Who Should Manage the Price Execution of a Livestreaming Supply Chain Considering Consumer Returns?

Xiaoxi Zhu and Meifei Gu

***Abstract*—In recent years, online livestreaming has progressively emerged as a prominent retail channel. With the growth of some leading livestreamers, the dispute over product pricing power between manufacturers and livestreamers has become a focal point of discussion. Meanwhile, the high return rate associated with livestreaming also poses a challenge to supply chain performance. In response, manufacturers seek to mitigate this risk by transferring the responsibility of return costs to the livestreamers. Inspired by these practical issues, this study constructs a supply chain system composed of a manufacturer and a livestreamer to examine how different combinations of pricing execution power and return business models affect profits, selling efforts, pricing strategies, and consumer surplus under the influence of spillover effects during livestreaming. The results indicate that: (1) Managing pricing execution always benefits the livestreamer, while whether the manufacturer can be benefited depends on the channel spillover effect coefficient. (2) The livestreamer with pricing power does not always execute at lower prices. She will set higher prices and invest more efforts with a relatively high channel spillover coefficient. (3) Transferring the responsibility for return costs to the livestreamer does not enhance the profits of supply chain stakeholders and may even diminish the livestreamer's selling efforts. The optimal return policy to incentivize livestreamer is the manufacturer bearing return costs. (4) When livestreamer manages price execution, the consumer surplus is also higher if the manufacturer bears return costs.**

***Index Terms*—livestreaming; pricing execution; product returns; channel spillover**

I. INTRODUCTION

IN recent years, livestreaming e-commerce has been maintaining a rapid development trend, providing consumers with a unique shopping experience and service through the intuitive display of products and real-time interaction. The livestreaming channel builds a bridge between manufacturers, livestreamers, and consumers. Manufacturers (brand owners) typically collaborate with third-party livestreamers or key opinion leaders in hopes of capitalizing on the livestreamer's large fan base, professional sales skills, and personal charisma to increase product exposure and intensify consumer purchasing intent.

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However, a series of management issues have emerged.The dispute over pricing execution power between the manufacturer and the livestreamer is one of the key challenges facing the livestreaming channel. As the position of livestreamers in the supply chain strengthens, their bargaining power correspondingly intensifies. Dominant livestreamers rely on their huge fan bases to negotiate prices with the manufacturers. In addition, some livestreamers and brands establish a "bottom price agreement" to manage the execution of product pricing, aiming to achieve the "lowest price across all platforms".

The spillover effect of the livestreaming channel on related products or channels of the brand is another key reason that why brands seek collaboration with livestreamers. The spillover effect may manifest as a positive promotional impact, that is, new consumers attracted by livestreaming are converted into purchasers in the online retail channel, which can be explained by the phenomenon of "free-rider" on the manufacturer's online retail channel. However, the spillover effect of livestreaming can also manifest as a negative erosive effect, where livestreaming attracts and diverts customers who would otherwise purchase through the online retail channel, thereby diminishing retail channel sales.

The characteristic of impulsive purchasing behavior among consumers also leads to a higher product return rate in the livestreaming channel. Livestreamers often promote products as limited-time offers or exclusive deals, creating a sense of urgency that induces impulsive purchases. According to the latest survey data from the global market research firm eMarketer, the value of goods returned by American online shoppers is projected to reach $412.64 billion by 2026. Furthermore, the average return rate of livestreaming e-commerce in China ranges from 30% to 50%, which is higher than the 10% to 15% observed in traditional e-commerce, according to statistics. Returns not only increase the operational costs of the supply chain but also pose a potential threat to the manufacturer's sales performance and brand reputation. To clarify responsibility attribution and avoid disputes, some manufacturers may stipulate in the contract that the livestreamer bears the return costs, which depends on the specific contractual agreements between the manufacturer and the livestreamer. This option may also discourage the livestreamers' selling incentive during the livestreaming.

Based on our observations of the above practical problems, we propose the following research questions:

(1) Does the party who manages the price execution profit more? Does the livestreamer always execute lower prices when pricing? What is the role of the channel spillover

coefficient?

(2) Does transferring responsibility for return costs to the livestreamer improve the manufacturer's profit? Can the potential return costs incentivize the livestreamer to exert more selling efforts?

(3) Will the livestreamer with pricing power charge higher prices under the pressure of return costs? Who is better positioned to improve consumer surplus by bearing the cost of returns?

To address the above questions, this paper constructs a livestreaming supply chain system consisting of a manufacturer and a livestreamer. Based on the consideration of livestreaming spillover effects, we focus on the impacts of different combinations of pricing models (manufacturer pricing and livestreamer pricing) and return business models (either the manufacturer or livestreamer bears the return costs) on optimal supply chain decisions and performance. The main findings are as follows: (1) During the execution of pricing by the livestreamer, she always profits more. Howerer, the manufacturer only gains more profit when the channel spillover coefficient is moderate. (2) The livestreamer does not always execute lower prices when pricing. When the channel spillover effect is significant, the livestreamer tends to set higher prices while also investing more in selling efforts. (3) Transferring the responsibility for return costs to the livestreamer does not always improve the manufacturer's profits and even weaken the livestreamer's selling efforts. The manufacturer bearing return costs can more effectively incentivize the livestreamer to invest more. (4) When the livestreamer with pricing execution power is responsible for return costs, she may execute a higher price to alleviate cost pressure when the commission rate is low. In this case, it is more beneficial for the manufacturer to bear the cost of returns to increase consumer surplus.

The remaining structure of this paper is outlined as follows: Section II reviews the literature relevant to this study. Section III describes the model. The optimal solution is presented in Section IV. Section V analyzes the impact of pricing power on supply chain stakeholders' profits, pricing decisions, and consumer surplus. In Section VI, we extend the model to investigate the impact of different return business models on supply chain performance. Section VII summarizes the study. The proof is given in the Appendix.

II. LITERATURE REVIEW

A. Livestreaming Business Model

With consumers increasingly taking the initiative in product and channel selection, supply chain channels focus on improving their functionality and efficiency to keep pace with intensifying channel competition [1]. Livestreaming can provide customers with more value [2] and improve interaction [3] [4]. However, it may also encourage impulsive purchasing behavior [5]. By providing merchants with a new sales channel, livestreaming stimulates supply chain members to investigate innovative channel strategies. The introduction of livestreaming channels has been addressed in part of the literature [6] [7] [8]. For instance, Zhang et al. [8] observed that livestreaming benefits both livestreaming service providers and e-commerce platforms. The introduction of a livestreaming channel isn't always advantageous, though. According to Zhang and Tang [6], the introduction of a livestreaming channel is not beneficial for sellers and may negatively impact the manufacturer's profits [1]. The selling efforts and service level of supply chain members also impact the decision-making and supply chain performance [9] [10] [11]. Sang [11] discussed the influence of the service level on profits and decision-making under different power structures. Additionally, merchant self-broadcasting is receiving increasing attention [12] [13]. For example, Zhang et al. [12] examined the effects of two forms of merchant self-broadcasting and celebrity livestreaming on supply chain members. The findings demonstrated that the commission rate and fixed signing bonus impact the selection of livestreaming mode.

Channel spillover plays an important role in livestreaming supply chains. Li et al. [14] investigated the optimal strategies of two competing retailers considering the channel spillover effect. The study suggested that when livestreaming generates significant spillover effects, neither brand has an incentive to increase sales. Niu et al. [15] observed that greater positive channel spillovers may enhance the KOL effect, making brand owners more likely to increase profits through KOL. However, spillover effects in dual-channel supply chains can be beneficial as well as detrimental [16] [17]. For instance, Yang et al. [18] investigated how two spillover effects affected the decisions made by two sellers and proved that, if an e-retailer partners with a KOL, the manufacturer may benefit from free-riding behavior if the spillover effect generated by the livestreaming is favorable.

B. Product Returns in Supply Chain Management

Product returns is a significant topic in the supply chain. Consumers often consider return shipping insurance to mitigate the inconvenience and costs associated with returns [19]. The decision to purchase shipment insurance will influence customers' post-purchase regret [20]. While shipping insurance alleviates the burden of return shipping costs for consumers, Chen et al. [21] discovered that shipping insurance may reduce market size, and sellers who offer it may set higher prices. Due to the increased supply chain costs caused by high return rates, there is a growing research focus on strategies for reducing return rates and managing returned products [22] [23] [24]. Zhao et al. [25] considered the impact of transparency efforts on decreasing returns, and the results indicated that when acceptance of remanufactured products is low, the negative impact of returns should be mitigated by transparency measures implemented by retailers. Selling refurbished products is one approach to managing returns. Borenich et al. [22] found that the manufacturer's decision to market refurbished products in the online channel is detrimental to retailers' efforts due to the higher residual value of returns in the online channel.

