Synchronization of Novel Hyperchaotic Systems by Applying a Linear Active Controller and Its Application to Generating One Time Passwords

P. Muthukumar, *Member, IAENG,* and J. Murugan

***Abstract*—In this manuscript, a novel four-dimensional hyperchaotic system is introduced. The dynamical properties of the proposed system are examined. A linear active control approach is established and necessary conditions are derived for synchronizing hyperchaotic systems. A new algorithm for generating one time passwords (OTPs) is developed by utilizing the proposed hyperchaotic system. The effectiveness of the suggested techniques are demonstrated through numerical simulations.**

***Index Terms*—Chaos, Active control, Synchronization, One time password.**

I. Introduction

CHAOS is one of the most prominent features of nonlinear dynamical systems whose state variables are highly dependent on their initial conditions. This dependency leads to the divergent behavior of such systems, a fact that reveals the great importance of detailed study regarding chaotic phenomena. Due to the widespread occurrence of chaos in several disciplines, including dynamo theory, cryptography, chemical reactions, robotics, networks, and so on [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]. Recently, the problem of chaos control and chaotic system synchronization has received a great deal of attention [12], [13], [14], [15], [16], [17]. Complexity is significantly larger in hyperchaotic systems than in chaotic systems [18], [19], [20], [21], [22].

Chaos-based cryptosystems have emerged as a critical topic in the field of secure communication [23], [24], [25], [26]. The generation of random numbers is a critical technology for substantial numerical simulations and information security. Physical random numbers, in particular, are critical in information security for achieving irreproducibility and unpredictability. TRNGs (true random number generators) are widely regarded as the most critical component of a cryptographic system. Since no deterministic cryptographic primitive can start generating more information gain than is readily accessible at the inputs [27], [28], [29], [30]. As a necessary consequence, the unpredictable nature and security of a cryptographic system are primarily dependent on the TRNG, helping to make it the most critical and vital component.

In network security technology, an identity authentication has play more vital role and is to prevent unauthorized users from accessing network data and services. On the other hand, it can ensure that both parties have enough information to authenticate. A one-time password [31], [32], [33] is a form of identity authentication. Based on the information, an entity determines whether or not the customer is offered a password, after which the computer validates the password. If the password entered corresponds to the customer, the customer's identity is validated. Otherwise, the computer rejects the request and the authentication fails.

Chaos control and synchronization have been intensively investigated during the last decade and have attracted increasing attention in recent years. Up to now, numerous methods have been proposed to cope with chaos synchronization, such as backstepping design, adaptive design, impulsive control method, sliding mode control, linear quadratic regulator and other control methods [34], [35], [36], [37], [38]. However most of the proposed methods need more single-state variable information of the master system. However, for instance, the more state variables transmitted to the slave system the more bandwidth and energy consumption in secure communication system as well as security reduction. For example, in a real engineering case, some state variables may be difficult or even cannot be detected.

Motivated by the preceding discussion, the components of chaotic system[39] is extended to create a new hyperchaotic system. The proposed system's basic dynamical properties are investigated using phase portraits, equilibria, and Lyapunov exponents. An active control technique is used as a control application to synchronize the hyperchaotic system, and their theoretical results are derived using Lyapunov stability theory. A further application in information security is the generation of OTP for authenticating online transactions and other authentication methods using a synchronised hyperchaotic system.

The remaining part of the research is structured as follows: Section II describes a novel hyperchaotic system and analyses and numerically investigates its dissipativity, equilibria, and Lyapunov exponent. In Section III, an active controller is designed to control the chaotic trajectories to the unstable equilibrium. Section IV describes the method of synchronizing hyperchaotic systems with active controllers and their

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numerical simulations. Section V proposes a new algorithm for generating OTP based on a hyperchaotic system. Furthermore, it has been demonstrated that the numerical example works well with the proposed OTP algorithm. Section VI concludes the paper.

II. System description and analysis

A novel hyperchaotic four dimensional dynamical system is expressed by the following differential equations:

$$\begin{aligned} \dot{x} &= y - z \\ \dot{y} &= ay - x^2 z \\ \dot{z} &= -z + x \\ \dot{w} &= bw - cz^2 \end{aligned} \tag{1}$$

where x, y, z, w are the state variables and a, b, c are the parameters of the system (1). The system (1) exhibits hyperchaos when $a = 0.625, b = 0.09$ and $c = 3$. The different phase portraits of the chaotic attractor corresponding to the system (1) is shown in Fig. 1

In the following subsections, we will discuss about dissipativity, equilibrium points, stability and Lyapunov exponents of the hyperchaotic system (1).

A. Dissipativity

The state space of the system (1) is four-dimensional and their vector field is denoted by

$$V = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} y - z \\ ay - x^2 z \\ -z + x \\ bw - cz^2 \end{pmatrix} \tag{2}$$

A necessary and sufficient condition for the the system (1) to be dissipative is $\nabla V < 0$,

$$\begin{aligned} \nabla V &= \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} \\ \nabla V &= a + b - 1 = -0.285 < 0 \end{aligned} \tag{3}$$

Therefore, the system (1) is dissipative and its exponential contraction is $e^{-0.285}$.

