Dynamics of Lotka-Volterra Cooperative Systems with Impulsive Controls and Delays

Zhao Jiang, and Ahmadjan Muhammadhaji

Abstract—**This study investigated an n-species cooperative Lotka-Volterra system with impulsive controls and time delays. In the case of constructing appropriate Lyapunov functionals and using inequality techniques, some conditions for the permanence and global attractiveness of the system are obtained. In addition, sufficient conditions for the existence of positive periodic solutions were established using the coincidence degree theory. Finally, numerical simulation results illustrate the effectiveness of our findings.**

Index Terms—**cooperative system; time delay; impulsive control; global attractivity; periodic solution.**

I. INTRODUCTION

COOPERATIVE systems have been favored by researchers as an imperative part of population dynamics in recent years [1-3]. Cooperative population is an important manifestation of natural biodiversity. Studying their formation and evolution mechanisms can help to better understand and protect biological communities in nature.

Complex ecological relationships, such as food web and energy flow, exist within cooperative populations. Through an in-depth study of these relationships, we can better understand the structure and function of the ecosystem. Biologists widely believe that emphasizing the cooperative evolutionary theory is more effective since many species achieve survival and reproduction through mutual benefit and win-win, and studying cooperative populations is the exact proof [4,5]. Humans may also learn lessons from cooperative populations, such as how to find a balance and solve various common environmental problems.

Using simple non-time-delay dynamic models often cannot effectively describe time-delay effects in time because many phenomena in ecology involve species interactions and evolution. Dynamic models with time delays can more accurately explain practical problems. In addition, many ecosystems have time-delay effects caused by various processes that accumulate over time, such as nutrient transport in the soil and tree growth. Scholars have conducted extensive research on population models with time delays and obtained many promising results [6-10]. As discussed in [4] and [9], cooperative models with time delays obtain sufficient dynamic conditions such as persistence. Furthermore, time delay is a common phenomenon that has wide applications in many fields, such as electronics, mechanics, and chemistry [7,8]. Therefore, studying ecological models with time delays can provide more comprehensive solutions and theoretical systems for related problems in these fields.

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Z. Jiang is a postgraduate student of College of Mathematics and System Sciences, Xinjiang University, Urumqi 830017, P.R. China (e-mail: xjujz980919@163.com).

A. Muhammadhaji is an associate professor of College of Mathematics and System Sciences, Xinjiang University, Urumqi 830017, P.R. China (Corresponding author to provide e-mail: ahmatjanam@aliyun.com).

The biological population model is a crucial research field in ecology and can help us better understand population dynamics and ecological interactions in ecosystems. Because populations do not exist in isolation, any population can be affected by various momentary effects causing sudden changes in system variables or growth patterns. For example, pesticide spraying, natural enemy release, and natural disasters.

The introduction of impulsive differential equation theory in population modeling studies to describe the phenomenon of rapid changes in certain states of a system at fixed or irregular moments has gained much attention from scholars [11-15]. There are various pulse disturbances in ecosystems, such as natural disasters and human activities, which have a very complex and diverse impact on the ecosystem. Impulse-controlled biological population models can better simulate the impact of these pulse disturbances on different population numbers and ecological interactions in ecosystems. Thus, impulse-controlled biological population models helping us better understand the essence and impact mechanisms of these disturbances. In [10], the authors considered the following impulsive delay Logistic model

$$\begin{aligned} \dot{z}(s) &= z(s)[a(s)-b(s)z(s-\tau)], \quad s \neq s_k \\ z(s_k^+) &= h_k z(s_k), \quad k=1,2\cdots. \end{aligned} \tag{1}$$

The uniform persistence and global attractiveness of the model were obtained using useful technologies.

In the real world, various ecological interactions in ecosystems are quite complex and often nonlinear. In addition, biological population models with impulsive perturbations can better simulate the effects of global climate change on different population sizes and ecological interactions in ecosystems. Furthermore, they can help us better understand the interactions among various groups in ecosystems, even to predict ecosystem responses to different disturbances, and they can address major challenges, such as global climate change.

Considering the complexity of time delays and interactions among populations, Stamova in [11] investigated the following delayed impulsive cooperation systems

$$\begin{aligned} \dot{y}_i(s) &= y_i(s)\big[c_i(s)-b_i(s)y_i(s) \\ &\quad -\tfrac{y_i(s-\eta_{ii}(s))}{e_i(s)+\sum_{j=1,j\neq i}^{n} d_j(s)y_j(s-\eta_{ij}(s))}\big], \\ y_i\left(s_k^+\right) &= y_i\left(s_k\right)+F_{ik}(y_i(s_k)), s=s_k. \end{aligned} \tag{2}$$

Some conditions for the dynamic properties of the system were obtained. Despite the good results of the work done, Li [13] pointed out that a mistake exists in the results and investigated the following system

$$\begin{aligned} \dot{y}_i(s) &= r_i(s)y_i(s)[1-b_i(s)y_i(s-\tau_{ii}(s)) \\ &\quad -\frac{y_i(s-\tau_{ii}(s))}{a_i(s)+\sum_{j=1,j\neq i}^{n}c_j(s)y_j(t-\tau_{ij}(s))}], s\neq s_k, \\ y_i(s_k) &= y_i(s_k^-)+I_{ik}y_i(s_k^-)). \end{aligned} \tag{3}$$

Using a new method, the conditions for the persistence of the above model were derived.

The cooperative relationship between populations cannot be ignored, and the following system was considered in [15]

$$\begin{aligned} \dot{y}_i(s) &= y_i(s)(a_i(s)-b_i(s)y_i(s) \\ &\quad +\sum_{j=1,i\neq j}^{n}c_{ij}(s)\frac{y_j(s-\tau_j(s))}{1+y_j(s-\tau_j(s))}), s\neq s_k, \\ y_i(s_k^+) &= (1+h_{ik})y_i(s_k). \end{aligned} \tag{4}$$

The permanence, existence, and global stability of the almost periodic positive solution of the system were obtained.