Nevertheless, returns in the supply chain are not always detrimental. Li and Liu [26] suggested that accepting returns may be beneficial to manufacturers, particularly when selling to competing retailers. The uncertainty introduced by product returns to the supply chain has attracted the attention of some scholars who have started to explore the optimal return policy [27] [28] [29] and supply chain decision-making under the influence of return policy [30] [31]. Considering the risk in supply chain, Yoo [28] investigated the relationship between product quality decisions and return policies. The analysis revealed that improving product quality is more crucial than offering a generous return policy. Li et al. [29] constructed a two-stage model to study the full refund and no refund policies in a dual-channel supply chain. The study obtained the manufacturer's optimal pricing and return strategies considering different return rates and customers' perceived value. Taleizadeh et al. [31] examined a closed-loop supply chain with a return policy serving as an incentive and explored how supply chain profitability can be improved under different remanufacturing scenarios.

C. Research Gaps

Based on the above analyses, it is evident that livestreaming channels have a significant influence on product promotion. The following gaps are filled by this study: (1) The impact of price power on the livestreaming supply chain is unclear. We investigate the attribution of pricing power between the manufacturer and the livestreamer. (2) This study also examines the impact of channel spillover effects of livestreaming. We show how different pricing execution strategies of the manufacturer and the livestreamer affects stakeholders' profits and consumer surplus. (3) Returns in the livestreaming supply chain affect the interests of supply chain members. Therefore, we discuss the issue of return cost allocation in the extension to investigate the impact of the livestreamer bearing the return costs on the supply chain performance.

III. The model

We construct a game-theoretical model involving a manufacturer and a livestreamer. Before the manufacturer (denoted by the subscript "m") established the livestreaming channel, the products were exclusively sold through the online retail channel. To boost sales, the manufacturer invites a livestreamer (denoted by the subscript "l") to join the newly established livestreaming channel. Compared to the traditional text and image descriptions of retail channels, the real-time and interactive nature of livestreaming makes it easier for consumers to obtain detailed product information, thereby enhancing their purchasing motivation. During the livestreaming, the livestreamer determines her selling efforts, denoted as "e". These efforts include activities such as patiently addressing consumer inquiries about the product, providing a comprehensive demonstration of its features and functions, and offering professional guidance or advice to assist consumers in their purchasing decisions. These actions stimulate consumer interest in making a purchase, consequently positively influencing the demand for the product, leading to an increase in a portion of demand, denoted as "ke". Parameter k captures the elasticity of the livestreamer's selling efforts on market demand. The demand function can be expressed as $q_L = \Delta - p + ke$, where "Δ" represents the fan base of the livestreamer.

Here, we consider the livestreaming channel's spillover effect, which manifests in the changes in demand for other products sold through the manufacturer's retail channel. Assuming the unit price of products in the incumbent retail channel is standardized to 1, the manufacturer can obtain a profit of γq_L due to the spillover effect of the livestreaming channel in the incumbent retail channel. Here, γ represents the spillover effect coefficient ($\gamma \in [-1,1]$), capturing the net change in sales revenue for the online retail channel per unit sale through the livestreaming channel [17] [32]. Specifically, when γ is located in the interval $[0,1]$, it indicates a positive channel spillover effect. In this scenario, consumers are more likely to purchase products from the manufacturer's own channel, influenced by factors such as increased trust in the livestreamer. In contrast, when γ is located in the range $[-1,0]$, it signifies a negative channel spillover effect. In this case, the livestreaming may negatively impact the manufacturer's online retail channel because of intensified product substitution competition.

The innovative aspect of this study is to explore the ownership of price execution in a livestreaming supply chain. It suggests that the determiner of the product's selling price p $(p > c_m)$ is not limited solely to the manufacturer m but could also be the livestreamer l. The livestreamer, leveraging a massive fan base and influence, has gained more significant bargaining power. This compels the manufacturer to consider her opinions on pricing during collaborations with a dominant livestreamer. In some cases, the manufacturer may even relinquish price execution power to the dominant livestreamer to fully capitalize on the livestreamer's substantial audience resources and boost product sales. Therefore, in this paper, we investigate two pricing models, namely Manufacturer Pricing (denoted by superscript "*MP*") and Livestreamer Pricing (denoted by superscript "*LP*"). In our study, we assume a proportional relationship between price p and cost c_m, meaning that as the cost increases, the product's pricing will correspondingly increase [33] [34] [35]. This assumption leads to the condition that $4h + (\alpha - 1)k^2\phi > 0$, which eases the analytical work. The summary of notations is presented in Table I.

The possibility of consumers returning products is considered during the shopping process. In the benchmark model, we make the assumption that the manufacturer is responsible for the costs associated with returns (denoted by superscript "*MR*"). This implies that the manufacturer needs to pay a return cost of t per unit. Considering the return rate of α for the livestreaming channel, the total return costs borne by the manufacturer can be expressed as $t\alpha$. We focus on the return model of the livestreaming channel and thus do not consider returns generated from the online retail channel. In the model extension, we will explore the scenario where the livestreamer bears the return costs. For the revenue distribution, the manufacturer pays the livestreamer at a commission rate of ϕ. We denote the unit production cost as c_m. The livestreamer also has to pay a certain cost of he^2 for selling efforts. This non-linear cost assumption more accurately captures the actual cost structure of selling efforts, where parameter h denotes the cost coefficient for the livestreamer in the process of investing in selling efforts.

IV. Equilibriums

In this section, we derive the equilibrium solutions for two key scenarios, i.e., the scenario where the manufacturer manages the price execution and bears the return costs, and the scenario where the livestreamer determines the price while the manufacturer bears the return costs.

TABLE I
NOTATIONS

Parameter	Definition
Δ	The basic market demand of the livestreaming channel
p	The selling price of the livestreaming channel
e	The selling efforts exerted by the livestreamer
k	The elasticity of market demand to selling efforts
q_L	The total demand of the livestreaming channel
γ	The channel spillover coefficient
ϕ	The commission rate
α	The return rate
h	The cost coefficient of selling efforts
c_m	The production cost per unit
t	The return cost per unit product
Π_i	The profit of the supply chain stakeholders ($i = m, l$, representing the manufacturer and the livestreamer)
Superscript	
MP-MR	The benchmark scenario of manufacturer pricing and bearing return
LP-MR	The scenario of livestreamer pricing and manufacturer bears return
MP-LR	The scenario of manufacturer pricing and livestreamer bears return
LP-LR	The scenario of livestreamer pricing and bearing return

A. Manufacturer Manages the Price Execution

When the manufacturer manages the price execution in the livestreaming channel, he also bears the costs associated with consumer returns. The livestreaming channel generates spillover to the online retail channel of the manufacturer. Consequently, the profit functions of the supply chain stakeholders are as follows:

$$max\Pi_m^{MR} = (1-\alpha)q_L((1-\phi)p - c_m) + \gamma q_L - \alpha t q_L \quad (1)$$

$$max\Pi_l^{MR} = (1-\alpha)p\phi q_L - he^2 \quad (2)$$

In Equation (1), the first term $(1-\alpha)q_L((1-\phi)p - c_m)$ represents the net profit of the manufacturer. The second term γq_L denotes the profit from the livestreaming channel's spillover to the manufacturer's online retail channel. The term $\alpha t q_L$ is the cost borne by the manufacturer for consumer returns. In Equation (2), the term $(1-\alpha)p\phi q_L$ signifies the total commission obtained by the livestreamer after returns, and he^2 is the cost associated with the livestreamer's selling efforts. We derive the equilibrium solution for scenario *MP-MR* in Lemma 1:

Lemma 1: The optimal price and selling effort level of the livestreamer are: $p^{MP-MR*} = \frac{2h((\alpha-1)(\phi-1)\Delta-(\alpha-1)c_m-\gamma+\alpha t)}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)}$ and $e^{MP-MR*} = \frac{k\phi(\Delta(\alpha+\phi-\alpha\phi-1)+(\alpha-1)c_m+\gamma-\alpha t)}{(\phi-1)(4h+(\alpha-1)k^2\phi)}$. The summary of demand and profits is presented in Table II.

B. Livestreamer Manages the Price Execution

The profit functions under the scenario of livestreamer pricing remain defined by Equations (1) and (2). However, the solution process differs because the livestreamer manages price execution (p) and selling efforts (e) simultaneously. Therefore, we obtain the following Lemma 2:

Lemma 2: The optimal price and selling effort level of the livestreamer are: $p^{LP-MR*} = \frac{2\Delta h}{4h+(\alpha-1)k^2\phi}$ and $e^{LP-MR*} = \frac{(1-\alpha)\Delta k\phi}{4h+(\alpha-1)k^2\phi}$. The summary of demand and profits is presented in Table II.

In Table II, $x_1 = \phi(\alpha-1)(2\Delta h + k^2(\gamma-\alpha t))$, $x_2 = c_m(\alpha-1)(2h+(\alpha-1)k^2\phi)$, $x_3 = \Delta(\alpha-1)(\phi-1)$ and $x_4 = (1-\alpha)c_m - \gamma + \alpha t$.

