known as a possible sequence in the D-Vine structure with n dimensions to the root of the tree. $f(u_1,\dots,u_n)$ which is a density is given by:

$$\prod_{k=1}^{n} f(u_k)\prod_{j=1}^{n-1}\prod_{i=1}^{n-j} C_{ij}|F(u_1|u_{i+1},...,u_{i+j-1}), F(u_{i+j}|u_{i+1},...,u_{i+j-1})|, \tag{5}$$

in which the trees is identified by index j, whereas i runs over the edges of each tree.

The D-Vine simulation techniques are very straightforward to apply [13]. A method advanced by Genest et al. [33] which is the maximum likelihood estimation (MLE) is employed for the D-Vine copula parameterization. This approach is chosen since it makes no assumptions about the margins' parametric form. Indeed, a poorly specified marginal parameter copula may affect estimation [33], [30]. According to Genest et al. [33], the MLE approach employs the empirical probability integral transform [0,1] to obtain uniform margins. This approach can be broken down into two steps:

1) Transform margins $\{(x_1^t,...,x_n^t)\}_{t=1}^T$ into uniform variables $\{(u_1^t,...,u_n^t)\}_{t=1}^T$ using the empirical Cumulative Distribution Functions (CDF).
2) Determine the copula parameters:

$$\hat{\theta} = argmax\sum_{t=1}^{T}\ln C\left(\hat{u}_1^t,...,\hat{u}_1^n\right).$$

C. Goodness-of-fit Analysis

In order to assess the accuracy of the test that is being used in a standard situation, a bivariate reference analysis need to be performed to confirm the selection of the best suitable D-Vine copula. Correspondingly, a fit test will be conducted to analyze the reliability of the diverse copula selection tests based on Genest et al. [34]. An alternative test is suggested by the authors by depending on the Cramer-Von Mises statistic, $\hat{S}_n$ [35]. Assuming the copula-underlying candidate, C_0 to be tested and let C be the optimum multivariate copula for empirical data modelling. Meanwhile, the parameter connected to the copula C is θ. The hypothesis that will be evaluated is as described in the following:

$$H_0: c\in C = C_0 : \theta\in\Theta.$$

We employed the Cramer-von-Mises statistic, S_n to calculate the distance between the prospective parametric copula C_n and the empirical copula $\hat{C}_n$. It is provided by

$$S_n = \int_{[0,1]^d} C_n(u)^2 dC_n(u), \tag{6}$$

in which the Kendall process is C_n represented by $\sqrt{n}(C_n - \hat{C}_n)$. $\hat{C}_n$ as the empirical copula of the uniform data $U_1,...,U_d$ can be written as follows:

$$\hat{C}_n(u) = \frac{1}{n}\sum_{i-1}^{n} 1_{(U_{i_1}\le U_1,...,U_{i_d}\le U_d)}.$$

The letter U denoted the empirical n-dimensional marginal distributions. A methodology from Genest et al. [35] which is the bootstrap method is then used to determine p-values, ignoring the distribution of S_n. If the observed value S_n is bigger than the distribution's $(1-\alpha)^{th}$ percentile, we rejected the null hypothesis. After fitting the vine copula modeling into our database, we usually look for the best model based on one or more parameters.

Following that, a multivariate framework is then implemented in the second step of the process in order to choose the most effective D-Vine copula. Two types of knowledge criteria are taken into account which are the Bayesian information criterion (BIC)[36] and the Akaike's information criterion (AIC) [37]. In estimation of the regression function, AIC indicates minimax-rate optimal rules. On the other hand, a consistent model selection rules is represent by the BIC.

In a research paper by Fang et al. [38] , they explored the viability of utilizing the AIC in a copula model selection between a range of potential candidate copula models. Their results demonstrated that the AIC approach is generally more effective and precise than the multiplier goodness-of-fit test. Additionally, both AIC and BIC criteria are characterized by loglikelihoods combined with a fixed penalty.

D. Risk Measures and Simulation Procedure

One of the primary aims of multivariate modelling is to accurately assess risk capital for a conventional insurance company. The risk capital is the term used to describe the financial security that must be maintained by a business in order to guarantee its continued existence even in the most adverse of circumstances [30]. For a certain risk tolerance threshold α and time period T, it is the amount of money kept on hand to cover losses. Using the probability space $(\Omega,\mathcal{F},P)$, a non-life insurance company can develop a comprehensive model for evaluating the CR in relation to non-life insurance risk for long-term care. Indeed, the model allows for calculating risk capital, which is obtained from risk aggregation under different copula constraints [31].

The risk capital model presented in this study focuses on a risk aggregation-specific methodology based on the D-Vine copula. In addition, the copula theory provides some of the terminology that we used in this study [13]. Data collected is aggregated and sorted into risk categories (different lines of businesses) and a time span of T. The risk category is denoted as

$$C_n = (F_1, ..., F_n) = X_1 + ... + X_n : X_1 \sim F_i, i = 1, ..., n, \tag{7}$$

when the distributions of marginal risk F_i are known. In this context, the variables X_i denote non-negative random variables that represent individual risks over a certain time period T. Furthermore, we made assumption that the aggregate loss ratio, S is greater than or equal to zero, where S is derived from a multivariate random vector of variables that are dependent on each other. Consequently, the aggregate loss ratio is determined by the manner in which various risks overlap. In the subsequent context, the variable X denotes the vector that reflects the claims for an insurance business or the aggregate loss ratios incurred by an organisation.

