A Novel Approach to Railway Capacity Estimation Incorporating Cross-Line Trains

Haibo Mu, Yun Shi, Chenning Liu, Wenguo Ma, Hongzhen Wang, Huo Chai

Abstract—**Estimating railway capacity is both valuable and challenging. This paper introduces a novel approach to high-railway capacity calculation by incorporating cross-line trains. Leveraging the concept of compressing timetable diagrams, we generate diverse structured diagrams randomly and subsequently create a train timetable without buffer time using a genetic algorithm framework. We establish ideal arrival times for cross-line trains and comprehensively analyze the impact of these trains by comparing ideal and actual arrival times. Unlike traditional models that only consider single-line train operations, our model provides a more comprehensive evaluation of railway capacity. The NSGA-II algorithm developed for this model is innovative and efficient, generating a high-quality Pareto front solution set that offers multiple decision-making schemes for stakeholders. The results show that a heuristic algorithm can obtain a feasible solution more quickly when the objective is nonlinear. Compared with the ideal time deviation of cross-line trains without considering the ideal time deviation, the ideal time deviation derived by the heuristic algorithm is reduced by 93.2%; the GAP of train occupancy time obtained with the heuristic algorithm is 3.58%.**

Keywords: railway capacity; compression timetable; cross-line trains; high-speed railway; mixed integer programming model

I. Introduction

In recent years, the development of high-speed railways has been rapid worldwide, particularly in China. High-speed railways play a crucial role in passenger transportation. With the accelerated construction of the Chinese "eight verticals and eight horizontals" high-speed railway network, an increasing number of passengers are attracted to high-speed railway travel, leading to increased tension regarding high-speed railway capacity [1]. Many cross-line trains have dramatically increased the complexity of railway operations, requiring timely and accurate calculations of railway capacity. Properly assessing railway capacity is crucial in effectively meeting the escalating passenger demand.

Over the past few decades, extensive research on railway capacity has led to various definitions, each tailored to specific research or application objectives. According to Krueger [2], railway capacity measures an infrastructure's ability to transport a predetermined traffic volume along a specific route, utilizing allocated resources and adhering to a predetermined service schedule. The International Union of Railways (UIC) [3] posited that the capacity of railway infrastructure depends upon its utilization. Four interrelated parameters are the number of trains, stability, heterogeneity, and average speed, which significantly affect this capacity. Abril et al. [4] identified four primary definitions of railway capacity: theoretical ability, practical ability, useability, and available ability. Mussone et al. [5] defined capacity as the maximum number of trains that can traverse the entire railway system within a given time while meeting management constraints. Despite these variations in defining railway capacity, the overarching aim of capacity analysis remains consistent: estimating the maximum number of trains that can be accommodated within a specified timeframe under existing railway infrastructure and operational plans. Some scholars use the period to define railway capacity, such as Heydar et al. [6] and Petering et al. [7] characterized railway capacity as the shortest time needed to accommodate a certain number of trains on a particular section of track during each cycle. To maintain consistency, we adopt the definition provided by Wang et al. [8], which defines *high-speed railway capacity* as the minimum occupation time of a railway line under the conditions of a given infrastructure and the number of trains within one day.

The timetable compression method is widely used to calculate capacity occupation. The fundamental idea behind the train timetable compression method is to preserve the order of trains running at each time interval while taking the time intervals between trains as a constraint. All running lines within a specific range of the train diagram are then translated and compressed along the time axis towards the origin until no further translation is possible. This process results in a "compressed" train operation diagram, which allows for calculating the minimum time required for all trains to pass through the interval [9]. As a result, the capacity utilization rate of the interval can be determined. Considering the free time under the train operation diagram structure, the minimum time objectively reflects the current train diagram's capacity utilization. Alex [10] employed the train diagram

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compression method to assess the capacity of a single-track railway. Lindner et al. [11] examined the application of UIC406 in evaluating capacity, identified the factors affecting passing capacity, and emphasized that evaluating passing capacity at stations is crucial, as it may become a limiting factor in overall passing capacity. Depending on the varying intervals of running line compression, the train diagram compression method can be categorized into two main approaches: UIC406 and CUI methods. Sameni et al. [12] contrasted the differences between the UIC406 and CUI methods and proposed some meso-indicators to enhance the UIC406 method as a decision-making tool in similar scenarios. Jamili et al. [13] considered the required buffer time during operation diagram compression and assessed its performance under specific robustness conditions. Jensen et al. [14] proposed a capability analysis framework that generated multiple train timetables under specified conditions using random generation or optimization strategies like branch-and-bound algorithm and tabu search heuristic, followed by calculating capacity utilization using the train timetable compression method. Wang et al. [15] applied the timetable compression method to ensure consistent running times for cross-line trains, incorporating additional running lines behind the compressed timetable graphs for encryption purposes.

Based on the provided train operation diagram, the simulation method is employed to infer the state of the train operation process and simulate the execution of the train diagram. The results can be analyzed to provide a better understanding of the characteristics of the train operation diagram and the utilization of transport resources, which will help to adjust the timetable further and ultimately generate a comprehensive train operation map for capacity calculations. Simulation methods are often investigated using commercial software for train operation. For example, Abril et al. [16] developed a tool called MOM, which integrates analytical and optimization approaches in this context. This comprehensive system assists railway managers in performing capacity studies to optimize their timetables and evaluate track and station capacity to meet customer demands. Dingler et al. [17] utilized Rail Traffic Controller simulation software to examine the impact of different combinations of intermodal, unit, manifest, and passenger trains on a hypothetical, signalized, single-track line with typical North American railroad subdivision characteristics. In addition, there are some microsimulation commercial software, such as Switzerland's OpenTrack [18], and Germany's RailSys [19].

As mentioned above, there have been many researches and results on the issue of railway capacity. However, in the compression of train timetables, most scholars compressed the cross-line trains together with the single-line trains and did not separate the single-line trains from the cross-line trains. The linkage of cross-line trains in the whole network determines their unique characteristics in the compression process, which is different from that of main-line trains, and the adjustment of the operating time of single-line trains may cause a change in the operating time of trains in the whole network. The capacity calculated by simple compression needs to be more accurate. Therefore, an improved timetable compression method is needed to consider cross-line trains.

The research presented in this paper makes the following two contributions to the optimization methodology of high-speed railway capacity:

(1) Based on train timetable compression, a train operation timetable without buffer time is developed within the framework of a genetic algorithm, thus proposing a mixed integer programming model tailored for high-speed railways. The model not only determines the arrival and departure times of trains at stations but also considers the impact of cross-line trains on other lines. The deviation between the actual arrival time of a cross-line train and its ideal arrival time is used to measure the ideal time deviation, thus simplifying the procedure. The method proposed in this paper minimizes the total travel time and ideal time deviation of cross-line trains.

(2) The paper uses the train departure sequence as the solution to the proposed model. An NSGA-II algorithm is developed to solve the model, along with a conflict resolution method specifically designed for intersecting lines, which represents an innovative application of the algorithm. The NSGA-II algorithm produces a high-quality Pareto front solution set, providing multiple decision-making options for decision-makers. This approach effectively evaluates the capacity of high-speed railways.