Furthermore, studying population dynamics with time delays can accurately predict future trends and identify management strategies needed to manage them [16-20]. For example, in disaster recovery, ecological restoration, and species conservation, it is often necessary to predict and control species recovery and population growth to maintain biodiversity. In summary, it is important to study models of biological populations with delayed and impulsive perturbations [21-25]. Although research on impulse models is still in full swing [15-23], cooperative models with impulsive perturbations have been less studied, and their dynamical properties have not been sufficiently studied. Therefore, the following delayed cooperative system with impulsive controls was proposed and considered in this study

$$\begin{aligned} \dot{N}_i(s) &= N_i(s)[r_i(s)-b_{ii}(s)N_i(s) \\ &\quad -\frac{N_i(s)}{a_i(s)+\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)N_j(s+\tau)d\tau}], \\ N_i(s_k^+) &= (1+h_{ik})N_i(s_k), \quad s=s_k, \quad k\in n, \end{aligned} \tag{5}$$

where $N_i(s)$ represents the density of the population and s_k represents the moment of impulse. Some conditions about permanence and global attractivity were obtained by constructing appropriate Lyapunov functions using some inequality techniques and differential equation knowledge. Moreover, the existence of positive periodic solutions of the considered system was obtained using the coincidence degree theory.

Through this study, we firmly believe that this work can explain the impact of time delay on the dynamic behavior of other species. We also claim that studying the relevant theories of the system (5) is very meaningful as it can expand previous results, acknowledge biological value [16-25], and provide some ideas for future research.

II. Preliminaries

In this paper, we use the following initial conditions for system (5):

$$N_i(s)=\Theta_i(s), \quad \forall t\in[-\gamma,0], \tag{6}$$

where $\Theta_i(s)(i=1,2,\dots n)$ are continuous nonnegative functions defined on $[-\gamma,0]$ and satisfying $\Theta_i(0)>0$, where $\gamma=\max\{\gamma_{ij}(i,j=1,2,\dots n)\}$.

Throughout this paper, for any bounded continuous function $p(s)$ defined on $[0,+\infty)$ we define

$$p^L=\min_{s\in[0,\infty]}p(s), \quad p^M=\max_{s\in[0,\infty]}p(s),$$

and for any Ω-periodic continuous function $P(s)$ defined on $[0,+\infty)$ we define.

$$\bar{P}=\frac{1}{\Omega}\int_0^{\Omega}P(s)ds.$$

For convenience, we define

$$\begin{aligned} \sup s_k^1 &= \sup(s_{k+1}-s_k)=\eta, \\ \inf s_k^1 &= \inf(s_{k+1}-s_k)=\theta, k\in\mathcal{Z}, \end{aligned}$$

and

$$h_{ik}^M=\sup_{k\in\mathcal{Z}}h_{ik}, \qquad h_{ik}^L=\inf_{k\in\mathcal{Z}}h_{ik}.$$

The followings are some basic assumptions in this paper.

(H1) $r_i(s)>0, b_{ij}(s)>0(i,j=1,2,\dots n)$ are all continuous bounded functions on $[0,+\infty)$, and $K_{ij}(\tau)(i,j=1,2,\dots n)$ are nonnegative integrable functions defined in $[-\gamma_{ij},0]$ satisfying $\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)d\tau=1$, $h_{i_k}>0$ and $\gamma_{ij}>0$ are constants.

(H2) $r_i(s)>0, b_{ij}(s)>0(i,j=1,2,\dots n)$ are all continuous bounded Ω-periodic functions on $[0,+\infty)$, and $K_{ij}(\tau)(i,j=1,2,\dots n)$ are nonnegative integrable functions defined in $[-\gamma_{ij},0]$ satisfying $\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)d\tau=1$, $h_{i_k}>0$ and $\gamma_{ij}>0$ are constants.

Now, we introduce the following useful lemmas.

Lemma 1 Let $N(s)=(N_1(s),N_2(s),\dots,N_i(s))^T$ is any solution of system (5), then $N_i(s)>0$.

Proof. Firstly, it can be easy to see that the ith equation of the system can be written in the following form

$$\dot{N}_i(s)=Q_i(s)N_i(s), s\neq s_k, i=1,2,\dots,n,$$

where

$$Q_i(s)=-\frac{N_i(s)}{a_i(s)+\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)N_j(s+\tau)d\tau} \\ r_i(s)-b_{ii}(s)N_i(s),$$

which yields

$$N_i(s)=\prod_{0<s_k<s}(1+h_{ik})N_i(0)\exp(\int_0^t Q_i(s)ds)>0.$$

Lemma 2[16] Assume that the sequence s_k satisfies $0\leq s_0\leq<s_1<s_2<\dots$, with $\lim_{k\to\infty}s_k=\infty$. Moreover, suppose that

(i) $m\in PC[R_+,R^n]$, $m(s)$ is left continuous at $s=s_k(k=1,2,\dots)$, $g\in C[R_+\times R^n,R^n]$, $g(s,u)$ is quasimonotone nondecreasing in u for each s for $k=1,2,\dots$, $\phi_k(u)\in C[R^n,R^n]$ and $\phi_k(u)$ is nondecreasing in u and

$$\begin{aligned} &Dm(s)\leq g(s,m(s)), \quad s\neq s_k,\ m(s_0)\leq u_0, \\ &m(s_k^+)\leq\phi_k(m(s_k)), \quad k=1,2,\dots; \end{aligned}$$

(ii) $q(s)$ is the maximum solution of impulsive differential system (7)

$$\begin{aligned} &\dot{u}(s)=g(s,u), \quad s\neq s_k,\ u(t_0)\leq u_0, \\ &u(s_k^+)=\phi_k(u(s_k)), \end{aligned} \tag{7}$$

existing on $[s_0,\infty)$. Then we have that $m(s)\leq q(s), s\geq s_0$.