B. Equilibrium points and stability

By making $\dot{x} = \dot{y} = \dot{z} = \dot{w} = 0$, the equilibrium point of the system (1) can be obtained as $E_1 = (0, 0, 0, 0)$, $E_2 = (\sqrt{a}, \sqrt{a}, \sqrt{a}, \frac{ac}{b})$ and $E_3 = (-\sqrt{a}, -\sqrt{a}, -\sqrt{a}, \frac{ac}{b})$.

Theorem 2.1: For $a = 0.625$, $b = 0.09$ and $c = 3$, all equilibria of the system (1) are unstable.

Proof: The Jacobian matrix of the system (1) is defined by

$$J = \begin{pmatrix} 0 & 1 & -1 & 0 \\ -2xz & a & -x^2 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -2cz & b \end{pmatrix}$$

Case 1: For equilibrium $E_1 = (0, 0, 0, 0)$,

$$J_1 = \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & a & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & b \end{pmatrix}$$

The Eigen values are calculated by $|\lambda I - J_1| = 0$ and their corresponding Eigen values are $\lambda_1 = -0.5000 + 0.8660i$, $\lambda_2 = -0.5000 - 0.8660i$, $\lambda_3 = 0.6250$, and $\lambda_4 = 0.0900$.

Here λ_3 and λ_4 is positive real number; λ_1 and λ_2 are complex with equal and non zero real parts. Therefore, the equilibrium $E_1 = (0, 0, 0, 0)$ is a saddle-focus. So the system (1) is unstable at E_1.

Case 2: For equilibrium $E_2 = (\sqrt{a}, \sqrt{a}, \sqrt{a}, \frac{ac}{b})$,

$$J_2 = \begin{pmatrix} 0 & 1 & -1 & 0 \\ -2a & a & -a & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & -2c\sqrt{a} & b \end{pmatrix}$$

The Eigen values are calculated by $|\lambda I - J_2| = 0$ and their corresponding Eigen values are $\lambda_1 = 0.0900$, $\lambda_2 = 0.1533 + 1.3455i$, $\lambda_3 = 0.1533 - 1.3455i$, $\lambda_4 = -0.6816$.

Here λ_1 and λ_4 are real and non zero of opposite signs. λ_2 and λ_3 are complex with equal non zero positive real parts. Therefore, the equilibrium E_2 is a saddle-focus point and the system (1) is unstable at E_2.

Case 3: For equilibrium $E_3 = (-\sqrt{a}, -\sqrt{a}, -\sqrt{a}, \frac{ac}{b})$,

$$J_3 = \begin{pmatrix} 0 & 1 & -1 & 0 \\ -2a & a & -a & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 2c\sqrt{a} & b \end{pmatrix}$$

Similarly, E_3 is also a saddle-focus point and the system (1) is unstable at E_3.

The above brief analysis shows that three equilibria of the proposed non-linear system (1) are all saddle-focus points. ■

C. Lyapunov exponents

The Lyapunov exponents are only quantitative measure for the existence of hyperchaos and identifying different behaviors of a dynamical system. The hyperchaotic nature of a system depends on the sign of its Lyapunov exponents and they are determined numerically by choosing the parameters $a = 0.625, b = 0.09, c = 3$ with initial condition $X(0) = (0.1, 0.5, 0.1, 0.5)$. The corresponding Lyapunov exponents of the system (1) are $\lambda_{L_1} = 0.1038$, $\lambda_{L_2} = 0.0821$, $\lambda_{L_3} = 0$ and $\lambda_{L_4} = -0.4685$.

Thus, the system (1) exhibits the hyperchaos since it has two positive Lyapunov exponents, which is depicted in Fig. 2. Further, the Lyapunov dimension of the system (1) is $D_L = j + \frac{1}{|\lambda_{L_{j+1}}|} \sum_{i=1}^{j} \lambda_{L_i} = j + \frac{1}{|\lambda_{L_4}|}(\lambda_{L_1} + \lambda_{L_2} + \lambda_{L_3}) = 3.3968$.

III. Control of the proposed hyperchaotic system

In this section, an active controller to be designed for controlling the proposed hyperchaotic system (1).
The controlled hyperchaotic system (1) is

$$\begin{aligned} \dot{x} &= y - z + \varphi_1 \\ \dot{y} &= ay - x^2 z + \varphi_2 \\ \dot{z} &= -z + x + \varphi_3 \\ \dot{w} &= bw - cz^2 + \varphi_4 \end{aligned} \tag{4}$$