II. Problem Description

A. Related Assumptions and Notation

TABLE I
SETS AND INDEXES

Sets	*Definitions*
S	*Set of stations*
L	*Set of trains*
Indices	
i, j	*Index of trains,* $i, j \in L$
s	*Index of stations,* $s \in S$
Parameters	
$t^{i}_{s-1,s}$	*Pure running time for train* i *from station* $s-1$ *to* s , $s-1, s \in S$
t_1	*Starting additional time for train*
t_2	*Stopping additional time for train*
I_{AA}	*Arrival headway when the pair of trains both stop*
I_{AP}	*Arrival headway when the front train skips and the latter train stops*
I_{PA}	*Arrival headway when the front train skips and the latter train stops*
I_{DD}	*Departure headway when the pair of trains both stop*
I_{DP}	*Departure headway when the front train skips and the latter train stops*
I_{PD}	*Departure headway when the front train stops and the latter train skips*
I_{PP}	*Departure headway when the pair of trains both skip*
h_i	*= 1 if train* i *has higher speed level* *= 0 otherwise*
M	*A dominantly large positive number*
$T_{\min}$	*Minimum dwell time*
$T_{\max}$	*Maximum dwell time*
$x_{i,s}$	*= 1 if train* i *is planned to stop at station* s *;* *= 0 otherwise*
A_p	*Lower bound of the rail line's operation time window*
D_p	*Upper bound of the rail line's operation time window*
Decision Variables	

$a_{s,i}$	*Continuous; arrival time variable of train* i *at station* S , $s \in S, i \in L$
$d_{s,i}$	*Continuous; departure time variable of train* i *at station* S , $s \in S, i \in L$
$O_{i,j}^{s-1,s}$	*Binary; = 1 if train* j *runs after train* i *between station* $s-1$ *and* s , $=0$ *otherwise;* $i, j \in L, s-1, s \in S$
$a_{s,\max}$	$a_{s,\max} = \max(a_{s,i})$
$d_{s,\min}$	$d_{s,\min} = \min(d_{s,i})$

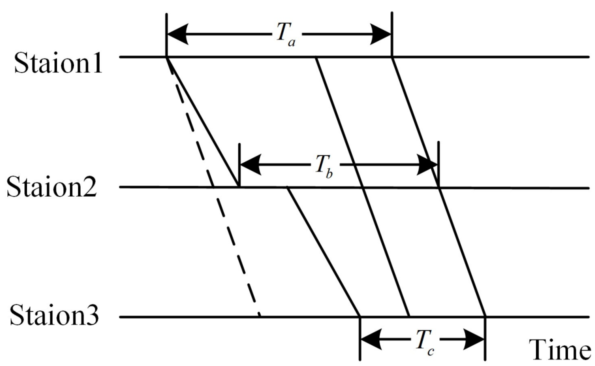
B. Problem Description

Chinese high-speed railway system employs a transport organization mode called "co-line matching operation of trains with different speed levels," which involves the simultaneous use of trains with varying speed grades on the same line and trains from different lines on the same track. This transport organization mode has many advantages but complicates transport organization and train timetable planning.

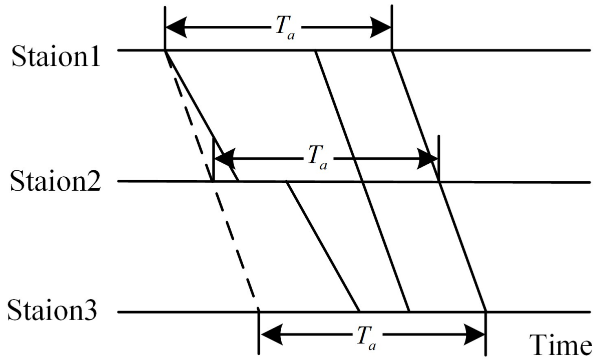
This paper analyzes the capacity of a double-track railway corridor at the macroscopic level. Cross-line trains are considered through the ideal time deviation of cross-line trains to develop a more efficient drawing scheme based on the operational plan of a high-speed railway passenger flow section, aiming to minimize train occupancy time in the diagram and reduce the ideal time deviation of cross-line trains. The resulting timetable, a compressed timetable, does not include any buffer time. The train occupation diagram's time and capacity utilization coefficients are calculated by compressing the timetable. The maximum number of running lines that can be accommodated within the current diagram structure is determined by dividing the number of running lines by the capacity utilization coefficient, representing the section's capacity.

The Train Occupancy Time

The train occupancy time refers to the total time occupied when designing the train running line, considering various factors such as stops at each station, additional time for acceleration and deceleration, time intervals between stations, and the overall running time of the train along the entire line. As depicted in Fig. 1, this time can be represented in two ways: one is by station, indicating the occupancy time at each station, and the other is by section, indicating the occupancy time for each line segment.



(a)



(b)

Fig. 1. Station occupancy time and interval occupancy time. (a) Station occupancy time; (b) Interval occupancy time.

The diagram presented in Fig. 1(a) illustrates station occupancy time, calculated by determining the difference between the latest departure and earliest arrival times for each train at a station. The maximum of these differences is then considered as the occupancy time for the train diagram. Fig. 1(b) presents interval occupancy time, calculated by selecting the first train as a base non-stop high-speed train and computing the variance between each train's latest departure time and the base train's departure time. The maximum difference is taken as the occupancy time for the train diagram. At Station 1, both methods yield identical results. However, at Stations 2 and 3, the calculation based on station occupancy time yields smaller values due to additional time spent stopping at stations and starting/stopping the train. Despite this, since the maximum value from each calculation is used, both methods yield equivalent results for the train diagram.

However, the results may differ if the plotting train running lines are modified and recalculated using two representation methods. For instance, as shown in Fig. 2, at station 1, the results of the two methods are identical; at station 2, the interval occupancy time representation method yields an enormous calculated occupancy time due to the influence of additional time incurred during train stops; at station 3, the combined impact of stop time and additional time results in an enormous calculated occupancy time using the interval occupancy time representation method as well. Ultimately, the maximum value of the occupancy time for each station or interval is selected. As shown in Fig. 2(a), T_b is chosen as the final calculation result at the second station, and in Fig. 2(b), T_c is selected as the final calculation result at the third station. The results obtained from the two calculation methods are not necessarily the same. Due to the influence of train stopping or overtaking at intermediate stations, the calculation results differ but are generally not significant, and they can also reflect the degree of utilization of the train diagram's capacity. In this paper, the station occupancy time representation method is employed to calculate the occupancy time of the train diagram.