On the other side, if the inequality takes the reverse direction, $l(s)$ be the minimum solution of impulsive differential equations (7) in $[s_0, +\infty)$, then $m(s_0^+) \geq u_0$ which means $m(s) \geq l(s), s \geq s_0$.

Lemma 3[10] Considering the following system (8)

$$\begin{aligned} \dot{N}(s) &= N(s)(a - bN(s)), s \neq s_k, \\ N(s_k^+) &= h_k N(s_k), k = 1, 2, .s, \end{aligned} \tag{8}$$

if $a > 0, b > 0, h_k^L > 1$, then for any positive solution $N(s)$ of system (8) satisfies

$$m_0 \leq \lim_{s\to\infty} \inf N(s) \leq \lim_{s\to\infty} \sup N(s) \leq M_0$$

where $m_0 = \frac{a\eta + \ln h_k^L}{b\eta h_k^L}$ and $M_0 = \frac{(a\theta + \ln h_k^M) h_k^M}{b\theta}$.

III. 3 Main results

Theorem 3.1 If (H1) holds, then system (5) is permanent.

Proof. Let $N(s) = (N_1(s), N_2(s), \ldots, N_i(s))^T$ is any positive solution of system (5), then we derive

$$\dot{N}_i(s) \leq N_i(s)[r_i^M - b_{ii}^L N_i(s)], \quad i = 1, 2, \ldots.$$

By Lemma 2, we get $N_i(s) \leq q_i(s)$, where $q_i(s)$ is any positive solution of the following system

$$\begin{aligned} \dot{q}_i(s) &= q_i(s)[r_i^M - b_{ii}^L q_i(s)], \quad s \neq s_k, \\ q_i(s_k^+) &= (1 + h_{ik}) q_i(s_k), \quad k = 1, 2, \ldots, n. \end{aligned}$$

Then, using Lemma 3, we get

$$\limsup_{s\to+\infty} N_i(s) \leq \limsup_{s\to+\infty} q_i(s) \leq M_i, \tag{9}$$

where $M_i = \frac{(r_i^M \theta + \ln(1 + h_{ik}^M))(1 + h_{ik}^M)}{b_{ii}^L \theta}$, thus we get

$$\dot{y}_i(s) \geq N_i(s)[r_i^L - b_{ii}^M y_i(s) - \frac{y_i(s)}{a_i^L}], \quad i = 1, 2, \ldots.$$

Next, we consider the following system

$$\begin{aligned} \dot{l}_i(s) &= l_i(s)[r_i^L - (b_{ii}^M + \frac{1}{a_i^L}) l_i(s)], \quad i = 1, 2, \ldots. \\ l_i(s_k^+) &= (1 + h_{ik}) l_i(s_k), \quad k = 1, 2, \ldots, n. \end{aligned}$$

Let $l_i(s)$ be a solution of the above system, then we get $N_i(s) \geq l_i(s)$. Moreover, using Lemma 3 we get

$$\liminf_{s\to+\infty} N_i(s) \geq \liminf_{s\to+\infty} l_i(s) \geq m_i = \frac{r_i^L \eta + \ln(h_{ik}^L + 1)}{(b_{ii}^M + \frac{1}{a_i^L})\eta(h_{ik}^L + 1)}. \tag{10}$$

Finally, from (9) and (10), one can easily get the permanence of system (5).

Theorem 3.2. If (H2) holds, then system (5) has at least a positive Ω-periodic solution.

Proof. First, we set

$$N_i(s) = \exp u_i(s), \quad i = 1, 2, \ldots, n.$$

Then system (5) is rewritten in the following form

$$\begin{aligned} \dot{u}_i(s) &= s_i(s) - b_{ii}(s) e^{u_i(s)} \\ &\quad - \frac{e^{u_i(s)}}{a_i(s) + \sum_{j\neq i}^n b_{ij}(s) \int_{-\gamma_{ij}}^0 K_{ij}(\tau) e^{u_j(s+\tau)} d\tau}, \\ u_i(s_k^+) &= \ln(1 + h_{ik}) + u_i(s_k). \end{aligned} \tag{11}$$

Next, we define X and Z as the normed vector spaces. Let $C(R, R^n)$ be the space of all continuous functions $u(s) = (u_1(s), u_2(s), \ldots, u_n(s)) : R \to R^n$. We take

$$X = Z = u(s) \in C(R, R^n),$$

where $u(s)$ is an Ω-periodic function with norm

$$\|u\| = \sum_{i=1}^n \max_{s\in[0,\Omega]} |u_i(s)|.$$

Then, X and Z are the Banach spaces.

Let

$$Y = \left\{ \left[\begin{pmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_n(s) \end{pmatrix}, \begin{pmatrix} g_{1_1} \\ g_{2_1} \\ \vdots \\ g_{n_1} \end{pmatrix}, \cdots, \begin{pmatrix} g_{1_q} \\ g_{2_q} \\ \vdots \\ g_{n_q} \end{pmatrix} \right] \left| \begin{array}{c} u_i(s) \in C_\omega (i = 1, 2 \cdots, n) \\ (g_{1_k}, g_{2_k}, \cdots, g_{n_k})^T \\ = (\Delta U_1(s_k), \cdots, \Delta U_n(s_k))^T \\ k = 1, 2, \ldots, n \end{array} \right. \right\},$$

where $U_i(s)$ is original function of $u_i(s)$.

For $k = 1, 2, \ldots q$, define

$$z = [u_0(s), z_1, \ldots, z_q],$$

where $u_0(s) = (u_1(s), u_2(s), \ldots, u_n(s))^T$, and $z_k = (g_{1_k}, g_{2_k}, \ldots, g_{n_k})^T$.

Then we define

$$\|z\| = \sum_{i=1}^n \max_{s\in[0,\Omega]} |u_i(s)| + \sum_{k=1}^q \|z_k\|.$$

One can see that $(Z, \|\cdot\|)$ is Banach space.