In contrast, Tang & Valdez [39] argues that the aggregate distribution at the business level can be determined by utilising the weighted average of each line's loss ratio, based on a predetermined proportion of earned premium. The total loss ratio is then calculated as follows:

$$S = \sum_{i}^{n} \lambda_{it} S_{it}, \tag{8}$$

where $\lambda_{it} = \frac{EP_{i,t}}{\sum_{i=1}^{n} EP_{i,t}}$ is the weight of the line i in the portfolio that is weighted on the earned premium in the period t in comparison to premium amounts of risk i. Note that the total of their portions is one. Consequently, the risk measure is practised to this presumption in the non-linear dependence model for risk capital evaluation. As an outcome, the CR is calculated using a statistical determination of capability prospect losses and quantile-based risk controls [30]. Significantly, the VaR is the greatest achievable loss that a corporation can sustain at a particular confidence level κ over a time horizon T. The α-VaR at a level $\alpha \in [0, 1]$ is defined as follows:

$$VaR_{1-\alpha}(X) = inf\{x \in R : P(X \leq x) \geq 1 - \alpha\}. \tag{9}$$

When the continuous and rigorously growing cdf $F(x)$ of the random variable X is satisfied, $VaR(X)$ is the sole x fulfilling $F(X) = P(X \leq x) = 1 - \alpha$, and $VaR_{1-\alpha}(X) = F_x^{-1}(1-\alpha)$. The VaR is the k-th quantile of the loss random variable X distribution.

Additionally, the Tail-Value-at-Risk (TVaR) measure provides insight into the characteristics of the upper tail of a distribution. The expression for the TVaR is given at a confidence level α within the range of 0 to 1 as below:

$$TVaR_\alpha(X) = \frac{1}{1-\alpha} \int_1^P VaR(X) \cdot dp. \tag{10}$$

For a continuous loss distribution function F_x, the assurance loss beyond the VaR is denoted by the TVaR:

$$TVaR_\alpha(X) = E_\alpha[X|X \geq VaR_P(X)] \tag{11}$$

Otherwise, the TVaR can be expressed as follows:

$$TVaR_\alpha(X) = VaR_\alpha(X) + \frac{1}{1-\alpha} E[(X - VaR_\alpha(X))^+] \tag{12}$$

Correspondingly, we forecast the N loss ratios using a multivariate model to generate the VaR and the TVaR measures of aggregated loss ratio, S at a confidence level α incorporating copulas. The steps are as follows:

1) For each individual loss ratio, fit a marginal distribution. Estimation of parameters is obtained.

2) Using the estimated cumulative distribution function, convert every parameter into uniformity $u_i \in (0; 1)$. For $i = 1, \ldots, n$, we write $u_i = \hat{F}_1(X_1), ..., u_n = \hat{F}_n(X_n)$.

3) For every pair of modified input variables, the corresponding copula $\hat{C}$ is fitted. Maximize the likelihood function to obtain at estimated variables $\hat{\theta}$.

4) To construct N iterations, replicate N times from the approximated copula. Estimating predicted aggregate loss ratios $\hat{S}_j = \sum \lambda \hat{S}_{ij}$ from N weighted portfolios to arrive at the CR.

III. Results and Discussion

This study used two different tools to assist with data analysis and visualization. We used Microsoft Excel and R programming. Specifically, Microsoft Excel was used in utilizing the preliminary data of earned premiums and incurred claims. It helps in the calculation of loss ratios for our data which is the four business lines of the non-life insurance companies. Meanwhile, R programming was mainly used in parameters estimation and structures visualization for the best D-Vine copula selection. It also assisted in risk measures calculation of the VaR and the TVaR.

A. Data Statistics

After analyzing the data presented in Section II, we compute the loss ratios by dividing the total insurance claims paid by the sum of earned premiums for each business line. As our investigation focuses on time series datasets, we opt to assess the stationarity of the data. Stationarity, a crucial concept in time series research, significantly influences the interpretation and forecasting of data. It asserts that the variable's value does not vary over time, indicating not a constant series but a consistent pattern of change within the series. Key summary statistics, such as the mean or variance, remain consistent throughout the time series.

Beyond its descriptive aspect, stationarity is fundamental for various analytical tools, statistical tests, and models. In the realm of time series modeling, the assumption of independence among individual data points is common for forecasting future points. The stability or stationarity of a dataset, where past cases remain constant, facilitates more straightforward time series modeling. Statistical modeling methods often rely on this assumption of stationarity, as deviations from it may lead to inaccurate and unreliable results, undermining both understanding and forecasting.

To address this issue, we utilized two types of tests. The first is the KPSS test, developed by Kwiatkowski et al. [40], and the second is the PP test, developed by Phillips & Perron [41]. To test the null hypothesis that an observed time series is stationary around a deterministic trend (trend stationary) versus the alternative hypothesis of a unit root, the KPSS in used. Conversely, to examine the null hypothesis that a time series is integrated of order against the alternative hypothesis of a unit root, the PP test is used.

The R programming software is employed to perform stationarity tests using the PP and KPSS tests. For the PP tests, the p-values consistently fall below 0.01, which strongly supports the rejection of the null hypothesis. This shows that the time series is integrated of order, in favor of the alternative hypothesis indicating a unit root. Conversely, the KPSS test p-values exceed 0.05, showing weak evidence against the null hypothesis that the observed time series around a deterministic trend is stationary. Therefore, the null hypothesis is accepted. Table I shows the detailed results. In conclusion, the dataset is considered stationary at all levels.