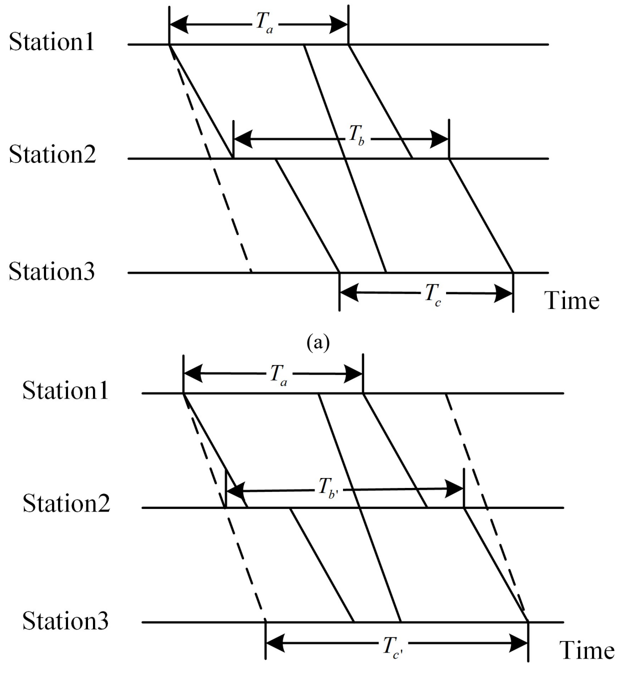


Fig. 2. Difference between station occupancy time and interval occupancy time. (a) Station occupancy time; (b) Interval occupancy time

Ideal time deviation of cross-line trains

The preparation process for the cross-line train diagram involves determining the arrival and departure times of the cross-line train at each station. The arrival time at a station can be calculated based on the departure time and the interval running time from the previous station. The interval running time can be determined once the stop plan and train type are established. In Fig. 3, the ideal arrival time for each station of a cross-line train is defined to accomplish this objective. The goal is to minimize the deviation between the cross-line train and the ideal time. It is then converted into minimizing the deviation between the cross-line train and the ideal arrival time at each station.

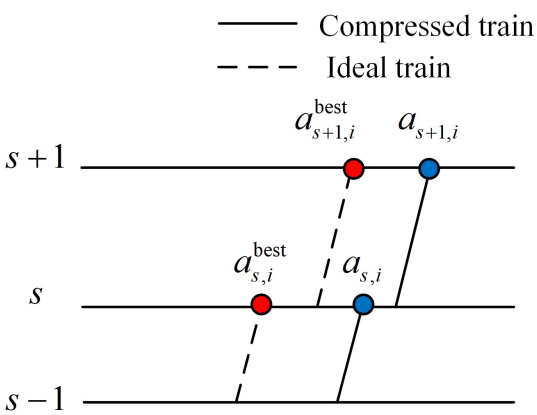


Fig. 3. Ideal time deviation of cross-line trains

III. Construction of the Mixed Integer Programming Model

A. *Modeling assumptions*

This study is based on the following four assumptions:

The operational plans for local trains and cross-line trains are known.

The station arrival-departure lines have sufficient capacity.

Each station is assumed to have the capability of simultaneous receiving and departure.

To better elaborate on the sequence changes of trains, here all trains are assumed to share the same route with the same origin and terminal station, which covers the whole rail line considered.

B. Objective function

Minimize the train occupancy time

In order to enhance the quality of railway transportation services, improve portability, and reduce passenger travel time, we establish the objective function of minimizing train occupancy time for the prepared train diagram:

$$Z_1 = \min(\max(a_{s,\max} - d_{s,\min}))\quad s \in S \tag{1}$$

Minimize the ideal time deviation of cross-line trains

Establish the objective function of minimizing the ideal time deviation of cross-line trains to reduce the change in cross-line train time:

$$Z_2 = \min \sum_{i\in L_f} \sum_{s\in r_i} \left| a_{s,i} - a_{s,i}^{\text{best}} \right| \tag{2}$$

C. Constraint condition

Operation time window constraint

Optimal scheduling of departure and arrival times is crucial for enhancing train operational efficiency, improving punctuality rates, and ensuring operational stability. This approach ensures trains reach their destinations within the stipulated timeframes, minimizing passenger waiting times and delays. The departure and arrival times for trains on this route must not exceed the predefined limits. The associated constraints can be mathematically expressed as follows:

$$\begin{cases} A_{\text{p}} \le d_{s,i} \le D_{\text{p}} \\ A_{\text{p}} \le a_{s,i} \le D_{\text{p}} \end{cases} \quad i \in L, s \in S \tag{3}$$

Section running time constraint

The train operating in the section must adhere to a specific running time. Considering the previous station's departure time, the section's pure running time, the scheduled stops, and the additional time for starting and stopping, the train arrival time at the subsequent station can be calculated. The constraint is expressed as follows:

$$\begin{gathered} a_{s,i} - d_{s-1,i} - x_{s-1,i}t_1 - x_{s,i}t_2 = t_{s-1,s}^i \\ i \in L, s, s\text{-}1 \in S \end{gathered} \tag{4}$$

Train headway constraint

The minimum train tracking interval time ensures that the last train can enter the section only after the previous train has wholly exited the section, considering the operational conditions and uncertainties of the trains. The interval time between trains traveling in the same direction must adhere to the minimum tracking interval time requirement to prevent collisions and other accidents, as shown in Fig. 4. The constraint is expressed as follows:

$$\begin{gathered} a_{s,j} - a_{s,i} - I_{AA} + M(3 - x_{s,i} - x_{s,j} - O_{i,j}^{s-1,s}) \ge 0 \\ i \in L, j \in L \setminus i, s, s\text{-}1 \in S \end{gathered} \tag{5}$$

$$\begin{gathered} a_{s,j} - a_{s,i} - I_{Ap} + M(2 - x_{s,i} + x_{s,j} - O_{i,j}^{s-1,s}) \ge 0 \\ i \in L, j \in L \setminus i, s, s\text{-}1 \in S \end{gathered} \tag{6}$$

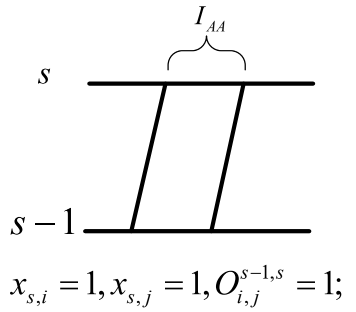
$$a_{s,j} - a_{s,i} - I_{PA} + M(2 + x_{s,i} - x_{s,j} - O_{i,j}^{s-1,s}) \geq 0$$
$$i \in L, j \in L \setminus i, s, s\text{-}1 \in S \tag{7}$$

$$d_{s,j} - d_{s,i} - I_{DD} + M(3 - x_{s,i} - x_{s,j} - O_{i,j}^{s,s+1}) \geq 0$$
$$i \in L, j \in L \setminus i, s, s+1 \in S \tag{8}$$

$$d_{s,j} - d_{s,i} - I_{DP} + M(2 - x_{s,i} + x_{s,j} - O_{i,j}^{s,s+1}) \geq 0$$
$$i \in L, j \in L \setminus i, s, s+1 \in S \tag{9}$$

$$d_{s,j} - d_{s,i} - I_{PD} + M(2 + x_{s,i} - x_{s,j} - O_{i,j}^{s,s+1}) \geq 0$$
$$i \in L, j \in L \setminus i, s, s+1 \in S \tag{10}$$

$$d_{s,j} - d_{s,i} - I_{PP} + M(1 + x_{s,i} + x_{s,j} - O_{i,j}^{s,s+1}) \geq 0$$
$$i \in L, j \in L \setminus i, s, s+1 \in S \tag{11}$$