TABLE II
EQUILIBRIUM RESULTS IN SCENARIOS MP-MR AND LP-MR

Scenarios	Optimal profits and demands
MP-MR	$\Pi_m^{MP-MR*} = \frac{(2h(\gamma+\Delta-\alpha(\Delta+t))+x_1+x_2)^2}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)^2}$
	$\Pi_l^{MP-MR*} = \frac{h\phi(x_3+x_4)(x_3-x_4)}{(1-\alpha)(\phi-1)^2(4h+(\alpha-1)k^2\phi)}$
	$q_L^{MP-MR*} = \frac{(\alpha-1)\phi(2\Delta h+k^2(\gamma-\alpha t)) + (\alpha-1)c_m(2h+(\alpha-1)k^2\phi) + 2h(\gamma+\Delta-\alpha(\Delta+t))}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)}$
LP-MR	$\Pi_m^{LP-MR*} = \frac{2\Delta h\phi(\alpha-1)(2\Delta h+k^2(\gamma-\alpha t)) + 2\Delta hc_m(\alpha-1)(4h+(\alpha-1)k^2\phi) + 4\Delta h^2(2\gamma+\Delta-\alpha(\Delta+2t))}{(4h+(\alpha-1)k^2\phi)^2}$
	$\Pi_l^{LP-MR*} = \frac{(1-\alpha)\Delta^2 h\phi}{4h+(\alpha-1)k^2\phi}$
	$q_L^{LP-MR*} = \frac{2\Delta h}{4h+(\alpha-1)k^2\phi}$

V. THE PERFORMANCE OF DIFFERENT PRICING MODELS IN THE LIVESTREAMING SUPPLY CHAIN

In this section, we first explore the impact of pricing power on the profits of manufacturer and livestreamer. Subsequently, we examine how the decisions of supply chain members differ between manufacturer pricing (*MP*) and livestreamer pricing (*LP*) scenarios. Finally, we investigate which pricing model is more advantageous for consumer surplus.

A. The Impact of Pricing Power on Stakeholders' Profits

To determine whether it is more advantageous for the manufacturer or the livestreamer to manage the price execution in terms of supply chain stakeholders' profits, we examine the optimal profits in scenarios *MP-MR* and *LP-MR*. Proposition 1 is obtained as follows:

Proposition 1: (1) For the manufacturer, if the channel spillover coefficient is located in the interval (γ_1, γ_2), his profits will increase when the livestreamer manages the price execution; otherwise, the manufacturer's profits will decrease. (2) For the livestreamer, the possession of pricing power always enhances her profit, i.e., $\Pi_l^{MP-MR*} < \Pi_l^{LP-MR*}$ always holds. Here, $\gamma_1 = (1-\alpha)c_m + \alpha t$ and $\gamma_2 = \frac{4\alpha h^2 t + 2hk^2\phi(\alpha-1)(\Delta(\alpha+\phi-\alpha\phi-1)+2\alpha t) + k^4\phi^2 t\alpha(\alpha-1)^2 - c_m(\alpha-1)(2h+(\alpha-1)k^2\phi)^2}{(2h+(\alpha-1)k^2\phi)^2}$.

Proposition 1 highlights the impact of pricing power on the profits of supply chain stakeholders. We employ the channel spillover coefficient γ as the critical threshold to capture the channel profits. Parameter γ can identify the condition under which the livestreamer pricing is superior to the manufacturer pricing in terms of manufacturer's profit. The manufacturer prefers to retain price execution ($\Pi_m^{MP-MR*} \geqslant \Pi_m^{LP-MR*}$) only when the channel spillover coefficient γ is either extremely low or high ($\gamma \leqslant \gamma_1$ or $\gamma \geqslant \gamma_2$). This phenomenon can be explained that, when γ is relatively unstable, the manufacturer can regulate the competitive relationship between the livestreaming channel and the online retail channel by managing price execution, thereby protecting his profits. However, when γ is at a moderate level ($\gamma_1 < \gamma < \gamma_2$), the manufacturer is more inclined to delegate price execution to the livestreamer ($\Pi_m^{MP-MR*} < \Pi_m^{LP-MR*}$). Another interesting observation from Proposition 1 is that the livestreamer always benefits more when she manages price execution. In this case, the livestreamer can leverage her influence to negotiate a more favorable price and thus gain more profits. In addition, livestreamer pricing does not always hurt the profiting of the manufacturer. This finding reveals the fundamental motivation for many dominant livestreamers to compete for pricing power in their collaborations with manufacturers.

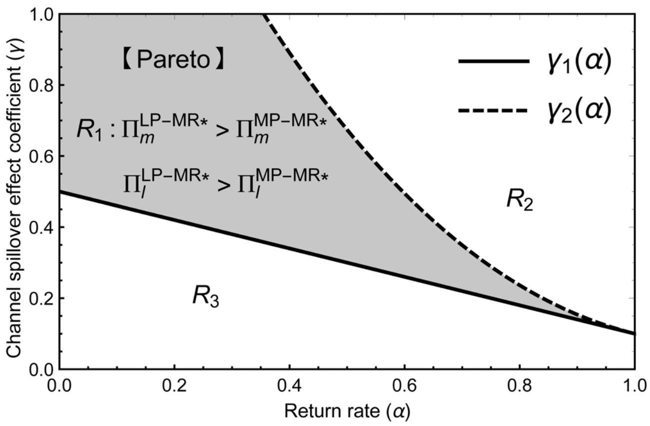

Fig. 1. Profits of supply chain stakeholders in scenarios MP-MR and LP-MR (with $\Delta = 15,\ k = 2,\ h = 2,\ t = 0.1,\ c_m = 0.5$ and $\phi = 0.1$)

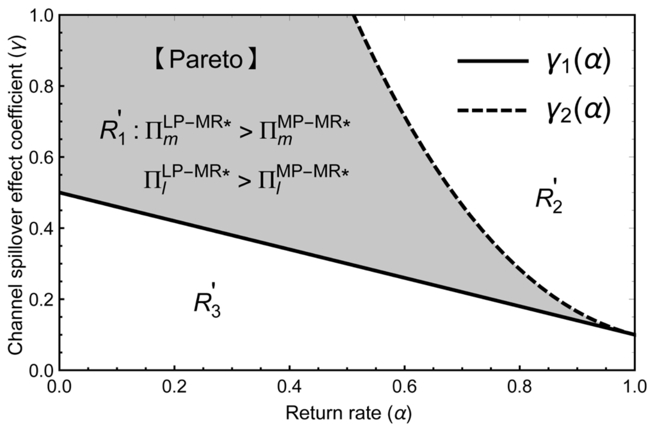

Fig. 2. Profits of supply chain stakeholders in scenarios MP-MR and LP-MR (with $\Delta = 15,\ k = 2,\ h = 2,\ t = 0.1,\ c_m = 0.5$ and $\phi = 0.2$)

In Fig. 1 and Fig. 2, we use two numerical examples to illustrate the key findings of Proposition 1. In both region R_2 and R_3, we have $\Pi_m^{LP-MR*} < \Pi_m^{MP-MR*}$ and $\Pi_l^{LP-MR*} > \Pi_l^{MP-MR*}$. Regions R_1 and R_1' depict the Pareto region, where the profits of both manufacturer and livestreamer can be improved in scenario *LP-MR*. In Fig. 2, with all other parameters remaining unchanged, it can be observed that an increase in the commission rate ($\phi = 0.1 \rightarrow 0.2$) leads to a larger Pareto region.

B. The Impact of Pricing Power on Optimal Decisions

Proposition 2: If $\gamma < (1-\alpha)c_m + \alpha t$, we have $e^{MP-MR*} > e^{LP-MR*}$ and $p^{MP-MR*} > p^{LP-MR*}$; otherwise, we have $e^{MP-MR*} \leq e^{LP-MR*}$ and $p^{MP-MR*} \leq p^{LP-MR*}$.

In Proposition 2, a relatively low channel spillover coefficient (i.e., $\gamma < (1-\alpha)c_m + \alpha t$) indicates a weaker influence of the livestreaming channel. The livestreamer has the motivation to invest more in scenario *MP-MR*. In this case, the manufacturer will execute a higher price. This finding can be explained as follows: when the channel spillover coefficient γ is relatively low, the manufacturer tends to rely more on the livestreaming channel to drive product sales. He may adopt a high-price strategy to optimize profits. As the recommender of products, the increased selling efforts of livestreamer can also partially mitigate the impact of price sensitivity on consumers. When γ is relatively high, which means the size/influence of livestreamer can bring more spillover to online retail channel, it can be observed that the livestreamer tends to set a higher price. This contradicts the common intuition that the livestreamer depresses pricing to attract consumers. In reality, there are also examples of higher pricing in livestreaming channels, which supports the findings. For instance, well-known livestreamer Jiaqi Li sold Shiseido products at a higher price than other platforms in his livestreaming room, causing consumer dissatisfaction. This finding also reveals the potential risk of channel price competition and unfairness that may arise from the pricing execution managed by livestreamer.