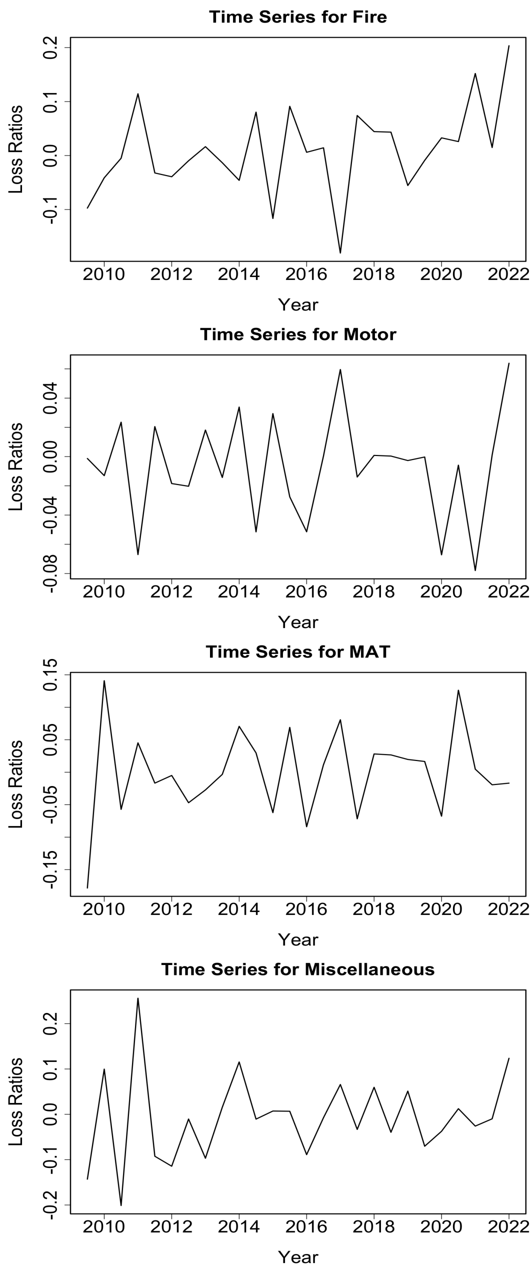

Fig. 3. Stationary Time Series of Fire, Motor, MAT and Miscellaneous Insurances Obtained from R Programming Software

TABLE I
STATIONARY TEST FOR EACH BUSINESS LINES' LOSS RATIOS

	Fire	Motor	MAT	Miscellaneous
PP test	0.006	0.009	0.008	0.001
KPSS test	0.426	0.052	0.184	0.161

PP test is significant at 1%, KPSS test is significant at 5%

Moreover, Figure 3 depicts the time series of each insurance once it has reached its stationary state. The Fire insurance time series in the figure illustrates that the time series does not involve any trends or seasonality. By trends meaning, it does not execute a long-term change in a time series level. For example, there exists an upward, which is an increase in level, or a downward, which is a decrease in level in terms of the changes.

Moreover, seasonality denotes a characteristic of a time series characterized by predictable or regular fluctuations in the data. Consequently, we infer that the Fire insurance time series exhibits stationarity, a conclusion that also holds for other insurance time series, with all data series acknowledged to be stationary at standard levels. Having established the stationarity of all data series, our analysis proceeded to calculate the CR for this study.

Next, Table II displays the four business lines descriptive statistics. From the table, we observed some degree of dispersion across all four variables. We noticed that the acquired data does not appear to be symmetrical. The presence of a positive coefficient of asymmetry supports this finding, which is the skewness that is more than 0 for three risk insurances: Fire, MAT, and Miscellaneous, and it falls somewhere in the range of 1.49, 0.12 and 0.92, respectively. A right-sided distribution asymmetry has been confirmed.

Conversely, Motor insurance shows -0.71 which is a negative skewness coefficient, indicating a leftward asymmetry in the distribution. The kurtosis coefficients for Motor and MAT insurances are both negative, specifically -0.03 and -0.78, respectively. In contrast, the kurtosis coefficients for the variables Fire and Miscellaneous are positive, specifically 1.87 and 0.13, respectively. The data reveals a departure from a normal distribution, underscoring the suitability of applying copula functions. The presence of diverse risks contributes to the observed variability in marginal distributions, resulting in distinct tail behavior variations.

TABLE II
SUMMARY OF DESCRIPTIVE STATISTICS FOR BUSINESS LINES' LOSS RATIOS

	Fire	Motor	MAT	Miscellaneous
Mean	0.37	0.71	0.40	0.45
Sd	0.12	0.07	0.06	0.08
Max	0.71	0.81	0.51	0.66
Min	0.19	0.55	0.28	0.34
Median	0.34	0.71	0.40	0.45
Kurtosis	1.87	-0.03	-0.78	0.13
Skewness	1.49	-0.71	0.12	0.92

When utilizing the copula, one of the crucial inputs to be considered is the correlation matrix. Table III describes the linear correlation matrix. In this study, the Pearson correlation was utilized in the datasets to analyze the linear relationship. It has come to our attention that the pair of business lines consisting of MAT and Fire insurances possesses the highest positive linear correlation coefficient, equal to 0.62 of all relationships.