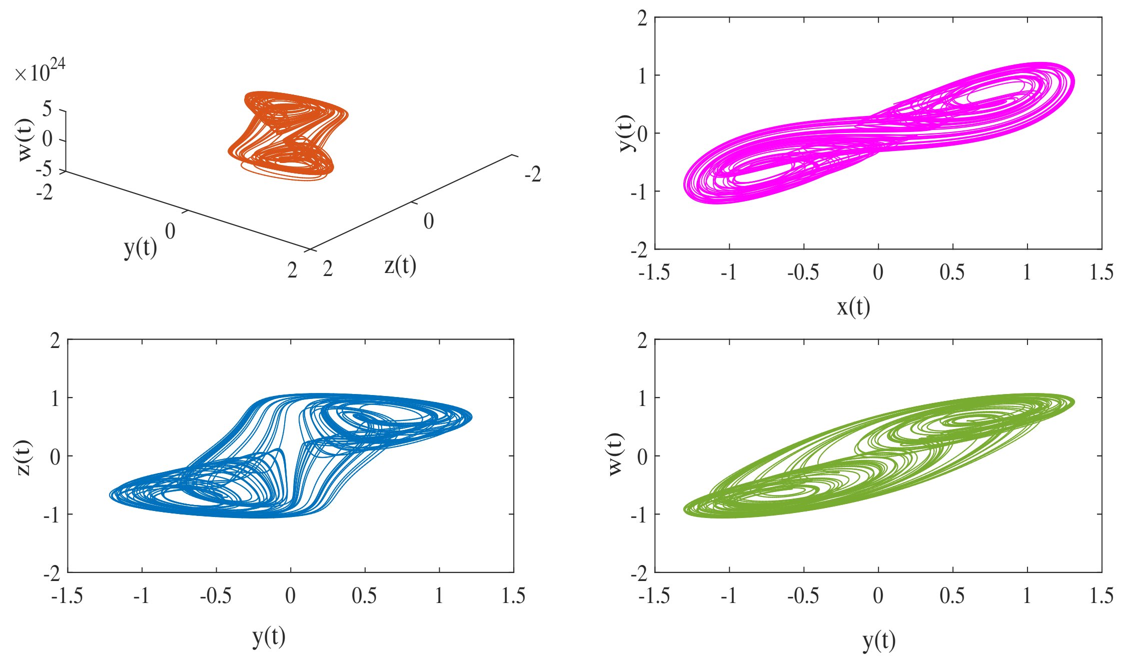

Fig. 1. Different phase portraits of the system (1).

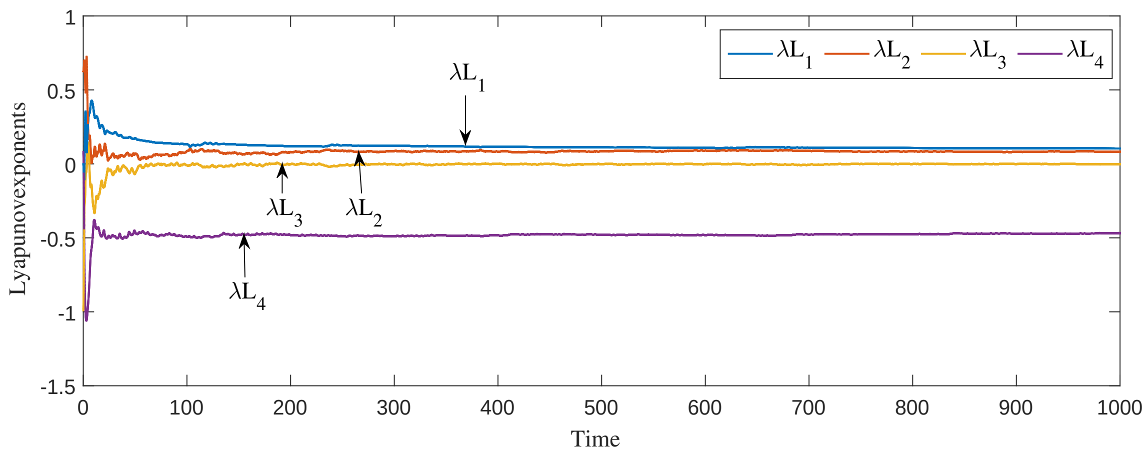

Fig. 2. Lyapunov Exponents of the system (1).

where φ_i's $(i = 1, 2, 3, 4)$ are the active controllers and it is described by

$$\begin{aligned} \varphi_1(t) &= -y + z + u_1(t) \\ \varphi_2(t) &= -ay + x^2 z + u_2(t) \\ \varphi_3(t) &= z - x + u_3(t) \\ \varphi_4(t) &= -bw + cz^2 + u_4(t) \end{aligned} \tag{5}$$

where $u_i(t)$ are linear feedback controller.

Substitute (5) in (4), the obtained closed loop system is

$$\dot{x} = u_1(t), \dot{y} = u_2(t), \dot{z} = u_3(t), \dot{w} = u_4(t) \tag{6}$$

Let as assume $u_1(t) = -k_1 x, u_2(t) = -k_2 y, u_3(t) = -k_3 z$ and $u_4(t) = -k_4 w$, where k_1, k_2, k_3 & k_4 are the feedback gains.