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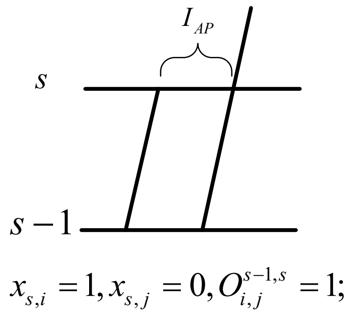

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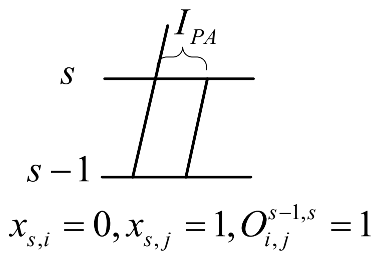

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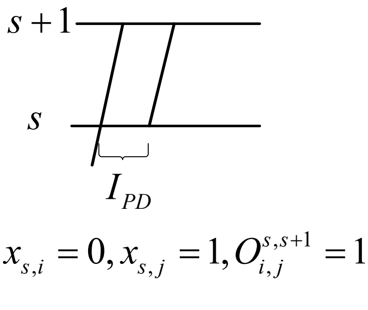

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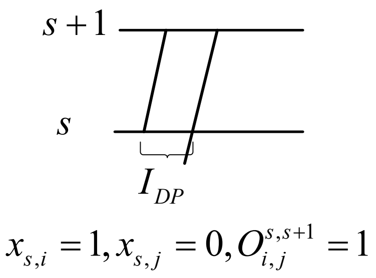

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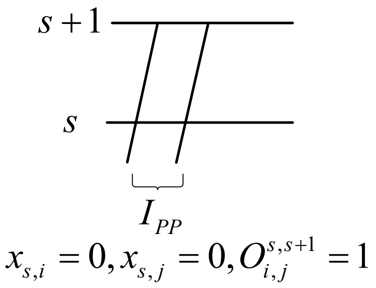

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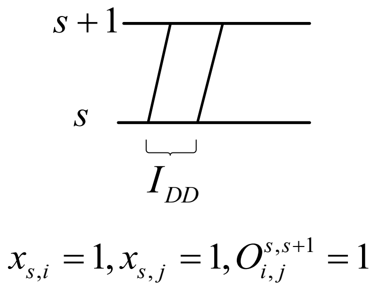


(g)

Fig. 4. Seven types of minimum safety interval. (a) Arrival headway when the pair of trains both stop; (b) Arrival headway when the front train skips and the latter train stops; (c) Arrival headway when the front train skips and the latter train stops; (d) Departure headway when the pair of trains both stop; (e) Departure headway when the front train skips and the latter train stops; (f) Departure headway when the front train stops and the latter train skips; (g) Departure headway when the pair of trains both skip.

Train stop time constraint

To maintain a balance between train speed, passenger demand, and safety, the duration of train stops should neither be shorter than the minimum allowable stop time nor exceed the maximum allowable stop time. This constraint can be expressed as follows:

$$T_{\min} x_{s,i} \leq a_{s,i} - d_{s,i} \leq T_{\max} x_{s,i},$$
$$s \in S, i \in L, j \in L \setminus i \tag{12}$$

Interval operation sequence constraints

Due to high-speed railways primarily consisting of double-track lines, overtaking is only permitted at stations to prevent conflicts and is not allowed at intervals between stations.

The constraint is expressed as follows:

$$O_{i,j}^{s-1,s} + O_{j,i}^{s-1,s} = 1$$
$$s-1, s \in S, i \in L \tag{13}$$

Overtaking constraint

Same-direction trains may overtake each other at stations due to speed differences within the interval. To enhance passing capacity, low-grade trains are prohibited from overtaking high-grade trains. The constraint is expressed as follows:

$$\sum_{j\in I} O_{i,j}^{s-1,s} - \sum_{j\in I} O_{i,j}^{s,s+1} \geq 0 \quad \text{if } h_i = 1, s-1, s, s+1 \in S, i \in L, j \in L \setminus i \tag{14}$$

Train overtaking qualification constraint

To guarantee service quality, it is mandated that only one train is permitted to overtake or be overtaken simultaneously. This constraint can be expressed as follows:

$$\sum_{j\in I} \left| O_{i,j}^{s-1,s} - O_{i,j}^{s,s+1} \right| \leq 1, \quad s-1, s, s+1 \in S, i \in L, j \in L \setminus i \tag{15}$$

IV. Algorithm Design

The model presented in this paper is a multi-objective mixed integer programming model, and the actual train schedule involves many trains. As a result, traditional algorithms struggle to solve the model, necessitating the design of heuristic algorithms. Due to the limitations of traditional genetic algorithms in solving multi-objective models, Deb et al. [20] introduced the NSGA-II algorithm. Compared to the traditional non-dominated sorting genetic algorithm, the NSGA-II algorithm improves complexity, running speed, and solution set convergence and is widely employed in addressing multi-objective optimization problems.

The improved train working compression diagram method can be divided into two primary components: determining the sequence of trains in each section and drawing the train diagram based on the given stop plan and train interval order. Once the departure sequence of trains within the interval is established, a compressed train timetable without buffer time can be constructed by considering factors such as train grade, stop the plan, minimum stop time, minimum tracking train interval time, and the pure running time of each grade of a train within the interval. Subsequently, railway capacity can be calculated. The key to drawing the compressed train diagram lies in arranging and combining the train departure sequence at each station. Moreover, determining the train departure sequence dictates the train's stopping pattern and tracking operation. The train will maintain its departure sequence, except for overtaking at intermediate stations. Therefore, this paper selects the train departure sequence as the solution for the model. For instance, as illustrated in Fig. 5, the chromosome structure signifies that the train departure sequence is Train 1, Train 2, Train 3, Train 4, and Train 5. In this paper, a corresponding NSGA-II algorithm is designed based on the problem's characteristics.

1	2	3	4	5

Fig. 5. Chromosome structure

NSGA-II algorithm solution process:

Step 1. Initialization.

Set the maximum genetic algebra to gen_{max}, the probability of crossover to P_c and the probability of mutation to P_m. According to the above chromosomal coding rule in the upper layer and the search space corresponding to the constraints, the algorithm randomly generates an initial population of size popsize. Set the number of iteration $\text{gen\_no} = 0$.

Step 2. Generate the initial population.