Subsequently, the finding demonstrates that there exist a strongest linear relationship between these two variables among other variables. It tells us that if the coefficient of MAT insurance increases, the coefficient of Fire insurance will also increase, and vice versa. Since these two variables move in the same direction, and theoretically, they are influenced by the same external forces. In addition, it is abundantly evident that the relationship of Fire and Motor insurances displays the highest negative coefficient, a value of -0.62. This informs us that Fire and Motor insurances have the weakest linear relationship among the other pairs.

In addition to that, we investigate the non-linear relation by employing the non-linear correlation matrix, commonly known as Kendall's tau. The degree and direction of the relationship between two variables can be determined with the help of Kendall's tau. Its correlation matrix is laid up

TABLE III
LINEAR CORRELATION PEARSON MATRIX

	Fire	Motor	MAT	Miscellaneous
Fire	1.0000	-0.6209	0.6235	-0.1764
Motor	-0.6209	1.0000	-0.5196	0.5583
MAT	0.6235	-0.5196	1.0000	0.1270
Miscellaneous	-0.1764	0.1270	0.55835	1.0000

Notes: The sign of the correlation coefficient (positive or negative) indicates the direction of the relationship. The absolute value defines the strength of the correlation.

*The p-values in table are at a significance level of 5%.

similarly in Table IV. According to the findings, the pair consisting of Motor and MAT insurances has the highest negative correlation matrix. This signifies that the variables are inversely related to one another or that as one variable increases, the other variable drops. On the other hand, the pair consisting of Motor and Miscellaneous insurances displays the largest positive correlation, which is 0.4, and suggests that when one variable increases, so does the other variable.

TABLE IV
NON-LINEAR CORRELATION KENDALL'S TAU MATRIX

	Fire	Motor	MAT	Miscellaneous
Fire	1.0000	-0.1743	0.3286	-0.1114
Motor	-0.1743	1.0000	-0.2593	0.4017
MAT	0.3286	-0.2593	1.0000	0.1567
Miscellaneous	-0.1114	0.4017	0.1567	1.0000

*The p-values in table are at a significance level of 5%.

B. Fitting Marginal Distribution

To utilize the copulas simulation technique, having access to the marginal distributions of the loss ratios for each business line is crucial. This section examines the challenges arising from this requirement, estimates the required parameters, and presents the selected distributions. We focus on the loss ratios of individual business lines, assuming they conform to a predefined distribution. Therefore, we expect the behavior of loss ratio distributions to mirror that of claim severity distributions. The loss ratio represents the proportion of each business line's contribution to the total underwriting loss.

In this regard, we proposed four common distributions that are commonly advocated in actuarial science so that they can be examined. The first one is the lognormal distribution, which is the most frequently utilized in actual practice. This distribution has been included as a hypothesis in the form of distributions that adjust to the degree of the loss. The second distribution is the Gamma distribution, known for its heavy tail weight. Following that is the Weibull distribution, characterized by a thin tail. Lastly, the log-logistic distribution, a variant approaching normal distribution, is considered. Each of these distributions has distinct tail thicknesses, defined by two parameters. Various statistical methods can be used to determine these distribution values. Consequently, we opted for the Maximum Likelihood Estimation (MLE) method due to its direct numerical analysis approach.

Table V presents the findings obtained by performing MLE on the theoretical distributions. The values chosen for each distribution's parameters are considered to be reasonable.

TABLE V
ESTIMATED MARGINAL DISTRIBUTION OBTAINED FROM FOUR DISTRIBUTION TEST

	Distribution	Parameter 1	Parameter 2	KS	AD
Fire	Log-logistic	Shape: 7.9645	Scale: 0.3401	0.1328	0.4631
Motor	Weibull	Shape: 13.6121	Scale: 0.7371	0.1627	0.3911
MAT	Gamma	Shape: 46.8938	Rate: 116.9769	0.0820	0.1365
Miscellaneous	Log-logistic	Shape: 10.2214	Scale: 0.4418	0.0933	0.3958