Theorem 3.1: The controlled hyperchaotic system (4) is globally and exponentially stabilized for all initial states $X(0) = (x(0), y(0), z(0), w(0)) \in R^4$ by the active control law (5).

Proof: Consider the quadratic Lyapunov function

$$V_1(X) = \frac{1}{2}(x^2 + y^2 + z^2 + w^2) \tag{7}$$

Here V_1 is a positive definite function on R^4. Based on Lyapunov stability theory, it is enough to prove that $\dot{V}_1$ is negative definite.

$$\begin{aligned} \dot{V}_1 &= x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} &(8) \\ &= -k_1 x^2 - k_2 y^2 - k_3 z^2 - k_4 w^2 \\ \dot{V}_1 &= -X^T P_1 X &(9) \end{aligned}$$

where $P_1 = \begin{pmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{pmatrix}$ for every $k_i > 0$, P_1 is positive definite and $\dot{V}_1$ is negative definite on R^4.

By Lyapunov stability theory, $x(t) \to 0, y(t) \to 0, z(t) \to 0, w(t) \to 0$ exponentially as $t \to \infty$. Hence, the system (4) is globally and exponentially stable. ∎

A. Numerical simulations

For $k_1 = k_2 = k_3 = 1$ and $k_4 = 5$, then (5) becomes

$$\begin{aligned} \varphi_1(t) &= -y+z-x \\ \varphi_2(t) &= -ay+x^2z-y \\ \varphi_3(t) &= z-x-z \\ \varphi_4(t) &= -bw+cz^2-5w \end{aligned} \tag{10}$$

Consequently, the controlled hyperchaotic system (4) gives

$$\dot{x} = -x, \dot{y} = -y, \dot{z} = -z, \dot{w} = -5w \tag{11}$$

The time variation of the controlled system (11) is depicted in Fig. 3. It shows that $x(t) \to 0$, $y(t) \to 0$, $z(t) \to 0$, $w(t) \to 0$ for every $t > 2$.

Result 3.2: The convergence rate is inversely proportional to feedback gains. If feedback gain increases then the convergence time response of control system (11) is decreases.

For instance, If $k_1 = k_2 = k_3 = 5$ and $k_4 = 25$, then $x(t) \to 0, y(t) \to 0, z(t) \to 0, w(t) \to 0$ for every $t \geq 1$.The time variation of the controlled system (11) is shown in Fig. 4.

It is clear that the time response of control system (11) by utilizing active controller is 2 times reduced by comparing Fig. 3 and Fig. 4.

IV. Synchronization of the hyperchaotic systems

In this section, the aim of study is to achieve asymptotically globally synchronization between two identical hyperchaotic systems using linear active control method. Also their numerical simulation results will be given.

A. Synchronization using the linear active control

Consider the system (1) as drive system and the following system as a response system, which is identical to the drive system

$$\begin{aligned} \dot{x_1} &= y_1 - z_1 + \psi_1 \\ \dot{y_1} &= ay_1 - x_1^2 z_1 + \psi_2 \\ \dot{z_1} &= -z_1 + x_1 + \psi_3 \\ \dot{w_1} &= bw_1 - cz_1^2 + \psi_4 \end{aligned} \tag{12}$$

where ψ_i's $(i = 1, 2, 3, 4)$ are the active controllers to be determined later.

Let $e_1 = x_1 - x,\ e_2 = y_1 - y,\ e_3 = z_1 - z$ and $e_4 = w_1 - w$ be the error variables.

Then the error system of (1) and (12) can be derived as

$$\begin{aligned} \dot{e_1} &= e_2 - e_3 + \psi_1 \\ \dot{e_2} &= ae_1 - x_1^2 z_1 + x^2 z + \psi_2 \\ \dot{e_3} &= e_1 - e_3 + \psi_3 \\ \dot{e_4} &= be_4 - cz_1^2 + cz^2 + \psi_4 \end{aligned} \tag{13}$$

Theorem 4.1: The identical hyperchaotic systems (1) and (12) will approach globally, exponentially and asymptotically synchronized with the following suitable active controller

$$\begin{aligned} \psi_1(t) &= u_1(t) \\ \psi_2(t) &= x_1^2 z_1 - x^2 z + u_2(t) \\ \psi_3(t) &= u_3(t) \\ \psi_4(t) &= cz_1^2 - cz^2 + u_4(t) \end{aligned} \tag{14}$$

where $u_i(t) = -k_i e_i$, $i = 1, 2, 3, 4$ and $k_i's$ are the feedback gains.

Proof: Substituting (14) in (13),

$$\begin{aligned} \dot{e_1} &= e_2 - e_3 - k_1 e_1 \\ \dot{e_2} &= ae_2 - k_2 e_2 \\ \dot{e_3} &= e_1 - e_3 - k_3 e_3 \\ \dot{e_4} &= be_4 - k_4 e_4 \end{aligned} \tag{15}$$

Consider the Lyapunov error candidate function as

$$V_2(E) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2) \tag{16}$$

where $E = (e_1, e_2, e_3, e_4) \in R^4$.