A departure sequence table is generated for each station for each individual in the current generation. The current train is a B-type stopping train, and the subsequent train is an A-type train. Overtaking occurs with a certain probability P_0. Due to the uncertainty of the overtaking station, the table is standard. Therefore, each station's train departure sequence table under the starting sequence generated by each iteration may not necessarily be the same. The type of train interval is determined based on the departure sequence table and the stopping plan for each station.

Step 3. Decoding.

According to equation (3) or equation (13), the departure time for the current station is generated, and then the arrival time for the next station is generated according to equation (4). This process is repeated in the order of intervals until all intervals are completed.

Step. 4 Adjusting train conflicts.

According to equations (5)- (12), evaluate whether the train headway constraint meets the requirements. If satisfied, proceed to Step 5; otherwise, conflict resolution is implemented. It involves calculating the translation amount, traversing the set of rear trains (trains requiring translation) of the forward train, and determining whether the rear train was overtaken in the previous interval. The running line is shifted to the right if the rear train is not overtaken. If the rear train was overtaken and the overtaking train was in the rear train set, the running line is translated as a whole. Otherwise, only the overtaking station and the subsequent running line are translated.

Step 5. Non-dominated sorting and congestion allocation.

The objective values of the objective functions for each individual are calculated using equations (1) and (2). The individuals in the population are then sorted based on their performance across multiple objective functions to obtain a set of non-dominated ranks. Within a non-dominated rank, individuals have no dominance relationship, meaning that no individual is superior to another individual in all objective functions. Non-dominated sorting helps maintain the diversity of the population and prevents convergence to local optimal solutions.

After non-dominated sorting, the crowding distance in the objective function space is calculated for each individual in the non-dominated layer. The crowding distance reflects the density around an individual, indicating whether other individuals have thoroughly explored the region where the individual is located. The crowding distance assists the algorithm in maintaining population balance and preventing excessive concentration of the population in specific local areas.

Step6. Evolutionary operations.

Evolutionary operations primarily involve selection, crossover, and mutation. The binary tournament selection mechanism is utilized to select individuals from the parent population for the next generation. The selection operation is based on the individuals' non-dominated rank and crowding distance to ensure algorithm convergence.

In this paper, a two-point crossover approach is applied. Two chromosomes are selected from the population, and two distinct positions on one chromosome are randomly chosen. The gene fragment between these two positions then replaces the corresponding gene fragment on the other chromosome. The crossover process is illustrated in Fig. 6. The starting and ending positions are randomly selected in the two parent chromosomes. Genes from the region of parent chromosome 1 are subsequently copied to the same position in offspring 1, and any missing genes in offspring 1 are filled in order from parent chromosome 2. A similar process is employed to generate another offspring.

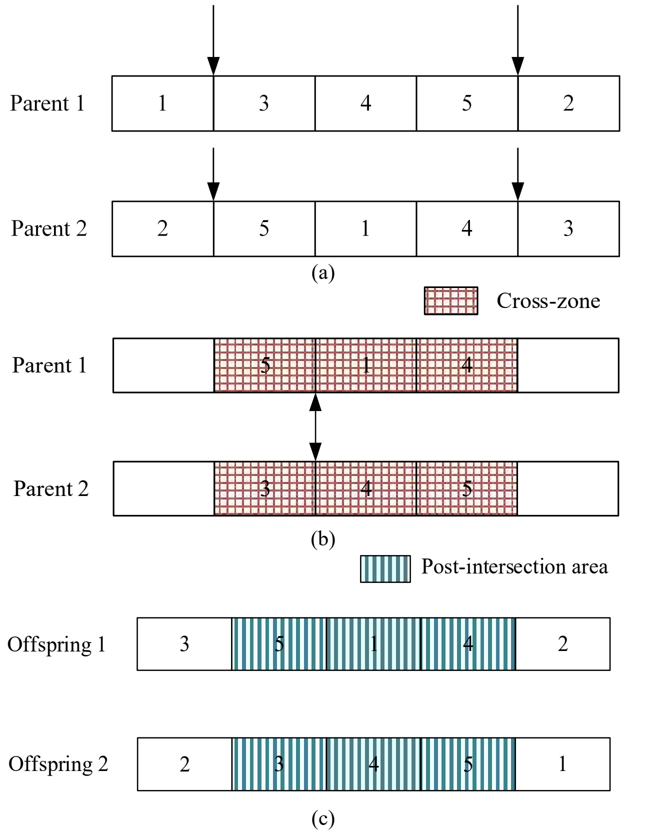


Fig.6. Chromosome double-point crossover. (a) Select the cross position; (b) Copied genes in the cross position. (c) Add uncrossed genes.

The mutation operation involves randomly selecting two gene positions on the chromosome and exchanging their respective gene values. This process is illustrated in Fig. 7.

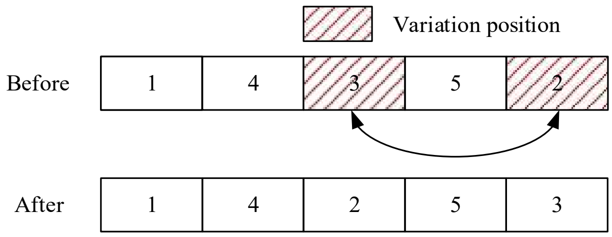


Fig.7. Genetic variation process.

Step7. Elitist strategy.

The initial population and newly generated individuals are combined to form a larger population. This merged population undergoes non-dominated sorting, and the crowding distance is calculated following the steps outlined in Step 5. The next generation's individuals are selected from the merged population. Based on their non-dominated sorting rank, the entire population is arranged sequentially into the new parent population until the limit is reached, with any remaining unqualified individuals being eliminated. Finally, the population is sorted based on crowding distance, and the new parent population size is adjusted to match the initial population size pop_size. Then, the number of iterations $gen\_no = gen\_no + 1$.

Step 8. Repeat steps 3-7 until the number of iterations gen_no reaches the upper limit gen_max.

V. Numerical Experiment

A. Experiment Scenario

A simulated section in the downstream direction is employed as a case study to evaluate the algorithm's efficacy. As illustrated in Fig.8, the stations range from 0 to 10, with station 0 and station 10 being cross-line stations connected to other lines. The trains are numbered from 0 to 29, with 16 to 29 being cross-line trains. This paper examines two types of trains with distinct speed levels, and the fundamental station data is presented in Table Ⅱ. The parameter values are displayed in Table Ⅲ.

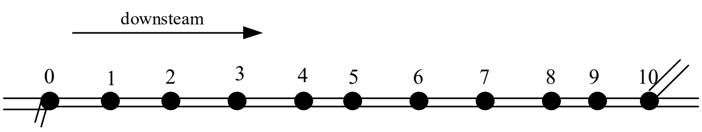


Fig.8. The simulated section.

