Train Service Design for Cross-line Operation during Peak Passenger Flow Period

Kewei Zhao, Changfeng Zhu, Xuelin Li, Shixiang Wan, Boyu Li, Junyi Zhao, and Yu Wang

***Abstract*—To solve the problem of passenger travel cost and full load rate balance before and after the implementation of cross-line operation in the peak period of urban rail transit (URT) passenger flow. This paper introduces the concept of section full load rate balance. According to the division of different types of passenger flow in the intersection mode of "X" and "Y" lines, based on the characteristics of passenger flow in peak passenger flow period, a mathematical model with cross-line operation mode turn-back station and operation frequency of train as decision variables is constructed to minimize passenger travel cost, full load rate balance objective function and enterprise operation cost. Taking the Changping Line and Line 8 in the Beijing URT system as examples, the validity of the model is verified. The results show that the operation mode of the cross-line train can alleviate the passenger flow pressure of the transfer station, reduce the overload of the original train on the line, improve the full load rate balance of the train, make the utilization of the transport capacity more balanced, and optimize the passenger ride experience. In addition, considering the operation of cross-line trains may increase the operating costs of enterprises.**

***Index Terms*—Urban rail transit, Cross-line operation, "X/Y" type routing, Peak passenger flow period, Full load rate balance**

I. Introduction

WITH the continuous development of urban rail transit (URT), the accessibility of the network has increased significantly, and the travel needs of passengers have diversified. The commuter flow during peak passenger flow period brings great challenges to the organization and operation of the rail transit system. It is necessary to design and formulate a flexible and feasible train operation plan so that the capacity resources can be reasonably allocated to direct passengers and transit passengers. Most of the existing studies focus on single-line operations, ignoring the travel demand of transit passengers. The cross-line operation mode has been proven to improve the transportation efficiency of transit passengers.

The rail transit system has expanded from the original single-line operation to the network operation, which has increased passenger accessibility. Due to the single movement direction of commuting passenger flow during the peak passenger flow period, the number of passengers in opposite directions is not balanced under the same section. At the same time, the travel distance of commuting passengers is long, and the transfer, it has brought great trouble to the operation of URT.

Existing research has confirmed that cross-line operation is an effective way to alleviate the transfer pressure and improve the service quality of transfer stations. Cross-line operation converts multiple types of transfer passengers into direct passengers, which may increase the waiting time of original direct passengers and others who still need to transfer after crossing the line. However, it is not advisable to improve the efficiency of system transportation by sacrificing the interests of some passengers. It is necessary to consider the interests of various passengers under cross-line conditions.

Through the operation of cross-line operation trains, the unbalanced spatial distribution of passengers is improved, and the unbalanced time distribution of passenger flow is alleviated by adjusting the operation routes of cross-line trains in different periods. Taking the turn-back station of cross-line trains and the operation frequency of different types of trains as decision variables, the mathematical model of cross-line operation during the peak period of passenger flow is constructed with the goal of passenger travel cost, full load rate balance, and enterprise operation cost.

II. Literature review

A. Cross-line operation

With the rapid construction of URT, problems have been such as a mismatch between transportation capacity and passenger flow demand and low passenger service level. It has become a research hotspot to optimize through network operation organization technology. Yang [1] constructed a model considering the influence of cross-line trains on line capacity. And also put forward the necessary conditions for

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implementing cross-line operations. Some scholars have combined cross-line with multi-group. The upper layer of [2] takes passenger time and enterprise cost as the optimization objectives, and the lower layer takes the balance of different train full-load rates as the optimization objective and constructs a bi-level programming model. Yang [3] and Zhu [4] proposed a passenger flow assignment model for cross-line operation and considered constraints such as transfer station capacity and train full load rate. Duan [5] section combines cross-line operation with a skip-stop strategy, considers parameters such as the number of people on and off the train to describe the train stop time, and takes the total cost of the enterprise and the passenger travel time as the optimization target to construct the research model. ITO [6], [7] studied the coordination of both parties involved in the cross-line operation of different standard lines, analyzed the conditions and feasibility of implementing cross-line operations, and provided suggestions for the implementation of cross-line operations. In terms of practical application, the more developed metropolitan areas have explored and practiced the cross-line operation mode, such as Dongwu East Line-Vice Capital Line in Tokyo [8] and Line 4-Daxing Line in Beijing, etc.

The realization of cross-line operation requires the support of certain technical conditions, including train types and related infrastructure. Based on the case of Sunderland Metro and Light Rail in the UK, [9] found that cross-line operation can increase the attraction of rail transit system, and the technical difficulties of cross-line operation in power supply, signal, and trains' models are analyzed. This paper will study cross-line operation from the perspective of operation management under the technical conditions of cross-line operation.

B. Full load rate balance

The balance of the train's full load rate is an important factor affecting the service level of passengers [10]. Ceder and Philibert [11] studied the timetable optimization model aiming at the balance of the full load rate of each train under the condition of passenger flow change. Li [12] added the too-high or too-low train full load rate as a penalty to the objective function of the train operation plan optimization.

The operation organization technology of multi-group was originally based on the multi-type operation plan of conventional buses [13].

It is also to solve the imbalance of spatial and temporal distribution of passenger flow and reduce the mismatch between transportation capacity and passenger flow demand. Rong [14] explored the influence of different train formations and departure intervals of different train formations on the balance of full load rate and found that the unbalanced departure of multi-group trains can improve the space-time balance of train full load rate. Considering multi-route and multi-group, [15] established an optimization model of the train routing plan, and found that the plan can effectively reduce the average full load rate of trains.

C. Research gap

Previous studies have the following characteristics:

(1) Only the passenger flow in a certain operating period is selected as the research object, and the influence of the direction of commuting passenger flow in different periods on rail transit is not considered.

(2) Although the existing literature has fully studied the entire track system network, the train is only operated on the single-line and does not consider cross-line trains.

(3) The existing models are based on the interval as the unit to construct the target of the train full load rate, which is only applicable to the single-line operation mode, and cannot effectively measure the train full load rate under the cross-line operation mode.

D. Major contributions

A general model of the cross-line operation of URT is constructed. Combined with the "X" and "Y" type routes in the rail transit system, the passengers on the line are divided into 2 categories and 8 subcategories. All direct and transfer passengers affected by cross-line operations are involved. A passenger travel model under cross-line operation is constructed by analyzing the passenger flow characteristics of different passengers.