We set L and N as operators satisfying

$$L \begin{bmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_n(s) \end{bmatrix} = \left[\begin{pmatrix} \dot{u}_1(s) \\ \dot{u}_2(s) \\ \vdots \\ \dot{u}_n(s) \end{pmatrix}, \begin{pmatrix} \Delta u_1(s_1) \\ \Delta u_2(s_1) \\ \vdots \\ \Delta u_n(s_1) \end{pmatrix}, \cdots, \begin{pmatrix} \Delta u_1(s_q) \\ \Delta u_2(s_q) \\ \vdots \\ \Delta u_n(s_q) \end{pmatrix} \right\},$$

where

$$\begin{aligned} DomL &= \left\{ (u_1(s), u_2(s), \cdots, u_n(s))^T \in X \mid \dot{u}_i(s) \in C_\Omega \right\} \\ &= \left\{ (u_1(s), u_2(s), \cdots, u_n(s))^T \in X \mid u_i(s) \in C'_\Omega \right\}, \end{aligned}$$

and

$$N \begin{bmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_n(s) \end{bmatrix} = \left[\begin{pmatrix} f_1(s) \\ f_2(s) \\ \vdots \\ f_n(s) \end{pmatrix}, \begin{pmatrix} \ln(1 + h_{1_1}) \\ \ln(1 + h_{2_1}) \\ \vdots \\ \ln(1 + h_{n_1}) \end{pmatrix}, \cdots, \begin{pmatrix} \ln(1 + h_{1_q}) \\ \ln(1 + h_{2_q}) \\ \vdots \\ \ln(1 + h_{n_q}) \end{pmatrix} \right],$$

where

$$f_i(s) = r_i(s) - \frac{e^{u_i(s)}}{a_i(s) + \sum_{j\neq i}^{n} b_{ij}(s)\int_{-\gamma_{ij}}^{0} K_{ij}(\tau)e^{u_j(s+\tau)}d\tau} - b_{ii}(s)e^{u_i(s)}.$$

Further, we define that $P : X \to X$ and $Q : Z \to Z$ satisfying

$$P\begin{pmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_n(s) \end{pmatrix} = \frac{1}{\omega}\begin{pmatrix} \int_0^\omega u_1(s)ds + \sum_{k=1}^{q} g_{1_k} \\ \int_0^\omega u_2(s)ds + \sum_{k=1}^{q} g_{2_k} \\ \vdots \\ \int_0^\omega u_n(s)ds + \sum_{k=1}^{q} g_{n_k} \end{pmatrix},$$

and

$$Q\left[\begin{pmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_n(s) \end{pmatrix}, \begin{pmatrix} g_{1_1} \\ g_{2_1} \\ \vdots \\ g_{n_1} \end{pmatrix}, \cdots, \begin{pmatrix} g_{1_q} \\ g_{2_q} \\ \vdots \\ g_{n_q} \end{pmatrix}\right]$$

$$= \left[\frac{1}{\omega}\begin{pmatrix} \int_0^\omega u_1(s)ds + \sum_{k=1}^{q} g_{1_k} \\ \int_0^\omega u_2(s)ds + \sum_{k=1}^{q} g_{2_k} \\ \vdots \\ \int_0^\omega u_n(s)ds + \sum_{k=1}^{q} g_{n_k} \end{pmatrix}, \left\{\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}\right\}_{k=1}^{q}\right].$$

One can see that $KerL = n$ and ImL is closed in Z, where

$$ImL = \left\{\left[\begin{pmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_n(s) \end{pmatrix}, \begin{pmatrix} g_{1_1} \\ g_{2_1} \\ \vdots \\ g_{n_1} \end{pmatrix}, \cdots, \begin{pmatrix} g_{1_q} \\ g_{2_q} \\ \vdots \\ g_{n_q} \end{pmatrix}\right] \middle| \begin{pmatrix} \int_0^\omega u_2(s)ds + \sum_{k=1}^{q} g_{1_k} = 0 \\ \int_0^\omega u_2(s)ds + \sum_{k=1}^{q} g_{2_k} = 0 \\ \vdots \\ \int_0^\omega u_n(s)ds + \sum_{k=1}^{q} g_{n_k} = 0 \end{pmatrix}\right\},$$

and

$$ImP = KerL, KerQ = ImL, \\ dimKerL = codimImL = n.$$

Thus, L is a Fredholm mapping of index zero.

Moreover, the following form is the generalized inverse of $K_p : \mathrm{Im}\,L \to \mathrm{Ker}\,P \cap \mathrm{Dom}\,L$:

$$\mathrm{K}_P Z = \begin{pmatrix} \int_0^s f_1(s)ds - \frac{1}{\Omega}\int_0^\Omega \int_0^s f_1(v)dvds \\ \quad + \sum_{0<s_k<s} g_{1_k} - \sum_{k=1}^{q} g_{1_k} \\ \int_0^s f_2(s)ds - \frac{1}{\Omega}\int_0^\Omega \int_0^s f_2(v)dvds \\ \quad + \sum_{0<s_k<s} g_{2_k} - \sum_{k=1}^{q} g_{2_k} \\ \vdots \\ \int_0^s f_n(s)ds - \frac{1}{\Omega}\int_0^\Omega \int_0^s f_n(v)dvds \\ \quad + \sum_{0<s_k<s} g_{n_k} - \sum_{k=1}^{q} g_{n_k} \end{pmatrix}.$$