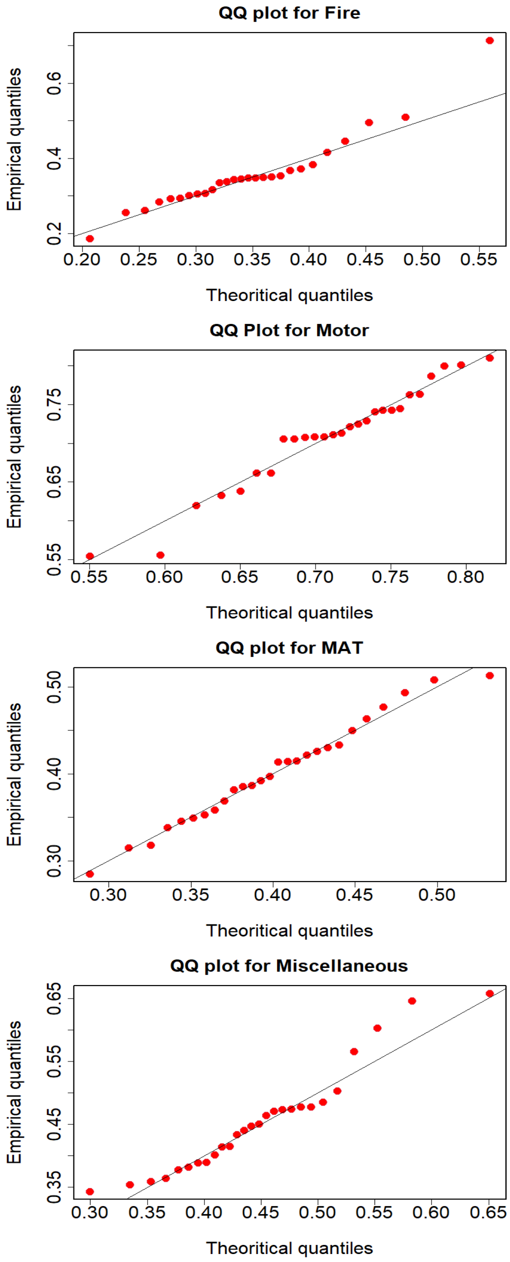

Fig. 4. QQ Plot of Selected Marginal Distributions for Four Business Lines' Loss Ratio Retrieved from R Programming Software

One of the most common challenges is figuring out how to choose the distribution that will provide the best match with our data set. In the statistical literature, we come across a few different graphic tests or visual comparisons. Our options are narrowed down based on the graphical representation of the probability QQ plot for each variable. According to the QQ plot depicted in Figure 4, the fitted distributions of the four variables have thicker tails at their endpoints. This denotes that appropriate distributions have been chosen for all of the variables. Subsequently, we evaluated the goodness-of-fit test utilizing the Kolmogorov-Smirnov and the Anderson-Darling tests.

The empirical and calculated parametric distributions are compared in this series of tests to determine how closely they correspond to one another. As a result, Table V is presented, which confirms the distribution estimates that were chosen for each variable. Notably, the absence of a significant difference between the two distributions is pointed out by a large p-value. According to the findings, the p-values indicate that the two insurances, Fire and Miscellaneous, have the distribution of the log-logistic. On the other hand, the Weibull distribution seems to be well fitted with the Motor insurance and it appeared that the gamma distribution is sufficient for altering the MAT insurance.

C. The Empirical Result

We aimed to utilize the multivariate copula to explore the interrelationships among variables and their dependent structure. To accurately gauge the effectiveness of the optimal D-Vine, it is essential to carefully choose six ideal combinations of bivariate copulas or $n(n-1)/2$ pairings of bivariate copulas. The selection of these bivariate copulas plays a crucial role in interpreting the results. To identify the best D-Vine copula, we estimated the bivariate copulas parameters. The initial step involves choosing which bivariate copula pairs will be positioned at level 1 of the tree. The optimal D-Vine combination is determined by establishing connections between pairings with the highest level of dependence in the initial tree. In this study, the specification tree of the D-Vine is progressively fine-tuned.

Moreover, the criteria for determining the tree structure involve assessing the AIC and BIC values for copula pairs. To simplify, we assigned numerical labels (1, 2, 3, and 4) to represent the types of insurance business lines: Fire, Motor, MAT, and Miscellaneous, respectively. As a result, we deem the permutation (1, 2, 3, 4) = (Fire, Motor, MAT, Miscellaneous) to be the most pertinent for the initial level of the D-Vine tree structure. This choice is based on the inclusion of the highest number of potential dependencies. The next step includes specifying the parametric structure of each copula within the model. The initial examination of the vine plot demonstrates the arrangement of different unique bivariate copula families, confirming the versatility of the copula D-Vine structure.

The initial lavel of the tree diagram incorporates three distinct pairs of copulas, each linking loss ratio variables according to their dependency levels. The D-Vine model is known to combine Elliptical and Archimedean copulas, both characterized by a single parameter. Our analysis indicates that the final D-Vine copula consists of specific copulas, including one Frank copula, two Gaussian copulas, two

TABLE VI
ESTIMATED PAIR-COPULAS AT EACH D-VINE COPULA MODEL TREE STRUCTURES (TREE 1, TREE 2 AND TREE 3)

	Copulas	Θ1	Θ2	Pair AIC	Pair BIC	$\hat{S}_n$
Tree 1						
MAT,Fire (1,3)	Survival Gumbel	5.28	-	-46.5266	-45.2307	0.0574
Motor,MAT (2,3)	Gaussian	-0.94	-	-34.8548	-33.5589	0.1385
Miscellaneous,Motor (2,4)	Gaussian	-0.83	-	-11.4176	-10.1218	0.2831
Tree 2						
(12,3)	Gumbel	-4.98	-	-13.7663	-12.4704	
(34,2)	Frank	-3.23	-	-41.6653	-40.3694	
Tree 3						
(14,23)	Gumbel	1.28	-	-1.67746	-0.3816	
AIC	[D-Vine]	-149.91				
BIC	[D-Vine]	-142.13				
Log-likelihood	80.95					

*The p-values in table are at a significance level of 10%.

Gumbel copulas, and one Survival Gumbel copula. The crucial step in the final phase of the procedure is to estimate the parameters of the bivariate copulas within the D-Vine structure.

The parameters of the D-Vine specification tree can be estimated step by step using the algorithm proposed by Aas et al. [13], as explained in Section II. The results of these estimations for the pair copulas chosen to create the resulting D-Vine copula are presented in Table VI. The empirical findings of the study indicate that four distinct bivariate copulas have negative parameters, which include the two Gaussian copulas, the Gumbel copula, and the Frank copula. Conversely, positive parameter values are observed in the other copulas, specifically the Survival Gumbel and Gumbel copulas.