In the general model, the concept of effective travel time and invalid travel time is introduced to describe the passenger travel cost. At the same time, the balance of the full load rate is considered. According to the size and flow direction of passenger flow, the operation period of the subway is divided into three parts: early peak period, flat peak period, and late peak period. According to the characteristics of passenger flow in different periods, different cross-line trains are operated; the full load rate of each section is more balanced by running cross-line trains with different routes. The operation of cross-line trains will better match the passenger flow demand; the frequency of cross-line and local trains and the selection of cross-line train turn-back stations are studied.

Finally, taking the passenger flow data of a working day of CP Line and Line 8 in the Beijing URT system as examples, the validity of the model is verified, and the influence of factors such as cross-line train turn-back station and operation frequency on travel cost and operation cost is investigated. The variation law of full load rate balance before and after optimization of the cross-line section is discussed.

III. PROBLEM DESCRIPTION

A. Routing Settings

The schematic diagram of cross-line operation on the "X/Y"-type operation of cross-line, as shown in Fig. 1. The two lines can be named Line 1 and Line 2 respectively, and some sections of Line 2 can be used as potential sections for cross-line operation. The starting and ending stations of Line 1 are L1{1} and L1{n'}.

The starting and ending stations of Line 2 are L2{1} and L2{n}, respectively. The intersection of Line 1 and Line 2 is the cross-line station L1{C} / L2{C}, and the cross-line train turn-back station is L1{C'}.

The two lines are divided into five sections, and the stations included in Z_{I} are from L1{1} to L1{C-1}, Z_{II}= L1{C}, …, L1{C'}, Z_{III}= L1{C'+1}, …, L1{n'}, Z_{IV}= L2{1}, …, L2{C}, Z_{V}= L2{C+1}, …, L2{n}.

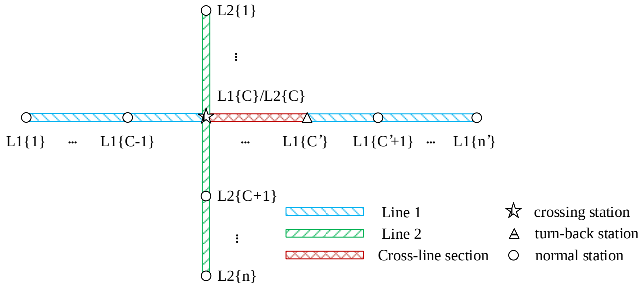

Fig. 1. "X/Y"-type cross-line operation

B. Definition of train type

Train 1 is the local train running on Line 1, Train 2 is the cross-line train running on the Z_{II} and Z_{IV} sections, and Train 3 is the local train running on Line 2. The driving sections of Train 1, Train 2, and Train 3 are Z_1, Z_2 and Z_3 respectively. Especially, for Train 3, when $\beta=0$, $Z_3=Z_{IV}$, that is "Y"-type; when $\beta=1$, $Z_3=Z_{IV}+Z_V$. As shown in Fig. 2.

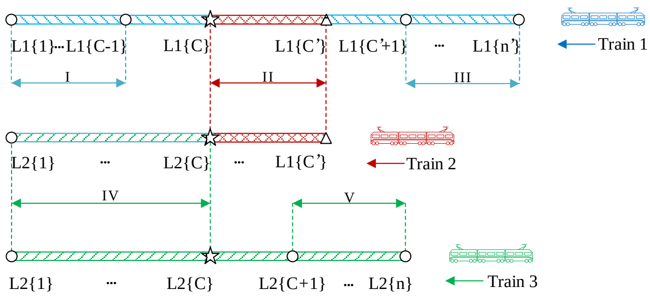

Fig. 2. The driving section of train 1, train 2 and train 3

IV. Model Construction

A. Objective function

A.1. Passenger travel cost

The effective time of passengers is the travel time of passengers. The farther the distance between the two places is, the longer the effective time of passengers is. The invalid time (*IT*) of passengers is the walking and waiting time of passengers in the urban rail system.

Passengers hope that the shorter the invalid time, the better. The travel cost is defined as the sum of the invalid time of all kinds of passengers.

The calculation methods of various passenger travel costs are introduced respectively. For each kind of passenger, the travel cost is *IT* cost. In this study, the passenger arrival obeys the uniform distribution [16], so the passenger *IT* is set to half of the train departure interval, and the train departure interval can be expressed by the operation frequency f_m.

Therefore, the *IT* of different kinds of passengers is described by the train operation frequency. It is worth mentioning that when two types of trains (such as Train 1 and Train 2) are running on the same track, the trains will interact with each other, so the passenger *IT* is calculated as $30/(f_1+f_2)$ [17].

TABLE I
RIDE CHOICE OF DIFFERENT OD PASSENGER FLOW UNDER CROSS-LINE OPERATION

Direct / Transit passengers	Quantity	OD distribution	Available train sorts
Direct passengers (Dp)	Dp1 ($\beta=0$)	OD is within Line 1, no O or D involving the station in Z_{II}.	Train 1
	Dp2 ($\beta=0$)	Z_{IV}	Train 2 or Train 3
	Dp3 ($\beta=0$)	Z_{II}	Train 1 or Train 2
	Dp4 ($\beta=0$)	Z_{II} and Z_{IV}	Train 2
	Dp5 ($\beta=1$)	OD is within Line 2, no O or D involving the station in Z_{IV}.	Train 2
Transit passengers (Tp)	Tp1 ($\beta=0$)	Z_I, Z_{III}, Z_{IV}	$\begin{cases}\text{Train 1}, n\in(\text{I,III})\\ \text{Train 2}, n=\text{IV}\\ \text{Train 3}, n=\text{IV}\end{cases}$
	Tp2 ($\beta=1$)	Z_I, Z_{III}, Z_V	$\begin{cases}\text{Train 1}, n\in(\text{I,III})\\ \text{Train 2}, n\in(\text{V})\end{cases}$
	Tp3 ($\beta=1$)	Z_{II}, Z_V	$\begin{cases}\text{Train 1}, n=\text{II}\\ \text{Train 2}, n=\text{II}\\ \text{Train 3}, n=\text{V}\end{cases}$

Table I shows the ride choice of different OD passenger flow under the cross-line operation.