Then we have

$$QNu(s) = \left[\frac{1}{\omega}\begin{pmatrix} \int_0^\Omega f_1(s)ds + \sum_{k=1}^{q} \ln(1+g_{1_k}) \\ \int_0^\Omega f_2(s)ds + \sum_{k=1}^{q} \ln(1+g_{2_k}) \\ \vdots \\ \int_0^\Omega f_n(s)ds + \sum_{k=1}^{q} \ln(1+g_{n_k}) \end{pmatrix}, \left\{\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}\right\}_{k=1}^{q}\right],$$

and $K_p(I-Q)N : X \to X$,

$$K_p(I-Q)N\begin{pmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_n(s) \end{pmatrix} = \begin{pmatrix} \int_0^\Omega f_1(s)ds + \sum_{0<t_k<t} \ln(1+g_{1_k}) \\ \int_0^\Omega f_2(s)ds + \sum_{0<t_k<t} \ln(1+g_{2_k}) \\ \vdots \\ \int_0^\Omega f_n(s)ds + \sum_{0<t_k<t} \ln(1+g_{n_k}) \end{pmatrix}$$

$$- \begin{pmatrix} (\frac{s}{\Omega} - \frac{1}{2})\int_0^s f_1(s)ds + \sum_{k=1}^{q} \ln(1+g_{1_k}) \\ (\frac{s}{\Omega} - \frac{1}{2})\int_0^s f_2(s)ds + \sum_{k=1}^{q} \ln(1+g_{2_k}) \\ \vdots \\ (\frac{s}{\Omega} - \frac{1}{2})\int_0^s f_n(s)ds + \sum_{k=1}^{q} \ln(1+g_{n_k}) \end{pmatrix}$$

$$- \begin{pmatrix} \frac{1}{\Omega}\int_0^\Omega\int_0^s f_1(s)ds + \sum_{k=1}^{q} \ln(1+g_{1_k}) \\ \frac{1}{\Omega}\int_0^\Omega\int_0^s f_2(s)ds + \sum_{k=1}^{q} \ln(1+g_{2_k}) \\ \vdots \\ \frac{1}{\Omega}\int_0^\Omega\int_0^s f_n(s)ds + \sum_{k=1}^{q} \ln(1+g_{n_k}) \end{pmatrix}.$$

From the above discussion and by using Arzela-Ascoli theorem , one can see that $QN(\bar{\Omega})$ is bounded and QN and $K_p(I-Q)N$ are continuous operators and $\overline{K_p(I-Q)N(\bar{\Omega})}$ is compact for any open bounded set $\Omega \subset X$.

Next, in order to find the open bounded subset $\Omega \subset X$, let $Lu(s) = \lambda Nu(s)$ with $\lambda \in (0,1)$ and from (11), we get

$$\begin{aligned} \dot{u}_i(s) &= \lambda f_i(s), \quad s \neq s_k. \quad i = 1, 2, \ldots, n, \\ u_i(s_k^+) &= \lambda \ln(1+h_{ik}) + u_i(s_k), \quad s = s_k. \end{aligned} \tag{12}$$

Let, $u(s)=(u_1(s),u_2(s),\ldots,u_n(s))\in X$ is a solution of system (12) for some parameter $\lambda\in(0,1)$. Then integrating system (6) over the interval $[0,\Omega]$, we have

$$\int_0^\Omega \big(r_i(s)-b_{ii}(s)e^{u_i(s)} - \frac{e^{u_i(s)}}{a_i(s)+\sum_{j\neq i}^n b_{ij}(s)\int_{-\gamma_{ij}}^0 K_{ij}(\tau)e^{u_j(s+\tau)}d\tau}\big)ds = -\frac{1}{\Omega}\ln(\prod_{k=1}^q(1+h_{i_k})),\qquad i=1,2,\ldots,n,$$

which yields $(1+\frac{1}{a_i^L b_{ii}})(\bar{s}_i\Omega+\Gamma_i\Omega)$

$$\int_0^\Omega \frac{e^{u_i(s)}}{a_i(s)+\sum_{j\neq i}^n b_{ij}(s)\int_{-\gamma_{ij}}^0 K_{ij}(\tau)e^{u_j(s+\tau)}d\tau} + b_{ii}(s)e^{u_i(s)}ds = \bar{r_i}\Omega+\Gamma_i\Omega,\qquad i=1,2,\ldots,n, \tag{13}$$

where

$$\Gamma_i=\frac{1}{\Omega^2}\ln(\prod_{k=1}^q(1+h_{i_k}))=\frac{1}{\Omega^2}\sum_{k=1}^q\ln(1+h_{i_k}).$$

From the solutions $u(s)=(u_1(s),u_2(s),\ldots,u_n(s))$, we have some constants $\xi_i,\eta_i\in[0,\Omega](i=1,2,\ldots,n)$ and satisfying

$$u_i(\xi_i)=\max_{s\in[0,\Omega]}u_i(s),\quad u_i(\eta_i)=\min_{s\in[0,\Omega]}u_i(s). \tag{14}$$

From (13) and (14), we get

$$\int_0^\Omega b_{ii}(s)e^{u_i(\eta_i)}ds\le\bar{r_i}\Omega+\Gamma_i\Omega.$$

Therefore, we find that

$$u_i(\eta_i)\le\ln\frac{\bar{r}_i\Omega+\Gamma_i\Omega}{\bar{b}_{ii}}=:A_i. \tag{15}$$

On the other hand, we have

$$\int_0^\Omega \big(b_{ii}(s)e^{u_i(s)}+\frac{e^{u_i(s)}}{a_i(s)}\big)ds\ge\bar{r_i}\Omega+\Gamma_i\Omega,\quad i=1,2,\ldots,n.$$

Thus, we further obtain

$$u_i(\xi_i)\ge\ln\frac{\bar{a}_i(\bar{r}_i\Omega+\Gamma_i\Omega)}{\bar{a}_i\bar{b}_{ii}+1}=:C_i. \tag{16}$$

From (15) and (16), we derive

$$\begin{aligned}&\int_0^\Omega|\dot{u}_i(s)|ds\\ &\le|\Gamma_i|\Omega+|r_i|\Omega+\int_0^\omega\big(b_{ii}(s)e^{u_i(s)} + \frac{e^{u_i(s)}}{a_i(s)+\sum_{j\neq i}^n b_{ij}(s)\int_{-\gamma_{ij}}^0 K_{ij}(s)e^{u_j(s+\tau)}d\tau}\big)ds\\ &\le|\Gamma_i|\Omega+|r_i|\Omega+\int_0^\omega\big(b_{ii}(s)e^{u_i(s)}+\frac{e^{u_i(s)}}{a_i(s)}\big)ds\\ &\le|\Gamma_i|\Omega+|r_i|\Omega+\frac{\bar{s}_i\Omega+\Gamma_i\omega}{\bar{b}_{ii}}\\ &=:D_i.\end{aligned}$$