Our findings reveal the presence of three distinct copulas at the first level of the tree, each characterized by a unique tail: the Survival Gumbel copula (5.28), and two Gaussian copulas (-0.94, -0.83). At the second level of the tree, the Gumbel copula (-4.98) and the Frank copula (-3.23) are utilized. Finally, the Gumbel copula (1.28) is employed at the third level of the tree. To validate the selection of the bivariate copulas in the initial level of the D-Vine tree, a robustness test based on Cramer-von Mises, denoted as $\hat{S}_n$, is conducted. The empirical results of the $\hat{S}_n$ statistics are summarized in VI. The p-values of $\hat{S}_n$ for the bivariate copulas do not reject the Survival Gumbel and two Gaussian copulas in the first tree.

Based on the data provided in the same table, the log-likelihood of the D-Vine model is 80.95. Additionally, it is crucial to evaluate the AIC and BIC criteria for the chosen copula pairs and identify the values that minimize these criteria to achieve the best D-Vine model. The AIC and BIC selection criteria were computed based on the fitted model to assess the accuracy of the vine copula structure for risk analysis, aiming for optimal results. Aas et al. [13] posit in their research that the goodness-of-fit test for vine copulas is reliable, and consequently, they apply these procedures to the bivariate blocks. The values of the AIC and BIC criteria are likewise displayed in Table VI.

Figure 5 illustrates the tree structure derived from the parametric D-Vine models. It is crucial to highlight that this tree reflects the best structure, demonstrating superior goodness-of-fit compared to other copulas examined. The flexibility of the D-Vine copula is evident in its ability to blend Elliptical and Archimedean copulas, even when handling low-frequency data.

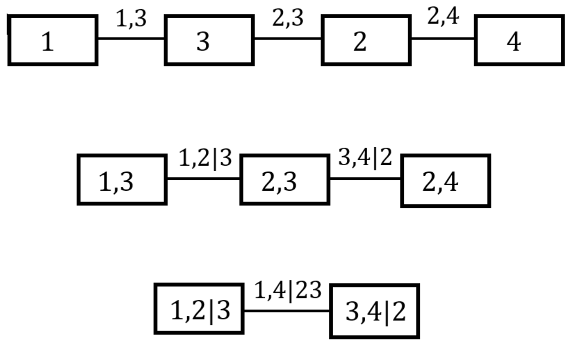

Fig. 5. The Optimal D-Vine Copula Tree in 4-dimensions Visualized by R Programming Software

D. Capital Requirements for the Non-life Insurance Industry

The application of our carefully chosen set of five copulas is instrumental in generating a dataset comprising 1,000 observations of loss ratios spanning across diverse business lines. Employing the risk aggregation model delineated in Subsection II-D, we construct a distribution for the aggregate loss ratio under the influence of the multivariate copula. The synthesis of simulated loss ratios involves the application of industry weights derived from earned premiums. This in-depth analysis of aggregate loss ratio data pertains specifically to an insurance company that aligns its business lines with industry standards, allowing for the creation of distinct distributions for the aggregate loss ratio under each copula.

Risk measures derived from the distribution of a random variable for total loss ratios are used to fine-tune the CR. As a result, the VaR and the TVaR portfolio forecasts are chosen based on each risk's relative weights at a given probability level. The CR is determined by applying the VaR and the TVaR risk measures at 95.5%, 97.5%, 98%, 99%, and 99.5%, respectively. These risk measures are computed empirically throughout the rest of this section by taking the simulated loss ratios and doing so in a relatively basic manner. Note that we computed the sample quantiles to obtain the VaR measure. Meanwhile, to achieve the TVaR measure, we computed the sample mean for the observed values higher than the corresponding quantile.

TABLE VII
ESTIMATED VaR AND TVaR DERIVED FROM D-VINE COPULA AND INDEPENDENT COPULA

		95.5%	97.5%	98%	99%	99.5%
D-Vine	VaR	10.8808	11.16109	11.24498	11.7083	12.0337
	TVaR	10.8909	11.1780	11.2836	11.7485	12.0919
Indep	VaR	10.9022	11.2146	11.3121	11.6610	11.8751
	TVaR	10.9115	11.2332	11.3333	11.68510	11.91428

In most cases, the distributions are based on a 1,000-simulated loss ratio. Hence, the VaR of 95.5 % corresponds to the 955th value. For the TVaR, we select the arithmetic average, or predicted value, of the values that come after the value that the VaR measure is based on. This means that the TVaR of 95.5% determined by taking the mean of the loss ratios distribution, specifically from the 956th to the 1000th ranked values. Therefore, Table VII presents data regarding the calculation of the CR using the VaR and the TVaR estimations incorporated by the D-Vine copula across different confidence levels.

Continuing with the previously described process, an alternate method of determining the amount of the the CR is proposed. The methodology employed in this study, which utilises a Monte Carlo simulation, neglects the underlying non-linear relationships among the risks. In order to enhance precision, the industry weights, which are established by the earned premium, are used to aggregate the simulated loss ratios for each business line. This is due to the construct of an aggregate loss ratio distribution based on the assumption of counter-monotony. Using this strategy, we analyzed the variation in the total amount of funds needed to maintain the same confidence levels while assuming that the results are independent. This analyses' findings are provided in the same table.