Kind Dp1:

$$IT^{Dp1}=\left(\sum_{i,j\in Z_1}q_{ij}-\sum_{i,j\in Z_{II}}q_{ij}\right)\cdot\frac{30}{f_1},\forall i\neq j \tag{1}$$

Kind Dp2:

$$IT^{Dp2}=\sum_{i,j\in Z_{IV}}q_{ij}\cdot\frac{30}{f_2+f_3},\forall i\neq j \tag{2}$$

Kind Dp3:

$$IT^{Dp3}=\sum_{i,j\in Z_{II}}q_{ij}\cdot\frac{30}{f_1+f_2},\forall i\neq j \tag{3}$$

Kind Dp4:

$$IT^{Dp4}=\frac{30}{f_2}\left[\sum_{i,j\in Z_2}q_{ij}-\sum_{i,j\in Z_{II}\cup Z_{IV}}\left(q_{Ci}+q_{iC}\right)\right],\forall i\neq j \tag{4}$$

Kind Dp5:

$$IT^{Dp5}=\left(\sum_{i,j\in Z_3}q_{ij}-\sum_{i,j\in Z_{IV}}q_{ij}\right)\cdot\frac{30}{f_3},\forall i\neq j \tag{5}$$

where t_{walk} is Walking time for passenger transfer, s is the unit time cost of passengers.

$$K_1\in(\text{Dp1, Dp2, Dp3, Dp4}) \tag{6}$$

$$K_2\in(\text{Tp1}) \tag{7}$$

$$K_3\in(\text{Dp5, Tp2, Tp3}) \tag{8}$$

Kind Tp1:

$$IT^{\mathrm{Tp1}}=\left[\sum_{i\in Z_{\mathrm{I}}\cup Z_{\mathrm{III}}}\sum_{j\in Z_{\mathrm{IV}}}q_{ij}+\sum_{i\in Z_{\mathrm{IV}}}\sum_{j\in Z_{\mathrm{I}}\cup Z_{\mathrm{III}}}q_{ij}-\sum_{i\in Z_{\mathrm{I}}\cup Z_{\mathrm{III}}}\left(q_{\mathrm{C}i}+q_{i\mathrm{C}}\right)\right]\cdot\left(\frac{30}{f_1}+\frac{30}{f_2+f_3}+t_{walk}\right) \quad (9)$$

Kind Tp2:

$$IT^{\mathrm{Tp2}}=\left(\sum_{i\in(Z_{\mathrm{I}}\cup Z_{\mathrm{III}})}\sum_{j\in Z_{\mathrm{V}}}q_{ij}+\sum_{i\in Z_{\mathrm{V}}}\sum_{j\in(Z_{\mathrm{I}}\cup Z_{\mathrm{III}})}q_{ij}\right)\cdot\left(\frac{30}{f_1}+\frac{30}{f_3}+t_{walk}\right) \quad (10)$$

Kind Tp3:

$$IT^{\mathrm{Tp3}}=\left[\sum_{i\in Z_{\mathrm{II}}}\sum_{j\in Z_{\mathrm{V}}}q_{ij}+\sum_{i\in Z_{\mathrm{V}}}\sum_{j\in Z_{\mathrm{II}}}q_{ij}-\sum_{i\in Z_{\mathrm{V}}}\left(q_{\mathrm{C}i}+q_{i\mathrm{C}}\right)\right]\cdot\left(\frac{30}{f_3}+\frac{30}{f_1+f_2}+t_{walk}\right) \quad (11)$$

$$Z=s\cdot\left(\sum_{k\in K_1\cup K_2}IT^k+\beta\cdot\sum_{k\in K_3}IT^k\right),\forall k\in K \quad (12)$$

The passenger travel cost in this study is the *IT*.

A.2. Full load rate balance

Considering that the operation of cross-line trains on URT will lead to an imbalance in the full load rate of local and cross-line trains to a certain extent. Under cross-line operation, the full load rate balance and service level of the train should be ensured as much as possible. The full load rate balance in each section is measured by the square sum of the difference between the full load rates of two adjacent trains.

$$Ind^{z}_{equalization}=\left(Q^{z}_{change}/\mathrm{Cap}\right)^2,\forall z\in Z \quad (13)$$

where Q^{z}_{change} is the difference of passenger capacity between two adjacent trains passing through z station in each section.

The research [10] shows that when the calculated value of the full load rate balance of the section is less than 0.5, the full load rate is more balanced.

The calculation method of the objective function value of the full load rate balance is as follows:

$$f_{equalization}=\frac{\sum_{z\in Z_2}Ind^{z}_{equalization}}{D} \quad (14)$$

where *D* is the number of sections that the cross-line train passes through.

A.3. Enterprise operation cost

According to the operation experience of URT in Beijing and other cities, 10 % -15 % of the backup trains are generally reserved.

Therefore, the actual number of vehicles in this study is 1.3 times that of the number of vehicles used, that is, μ =1.3. The sum of train use cost and train operating cost is the operating cost.

$$C=\mu\cdot\tau\cdot\sum_{m=1}^{3}\left[\frac{f_m\cdot T_m}{60}\right]+\sum_{m=1}^{3}\psi\cdot L_m\cdot f_m \quad (15)$$

where T_m is the whole operation cycle of the train, including dwell time, section running time, and return time, τ represents the depreciation cost per unit time of the train, ψ is the cost of train operation per kilometer, and indicates the train running distance.

B. Constraint conditions

B.1. Constraints related to through capacity

Due to the increase of cross-line trains, the operation frequency of local trains on this line will be affected, that is f_1, f_3; in the cross-line section, when the cross-line trains have the same return station as the original route, different types of train operation can be regarded as alternating operation [18].Therefore, the operation frequency should satisfy the formula.

$$f_m\le\left(\left\lfloor\frac{60}{f_2\cdot t_2^{\min}}\right\rfloor-1\right)\cdot\left\lfloor\frac{f_2\cdot T_2}{60}\right\rfloor\cdot\frac{60}{T_2} \quad (16)$$

$$m\in\begin{cases}\{1\},1[\mathrm{C'}]\ne 1[\mathrm{n'}]\\ \{1,2\},1[\mathrm{C'}]=1[\mathrm{n'}]\end{cases} \quad (17)$$

where $t_l^{\min}$ represents the minimum headway for Line *l*, *l*=1, 2; The symbol $\lfloor\ \rfloor$ represents rounded down.