By further calculation, it can be concluded that

$$u_i(s)\le u_i(\eta_i)+\int_0^\Omega|\dot{u}_i(s)|ds\le\bar{A}_i+D_i:=R_i, \tag{17}$$

and

$$u_i(s)\ge u(\varepsilon_i)-\int_0^\Omega|\dot{u}_i(s)|ds\ge\bar{C}_i-D_i:=K_i. \tag{18}$$

Thus, we get

$$\max_{s\in[0,\Omega]}|u_i(s)|\le\max(R_i,K_i):=B_i. \tag{19}$$

Let $0<B<+\infty$ be big enough, satisfying $|u_1^*|+|u_2^*|+\cdots+|u_n^*|<B$ and $B>B_1+B_2+\cdots+B_n$. Let $\Omega^*\subset X$ be a bounded open set satisfying:

$$\Omega^*=\{u\in X:\|u\|<B\}.$$

In order to verify all the conditions of the coincidence degree theory, we define

$$h_\mu(x)=\mu\mathrm{QN}u+(1-\mu)\mathrm{G}u,\mu\in[0,1],$$

then, we can get $0\notin h_\mu(\Omega^*\cap\operatorname{Ker}L)$.

Thus, according to the homology invariance and similar methods in [20], we have

$$\begin{aligned}&\deg\{JQN,\Omega^*\cap\operatorname{Ker}L,(0,0,\ldots,0)\}\\ =&\deg\{G,\Omega^*\cap\operatorname{Ker}L,(0,0,\ldots,0)\}\\ \neq&0.\end{aligned}$$

Finally, one can see that Ω^* satisfies all the conditions of coincidence degree theory. Therefore, system (11) has an Ω-periodic solution $u^*(s)=(u_1^*(s),u_2^*(s),\ldots,u_n^*(s))$. Therefore, system (5) has a positive Ω-periodic solution.

Theorem 3.3. If (H1) holds and $G_i>0(i=1,2,\cdots,n)$, the system (6) is globally attractive. Where

$$G_i=b_{ii}^L+\frac{a_i^L}{(a_i^M+\sum_{j\neq i}^n b_{ij}^M M_j)^2}-\frac{\sum_{j\neq i}^n b_{ji}^M}{(a_i^L+\sum_{j\neq i}^n b_{ji}^L m_j)^2}>0.$$

Proof. Assume that $(W_1(s),\cdots,W_n(s)$ and $(N_1(s),\cdots,N_i(s))$ are any two positive solutions of system (5), then there exist real numbers $S^*>0$ and $M>m>0$ for $s\ge S^*$ such that

$$m\le W_i(s),N_i(s)\le M.$$

Define a function as follows

$$V_1(s)=\sum_{i=1}^n|\ln W_i(s)-\ln N_i(s)|, \tag{20}$$

then we obtain

$$
\begin{aligned}
&D^{+}V_1(t)\\
&=\sum_{i=1}^{n}\operatorname{sgn}(W_i(s)-N_i(s))(\frac{\dot{W}_i(s)}{W_i(s)}-\frac{\dot{N}_i(s)}{N_i(s)})\\
&=\sum_{i=1}^{n}\operatorname{sgn}(W_i(s)-N_i(s))\big(-b_{ii}(s)(W_i(s)-N_i(s))\\
&\quad-(\frac{W_i(s)}{a_i(s)+\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)W_j(s+\tau)d\tau}\\
&\quad-\frac{N_i(s)}{a_i(s)+\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)N_j(s+\tau)d\tau}))\\
&\leq\sum_{i=1}^{n}\big[-b_{ii}(s)|W_i(s)-N_i(s)|\\
&\quad-[\frac{a_i(s)|W_i(s)-N_i(s)|}{a_i(s)+\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)W_j(s+\tau)d\tau}\\
&\quad\times\frac{1}{a_i(s)+\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)N_j(s+\tau)d\tau}\\
&\quad+\frac{\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)|W_j(s+\tau)-N_j(s+\tau)|d\tau}{a_i(s)+\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)W_j(t+\tau)d\tau}\\
&\quad\times\frac{1}{a_i(s)+\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)N_j(t+\tau)d\tau})\big]\\
&\leq\sum_{i=1}^{n}\big[-(b_{ii}(s)+\frac{a_i^L}{(a_i^M+\sum_{j\neq i}^{n}b_{ij}^M M_j)^2})|W_i(s)-N_i(s)|\\
&\quad+\frac{\sum_{j\neq i}^{n}b_{ij}(s)\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)|W_j(s+\tau)-N_j(s+\tau)|d\tau}{(a_i^L+\sum_{j\neq i}^{n}b_{ij}^L m_j)^2}\big].
\end{aligned}
$$

Next, we let

$$
\begin{aligned}
V_2(s)=&\sum_{i=1}^{n}\sum_{j\neq i}^{n}\int_{-\gamma_{ij}}^{0}\frac{1}{(a_i^L+\sum_{j\neq i}^{n}b_{ij}^L m_j)^2}K_{ij}(\tau)\\
&\times\int_{s+\tau}^{s}b_{ij}(\sigma)|W_j(\sigma)-N_j(\sigma)|d\sigma d\tau.
\end{aligned}
$$