Next, we further investigate which pricing model can better stimulate the livestreamer to increase selling efforts. We analyze the optimal selling efforts in Corollary 1.

Corollary 1:

(1) By examining the sensitivity of return rate α on selling efforts, we have $\frac{\partial e^{MP-MR*}}{\partial \alpha} - \frac{\partial e^{LP-MR*}}{\partial \alpha} < 0$ if $\gamma < \gamma_3$ and $\frac{\partial e^{MP-MR*}}{\partial \alpha} - \frac{\partial e^{LP-MR*}}{\partial \alpha} \geq 0$ if $\gamma \geq \gamma_3$.

(2) By analyzing the sensitivity of commission rate ϕ on selling efforts, we have $\frac{\partial e^{MP-MR*}}{\partial \phi} - \frac{\partial e^{LP-MR*}}{\partial \phi} > 0$ if $\gamma < \gamma_1$ and $\frac{\partial e^{MP-MR*}}{\partial \phi} - \frac{\partial e^{LP-MR*}}{\partial \phi} \leq 0$ if $\gamma \geq \gamma_1$. Here, $\gamma_1 = (1-\alpha)c_m + \alpha t$ and $\gamma_3 = \frac{4h(c_m - t) + k^2 t\phi}{k^2\phi}$.

Corollary 1 identifies how the parameter γ (γ_1 and γ_3) affects the sensitivity of return rate α and commission rate ϕ on selling efforts in two pricing models. Specifically, if the channel spillover coefficient γ is relatively low ($\gamma < \gamma_3$), an increase in the return rate α more effectively enhances the livestreamer's selling efforts in scenario *LP* than in scenario *MP*. However, when $\gamma \geq \gamma_3$, the opposite result is obtained. In contrast, the sensitivity of commission rate ϕ on selling efforts exhibits a reverse trend. That is to say, when the parameter γ is below threshold γ_1, increasing the commission rate ϕ is more effective in enhancing selling efforts of the livestreamer in scenario *MP*.

Fig. 3 and Fig. 4 provide numerical insights into the sensitivity of return rate α and commission rate ϕ on selling efforts. In specific, the selling effort of the livestreamer decreases as the return rate rises, while it raises as the commission rate ϕ increases. It can also be observed that in scenario *MP-MR*, for a given return rate α, a lower channel spillover coefficient leads to higher selling efforts. This observation remains consistent when considering the same commission rate ϕ. Furthermore, as depicted in Fig. 4, when the channel spillover coefficient is -0.7, increasing commission rate in scenario *MP* more effectively incentivizes the livestreamer to improve selling efforts than in scenario *LP*.

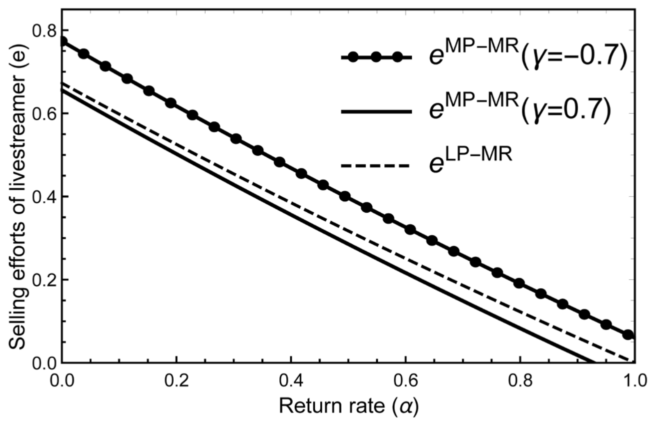

Fig. 3. Selling efforts in scenarios MP-MR and LP-MR (with $\Delta = 10$, $k = 1.8,\ h = 1.5,\ t = 0.1,\ c_m = 0.5$ and $\phi = 0.2$)

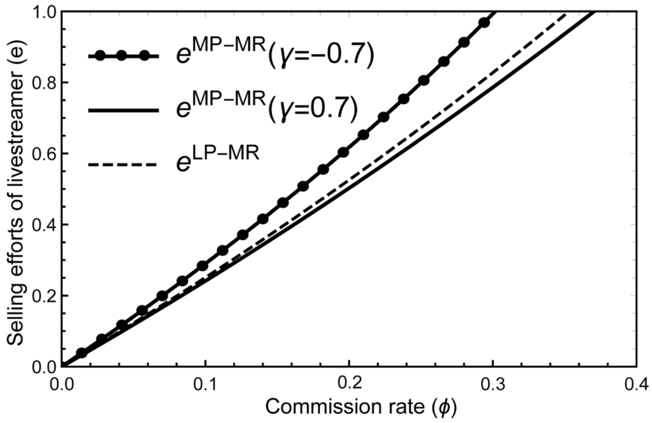

Fig. 4. Selling efforts in scenarios MP-MR and LP-MR (with $\Delta = 10$, $k = 1.8,\ h = 1.5,\ t = 0.1,\ c_m = 0.5$ and $\alpha = 0.2$)

In scenario *LP-MR*, we further obtain the findings presented in Corollary 2:

Corollary 2: In the scenario when the livestreamer manages the price execution (*LP*): (1) For the selling efforts, we have $\frac{\partial e^{LP-MR*}}{\partial \alpha} < 0$ and $\frac{\partial e^{LP-MR*}}{\partial \phi} > 0$; (2) For the selling prices, we have $\frac{\partial p^{LP-MR*}}{\partial \alpha} < 0$ and $\frac{\partial p^{LP-MR*}}{\partial \phi} > 0$.

Corollary 2 indicates that in scenario *LP-MR*, both selling efforts and selling prices decline as the return rate α increases. This implies that if the pricing execution is managed by the livestreamer, she is likely to adopt a more attractive pricing strategy, i.e., a lower price to retain consumers when confronted with a higher return rate. Simultaneously, the high return rate diminishes the livestreamer's motivation to invest more in selling efforts. Corollary 2 also suggests that the commission rate ϕ has a positive effect on incentivizing the livestreamer to enhance both selling efforts and selling price. This can be explained that an increase in the commission rate implies higher revenue for the livestreamer for each product sold. Consequently, if the price is higher, the livestreamer's profits could also be greater, and this motivating her to intensify selling efforts.

C. The Impact of Pricing Power on Consumer Surplus

Consumer surplus captures the difference between the maximum price a consumer is willing to pay and the actual price paid, indicating the additional value consumers derive from purchasing a product [36]. It can be measured by the following equation:

$$CS = \int_{p_{min}}^{p_{max}} q_L dp = \frac{(\Delta - p + ke)^2}{2} \tag{3}$$

Proposition 3: Consumer surplus can be enhanced when manufacturer manage price execution if $\gamma < \min\{\gamma_1, \gamma_4\}$ or $\gamma > \max\{\gamma_1, \gamma_4\}$; otherwise, consumers prefer livestreamer pricing as $CS^{MP-MR*} \leq CS^{LP-MR*}$ if $\min\{\gamma_1, \gamma_4\} \leq \gamma \leq \max\{\gamma_1, \gamma_4\}$.

The relationship between γ_1 and γ_4 depends on α, i.e. $\gamma_1 < \gamma_4$ if $0 < \alpha < 1 - \frac{2h}{k^2\phi}$ and $\gamma_1 \geq \gamma_4$ if $1 - \frac{2h}{k^2\phi} \leq \alpha < 1$. Here, $\gamma_1 = (1-\alpha)c_m + \alpha t$ and $\gamma_4 = \frac{(1-\alpha)(2h+(\alpha-1)k^2\phi)c_m + 4\Delta h(\alpha(-\phi)+\alpha+\phi-1) + 2\alpha h t + (\alpha-1)\alpha k^2 t\phi}{2h+(\alpha-1)k^2\phi}$.

In Proposition 3, we demonstrate the conditions under which consumer surplus can be improved in both pricing models. In specific, when the channel spillover coefficient γ is relatively high or low, manufacturer managing price execution will result in greater consumer surplus. In contrast, at a relatively stable level of channel spillover effects, livestreaming pricing tends to enhance consumer surplus. This proposition reveals the complex interaction between channel spillover and pricing power in affecting consumer surplus. Understanding these dynamic mechanisms can help supply chain members formulate more effective pricing and marketing strategies.

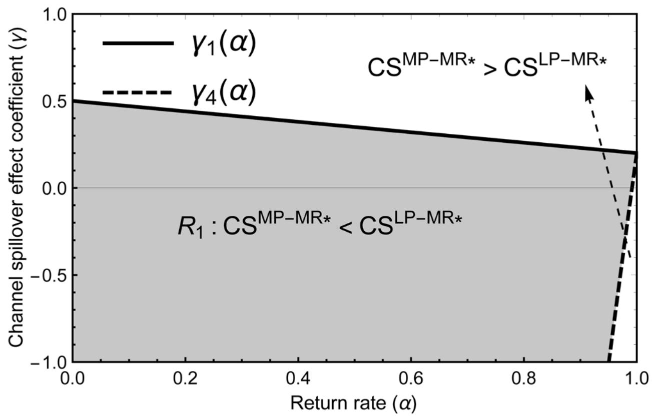

Fig. 5. Consumer surplus in scenarios MP-MR and LP-MR (with $\Delta = 15$, $k = 3,\ h = 2,\ t = 0.2,\ c_m = 0.5$ and $\phi = 0.2$)

Fig. 5 illustrates the cross effects of channel spillover coefficient γ and return rate α on consumer surplus. It demonstrates that when both the channel spillover coefficient and the return rate are relatively low, livestreamer managing price execution is more likely to improve consumer surplus.