The D-Vine distribution is computed to be 10.8808 for the VaR of 95.5%. This indicates that for the value of 10.8808, we need to multiply it by the total earned premium, which is RM14,237,200. This value is obtained from the CEIC database of the total earned premium for the year 2021. After doing so, we accomplished the amount of RM154,912,552.90, which we may summarize into the figure RM154 million. The amount of RM154 million is our required CR for the insurance company to continue maintaining and operating its current business line. To conclude, we have a confidence level of 95.5% and are certain that if the capital of the insurance company is as much as RM154 million, the insurance company will be able to keep its business running. They will have no trouble handling the situation of many insurers claiming insurance simultaneously if a natural disaster occurs by chance, such as an earthquake or flood.

Nevertheless, the size of the influence can change depending on the risk measure that was utilized as well as the copula that was selected. In addition, the computation is conducted on the TVaR, which results in the amount of RM155 million. The TVaR will, as was to be expected, be slightly greater than the VaR. This is due to the fact that the TVaR describes the highest potential level of market risk. It is evident that the TVaR is always higher than the VaR and that the TVaR produces outcomes that are more appropriate than the VaR. As a result of this analysis, we realized that different values and types of risk measures would eventually affect the forecasting of CR using real-life data.

Moreover, the computation for the scenario of independence mirrors precisely that of the D-Vine calculation. The sole distinction lies in the multiplication by the total earned premium for the individual business line in the case of independence. This stems from the autonomy of the business lines, resulting in elevated VaR and TVaR figures —specifically, RM155.2 million and RM155.3 million, respectively —at a confidence level of 95.5%. Remarkably, it was observed that both VaR and TVaR are higher in the independence model compared to the D-Vine dependent scenario.

This observation holds significance, given that, to uphold tail dependence, a D-Vine copula is typically favored over a counter-monotonicity instance. Furthermore, the prescribed CR ranges between RM20 million to RM100 million, in accordance with Section I of the Insurance Act of 1996. In line with our computations, the CR derived from our D-Vine modeling amounts to RM154 million, surpassing the required minimum CR. Consequently, we can infer that the obtained amount from the D-Vine model aligns with the specified requirements.

IV. Conclusion

In this paper, we introduce a hybrid model that combines the D-Vine copula with the risk measures of Value at Risk (VaR) and Tail Value at Risk (TVaR). This model is used to calculate the capital requirement (CR) for four business lines within the non-life insurance, focusing on empirical evidence from the Malaysian insurance industry in Malaysia.

In essence, we consider up to four dimensions. However, expanding the scope is feasible by incorporating a larger dimension. Through the combination of associated risks, the potential insurer's portfolio risks could be reduced. Therefore, the results will be very useful for a more structured risk management.

References

[1] A. Sklar, "Fonctions de répartition à n dimensions et leurs marges," *Publications de l'Institut de Statistique de l'Université de Paris*, vol. 8, pp. 229–231, 1959.

[2] M. Fréchet, "Sur les tableaux de corrélation dont les marges sont données," *Ann. Univ. Lyon, 3ˆ e serie, Sciences, Sect. A*, vol. 14, pp. 53–77, 1951.

[3] R. Féron, "Sur les tableaux de corrélation dont les marges sont données. cas de l'espace à trois dimensions," in *Annales de l'ISUP*, no. 1, pp. 3–12, 1956.

[4] G. Kimeldori and A. Sampson, "Uniform representations of bivariate distributions," *Communications in Statistics–Theory and Methods*, vol. 4, no. 7, pp. 617–627, 1975.

[5] W. Whitt, "Bivariate distributions with given marginals," *The Annals of Statistics*, vol. 4, no. 6, pp. 1280–1289, 1976.

[6] B. Schweizer and A. Sklar, "Operations on distribution functions not derivable from operations on random variables," *Studia Mathematica*, vol. 52, no. 1, pp. 43–52, 1974.