Then the time derivative of V_2 can be written as

$$\begin{aligned} \dot{V_2} &= e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} + e_4\dot{e_4} \\ &= -k_1e_1^2 - (k_2 - a)e_2^2 - (k_3 + 1)e_3^2 \\ &\quad -(k_4 - b)e_4^2 + e_1e_2 \\ &= -[k_1e_1^2 + (k_2 - a)e_2^2 + (k_3 + 1)e_3^2 \\ &\quad +(k_4 - b)e_4^2 - e_1e_2] \\ \dot{V_2} &= -E^T P_2 E \end{aligned} \tag{17}$$

where $P_2 = \begin{pmatrix} k_1 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & k_2 & 0 & 0 \\ 0 & 0 & k_3+1 & 0 \\ 0 & 0 & 0 & k_4 - b \end{pmatrix}$ is a positive definite matrix. It shows that $\dot{V_2} < 0$ for all $k_i > 0$ on R^4. Then, $\lim\limits_{t\to\infty} e(t_i) = 0, i = 1, 2, 3, 4$, based on Lyapunov stability theory.

Hence the systems (1) and (12) are globally, exponentially and asymptotically synchronized. ■

B. Numerical simulations

If $k_1 = 2, k_2 = 3, k_3 = 4$ and $k_4 = 20$, then the controller (14) becomes

$$\begin{aligned} \psi_1(t) &= -2e_1 \\ \psi_2(t) &= x_1^2 z_1 - x^2 z - 3e_2 \\ \psi_3(t) &= -4e_3 \\ \psi_4(t) &= cz_1^2 - cz^2 - 20e_4 \end{aligned} \tag{18}$$

Then the error system (15) can be written as

$$\begin{aligned} \dot{e_1} &= -2e_1 + e_2 - e_3 \\ \dot{e_2} &= -3.625e_2 \\ \dot{e_3} &= e_1 - 5e_3 \\ \dot{e_4} &= -19.91e_4 \end{aligned} \tag{19}$$

The time variation of the error system (19) using the linear active controller is depicted in Fig. 4. It shows that, the error between the systems (1) and (12) are completely synchronized.

Result 4.2: The convergence rate is inversely proportional to feedback gains. If feedback gain decreases then convergence rate of time response of error system (19) is increases.

For instance, If $k_1 = 1, k_2 = 1.5, k_3 = 2$ and $k_4 = 10$, then $e_1(t) \to 0, e_2(t) \to 0, e_3(t) \to 0, e_4(t) \to 0$ for every $t \geq 8$. The time variation of the error system (19) is shown in Fig. 6.

It is clear that the rate of convergence of time response of error system (1) and (19) utilizing active controller is 3 times increased by comparing Fig. 5 and Fig. 6.

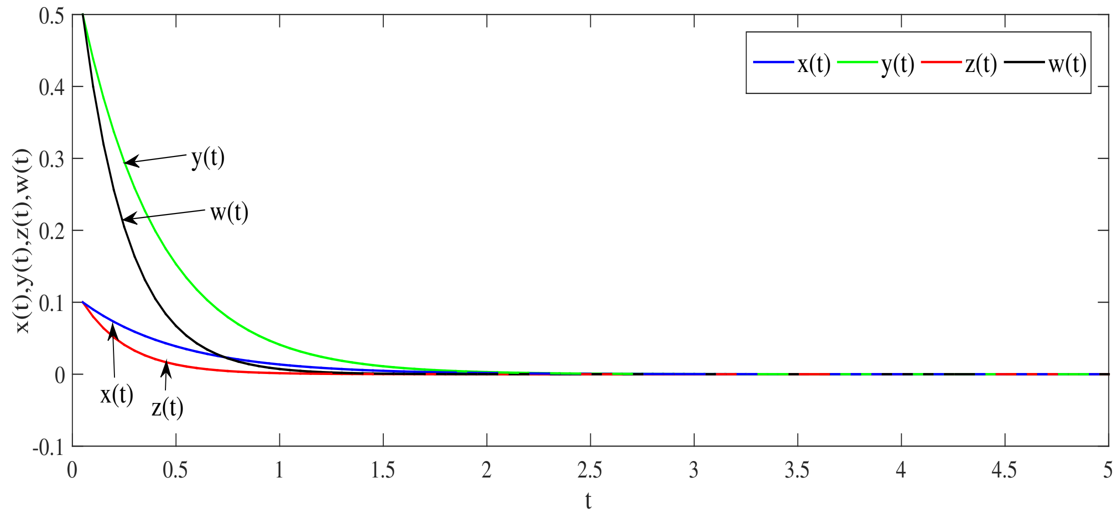

Fig. 3. Time history of the controlled hyperchaotic system (4) when $k_1 = k_2 = k_3 = 1$ and $k_4 = 5$.