However, when the return rate is relatively high, manufacturer pricing is more favorable for enhancing consumer surplus. This can be explained that manufacturer pricing may increase consumer trust in products compared to livestreamer pricing in markets with higher return rates.

VI. Extensions on Return Policy

In this section, we extend our model by considering the case where the livestreamer assumes the responsibility for return costs (denoted by the superscript "*LR*"). We investigate whether transferring the responsibility of return costs to the livestreamer is advantageous for the interests of the supply chain members in both scenarios *MP* and *LP*. Additionally, we explore the impact of return cost allocation on key performance in the livestreaming supply chain.

A. The Impact of Different Return Policies in Scenario MP

When the return costs are borne by the livestreamer, the livestreamer incurs a total cost of $\alpha t q_L$. Therefore, the profit function expressions for the supply chain members are as follows:

$$max\Pi_m^{LR} = (1-\alpha)q_L((1-\phi)p - c_m) + \gamma q_L \quad (4)$$

$$max\Pi_l^{LR} = (1-\alpha)p\phi q_L - he^2 - \alpha t q_L \quad (5)$$

We derive the equilibrium solution for scenario *MP-LR* and obtain the following Lemma 3:

Lemma 3: The optimal price for the manufacturer and the optimal selling efforts level of the livestreamer are: $p^{MP-LR*} = \frac{2(1-\alpha)hc_m - 2h(\Delta(\alpha+\phi-\alpha\phi-1)+\gamma)+\alpha k^2 t(\alpha+\phi-\alpha\phi-1)}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)}$ and $e^{MP-LR*} = \frac{k(\phi(\Delta(\alpha+\phi-\alpha\phi-1)+\gamma)+(\alpha-1)\phi c_m - 2\alpha t(\phi-1))}{(\phi-1)(4h+(\alpha-1)k^2\phi)}$.

The summary of demand and profits is presented in Table III.

In Table III, $x_5 = (\alpha-1)k^2(\alpha t(\phi-1)-\gamma\phi)$, $x_6 = (\alpha-1)c_m$, $x_7 = \phi(\Delta(\alpha+\phi-\alpha\phi-1)+\gamma)$ and $x_8 = 2h((\alpha-1)\Delta\phi+\alpha t)$.

TABLE III
EQUILIBRIUM RESULTS IN SCENARIOS MP-LR AND LP-LR

Scenarios	Optimal profits and demands
MP-LR	$\Pi_m^{MP-LR*} = \frac{(2h((\alpha-1)\Delta(\phi-1)+\gamma)+x_2-x_5)^2}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)^2}$
	$\Pi_l^{MP-LR*} = \frac{h(\gamma+x_3+x_6)(2\alpha t(1-\phi)+x_6\phi+x_7)}{(\alpha-1)(\phi-1)^2(4h+(\alpha-1)k^2\phi)}$
	$q_L^{MP-LR*} = \frac{2h((\alpha-1)\Delta(\phi-1)+\gamma) + c_m(\alpha-1)(2h+(\alpha-1)k^2\phi) - k^2(\alpha-1)(\alpha t(\phi-1)-\gamma\phi)}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)}$
LP-LR	$\Pi_m^{LP-LR*} = \frac{x_8((1-\alpha)k^2\phi(\alpha t(\phi-1)-\gamma\phi)) + 2hx_8(2\gamma\phi+t(\alpha-\alpha\phi)+x_3\phi) + x_8(x_6\phi(4h+(\alpha-1)k^2\phi))}{\phi^2(\alpha-1)(4h+(\alpha-1)k^2\phi)^2}$
	$\Pi_l^{LP-LR*} = \frac{h((\alpha-1)\Delta\phi+\alpha t)^2}{\phi(1-\alpha)(4h+(\alpha-1)k^2\phi)}$
	$q_L^{LP-LR*} = \frac{2h((\alpha-1)\Delta\phi+\alpha t)}{\phi(\alpha-1)(4h+(\alpha-1)k^2\phi)}$

With these equilibrium results, we investigate the impact of the livestreamer bearing return costs on the profits of supply chain members in the manufacturer pricing scenario (MP). The findings are summarized in the following proposition:

Proposition 4:

(1) For the manufacturer,

- when $0 < \alpha < 1 - \frac{2h}{k^2\phi}$ or $1 - \frac{2h}{k^2} < \alpha < 1$, we have $\begin{cases} \Pi_m^{MP-LR*} < \Pi_m^{MP-MR*} if \gamma < \gamma_5; \\ \Pi_m^{MP-LR*} \geqslant \Pi_m^{MP-MR*} if \gamma \geqslant \gamma_5. \end{cases}$
- when $1 - \frac{2h}{k^2\phi} \leq \alpha \leq 1 - \frac{2h}{k^2}$, we have $\begin{cases} \Pi_m^{MP-LR*} > \Pi_m^{MP-MR*} \ if\ \gamma < \gamma_5; \\ \Pi_m^{MP-LR*} \leq \Pi_m^{MP-MR*} \ if\ \gamma \geqslant \gamma_5. \end{cases}$

(2) For the livestreamer, when $\gamma < \gamma_6$, we have $\Pi_l^{MP-LR*} > \Pi_l^{MP-MR*}$; otherwise, $\Pi_l^{MP-LR*} \leq \Pi_l^{MP-MR*}$.

Here, $\gamma_5 = \frac{4\Delta h(\alpha+\phi-\alpha\phi-1)+\alpha k^2 t(\alpha-1)(2\phi-1) + 2\alpha h t - 2(\alpha-1)c_m(2h+(\alpha-1)k^2\phi)}{4h+2(\alpha-1)k^2\phi}$ and $\gamma_6 = (\alpha-1)\Delta(\phi-1)^2 - (\alpha-1)c_m + \frac{\alpha t\phi}{2}$.

Proposition 4 elaborates the impact of the return policy on supply chain stakeholders' profits in scenario *MP*. Transferring return costs to livestreamer does not necessarily enhance the manufacturer's profits. The performance is jointly affected by the return rate and channel spillover effects. For instance, when $1 - \frac{2h}{k^2\phi} < \alpha < 1 - \frac{2h}{k^2}$ and $\gamma > \gamma_5$, manufacturer pricing has the potential to enhance his profits. More interestingly, as shown in Proposition 4(2), the livestreamer bearing return costs does not always harm her profits. In scenario *MP*, when γ is sufficiently low ($\gamma < \gamma_6$), the livestreamer can actually benefit more by assuming return costs. This proposition deserves further explanation. A lower γ indicates that consumer purchasing behavior predominantly focuses on the livestreaming channel, with less conversion to the online retail channel. Bearing the return costs may incentivize the livestreamer to optimize livestreaming content and interactive efficiency to attract more consumers, thereby boosting sales and her profits.

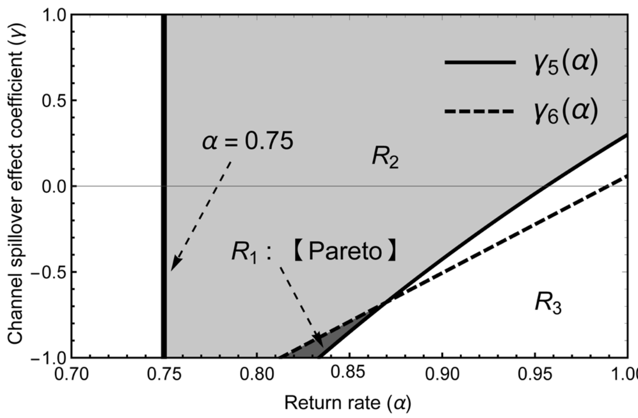

Fig. 6. Profits of supply chain stakeholders in scenarios MP-LR and MP-MR (with $\Delta = 10$, $k = 4$, $h = 2$, $t = 0.6$, $c_m = 0.8$ and $\phi = 0.2$)

Fig. 6 provides an intuitive illustration of the key findings of Proposition 4. It can be observed that when the manufacturer manages the price execution (*MP*), only when γ and α are relatively low, the livestreamer bearing the return costs can achieve a Pareto improvement in the profits of both supply chain members in region R_1 ($\Pi_m^{MP-LR*} >$

Π_m^{MP-MR*} and $\Pi_l^{MP-LR*} > \Pi_l^{MP-MR*}$). Region R_2 indicates $\Pi_m^{MP-LR*} > \Pi_m^{MP-MR*}$ and $\Pi_l^{MP-LR*} < \Pi_l^{MP-MR*}$, and R_3 region indicates $\Pi_m^{MP-LR*} < \Pi_m^{MP-MR*}$ and $\Pi_l^{MP-LR*} > \Pi_l^{MP-MR*}$. Additionally, we can also observe that when the commission rate increases ($\phi = 0.2 \rightarrow 0.25$), the Pareto region expands. It indicates that an increase in the commission rate can enhance the possibility of improving supply chain members' profits in scenario *LR*.