TABLE Ⅱ
STATION BASIC DATA

Section	*0-1*	*1-2*	*2-3*	*3-4*	*4-5*
Station spacing (km)	*7*	*13*	*49*	*65*	*21*
Class A train Running time (min)	*1*	*2*	*8*	*11*	*4*
Class B train Running time (min)	*2*	*3*	*12*	*16*	*5*
Station spacing (km)	*5-6*	*6-7*	*7-8*	*8-9*	*9-10*
Class A train Running time (min)	*22*	*42*	*42*	*52*	*7*
Class B train Running time (min)	*4*	*7*	*7*	*4*	*1*
Station spacing (km)	*5*	*10*	*10*	*6*	*2*

TABLE III
PARAMETER VALUES

I_{AP}	I_{PA}	I_{DD}	I_{DP}
3	*6*	*5*	*6*
P_0	T_{min}	T_{max}	A_p
0.5	*2*	*20*	*360*
I_{PD}	I_{PP}	t_1	popsize
2	*4*	*2*	*50*
D_p	P_m	t_2	gen_max
1440	*0.01*	*3*	*150*

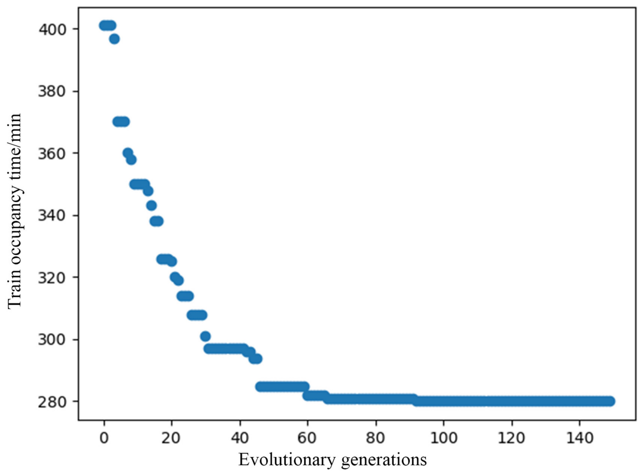
B. Results Analysis

The algorithm presented in this paper is implemented in Python. The computational experiment was conducted on a personal computer equipped with an Intel Core i7-7500U

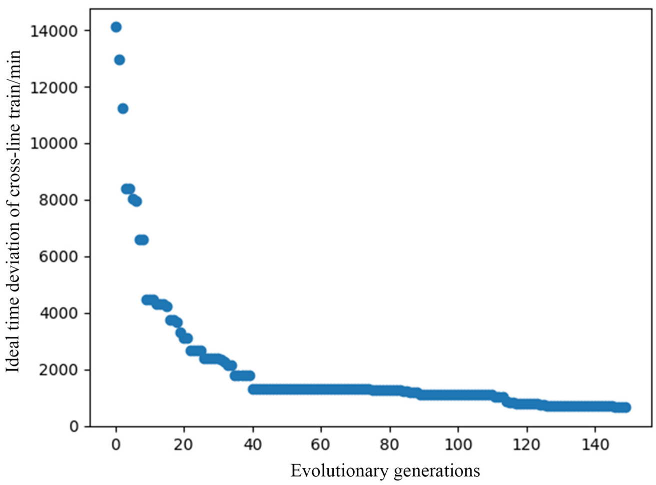
2.70 GHz CPU and 12 GB of memory. Fig. 9 illustrates the objective values calculated by the NSGA-II algorithm at each iteration during the first run. As demonstrated, the objective values increase with the number of iterations and eventually converge around the 140th iteration. Table Ⅲ presents the objective values of the Pareto optimal solution set. Additionally, Fig. 10 provides a visual representation of the obtained Pareto front.

TABLE Ⅲ
OBJECTIVE VALUES OF THE PARETO SOLUTION SET MIN

Solution	*Train occupancy time*	*Ideal time deviation of cross-line trains*	*Solution*	*Train occupancy time*	*Ideal time deviation of cross-line trains*
1	*280*	*7716*	*26*	*396*	*868*
2	*410*	*658*	*27*	*371*	*1526*
3	*287*	*6626*	*28*	*336*	*2616*
4	*312*	*3933*	*29*	*295*	*5726*
5	*297*	*5596*	*30*	*358*	*2014*
6	*300*	*5036*	*31*	*281*	*7686*
7	*318*	*3724*	*32*	*351*	*2210*
8	*307*	*4384*	*33*	*367*	*1754*
9	*326*	*3276*	*34*	*392*	*1135*
10	*282*	*7316*	*35*	*394*	*1016*
11	*310*	*4256*	*36*	*379*	*1376*
12	*291*	*6396*	*37*	*409*	*760*
13	*321*	*3556*	*38*	*404*	*788*
14	*361*	*1790*	*39*	*368*	*1674*
15	*294*	*5956*	*40*	*348*	*2364*
16	*340*	*2496*	*41*	*345*	*2486*
17	*284*	*7126*	*42*	*334*	*2886*
18	*305*	*4806*	*43*	*359*	*1955*
19	*292*	*6186*	*44*	*335*	*2756*
20	*330*	*2986*	*45*	*401*	*858*
21	*373*	*1410*	*46*	*381*	*1374*
22	*388*	*1182*	*47*	*284*	*7126*
23	*354*	*2194*	*48*	*331*	*2946*
24	*384*	*1285*	*49*	*347*	*2436*
25	*325*	*3458*	*50*	*304*	*4826*



(a)



(b)

Fig.9. NSGA-Ⅱ algorithm convergence. (a) Train occupancy time; (b) Interval occupancy time.