The operation frequency of each section of the cross-line operation should meet the constraint of the maximum operation frequency. Due to the limitation of conditions, it generally does not exceed 30 trains/hour [14].

$$\begin{cases}f_1+f_2\le f_{\max}\\ f_2+f_3\le f_{\max}\end{cases} \quad (18)$$

B.2. Constraints related to turn-back station

Since only one site is selected in Zone II as a turn-back station for cross-line operations, the formula is expressed as follows:

$$\sum_{i\in Z_{\mathrm{II}}}x_{1[i]}=1 \quad (19)$$

B.3. Cross-line train turn-back capacity constraint

Due to technical constraints, train retrace operation requires a certain time, so the number of retraces of cross-line train in unit time should be greater than the train operation frequency.

$$f_2\le\frac{60}{t^{retrace}} \quad (20)$$

where $t^{retrace}$ indicates the time required for the Train 2 turn-back operation.

B.4. Full load rate constraint

When the operating frequency of urban rail transit is higher, its transportation capacity is greater, and the number of passengers transported is more, but it is also accompanied by an increase in operating costs [17]. The Full load rate constraint can balance the relationship between passenger service level and operating costs [19]. The relevant constraints are as follows:

$$\begin{cases} \eta_{\min} \le \dfrac{Q_n}{\text{Cap}\cdot f_1} \le \eta_{\max}, n \in \{\text{I,III}\} \\ \eta_{\min} \le \dfrac{Q_n}{\text{Cap}\cdot (f_1+f_2)} \le \eta_{\max}, n = \text{II} \\ \eta_{\min} \le \dfrac{Q_n}{\text{Cap}\cdot (f_2+f_3)} \le \eta_{\max}, n = \text{IV} \\ \eta_{\min} \le \dfrac{Q_n}{\text{Cap}\cdot f_3} \le \eta_{\max}, n = \text{V} \end{cases} \tag{21}$$

The full load rate of the cross-line train needs to meet the following formula:

$$\eta_{\min} \le \frac{\sum_{i,j\in \text{Z}_2} q_{ij} - \sum_{i\in \text{Z}_{\text{II}}\cup \text{Z}_{\text{IV}}} (q_{\text{C}i} + q_{i\text{C}})}{\text{Cap}\cdot f_2} \le \eta_{\max} \tag{22}$$

where Q_n is the maximum passenger demand of the adjacent stations in Area n during the study period. Cap is the passenger capacity of the train, that is, the maximum number of passengers that each train can accommodate.

B.5. Train number constraint

Since the train resources in the rail transit system are limited, the total number of trains used on each line should not exceed the total number of available trains provided by the operator. The number of trains should meet the following.

$$\sum_{m=1}^{3} \left\lceil \frac{f_m \cdot T_m}{60} \right\rceil \le \sum_{l=1}^{2} nu_l \tag{23}$$

where nu_l is the number of trains running on Line *l*. $l \in (1,2)$.

B.6. Non-negative constraint of operation frequency

$$f_1, f_2, f_3 \in Z^+ \tag{24}$$

In summary, the mathematical model of full load rate balance based on an "X/Y"-type route can be expressed as:

$$\begin{cases} \min Z, f_{equalization}, C \\ s.t. \end{cases} \tag{25}$$

C. Algorithm design

In the URT system, the train stop stations are limited, so the stations that can be used as cross-line train turn-back stations are limited. The specific steps are as follows:

(1) Input the initial parameters.
(2) Determine the type route, let $i = 0$.
(3) Use the Floyd algorithm to search for the shortest path.
(4) All-or-none principle is used to allocate passenger flow.
(5) Determine the cross-line train turn-back station.
(6) Allocate the passenger flow of the new scheme.
(7) Use the solver to calculate the objective function.

This method is used to traverse all stations to determine the optimal turn-back station and operation frequency.

V. Case Study

In this study, some sections of CP Line and Line 8 in the Beijing URT system are selected as cases. The research section of Line 8 is between ZXZ and NLG, and the research section of the CP line is between CPX and XEQ. By the end of 2023, the section of Line 8 will contain six transfer stations, and the section of CP line will contain two transfer stations.

The number of each station is shown in Table II and Table III . The section between 1 station and 2 station is set as <1>, the section between 2 station and 3 station is set as <2>. The section between 10 station and 13 station is set as <12>, the section between 13 station and 14 station is set as <13>, and so on.

A. Peak passenger flow characteristics

Based on the data of more than 2.9 million swiping cards on a working day of Beijing Metro in December 2020, three time periods of 7:00-8:00, 3:00-4:00, and 5:00-6:00 on Line 8-CP line were selected as the research objects (morning peak period, flat peak period and evening peak period).

After data processing, the passenger flow OD data in the morning and evening peak periods are obtained, and the passenger flow OD heat map shown in Fig. 3 and Fig. 4 are drawn. It can be found that the commuter passenger flow has obvious directionality and has a greater number of passengers at the transfer stations.