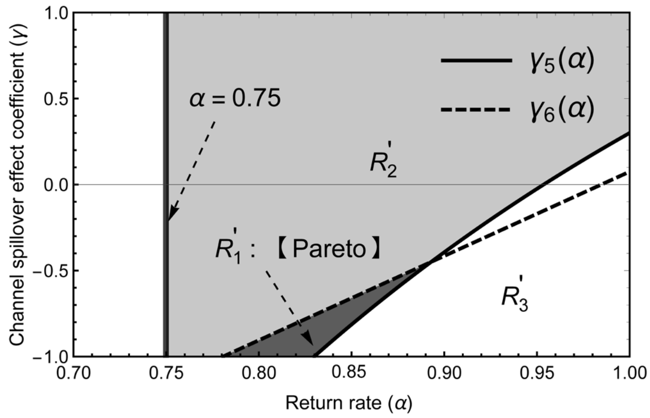

Fig. 7. Profits of supply chain stakeholders in scenarios MP-LR and MP-MR (with $\Delta = 10$, $k = 4$, $h = 2$, $t = 0.6$, $c_m = 0.8$ and $\phi = 0.25$)

Proposition 5: The responsibility of bearing return costs diminishes the selling efforts invested by the livestreamer and concurrently results in a lower selling price, namely, $e^{MP-LR*} < e^{MP-MR*}$ and $p^{MP-LR*} < p^{MP-MR*}$.

Proposition 5 illustrates that assigning responsibility for return costs to the livestreamer diminishes her incentive to enhance selling effort ($e^{MP-LR*} < e^{MP-MR*}$). This can be explained from two perspectives. Firstly, the livestreamer may perceive the allocation of return cost responsibility as unfair, potentially reducing her enthusiasm. Additionally, when the livestreamer bears return costs, she diminishes the selling effort dedicated to increasing sales volume to mitigate her losses. Because the increase in sales volume may be accompanied by additional return risks. Another observation from Proposition 5 reveals how pricing strategies are constrained and influenced by the cost strategy in scenario *MP*. For the manufacturer, he alleviates his cost pressure by transferring return costs to the livestreamer. Consequently, the manufacturer may reduce price to attract more consumers. On the contrary, when manufacturer bears return costs, he tends to set a higher price to cover potential return expenses.

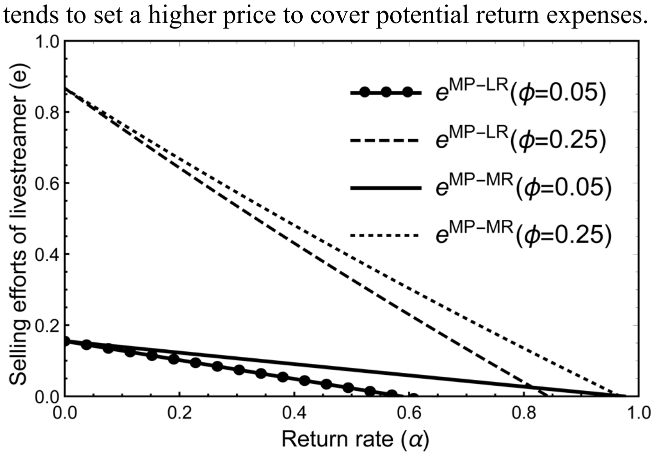

Fig. 8. Selling efforts in scenarios MP-LR and MP-MR (with $\Delta = 12$, $k = 2$, $h = 2$, $t = 0.2$, $c_m = 0.6$ and $\gamma = 0.5$)

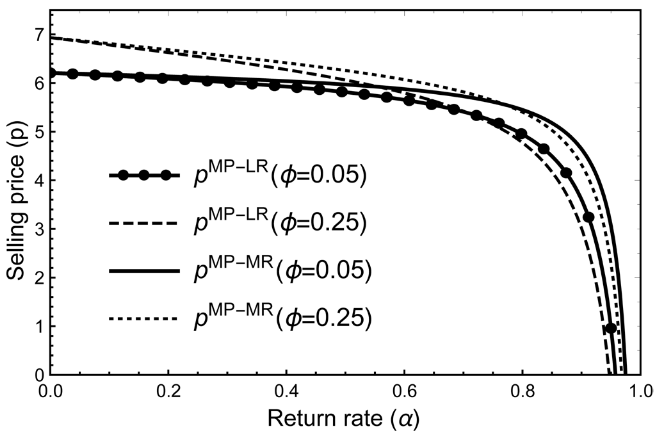

Fig. 9. Selling prices in scenarios MP-LR and MP-MR (with $\Delta = 12$, $k = 2$, $h = 2$, $t = 0.2$, $c_m = 0.6$ and $\gamma = 0.5$)

Fig. 8 and Fig. 9 depict the selling effort and prices of the two return business models under different commission rate levels. It can be observed that selling effort and price decrease with the increase of return rate. With a same commission rate level, we have $e^{MP-LR*} < e^{MP-MR*}$ and $p^{MP-LR*} < p^{MP-MR*}$. We further observe in Fig. 8 that as the commission rate ϕ increases, the selling efforts also rise, with the return rate becoming more sensitive on selling effort. In other words, high commission rate is an effective incentive mechanism that can easily increase the selling efforts of the livestreamer, especially under the condition of a high return rate.

Corollary 3:
(1) For the sensitivity of return rate on selling efforts, we obtain $\frac{\partial e^{MP-LR*}}{\partial \alpha} - \frac{\partial e^{MP-MR*}}{\partial \alpha} < 0$ if $0 < \phi < \frac{4h}{k^2}$ and $\frac{\partial e^{MP-LR*}}{\partial \alpha} - \frac{\partial e^{MP-MR*}}{\partial \alpha} \geq 0$ if $\frac{4h}{k^2} \leqslant \phi < 1$.
(2) For the sensitivity of commission rate on selling efforts, we obtain $\frac{\partial e^{MP-LR*}}{\partial \phi} - \frac{\partial e^{MP-MR*}}{\partial \phi} > 0$ if $h < h_1$ and $\frac{\partial e^{MP-LR*}}{\partial \phi} - \frac{\partial e^{MP-MR*}}{\partial \phi} \leq 0$ if $h \geq h_1$. Here, $h_1 = \frac{1}{4}k^2(\alpha - 1)(\phi^2 - 4\phi + 2)$.

Corollary 3(1) indicates that when the commission rate is relatively low ($0 < \phi < \frac{4h}{k^2}$), an increase in the return rate in scenario *MP-MR* is more effective in enhancing the livestreamer's selling efforts. The commission rate may influence the sensitivity of the livestreamer to costs. When the commission rate is relatively low, return costs impose a substantial burden on the livestreamer. When the manufacturer bears return costs, the risk perception for the livestreamer is reduced. As a result, the livestreamer can be more motivated to increase selling efforts, as she is not concerned about incurring extra costs due to returns. The second finding suggests that the sensitivity of the commission rate on selling efforts depends on the selling effort cost threshold $h = \frac{1}{4}k^2(\alpha - 1)(\phi^2 - 4\phi + 2)$. When h is sufficiently low ($h < \frac{1}{4}k^2(\alpha - 1)(\phi^2 - 4\phi + 2)$), increasing the commission rate ϕ is more favorable for enhancing the selling efforts of the livestreamer in scenario *MP-LR* compared to scenario *MP-MR*.

B. The Impact of Different Return Policies in Scenario LP

In scenario *LP-LR*, the profit functions of the supply chain stakeholders remain as Equations (4) and (5). Employing similar solution methods, we obtain the following Lemma 4:

Lemma 4: The optimal price for the manufacturer and the optimal selling efforts the livestreamer are: $p^{LP-LR*} = \frac{(\alpha-1)\phi(\alpha k^2 t-2\Delta h)+2\alpha h t}{(1-\alpha)\phi(4h+(\alpha-1)k^2\phi)}$ and $e^{LP-LR*} = -\frac{k((\alpha-1)\Delta\phi+\alpha t)}{4h+(\alpha-1)k^2\phi}$. The summary of demand and profits is presented in Table III.

By further investigating the profits of supply chain stakeholders in scenarios *LP-LR* and *LP-MR*, we obtain the following Proposition 6:

Proposition 6:

(1) For the manufacturer in scenario *LP*, when the channel spillover coefficient is sufficiently high, the manufacturer prefers to undertake the return costs on his own. In other words, when $\gamma > \gamma_7$, we have $\Pi_m^{LP-LR*} < \Pi_m^{LP-MR*}$.