[7] B. Schweizer and E. F. Wolff, "On nonparametric measures of dependence for random variables," *The Annals of Statistics*, vol. 9, no. 4, pp. 879–885, 1981.
[8] G. Kouaiba and D. Mentagui, "Modeling of market risk by elliptical copulas under the condition of coherence: Theory and application," *IAENG International Journal of Applied Mathematics*, vol. 52, no. 1, pp. 116–121, 2022.
[9] C. Savu and M. Trede, "Hierarchies of archimedean copulas," *Quantitative Finance*, vol. 10, no. 3, pp. 295–304, 2010.
[10] D. Berg and K. Aas, "Models for construction of multivariate dependence: A comparison study," *The European Journal of Finance*, vol. 15, no. 7, pp. 639–659, 2009.
[11] T. Bedford and R. M. Cooke, "Vines–a new graphical model for dependent random variables," *The Annals of Statistics*, vol. 30, no. 4, pp. 1031–1068, 2002.
[12] M. Eling and K. Jung, "Risk aggregation in non-life insurance: Standard models vs. internal models," *Insurance: Mathematics and Economics*, vol. 95, pp. 183–198, 2020.
[13] K. Aas, C. Czado, A. Frigessi, and H. Bakken, "Pair-copula constructions of multiple dependence," *Insurance: Mathematics and Economics*, vol. 44, no. 2, pp. 182–198, 2009.
[14] E. Brechmann, C. Czado, and S. Paterlini, "Flexible dependence modeling of operational risk losses and its impact on total capital requirements," *Journal of Banking & Finance*, vol. 40, pp. 271–285, 2014.
[15] A. Min and C. Czado, "Bayesian inference for multivariate copulas using pair-copula constructions," *Journal of Financial Econometrics*, vol. 8, no. 4, pp. 511–546, 2010.
[16] M. B. Righi, S. G. Schlender, and P. S. Ceretta, "Pair copula constructions to determine the dependence structure of treasury bond yields," *IIMB Management Review*, vol. 27, no. 4, pp. 216–227, 2015.
[17] T. Bedford and R. M. Cooke, "Probability density decomposition for conditionally dependent random variables modeled by vines," *Annals of Mathematics and Artificial intelligence*, vol. 32, pp. 245–268, 2001.
[18] H. Peng, Z. Yang, C. Xin, *et al.*, "Research on integrated risk of insurance company in my country based on r-vine copula model," *Academic Journal of Humanities & Social Sciences*, vol. 5, no. 2, pp. 74–85, 2022.
[19] K. Aas, "Pair-copula constructions for financial applications: A review," *Econometrics*, vol. 4, no. 4, p. 43, 2016.
[20] J. Dissmann, E. C. Brechmann, C. Czado, and D. Kurowicka, "Selecting and estimating regular vine copulae and application to financial returns," *Computational Statistics & Data Analysis*, vol. 59, pp. 52–69, 2013.
[21] O. Okhrin, A. Ristig, and Y.-F. Xu, "Erratum to: Copulae in high dimensions: An introduction," *Applied Quantitative Finance*, pp. E1–E1, 2017.
[22] B. Zhi, X. Wang, and F. Xu, "Portfolio optimization for inventory financing: Copula-based approaches," *Computers & Operations Research*, vol. 136, p. 105481, 2021.
[23] Y. Sun and K. Wang, "The implication of copula-based models for crop insurance and reinsurance under systemic risk," *Frontiers in Environmental Science*, vol. 10, p. 916494, 2022.
[24] S. Latif and S. P. Simonovic, "Parametric vine copula framework in the trivariate probability analysis of compound flooding events," *Water*, vol. 14, no. 14, p. 2214, 2022.
[25] L. Dalla Valle, M. E. De Giuli, C. Tarantola, and C. Manelli, "Default probability estimation via pair copula constructions," *European Journal of Operational Research*, vol. 249, no. 1, pp. 298–311, 2016.
[26] L. Yang and C. Czado, "Two-part d-vine copula models for longitudinal insurance claim data," *Scandinavian Journal of Statistics*, vol. 49, no. 4, pp. 1534–1561, 2022.
[27] P. V. Tófoli, F. A. Ziegelmann, O. Candido, and P. L. Valls Pereira, "Dynamic d-vine copula model with applications to value-at-risk (var)," *Journal of Time Series Econometrics*, vol. 11, no. 2, p. 20170016, 2019.
[28] O. d. Bandt and G. Overton, "Why do insurers fail? a comparison of life and non-life insolvencies using a new international database," in *90th International Atlantic Economic Virtual Conference. IAES*, 2020.
[29] A. Garayeta, J. I. De la Peña, and E. Trigo, "Towards a global solvency model in the insurance market: a qualitative analysis," *Sustainability*, vol. 14, no. 11, p. 6465, 2022.
[30] H. Mejdoub and M. B. Arab, "Impact of dependence modeling of non-life insurance risks on capital requirement: D-vine copula approach," *Research in International Business and Finance*, vol. 45, pp. 208–218, 2018.
[31] A. J. McNeil, R. Frey, and P. Embrechts, "Quantitative risk management: Concepts," *Economics Books*, 2015.
[32] H. Manner *et al.*, *Estimation and model selection of copulas with an application to exchange rates*. Citeseer, 2007.
[33] C. Genest, K. Ghoudi, and L.-P. Rivest, "A semiparametric estimation procedure of dependence parameters in multivariate families of distributions," *Biometrika*, vol. 82, no. 3, pp. 543–552, 1995.
[34] C. Genest, J.-F. Quessy, and B. Rémillard, "Goodness-of-fit procedures for copula models based on the probability integral transformation," *Scandinavian Journal of Statistics*, vol. 33, no. 2, pp. 337–366, 2006.
[35] C. Genest, B. Rémillard, and D. Beaudoin, "Goodness-of-fit tests for copulas: A review and a power study," *Insurance: Mathematics and Economics*, vol. 44, no. 2, pp. 199–213, 2009.
[36] G. Schwarz, "Estimating the dimension of a model," *The Annals of Statistics*, pp. 461–464, 1978.
[37] H. Akaike, "A new look at the statistical model identification," *IEEE Transactions on Automatic Control*, vol. 19, no. 6, pp. 716–723, 1974.
[38] Y. Fang, L. Madsen, and L. Liu, "Comparison of two methods to check copula fitting," *IAENG International Journal of Applied Mathematics*, vol. 44, no. 1, pp. 53–61, 2014.
[39] A. Tang and E. A. Valdez, "Economic capital and the aggregation of risks using copulas," *Available at SSRN 1347675*, 2009.
[40] D. Kwiatkowski, P. C. Phillips, P. Schmidt, and Y. Shin, "Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?," *Journal of Econometrics*, vol. 54, no. 1-3, pp. 159–178, 1992.
[41] P. C. Phillips and P. Perron, "Testing for a unit root in time series regression," *biometrika*, vol. 75, no. 2, pp. 335–346, 1988.

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