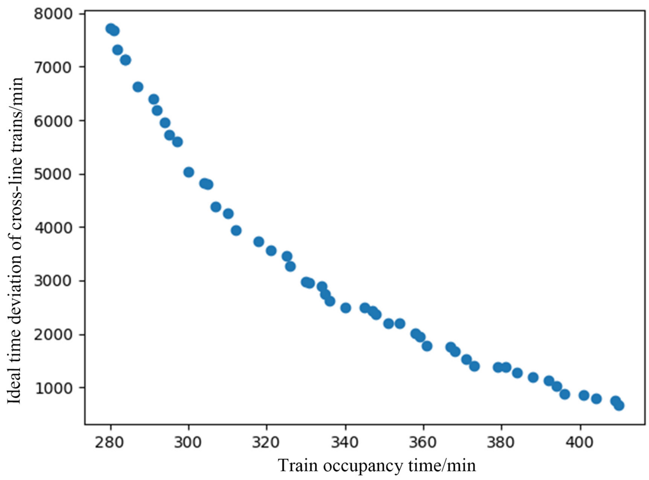


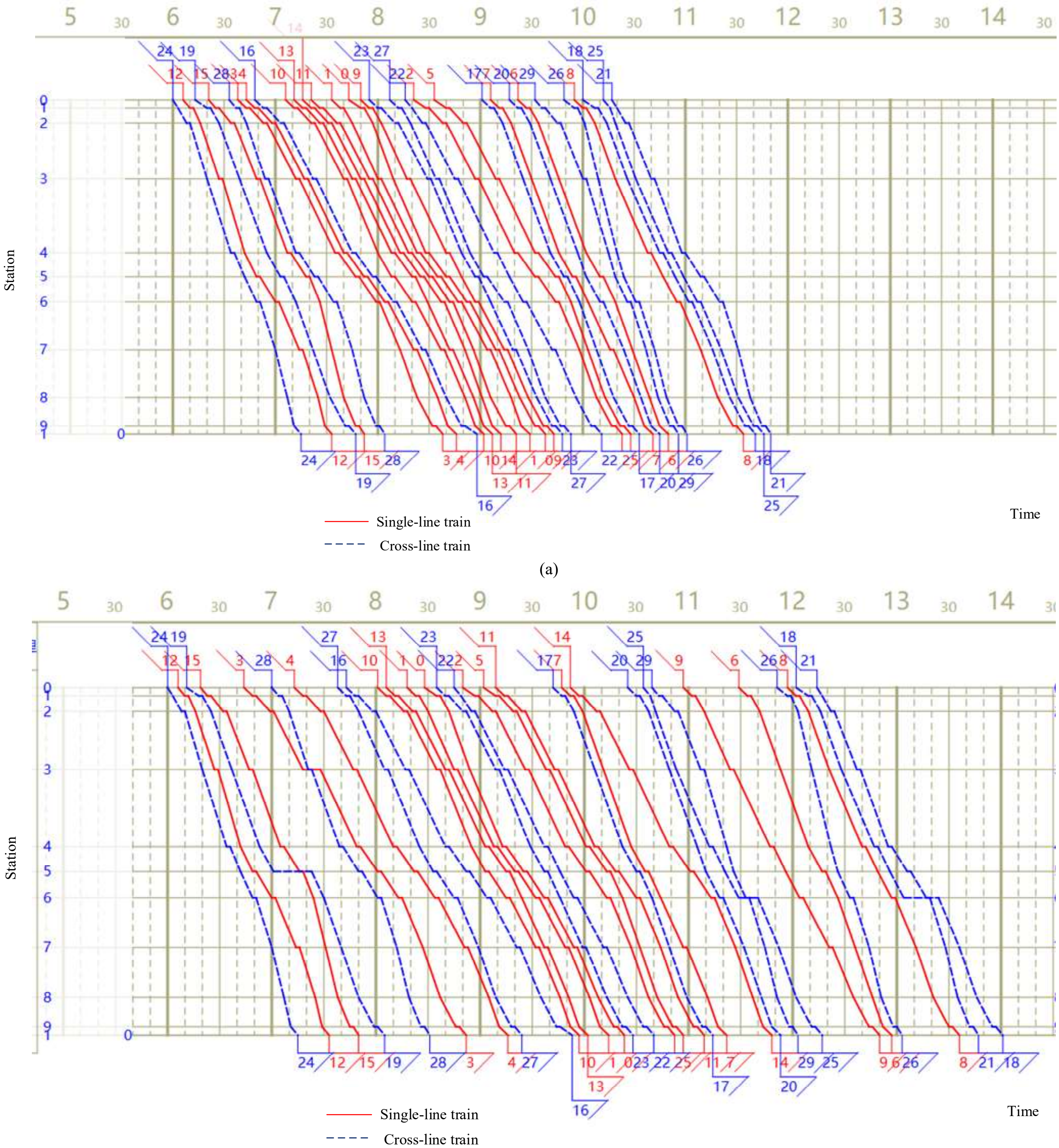
Fig.10. Pareto front

Based on the results in Table Ⅲ and Fig. 10, the following views can be observed:

The minimum train occupancy time of Solution 1 is 280 minutes. The minimum deviation from the ideal time of cross-line trains in Solution 2 is 410 minutes.

The algorithm effectively solves the model presented in the paper, as demonstrated by all solutions converging to the Pareto front. All solutions converge to the Pareto front, demonstrating the algorithm's ability to solve the model presented in the paper effectively—a more minor train occupancy time results in a greater carrying capacity. Therefore, Fig. 10 also reflects the relationship between railway capacity and cross-line trains.

Fig. 11 illustrates the train diagrams corresponding to Solution 1 and Solution 2, with red representing local trains and blue representing cross-line trains.



(a) (b)

Fig.11. Comparison of diagram. (a) Solution 1 diagram; (b) Solution 2 diagram.

Fig. 12 compares the train travel times for different solutions. Fig. 13 illustrates the ideal time deviation for each cross-line train for Solution 1 and Solution 2.

Some of the conditions were changed to analyze the train capacity. Fig. 14 demonstrates the effect of the proportion of high-class and low-class trains on train occupancy time. Table 4 illustrates the impact of different speed differentials on train occupancy time.

The total number of Class-A trains and Class-B trains is 30. Fig. 14 shows that, under different train mixing situations, when the number of Class-A trains increases from 0 to 30, the train occupancy time increases and then decreases, and when the number of Class-A trains is 15, i.e., Class-A and Class-B each occupy 50 percent of the total number, the train occupancy time is the largest, which is due to severe train mixing disturbances at this time. A single type of train occupies less time. Therefore, the proportion of high-class and low-class trains should be reasonably adjusted to use the passing capacity fully. As can be seen from Table Ⅳ, the speed difference between different types of trains also impacts the passing capacity, the speed difference is reduced by 66.7%, the train occupancy time is reduced by 20.3% accordingly. It can be seen that when trains of different speed types are running, priority should be given to the train operation mode with more minor speed difference.

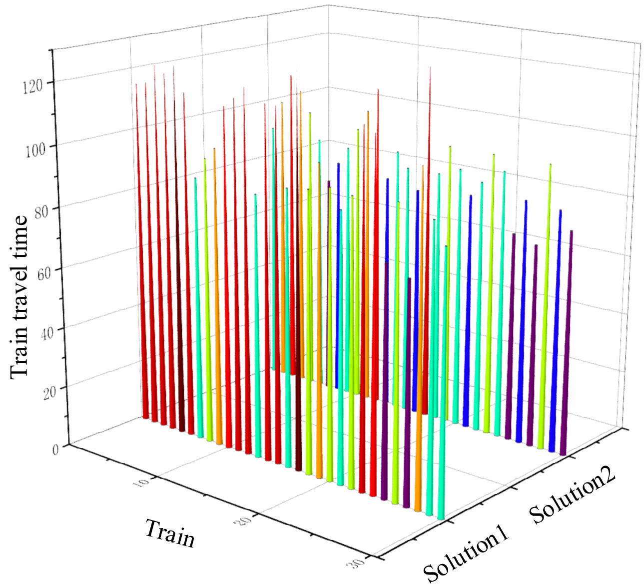


Fig.12. Comparison of train travel time.