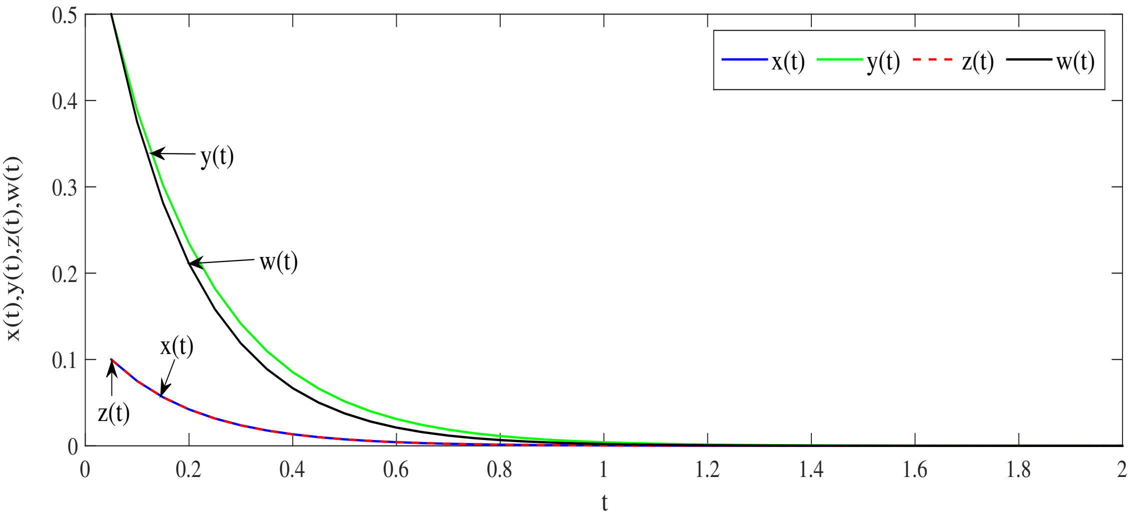

Fig. 4. Time history of the controlled hyperchaotic system (4) when $k_1 = k_2 = k_3 = 5$ and $k_4 = 25$.

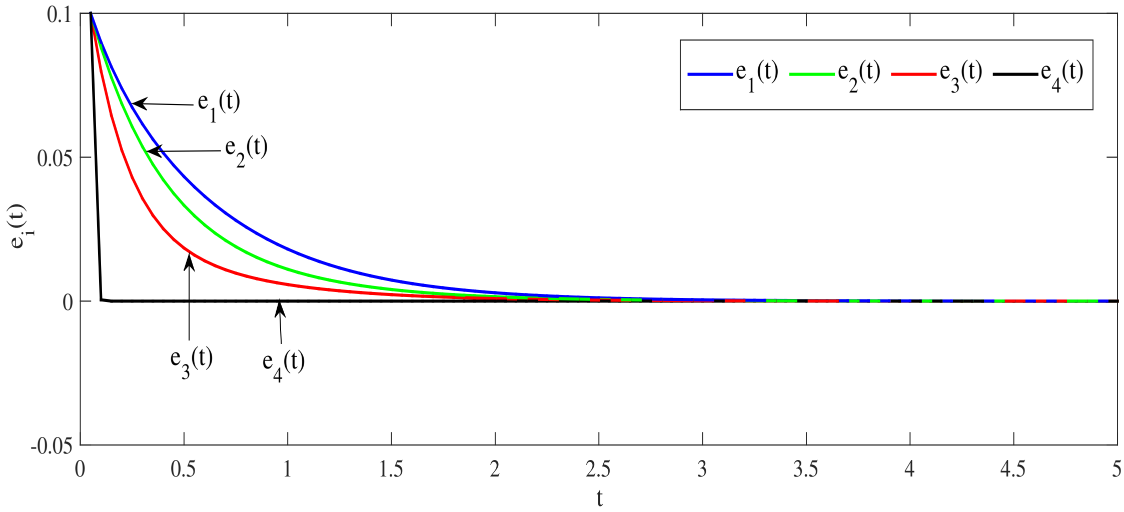

Fig. 5. Synchronization errors under the control law (14) when $k_1 = 2, k_2 = 3, k_3 = 4$ and $k_4 = 20$.