Finally, we let $V(s)=V_1(s)+V_2(s)$, then we obtain

$$
\begin{aligned}
D^{+}V(t)\leq&-\sum_{i=1}^{n}\Big(b_{ii}(s)+\frac{a_i^L}{(a_i^M+\sum_{j\neq i}^{n}b_{ij}^M M_j)^2}\Big)\\
&\times|W_i(s)-N_i(s)|+\sum_{i=1}^{n}\sum_{j\neq i}^{n}b_{ij}^M\\
&\times\frac{1}{(a_i^L+\sum_{j\neq i}^{n}b_{ij}^L m_j)^2}|W_j(s)-N_j(s)|\\
\leq&-\sum_{i=1}^{n}\Big(b_{ii}^L+\frac{a_i^L}{(a_i^M+\sum_{j\neq i}^{n}b_{ij}^M M_j)^2}\\
&-\frac{\sum_{j\neq i}^{n}b_{ji}^M}{(a_i^L+\sum_{j\neq i}^{n}b_{ji}^L m_j)^2}\Big)|W_i(s)-N_i(s)|\\
\leq&-\sum_{i=1}^{n}G_i|W_i(s)-N_i(s)|.
\end{aligned}
$$

In addition, for $s=s_k$, we get

$$
V(s_k^+)=V_1(s_k^+)+V_2(s_k^+)+\cdots+V_i(s_k^+)=V(s_k). \quad (21)
$$

Thus for $s>S_1$, we get

$$
D^{+}V(s)\leq-\sum_{i=1}^{n}G_i|W_i(s)-N_i(s)|.
$$

Integrating from s to S_1 on both sides of (21), we obtain

$$
V(s)-V(S_1)\leq-\sum_{i=1}^{n}G_i\int_{S_1}^{t}|W_i(s)-N_i(s)|ds,
$$

and

$$
V(s)+\sum_{i=1}^{n}\int_{S_1}^{s}|W_i(s)-N_i(s)|ds\leq V(S_1)<+\infty, \quad (22)
$$

which yields

$$
\sum_{i=1}^{n}\int_{S_1}^{s}|W_i(s)-N_i(s)|<+\infty.
$$

Similarly, by adopting the method in [7], and by Barbalat's lemma, it follows that for $i=1,2,\ldots,n$

$$
\lim_{s\to+\infty}|W_i(s)-N_i(s)|=0.
$$

This completes the proof.

Next, from Theorem 3.1, Theorem 3.2 and Theorem 3.3, we have the following corollary.

Corollary 3.1. If (H2) holds and $G_i>0(i=1,2,\cdots,n)$, then system (6) is permanent and has a global attractive positive Ω-periodic solution. Where

$$
G_i=b_{ii}^L+\frac{a_i^L}{(a_i^M+\sum_{j\neq i}^{n}b_{ij}^M M_j)^2}-\frac{\sum_{j\neq i}^{n}b_{ji}^M}{(a_i^L+\sum_{j\neq i}^{n}b_{ji}^L m_j)^2}>0.
$$

In system (5), if $h_{ik}=0$, then then system (5) is reduced to the following non-impulsive n-species non-autonomous cooperative Lotka-Volterra system with time delays

$$
\begin{aligned}
\dot{N}_i(s)=&N_i(s)\big[r_i(s)-b_{ii}(s)N_i(s)\\
&-\frac{N_i(s)}{a_i(s)+\sum_{l=1}^{m}b_{jl}(s)\int_{-l\tau}^{0}K_j(\tau)N_j(s+\tau)d\tau}\big].
\end{aligned} \quad (23)
$$

Accordingly, the assumptions (H1) and (H2) turn to

(H3) $r_i(s)>0, b_{ij}(s)>0(i,j=1,2,\ldots n)$ are all continuous bounded functions on $[0,+\infty)$, and $K_{ij}(\tau)(i,j=1,2,\ldots n)$ are nonnegative integrable functions defined in $[-\gamma_{ij},0]$ satisfying $\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)d\tau=1$, $\gamma_{ij}>0$ are constants.

(H4) $r_i(s)>0, b_{ij}(s)>0(i,j=1,2,\ldots n)$ are all continuous bounded Ω-periodic functions on $[0,+\infty)$, and $K_{ij}(\tau)(i,j=1,2,\ldots n)$ are nonnegative integrable functions defined in $[-\gamma_{ij},0]$ satisfying $\int_{-\gamma_{ij}}^{0}K_{ij}(\tau)d\tau=1$, $\gamma_{ij}>0$ are constants.

Then also from Theorem 3.1, Theorem 3.2 and Theorem 3.3, we have the following two corollaries.

Corollary 3.2 If (H3) holds and $G_i>0$, then system (23) is permanent and globally attractive.

Corollary 3.3 If (H4) holds and $G_i>0$, then system (23) is permanent and has a global attractive positive Ω-periodic solution.

4 Numerical examples

In this section, two examples are given to illustrate the obtained theoretical results in this paper. In order to facilitate understanding and calculation, in the following examples, for $n = 2$, we consider the differential equations with impulses.

Example 4.1

$$
\begin{aligned}
&\dot{N}_1(s) = N_1(s)[1.4 + 0.2\cos(s) - (1.8 + 0.2\cos(s))N_1(s) \\
&\quad - \frac{N_1(s)}{1.35+0.15\cos(s)+(1.95+0.05\cos(s))\int_{-\gamma_{12}}^{0} K_2(\tau)N_2(s+\tau)d\tau}], \\
&N_1(s_k^+) = (1.25 + 0.15\cos k)N_1(s_k), \quad s = s_k. \quad k \in n \\
&\dot{y}_2(s) = y_2(s)[0.7 + 0.1\cos(s) - (1.7 + 0.1\cos(s))y_2(s) \\
&\quad - \frac{N_2(s)}{1.6+0.1\cos(s)+(0.85+0.05\cos(s))\int_{-\gamma_{21}}^{0} K_1(\tau)N_1(s+\tau)d\tau}], \\
&N_2(s_k^+) = (1.65 + 0.15\cos k)N_2(s_k), \quad s = s_k, \quad k \in n.
\end{aligned} \tag{24}
$$

It is easy to get that $\theta = \frac{1}{2}, \eta = 1$. By direct calculation, we can obtain that

$$G_1 \approx 0.831 > 0, \quad G_2 \approx 0.413 > 0.$$

Obviously, the assumptions of Corollary 3.1 are all satisfied. Therefore, system (24) is permanent and has a globally attractive positive 2π-periodic solution. The corresponding numerical simulations are given in Fig. 1. and Fig. 2.