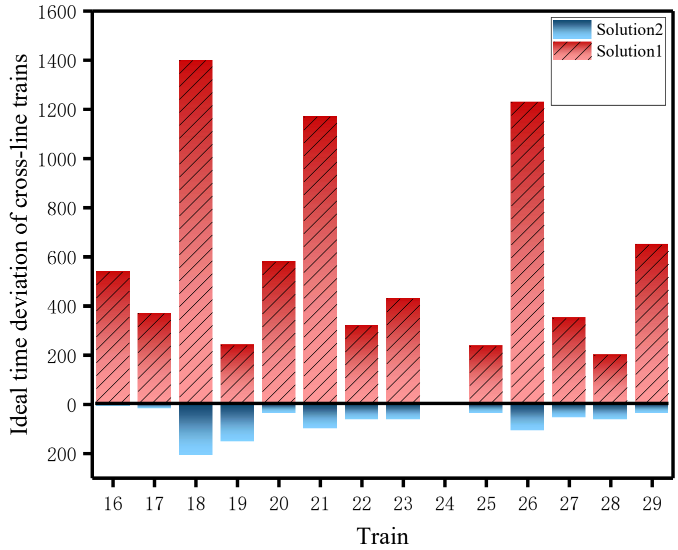


Fig.13. Comparison of Ideal Time Deviation of Cross-line Trains

The results of solving the single objective, respectively, using the Gurobi solver and the NSGA-II algorithm, are shown in Table Ⅴ. The results illustrate that a heuristic algorithm can obtain a feasible solution more quickly when the objective is nonlinear. In addition, compared with the ideal moment deviation of cross-line trains without considering the ideal moment deviation, the ideal moment deviation derived by the heuristic algorithm is reduced by 93.2%; the train occupancy time GAP obtained by the heuristic algorithm is 3.58%, which is more accurate, and the difference between calculating the ideal moment deviation of the cross-line trains and that of the Gurobi's solution is larger, which is since the heuristic algorithm generates the time of train stops by the minimum stopping time, while the moments in the ideal timetable are not necessarily generated according to the minimum time. Thus, there is always a deviation between the generated results and the ideal time, but the deviation is within acceptable limits.

The results indicate that the algorithm can solve the model and optimize train timetables to improve railway capacity utilization by reducing train occupancy times while maintaining smooth operations. This demonstrates the algorithm's effectiveness in solving the proposed model.

Overtaking also increases for some trains under specific solutions. Fig. 12 shows that the train travel times of Solution 1 and Solution 2 are the same, except some trains are overtaken, resulting in increased stop times. For example, in the Solution 2 diagram, train 18 is overtaken by train 21 at station 6.

The method proposed in this paper can also be used to analyze the structure of the train operation diagram. As shown in Fig. 11(a), the centralized laying of trains in one time period can effectively reduce the time occupied by trains and improve the passing capacity, e.g., centralized laying of Train 14, Train 3, and Train 2; however, in the actual operation of high-speed railways, it is impossible to run only a large number of trains for a long period, and no trains are operated in the rest of the time, which does not conform to the traveling pattern of passengers. Capacity that does not take into account the passenger transport demand is not an effective passing capacity. Therefore, combined with the passenger transport demand, the relatively concentrated laying method is more suitable for the current stage of the Chinese high-speed railway.

In addition, the mixing operation of trains at different speeds also impacts passing capacity. In Fig. 11(b), to fully consider the degree of deviation of the cross-line trains from their ideal time, the cross-line trains have more contact with the single-line trains, which causes a large number of high-speed trains to mix with the low-speed trains, resulting in an increase of 46.4 percent in the occupancy time of the trains, which significantly reduces the passing capacity.

The result demonstrates the trade-off between optimization targets: minimizing train occupancy time increases cross-line train time deviation while minimizing deviation reduces flexibility and increases occupancy time. Pareto optimality is achieved as no solution dominates across all targets. Overtaking also increases for some trains under specific solutions.

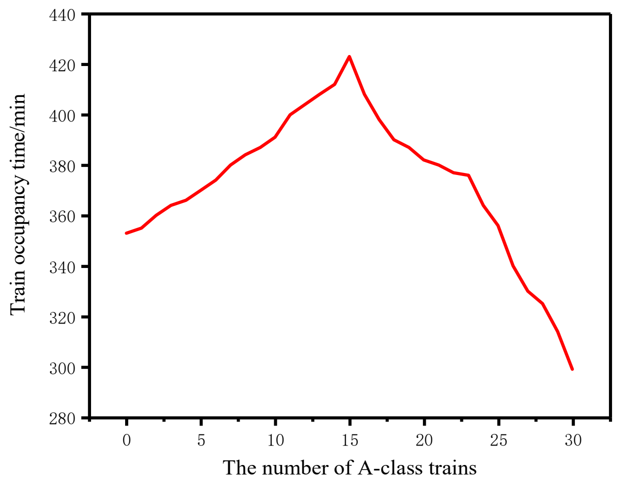


Fig.14. The number of A-class trains and their corresponding train occupancy time.

TABLE IV
EACH SPEED MATCHING SCHEME TAKES UP THE TIME OF RAILWAY DIAGRAM

Train speed differentials /km/h	*Train occupancy times/min*
50	*361*
100	*400*
150	*453*

TABLE V
OBJECTIVE VALUES OF THE PARETO SOLUTION SET

Method	*The calculation time of the first feasible solution/s*	*Train occupancy times/min*	*Ideal time deviation of cross-line trains/min*	*The final result calculation time*
Gurobi(aim1)	*10*	*450*	*0*	*300s*
Gurobi(aim2)	*360*	*270*	*9750*	*3600s*
NSGA- II	*3*	*280*	*658*	*1800s*

VI. CONCLUSIONS

(1) This paper presents a novel approach to high-speed railway capacity calculation by leveraging the concept of compressing train operation diagrams. Various structural diagrams are randomly generated, and a genetic algorithm framework subsequently is used to generate a train timetable without buffer time. An ideal arrival time is established for cross-line trains, and the impact of cross-line trains is comprehensively considered by analyzing the discrepancy between the ideal and actual arrival times.

(2) Through the mixed integer programming model and numerical experiment, we find that when the minimum train occupancy time is solely pursued, the train arrival interval is fully compressed, redundancy between trains is reduced, and the deviation between cross-line train time and ideal time will be more significant. When the ideal time deviation of cross-line trains is minimized, the cross-line train time is relatively fixed and has a small adjustable range. However, other train times on the line are more flexible, and redundant time between trains increases, increasing train occupancy time. Improving all targets simultaneously is not possible.

(3) The designed NSGA-II algorithm is used to solve the model and the train departure sequence is considered as the solution of the model. The NSGA-II algorithm generates a high-quality set of Pareto frontier solutions, which provides the decision maker with a variety of decision options.

(4) The method proposed in this paper provides an effective method to evaluate the capacity of high-speed railways by quantitatively considering cross-line train operations, which can be used to analyze the structure of the train operation diagram, as well as the influence of the speed difference and the way of laying diagram on the train passing capacity.We found that when two-speed types of trains are running on the line, increasing the number of high-level trains, the capacity decreases first and then increases. When the high- and low-level trains ratio is 1 1, the train heterogeneity is severe, and the capacity is the smallest. The capacity is smaller when the speed difference between the two trains is more enormous.

In the next stage, the research will take train operation planning integration as the goal, and deeply study the high-speed rail capacity issues, to strengthen the understanding and support for high-speed rail capacity improvement.

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