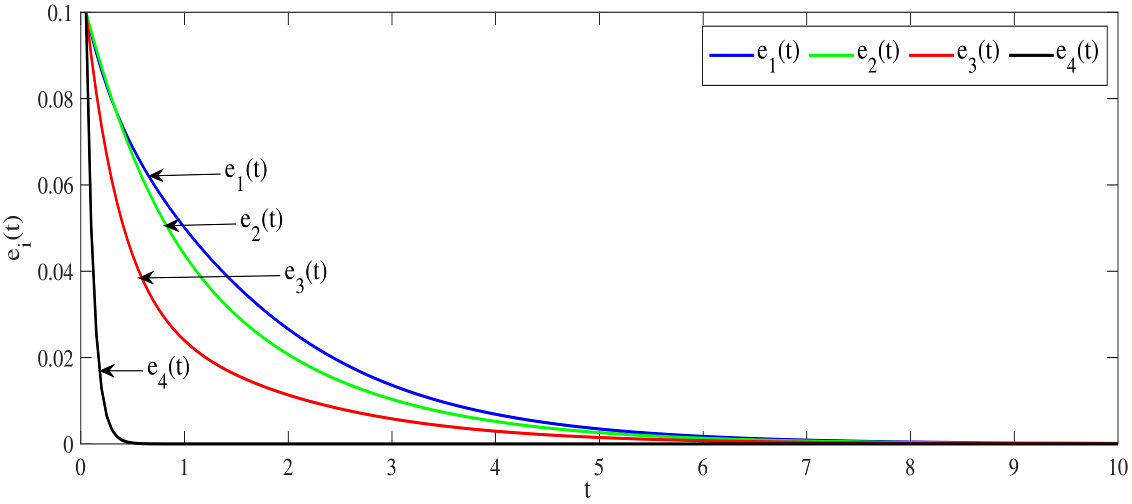

Fig. 6. Synchronization errors under the control law (14) when $k_1 = 1, k_2 = 1.5, k_3 = 2$ and $k_4 = 10$.

V. Applications of the Proposed Hyperchaotic System

In this section, the solutions of the proposed hyperchaotic system is utilized to generate OTP for authenticating transactions. The details of the proposed OTP generating algorithm are described as follows:

1) Consider the hyperchaotic system (1) for generation of OTP.
2) Compute the solutions of the system (1) at time $t \in (0, T]$.
3) Calculate $Y(t)$ for every t with step size h by $Y(t) = x(t) + y(t) + z(t) + w(t)$.
4) Compose the random numbers S_R between 0-9 for every t by $S_R = \lfloor Y(t) * 1000 \rfloor \ (mod\ 10)$ where $\lfloor x \rfloor$ is the $floor(x)$.
5) Split the sequence of random numbers S_R into blocks of random numbers. Each block consisting of n random numbers.
6) A block with n random numbers between 0-9 is described as OTP.

For demonstration, we take $h = 0.05, T = 2.50$. The solutions $x(t), y(t), z(t)$ and $w(t)$ of the system (1) for the interval $(0, 2.50]$ are computed. Further, $Y(t)$ and S_R are calculated. The computed and calculated values are tabulated in TABLE 1. The sequence of first 50 random numbers S_R is depicted in Fig. 7, which are listed as follows.

0	6	4	5	8	3	0	9	9	2
6	2	9	7	7	7	9	1	3	5
8	0	1	1	9	6	0	2	0	4
4	9	8	1	6	4	4	5	5	6
5	3	8	1	1	7	0	9	4	5.

For practical point of view, we take $n = 5$, thus the 5-digit OTP shows in TABLE I and the set of 10 random OTP has been generated successfully based on the proposed algorithm.

A. Security analysis of the proposed algorithm

By comparing the level of security, the proposed OTP generating algorithm has more secure than random number generation algorithm due to the hardness of finding the solution oh hayperchaotic system (1).

Result 5.1: Nowadays, time-based and hash-based OTP's are vulnerable to brute force assaults, forging attacks. This raises the research question, "Is there another method that could be used to generate a highly secure OTP system that is less vulnerable to such attacks?" Chaotic maps, which are well known for their very unpredictable behavior, were chosen to see if they might be utilized to generate highly random OTPs.

Furthermore, instead of 4-digit OTPs, our goal is to create a system that generates 5-digit OTPs with a restricted validity duration, making it extremely impossible for a standard computer to crack the OTP using Brute force.

VI. Conclusions

This paper has introduced a novel fascinating four-dimensional hyperchaotic system. An appropriate active controller has been designed to control and synchronize the hyperchaotic system. Numerical simulations are given to validate the correctness of the theoretical results of control and synchronization of hyperchaotic systems. In addition, OTPs-which are more secure than static passwords are generated using the proposed hyperchaotic system.