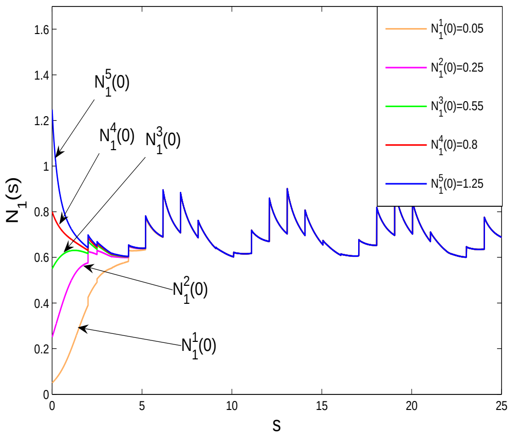

Fig. 1. The global attractivity, periodic solution and permanence of $N_1(s)$. Here, we take different initial values $N_1^i(0)$, i=1,2,3,4,5.

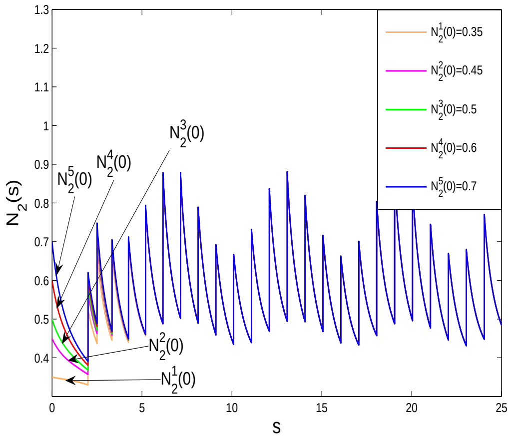

Fig. 2. The global attractivity, periodic solution and permanence of $N_2(s)$. Here, we take different initial values $N_2^i(0)$, i=1,2,3,4,5.

The following example is a special case of Example 4.1.

Example 4.2

$$
\begin{aligned}
&\dot{y}_1(s) = y_1(s)[1.4 + 0.2\cos(s) - (1.8 + 0.2\cos(s))y_1(s) \\
&\quad - \frac{y_1(s)}{1.35+0.15\cos(s)+(1.95+0.05\cos(s))\int_{-\gamma_{12}}^{0} K_2(\tau)y_2(s+\tau)d\tau}], \\
&\dot{y}_2(s) = y_2(s)[0.7 + 0.1\cos(s) - (1.7 + 0.1\cos(s))y_2(s) \\
&\quad - \frac{y_2(s)}{1.6+0.1\cos(s)+(0.85+0.05\cos(s))\int_{-\gamma_{21}}^{0} K_1(\tau)y_1(s+\tau)d\tau}].
\end{aligned} \tag{25}
$$

From Example 4.1, one can see that, the assumptions of Corollary 3.3 are satisfied. Then system (25) is permanent and has a globally attractive positive 2π-periodic solution. The corresponding numerical simulations are given in Fig.3. and Fig.4.

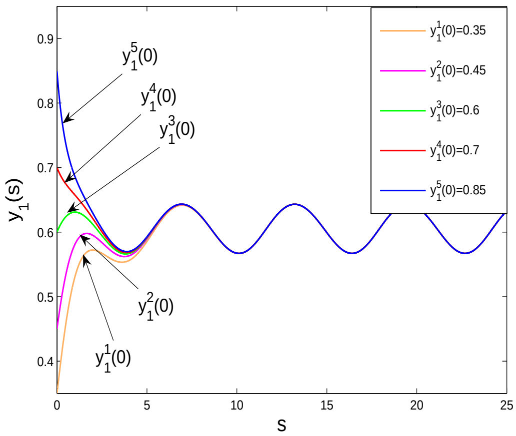

Fig. 3. The global attractivity, periodic solution and permanence of $y_1(s)$. Here, we take different initial values $y_1^i(0)$, i=1,2,3,4,5.

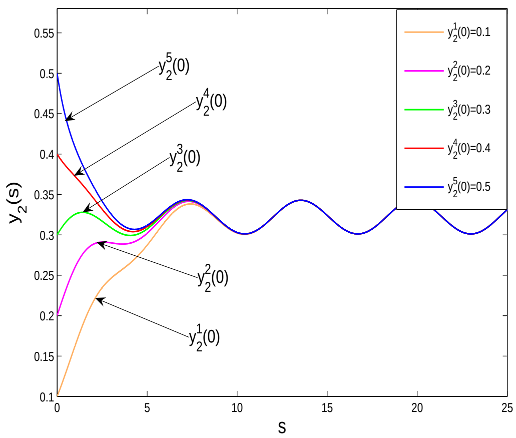

Fig. 4. The global attractivity, periodic solution and permanence of $y_2(s)$. Here, we take different initial values $y_2^i(0)$, i=1,2,3,4,5.

IV. Conclusion

Considering differential equations with impulses for certain species can provide valuable insights. For example, fishermen maximizing their benefits by implementing a ban on fishing and artificially releasing larvae for a specific period makes sense. However, among the many previously studied population dynamical impulse models, cooperation models are rare, and most of them only study the relationship between one or two populations, which also lack relevant numerical examples to illustrate the obtained results. Therefore, in this study, we investigated a class of n-species cooperation models (5) and used appropriate numerical examples to illustrate them. In this study, by utilizing useful inequality techniques, the impulsive comparison principle, and the Lyapunov method, some conditions on global attractivity, permanence, and the existence of positive periodic solutions were obtained.

Moreover, because the model (5) can be seen as a specialization of the model in [7-11], the theoretical results obtained in this study can be seen as an extension and complement to previous work.

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