(2) For the livestreamer, we always have $\Pi_l^{LP-LR*} < \Pi_l^{LP-MR*}$. Here, $\gamma_7 = \frac{2h(\alpha t(\phi-1))+(\alpha-1)k^2\phi((1-\alpha)\Delta\phi+\alpha t(\phi-1)) +\left((1-\alpha)\phi c_m(4h+(\alpha-1)k^2\phi)\right)-4(\alpha-1)\Delta h\phi^2}{\phi(4h+(\alpha-1)k^2\phi)}$.

Proposition 6 provides the conditions under which the manufacturer's profit improves in scenario *LP*. Notably, it reveals that in this scenario, assigning the return responsibility to the livestreamer does not necessarily lead to higher profits for the manufacturer. Specifically, when γ is sufficiently high, the manufacturer tends to profit more when he bears the return costs. In this case, the manufacturer can offset the return costs incurred in the livestreaming channel by increasing sales in the online retail channel. Conversely, when γ is relatively low, we obtain an opposite conclusion. It is worth noting that it is always detrimental to the livestreamer to bear the return costs. This is different from the finding in Proposition 4 (2) in scenario *MP*. We can explain this interesting finding from two perspectives. On one hand, in scenario *LP-LR*, the return cost directly reduces the profit margin of the livestreamer. On the other hand, the lack of insight into market and product feedback might lead to inaccurate or unreasonable pricing executed by the livestreamer, resulting in reduced profits. Consequently, the livestreamer who possesses pricing power may be inclined to avoid assuming the burden of return costs to protect her profitability.

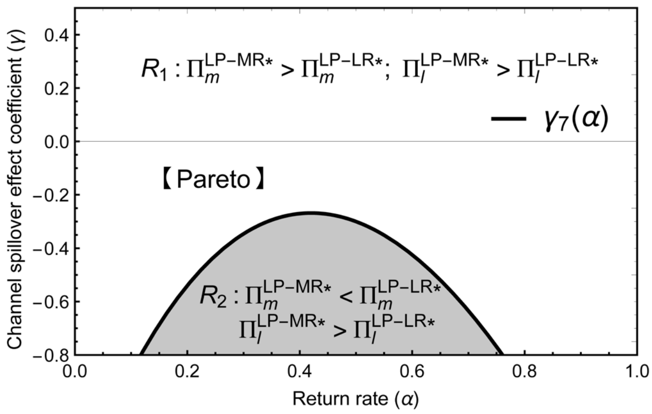

Fig. 10. Profits of supply chain stakeholders in scenarios LP-LR and LP-MR (with $\Delta = 10,\ k = 4,\ h = 2,\ t = 0.6,\ c_m = 0.8$ and $\phi = 0.15$)

We summarize the key findings of Proposition 6 in Fig. 10 and Fig. 11. When the livestreamer manages the price execution, in regions R_1 and R_1', we have $\Pi_m^{LP-MR*} > \Pi_m^{LP-LR*}$ and $\Pi_l^{LP-MR*} > \Pi_l^{LP-LR*}$. It indicates that the manufacturer bearing the cost of returns leads to a Pareto improvement to the profits of the supply chain members. It can be observed that there is a significant increase in the Pareto improvement region in profits with the increase of commission rate ($\phi = 0.15 \to 0.25$). In regions R_2 and R_2', when γ is relatively low, it is possible to improve the manufacturer's profits in scenario *LR*.

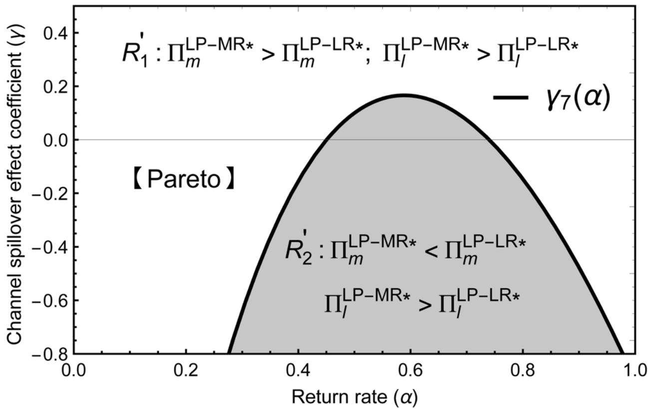

Fig. 11. Profits of supply chain stakeholders in scenarios LP-LR and LP-MR (with $\Delta = 10,\ k = 4,\ h = 2,\ t = 0.6,\ c_m = 0.8$ and $\phi = 0.25$)

Proposition 7:

(1) In scenario *LP*, livestreamer bearing return costs weakens the selling efforts and reduces market demand, i.e., $e^{LP-LR*} < e^{LP-MR*}$ and $q_L^{LP-LR*} < q_L^{LP-MR*}$.

(2) When the livestreamer manages price execution and the commission rate is low, the responsibility for return costs leads her to execute higher price. In other words, when $0 < \phi < \frac{2h}{(1-\alpha)k^2}$, we have $p^{LP-LR*} > p^{LP-MR*}$.

Proposition 7 illustrates that in scenario *LP*, the responsibility for return costs will weaken the livestreamer's selling efforts. This result is similar to the finding in Proposition 5 in scenario *MP*. In other words, assigning return costs to the livestreamer is detrimental to enhancing her selling efforts regardless of the pricing power. This finding suggests that, to better incentivize livestreamer to exert more selling efforts, the optimal return policy is for the manufacturer to bear the return costs. The second finding in Proposition 7 indicates the relationship between the prices executed by the livestreamer and the commission rate. When the commission rate is relatively low, the livestreamer tends to execute a higher price when confronted with the responsibility of bearing return costs. A lower commission rate results in smaller net profits per unit for the livestreamer. To address potential return losses, she chooses to execute a higher price, thereby retaining more profit margin per transaction.

Proposition 8: In the case of livestreamer pricing, it is always beneficial for the improvement of consumer surplus when the manufacturer bears the cost of returns. In other words, $CS^{LP-LR*} - CS^{LP-MR*} < 0$ always holds true.

Proposition 8 explains that when the livestreamer manages price execution, manufacturer bearing return costs can always enhance consumer surplus. If the livestreamer executes selling price, it is more likely to allocate more resources to product marketing, while neglecting service issues after the return. Even in reality, some livestreamers adopt improper practices to prevent returns, such as exaggerating the advantages of the product or concealing its disadvantages. These behaviors are detrimental to consumers and lead to a reduction in consumer surplus. In contrast, consumers tend to have greater trust when the manufacturer is responsible for returns, believing that the manufacturer will actively address return issues, thereby positively impacting consumer surplus. Therefore, in scenario *LP*, consumers will prefer the manufacturer to bear the return costs, as it allows them to obtain more consumer surplus.

VII. Conclusion

The debate over pricing power in the livestreaming supply chain has become a prominent topic of discussion. Managing price execution is crucial as it directly influences profit distribution and pricing strategy between the manufacturer and the livestreamer. Additionally, factors such as consumer impulse purchases, live product demonstrations, and actual differences contribute to the high return rates observed in livestreaming. In this context, we establish a livestreaming supply chain system composed of a manufacturer and a livestreamer. We investigate how different pricing schemes and return business models affect supply chain stakeholders' profits, the livestreamer's selling efforts, optimal prices, and consumer surplus under the influence of channel spillover effects. Our research provides strategic insights into pricing power allocation strategies and return policies in the livestreaming supply chain. Specifically, the managerial insights derived from our study are summarized as follows:

(1) For the manufacturer, delegating pricing execution to the livestreamer is not always harmful. The results are highly related to the channel spillover coefficient. For the livestreamer, managing pricing execution always improves her profits. Therefore, the spillover relationships between channels provide important insights for supply chain stakeholders in formulating cooperative strategies.

(2) When the spillover effect of the livestreaming channel to the incumbent retail channel is significant, the livestreamer should manage the pricing execution by setting higher selling prices, and she also invests more efforts to drive sales. This suggests that the manufacturer should pay more attention to the price differentiation and conflicts between channels when opening live channels.

(3) Transferring the responsibility for return costs to the livestreamer is not conducive to improving the profits of supply chain members, and it will weaken the selling efforts invested by the livestreamer. Thus, the optimal return policy to incentivize the livestreamer is for the manufacturer to bear return costs. In the case of manufacturer pricing, the livestreamer bearing the cost of returns leads the manufacturer to adopt a low-price strategy.

(4) Confronted with the responsibility of return costs, when the commission rate is relatively low, the livestreamer will execute higher prices to alleviate cost pressures. In this case, manufacturer bearing return costs will improve consumer surplus and increase sales volume of livestreaming channel.

The limitations of this study provide possible directions for future research. Firstly, the revenue relationship between the livestreamer and the manufacturer may have more complex cooperative mechanisms such as betting agreements in addition to the commission system, which is worthy of being one of the subsequent research directions. Additionally, the return cost is assumed to be linear in this paper, while the practical return cost may have scale effects, and the unit return cost may decrease as the volume of returns increases.