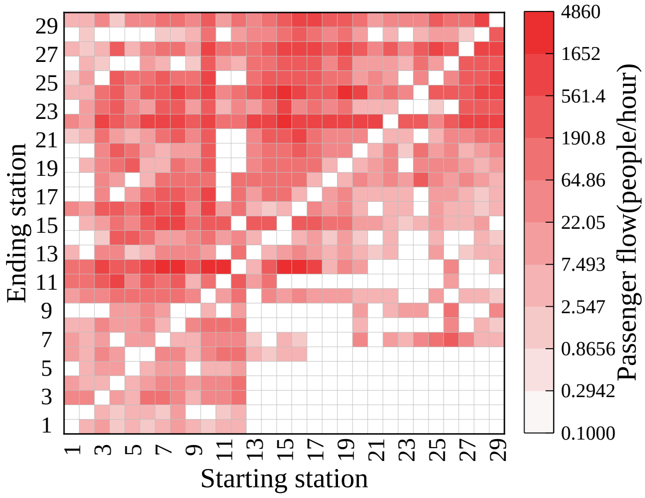

Fig. 3. Passenger flow OD of CP Line-Line 8 in the morning peak period

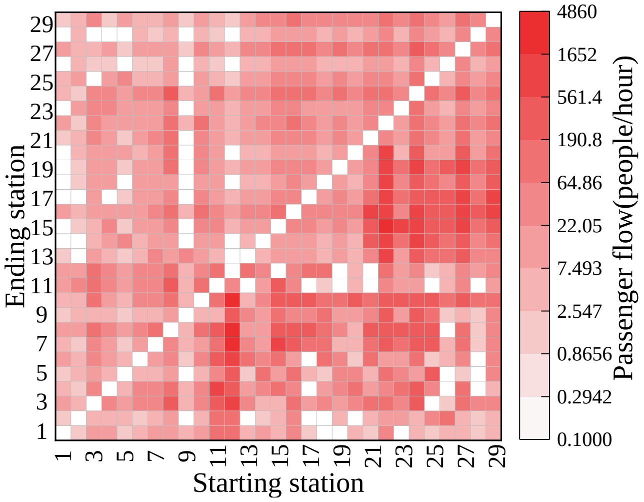

Fig. 4. Passenger flow OD of CP Line-Line 8 in the evening peak period

In Fig. 5, the passenger flow in the upward direction (CPX to NLG direction) during the morning peak period is much larger than that in the downward direction (NLG to CPX direction).

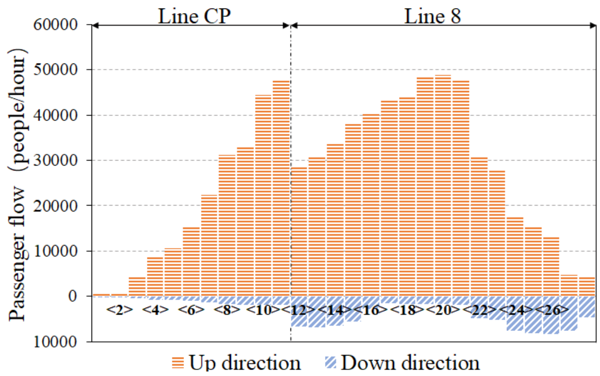

Fig. 5. The passenger flow of each section of morning peak period

In Fig. 6, the passenger flow in the upward and downward directions is relatively balanced, and the change trend of passenger flow presents a spindle shape.

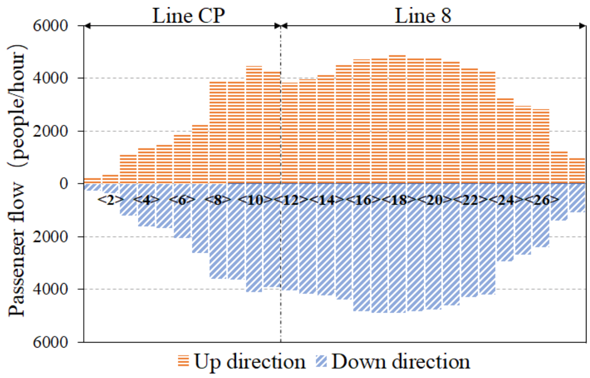

Fig. 6. The passenger flow of each section of flat peak period

In Fig. 7, the distribution of passenger flow in the evening peak period is opposite to that in the morning peak period. The commuting passenger flow diverges outward from the central area of the city. The passenger flow in the direction of leaving the city is significantly larger than that in the direction of entering the city, and the tidal characteristics of the line are significant.

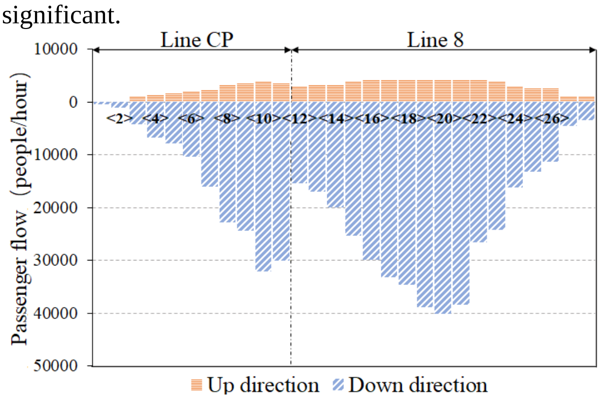

Fig. 7. The passenger flow of each section of evening peak period

Due to the large difference in passenger flow demand in different directions of tidal lines, and the direction with large passenger travel demand has higher transfer demand, the corresponding cross-line trains can be considered, that is, based on the passenger flow demand of each time period, the corresponding cross-line operation scheme is formulated to change the supply of transport capacity in different periods, relieve the operating pressure of the tight section, reduce the waiting time of passengers, and improve the service level of the line.

B. Cross-track scheme

The Floyd algorithm is used to iteratively obtain the shortest path between the two stations. Through the passenger flow distribution of all or none principle on the shortest path, obtain the proportion of transfer passenger flow in all directions during peak periods.

According to the statistics of passenger flow, the passenger flow in each transfer direction of CP line and Line 8 during the morning peak period is shown in Fig. 8. The number of upward and downward transfers in the two transfer directions of CPX and NLG accounts for 72.23 % of the total transfer number (70.05 % and 2.18 %), so the migration direction between CPX and NLG is the main migration direction. The passenger transfer from CPX to NLG accounts for 70.05 % of the total, so CPX is selected as the departure station of the cross-line train. According to the model, the study section is divided into II, III, IV, and V. The IV and V are fixed, and the II and III change according to the location of the cross-line train turn-back station. The route diagram of the cross-line train operation scheme in the morning peak period is as follows.