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TABLE I
SEQUENCE OF OTP

Time (t)	$x(t)$	$y(t)$	$z(t)$	$w(t)$	$Y(t)$	SR	5-digit OTP
0.05	0.1000	0.5000	0.1000	0.5000	1.2000	0	
0.10	0.1200	0.5156	0.1000	0.5008	1.2364	6	
0.15	0.1408	0.5316	0.1010	0.5015	1.2749	4	**06458**
0.2	0.1623	0.5481	0.1030	0.5022	1.3156	5	
0.25	0.1846	0.5651	0.1060	0.5029	1.3586	8	
0.30	0.2075	0.5826	0.1099	0.5035	1.4035	3	
0.35	0.2312	0.6006	0.1148	0.5039	1.4505	0	
0.40	0.2555	0.6190	0.1206	0.5042	1.4993	9	**30992**
0.45	0.2804	0.6380	0.1273	0.5043	1.5500	9	
0.50	0.3059	0.6574	0.1350	0.5041	1.6024	2	
0.55	0.3320	0.6773	0.1435	0.5037	1.6565	6	
0.60	0.3587	0.6977	0.1530	0.5029	1.7123	2	
0.65	0.3860	0.7185	0.1632	0.5016	1.7693	9	**62977**
0.70	0.4137	0.7398	0.1744	0.4999	1.8278	7	
0.75	0.4420	0.7614	0.1863	0.4976	1.8873	7	
0.80	0.4707	0.7834	0.1991	0.4946	1.9478	7	
0.85	0.5000	0.8056	0.2127	0.4909	2.0092	9	
0.90	0.5296	0.8282	0.2271	0.4863	2.0712	1	**79135**
0.95	0.5597	0.8509	0.2422	0.4807	2.1335	3	
1.00	0.5901	0.8736	0.2581	0.4741	2.1959	5	
1.05	0.6209	0.8965	0.2747	0.4663	2.2584	8	
1.10	0.652	0.9192	0.2920	0.4570	2.3202	0	
1.15	0.6833	0.9417	0.3100	0.4463	2.3813	1	**80119**
1.20	0.7149	0.9636	0.3286	0.0.4339	2.4413	1	
1.25	0.7467	0.9856	0.3480	0.4197	2.5000	9	
1.30	0.7785	1.0067	0.3679	0.4034	2.5565	6	
1.35	0.8105	1.0270	0.3884	0.3849	2.6108	0	
1.40	0.8424	1.0464	0.4095	0.3640	2.6623	2	**60204**
1.45	0.8743	1.0645	0.4312	0.3405	2.7105	0	
1.50	0.9059	1.0813	0.4533	0.3141	2.7546	4	
1.55	0.9373	1.0965	0.4760	0.2847	2.7945	4	
1.60	0.9684	1.1099	0.4990	0.2520	2.8293	9	
1.65	0.9989	1.1211	0.5225	0.2158	2.8583	8	**49816**
1.70	1.0288	1.1301	0.5463	0.1758	2.8810	1	
1.75	1.0580	1.1365	0.5704	.1318	2.8967	6	
1.80	1.0863	1.1401	0.5948	0.0836	2.9048	4	
1.85	1.1136	1.1406	0.6194	0.0309	2.9045	4	
1.90	1.1396	1.1379	0.6441	-0.0265	2.8951	5	**44556**
1.95	1.1643	1.1316	0.6689	-0.0888	2.8760	5	
2.00	1.1875	1.1216	0.6937	-0.1563	2.8465	6	
2.05	1.2089	1.1078	0.7183	-0.2292	2.8058	5	
2.10	1.2283	1.0899	0.7429	-0.3077	2.7534	3	
2.15	1.2457	1.0679	0.7671	-0.3918	2.6889	8	**53811**
2.20	1.2607	1.0418	0.7911	-0.4819	2.6117	1	
2.25	1.2733	1.0115	0.8146	-0.5779	2.5215	1	
2.30	1.2831	0.9770	0.8375	-0.6800	2.4176	7	
2.35	1.2901	0.9386	0.8598	-0.7883	2.3002	0	
2.45	1.2940	0.8964	0.8813	-0.9027	2.1690	9	**70945**
2.45	1.2948	0.8506	0.9019	-1.0233	2.0240	4	
2.50	1.2922	0.8016	0.9216	-1.1499	1.8655	5	

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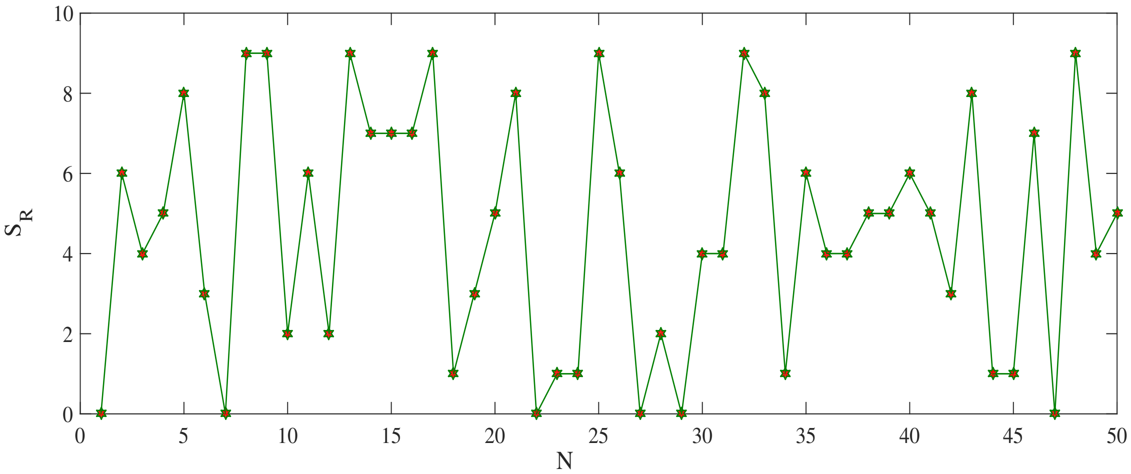

Fig. 7. Graph of OTP

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