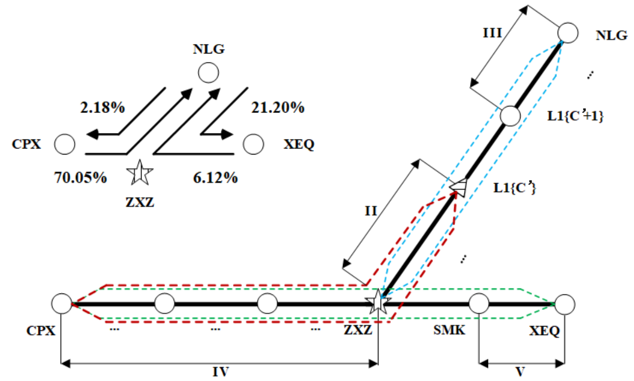

Fig. 8. Cross-line operation scheme routing in the morning peak period

According to the statistics of passenger flow, the passenger flow in each transfer direction of CP line and Line 8 during the evening peak period is shown in Fig. 9. The number of upward and downward transfers in the two transfer directions of CPX and NLGX accounts for 82.92 % of the total transfer number (55.12 % and 27.80 %), so the migration direction between CPX and NLGX is the main migration direction. The passenger transfer from NLGX to CPX accounts for 55.12 % of the total, so NLG is selected as the departure station of the cross-line train. According to the model, the study section is divided into I, II, III, and IV. The I and IV are fixed, and the II and III change according to the location of the cross-line

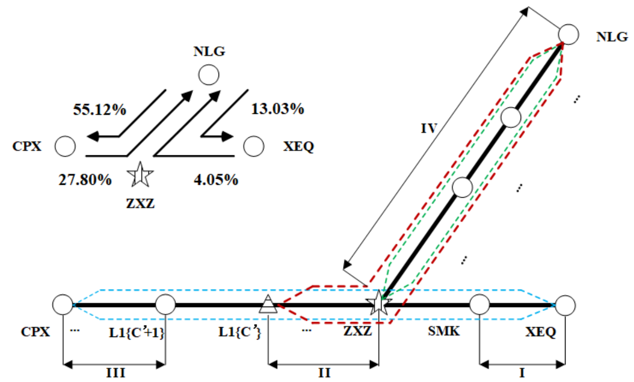

Fig. 9. Cross-line operation scheme routing in the evening peak period

train turn-back station.

The route diagram of the cross-line train operation scheme in the evening peak period is as follows:

s = 4RMB / min , τ = 4RMB / min , ψ =105RMB/km, cap = 1460person, $\eta_{\min} / \eta_{\max} = 0.1/1.3$, $nu_1 / nu_2 = 20/20$, $t_{walk} = 6$ min.

C. Optimal results

The overall objectives of different return stations are calculated and standardized to the objective function value (passenger travel cost, full load rate balance, enterprise operation cost). When the return station is ZXZ station, the frequency of Train 2 is 0; therefore, this situation can be regarded as the original scheme, that is non-cross-line operation.

Table II and Table III show the results of various indicators during the morning and evening peak periods, showing the operation frequency and target values of different trains when the cross-line trains choose different stations on Line 8 as turn-back stations. Table II and Table III are shown at the end of this page.

D. Analysis of optimization results

In the morning and evening peak hours, the different train departure frequencies and costs corresponding to different stations are selected as shown in Fig. 10 and Fig. 11.

Operating cross-line trains in the morning and evening peak periods will reduce the transport capacity of the local trains on the line.

The specific performance is that the operation frequency of the local train on the line is reduced.

With the increase of the cross-line train operation section and the operation frequency, the impact on the operation frequency of the local trains on the line will gradually decrease. It also shows that the operation frequency of the cross-line train is gradually approaching the passenger demand.

TABLE II
CROSS-LINE OPERATION RESULTS AT DIFFERENT TURN-BACK STATIONS DURING THE MORNING PEAK PERIOD

Number	Turn-back station	Total objective (standardization)	Z	$f_{equalization}$	C	f_1	f_2	f_3
10	ZXZ	-0.74	135014.42	31.78	87855.40	15	0	14
13	YZL	1.19	139070.34	37.84	88125.42	15	1	13
14	PXF	1.92	141988.15	35.38	89065.95	14	2	13
15	HLG	5.10	151530.27	33.24	91995.86	12	4	12
16	HY	2.63	138329.23	31.69	93605.97	11	5	12
17	YX	1.05	135895.18	32.34	91140.67	11	6	11
18	XXK	-1.66	131733.20	28.37	88551.89	12	6	9
19	YTZ	1.62	136530.43	29.57	92979.45	10	6	12
20	LCQ	0.87	137405.06	28.23	91333.71	10	6	11
21	SLG	-1.57	136932.49	24.46	87442.47	11	5	10
22	ALP	-0.92	136134.84	21.26	90288.01	10	6	10
23	ATZ	-1.72	135093.80	18.35	89989.21	9	6	11
24	BTC	-0.80	137740.61	20.84	89892.83	10	5	11
25	AHQ	-3.06	134495.26	19.25	87072.44	9	6	9
26	ADL	-2.20	136671.48	17.94	88311.24	10	5	10
27	GLD	-2.45	130363.24	15.71	91590.55	10	5	10
28	SCH	-1.25	134470.36	19.96	90812.98	9	5	11
29	NLG	1.99	141086.06	18.49	95094.36	5	7	12

TABLE III
CROSS-LINE OPERATION RESULTS AT DIFFERENT TURN-BACK STATIONS DURING THE EVENING PEAK PERIOD

Number	Turn-back station	Total objective (standardization)	Z	$f_{equalization}$	C	f_1	f_2	f_3
10	ZXZ	-0.36	134990.74	31.65	87825.93	15	0	14
9	GHC	1.71	139019.53	36.58	88066.61	15	1	13
8	SH	3.37	141926.11	34.14	89807.14	14	3	12
7	SHG	-1.31	135892.55	23.26	87920.79	11	6	11
6	NS	-1.14	136534.46	20.72	88468.97	12	6	10
5	BSW	-0.91	131728.67	18.25	91375.54	10	6	12
4	CPD	-2.40	130407.61	15.86	90293.98	10	6	11
3	CP	-2.29	135092.64	20.64	87408.41	12	5	9
2	SSL	0.45	136115.34	23.88	90262.94	10	6	10
1	CPX	2.89	141062.15	24.76	91565.48	9	6	11

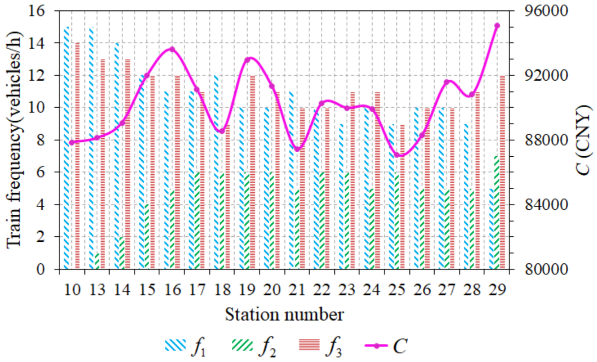

Fig. 10. Operation frequency and cost of different turn-back stations in morning peak period

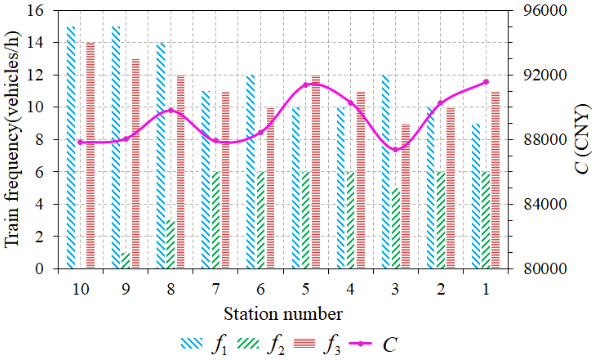

Fig. 11. Operation frequency and cost of different turn-back stations in evening peak period

By comparing the schemes of each turn-back station with without cross-line operation, the turn-back stations are divided into three categories:

grade 1: Among the three target values, two are better than the none cross-line operation.

grade 2: Among the three target values, one is better than the none cross-line operation.

grade 3: The three target values are worse than the non-cross-line operation.

For the cross-line scheme under the morning peak period, there are 4, 9, and 4 stations in the grade 1, grade 2, and grade 3 turn-back stations, respectively. The cross-line schemes of turning back at XXK station and GLD station, Z and $f_{equalization}$ the two target values are better than the original scheme. The cross-line scheme of turning back at SLG station and AHQ station, C and $f_{equalization}$ the two target values are better than the original scheme. Because the research in this paper focuses on passengers, and GLD station is better than XXK station in the two target values, so the cross-line train turning back station is set as GLD station without special explanation.

For the cross-line scheme under the evening peak section, there are 3, 5 and 1 stations in the grade 1, grade 2, and grade 3 turn-back stations respectively. The cross-line schemes of turn-back at BSW station and CPD station, Z and $f_{equalization}$ the two target values are better than the original scheme. The cross-line scheme of turning back at CP station, C and $f_{equalization}$ the two target values are better than the original scheme, and the cross-line train turning back station is designated as CPD station unless there is no special explanation in the follow-up.

E. Full load rate balance

E.1. Full load rate balance of each scheme

In this part, the reasons for the change of the objective function of the balance of the full load rate are discussed when the cross-line train turns back at different turn-back stations. As shown in Fig. 12 and Fig. 13.

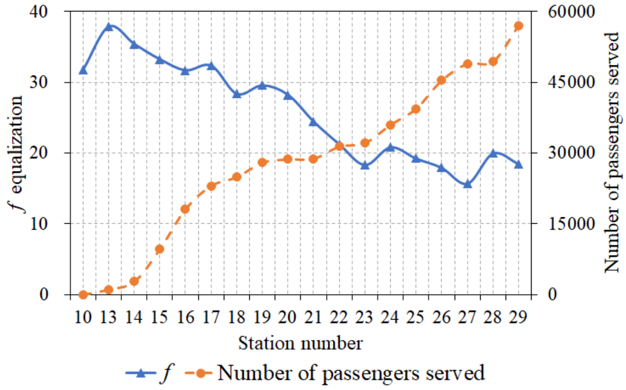

Fig. 12. The change of the objective function of the full load rate balanc of cross-line route and number of passengers served of Train 2 in morning peak period

The objective function of full load rate balance increases first and then decreases gradually at the YZL station. The reason for the increase is that the new cross-line train will affect f_3 of CP Line. At the same time, in the passenger flow distribution link, it has been assumed that passengers prefer direct trains and transfer passengers on Line 8 and CP are more willing to wait for cross-line trains with longer departure intervals, which greatly increases the waiting time of transfer passengers, further leading to the increase of $f_{equalization}$. Compared with non-cross-line trains, $f_{equalization}$ increases by 19.07 %, which leads to an increase in the overall objective function value. The reason for the decline is that with the increase of cross-line train operation section and operation frequency, the number of passengers served by cross-line trains in II also begins to increase, which alleviates the pressure of transfer stations and makes the full load rate of cross-line sections more balanced. Finally, compared with the original scheme, the turn-back operation of cross-line trains at GLD station reduces $f_{equalization}$ by 50.57 %.

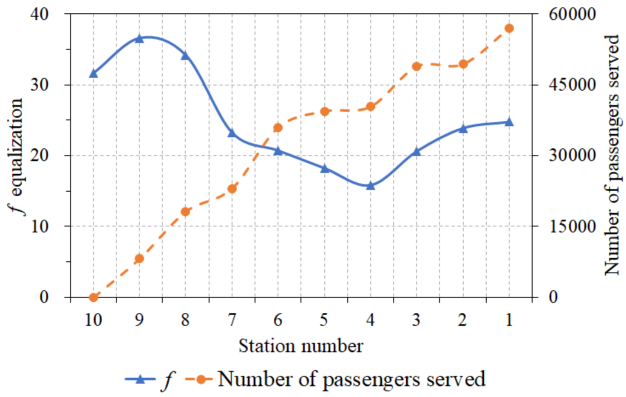

Fig. 13. The change of the objective function of the full load rate balanc of cross-line route and number of passengers served of Train 2 in evening peak period

The reasons for the increase and decrease of $f_{equalization}$ are similar to those during the morning peak period. When the turn-back station is GHC station, the frequency of cross-line trains is low, and the operation of cross-line trains will affect the f_3 of Line 8. At the same time, considering the preference of passengers for direct train travel, $f_{equalization}$ is increased by 15.58 % compared with non-cross-line trains. With the increase of cross-line section and f_2, more passengers with transfer demand choose to take cross-line trains. Finally, compared with the original scheme, $f_{equalization}$ of CPD station is reduced by 49.89 %.

In general, the effect of operating cross-line trains is related to the travel demand of passengers. When the cross-line demand of passengers is strong, operating corresponding cross-line trains is beneficial to the balance of full load rate.

E.2. Full load rate balance of each cross-line operation section

On each section of the cross-line train (Train2), the change trend of $Ind_{equalization}$ before and after improved are shown in Fig. 14 and Fig. 15:

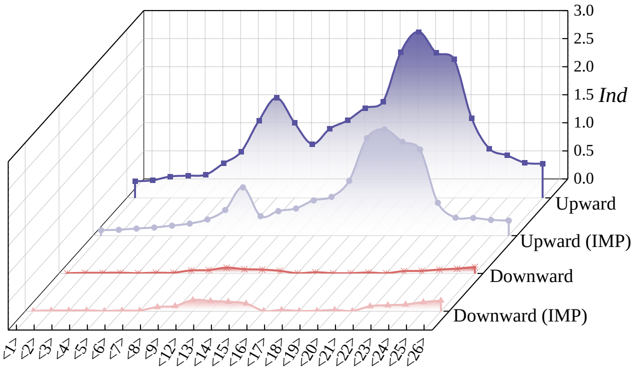

Fig. 14. The *Ind* before and after improvement of each cross-line section in the morning peak period

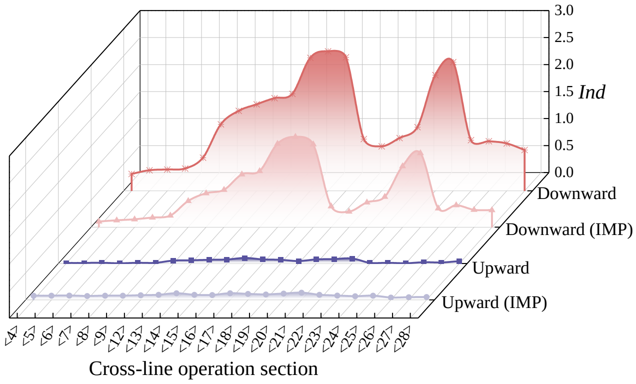

Fig. 15. The *Ind* before and after improvement of each cross-line section in the evening peak period

From the above figures, in the morning peak period, $Ind_{equalization}$ of each section of the cross-line section in different directions varies greatly, and the maximum difference is 2.54 times the average difference. The maximum value in the upward direction is 2.96, and the maximum value in the downward direction is 0.13; When $Ind_{equalization} \leq 0.50$, the difference of section full load rate was small, the full load rate of each section in the upward direction was extremely uneven, and the downward direction was relatively balanced. $Ind_{equalization}$ average upward direction after improvement is reduced by 0.61, and the maximum is reduced by 1.07, which is significantly improved compared with the full load rate balance before improvement. Although the downward direction has increased after improvement, the maximum value is 0.20, still less than 0.50, which is in a balanced state.

In the evening peak period, the maximum $Ind_{equalization}$ value of the downward direction before improvement is 2.58, the average value is 1.24, and the full load rate of the section is extremely uneven. The maximum value of the upward direction is 0.09, and the full load rate is more balanced. After improvement, the downward direction $Ind_{equalization}$ is reduced by 0.61 on average, and the maximum is reduced by 1.01. Although the section full load rate is still unbalanced, it is reduced by 49.5 % on average compared with that before improvement. Although in the upward direction, $Ind_{equalization}$ has increased, the maximum value is 0.13, and the full load rate is still at a relatively balanced level.

VI. Discussion and Conclusion

A. Discussion

Based on the characteristics of passenger flow during peak periods, focusing on the cross-line operation and the balance of full load rate. Different from the single-line operation studied and analyzed in the past, this study extends the research object to the cross-line operation between different lines.

The results show that cross-line operation can improve the quality of passenger travel and reduce passenger travel costs with transfer demand. The research on the balance of full load rate shows that the operation of cross-line trains in line with the flow direction of main commuter passengers during peak periods, will greatly improve the balance of full load rate in each section and improve the quality of train service.

However, compared with the related research on train timetables, the research on cross-line operation in this study is more macroscopic, and the research focus is on determining the turn-back station and frequency of cross-line trains. In the future, the micro train operation plan needs to be further considered.

B. Conclusion

This paper mainly studies the problem of full load rate balance of cross-line operation of URT. A general model is proposed for the "X/Y"-type route to solve the problem of cross-line train turn-back station selection and operation frequency. The model is applied to the actual case of the Beijing subway.

By analyzing the characteristics of commuting passenger flow in morning and evening peak periods, the cross-line train operation routing and operation frequency are determined. The optimization results of passenger travel cost, full load rate balance, and enterprise operation cost are obtained. The cross-line train turn-back station is divided into three levels so that the operator can determine the train turn-back station according to the actual situation.

The experimental results show that: First, from the perspective of passengers, the operation of cross-line trains can effectively improve the balance of full load rate and reduce passenger travel costs. Second, from the perspective of the operating enterprise, the operation of the cross-line train will increase the transportation cost, but it can make the section full load rate more balanced, significantly improve the service quality of the high full load rate section, improve the passenger travel experience, and reduce the passenger travel cost.

In addition, this paper also studies the influence of the operation of cross-line train on the operation frequency of the local train, the change trend of the passenger capacity of cross-line train when different turn-back stations are selected, and the change of the full load rate balance of each section before and after the operation of cross-line trains. The relevant research results have guiding significance for the construction of efficient and fast URT. The main findings of this study are as follows:

(1) When running cross-line trains, the transport capacity of the local train will be compressed to varying degrees. Specifically, when the cross-line operation scheme gradually approaches the optimal solution, the operation frequency of the two kinds of local trains are reduced by 27.38 % and 30.95 % on average. When the cross-line train fills this part of the vacancy, it reduces the travel cost of transfer passengers.

(2) With the extension of the cross-line train operation section and the increase of the operation frequency, more passengers choose to take the cross-line train, which reduces the objective function value of section full load rate balance by 50.23 %, and significantly improves the full load rate balance of each section.

(3) Compared with non-cross-line operation, cross-line operation can better allocate train resources. When running the same number of trains in the route, the quality of service of operating cross-line trains is better. At the same time, the balance of travel full load rate of operating cross-line trains is better than that of non-cross-line trains.

(4) Cross-line trains can improve the balance of full load rate during the peak periods of passenger flow. Especially in the section with a serious imbalance of full load rate, during the early peak period, $Ind_{equalization}$ is reduced by 36.07 %, during the late peak period is reduced by 47.36 %. The improvement effect is obvious.

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