Appendix

Proof of Lemma 1.

We solve the first order partial derivatives of Π_m^{MP-MR} with respect to p^{MP-MR} and Π_l^{MP-MR} with respect to e^{MP-MR} for scenario *MP-MR* as follows:

$$\frac{\partial \Pi_m^{MP-MR}}{\partial p^{MP-MR}} = (\alpha-1)(\phi-1)(\Delta+ek-p^{MP-MR}) + \alpha t - \gamma + (1-\alpha)(c_m+(\phi-1)p^{MP-MR})$$

$$\frac{\partial \Pi_l^{MP-MR}}{\partial e^{MP-MR}} = (1-\alpha)k\phi p^{MP-MR} - 2he$$

The second order conditions show $\frac{\partial^2 \Pi_m^{MP-MR}}{\partial (p^{MP-MR})^2} = -2(1-\alpha)(1-\phi) < 0$ and $\frac{\partial^2 \Pi_l^{MP-MR}}{\partial (e^{MP-MR})^2} = -2h < 0$. Subsequently, by solving the system of equations for $\frac{\partial \Pi_m^{MP-MR}}{\partial p^{MP-MR}} = 0$ and $\frac{\partial \Pi_l^{MP-MR}}{\partial e^{MP-MR}} = 0$, we obtain:

$$p^{MP-MR*} = \frac{2h((\alpha-1)(\phi-1)\Delta-(\alpha-1)c_m-\gamma+\alpha t)}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)}$$

$$e^{MP-MR*} = \frac{k\phi(\Delta(\alpha+\phi-\alpha\phi-1)+(\alpha-1)c_m+\gamma-\alpha t)}{(\phi-1)(4h+(\alpha-1)k^2\phi)}$$

Since we assume that the price p^{MP-MR*} is directly proportional to the production costs c_m, we derive the condition $\frac{\partial p^{MP-MR*}}{\partial c_m} = \frac{2h}{(1-\phi)(4h+(\alpha-1)k^2\phi)} > 0$, further obtaining the condition $4h+(\alpha-1)k^2\phi > 0$.

Submitting p^{MP-MR*} and e^{MP-MR*} into demand and profit functions, the equilibrium results are given by

$$q_L^{MP-MR*} = \frac{\begin{array}{c}\phi(\alpha-1)\big(2\Delta h+k^2(\gamma-\alpha t)\big)\\ +c_m(\alpha-1)(2h+(\alpha-1)k^2\phi)\\ +2h(\gamma+\Delta-\alpha(\Delta+t))\end{array}}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)}$$

$$\Pi_m^{MP-MR*} = \frac{(2h(\gamma+\Delta-\alpha(\Delta+t))+x_1+x_2)^2}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)^2}$$

$$\Pi_l^{MP-MR*} = \frac{h\phi(x_3+x_4)(x_3-x_4)}{(1-\alpha)(\phi-1)^2(4h+(\alpha-1)k^2\phi)}$$

Here, $x_1 = \phi(\alpha-1)(2\Delta h+k^2(\gamma-\alpha t))$, $x_2 = c_m(\alpha-1)(2h+(\alpha-1)k^2\phi)$, $x_3 = \Delta(\alpha-1)(\phi-1)$ and $x_4 = (1-\alpha)c_m-\gamma+\alpha t$.

Proof of Lemma 2 to Lemma 4.

The proofs for Lemmas 2-4 are similar to Lemma 1, and we choose to omit them.

Proof of Proposition 1.

(1) By observing $\Pi_m^{MP-MR*} - \Pi_m^{LP-MR*}$, we cannot directly determine whether the sign is positive or not. With respect to γ, we solve $\Pi_m^{MP-MR*} - \Pi_m^{LP-MR*} = 0$ and obtain

the following two roots:

$$\gamma_1 = (1-\alpha)c_m + \alpha t$$

$$\gamma_2 = \frac{\begin{gathered}4\alpha h^2 t + 2hk^2\phi(\alpha-1)(\Delta(\alpha+\phi-\alpha\phi-1)+2\alpha t)\\ +k^4\phi^2 t\alpha(\alpha-1)^2 - c_m(\alpha-1)(2h+(\alpha-1)k^2\phi)^2\end{gathered}}{(2h+(\alpha-1)k^2\phi)^2}$$

Since we observe that $\frac{\partial^2(\Pi_m^{MP-MR*}-\Pi_m^{LP-MR*})}{\partial\gamma^2} = \frac{2(2h+(\alpha-1)k^2\phi)^2}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)^2} > 0$. $\Pi_m^{MP-MR*} - \Pi_m^{LP-MR*}$ is a convex function with respect to γ. Further comparing the difference of the two roots γ_1 and γ_2, we find that $\gamma_1 - \gamma_2 = \frac{2(\alpha-1)^2\Delta hk^2(\phi-1)\phi}{(2h+(\alpha-1)k^2\phi)^2} < 0$, hence $\gamma_1 < \gamma_2$. We can deduce the conclusion $\Pi_m^{MP-MR*} > \Pi_m^{LP-MR*}$ if $\gamma < \gamma_1$ or $\gamma > \gamma_2$ and $\Pi_m^{MP-MR*} \le \Pi_m^{LP-MR*}$ if $\gamma_1 \le \gamma \le \gamma_2$.

(2) By comparing the profits of the livestreamer in scenarios *MP-MR* and *LP-MR*, we can obtain that $\Pi_l^{MP-MR*} - \Pi_l^{LP-MR*} = \frac{h\phi((\alpha-1)c_m+\gamma-\alpha t))^2}{(\alpha-1)(\phi-1)^2(4h+(\alpha-1)k^2\phi)} < 0$

Proof of Proposition 2.

To investigate the variation in selling efforts of the livestreamer in scenarios *MP-MR* and *LP-MR*, we derive the difference of e^{MP-MR*} and e^{LP-MR*}.

$$e^{MP-MR*} - e^{LP-MR*} = \frac{k\phi((\alpha-1)c_m+\gamma-\alpha t)}{(\phi-1)(4h+(\alpha-1)k^2\phi)}$$

The sign of $e^{MP-MR*} - e^{LP-MR*}$ is decided by the term $(\alpha-1)c_m+\gamma-\alpha t$. By solving $(\alpha-1)c_m+\gamma-\alpha t = 0$, we obtain the threshold $(1-\alpha)c_m+\alpha t$. Because $\frac{\partial(e^{MP-MR*}-e^{LP-MR*})}{\partial\gamma} = \frac{k\phi}{(\phi-1)(4h+(\alpha-1)k^2\phi)} < 0$, we can observe that $e^{MP-MR*} - e^{LP-MR*}$ decreases with the increase of γ. Therefore, we have $e^{MP-MR*} > e^{LP-MR*}$ if $\gamma < (1-\alpha)c_m+\alpha t$ and $e^{MP-MR*} \le e^{LP-MR*}$ if $\gamma \ge (1-\alpha)c_m+\alpha t$.

Similarly, the relationship between p^{MP-MR*} and p^{LP-MR*} can also be proved.

Proof of Corollary 1.

The first-order derivatives of e^{MP-MR*} and e^{LP-MR*} on α are given as:

$$\frac{\partial e^{MP-MR*}}{\partial\alpha} = \frac{k\phi(4hc_m - 4h(\Delta(\phi-1)+t)+k^2\phi(t-\gamma))}{(\phi-1)(4h+(\alpha-1)k^2\phi)^2}$$

$$\frac{\partial e^{LP\text{-}MR*}}{\partial\alpha} = -\frac{4\Delta hk\phi}{(4h+(\alpha-1)k^2\phi)^2}$$

Further, we have

$$\frac{\partial e^{MP-MR*}}{\partial\alpha} - \frac{\partial e^{LP-MR*}}{\partial\alpha} = \frac{k\phi(4h(c_m-t)+k^2\phi(t-\gamma))}{(\phi-1)(4h+(\alpha-1)k^2\phi)^2}$$

The term $4h(c_m-t)+k^2\phi(t-\gamma)$ decides the sign of $\frac{\partial e^{MP-MR*}}{\partial\alpha} - \frac{\partial e^{LP-MR*}}{\partial\alpha}$. Therefore, with γ as the threshold, we have $\gamma_3 = \frac{4h(c_m-t)+k^2t\phi}{k^2\phi}$. It can be concluded that $\frac{\partial e^{MP-MR*}}{\partial\alpha} - \frac{\partial e^{LP-MR*}}{\partial\alpha} < 0$ if $\gamma < \gamma_3$; otherwise, $\frac{\partial e^{MP-MR*}}{\partial\alpha} - \frac{\partial e^{LP-MR*}}{\partial\alpha} \ge 0$.

Similarly, we can derive that $\frac{\partial e^{MP-MR*}}{\partial\phi} - \frac{\partial e^{LP-MR*}}{\partial\phi} > 0$ if $\gamma < (1-\alpha)c_m+\alpha t$ and $\frac{\partial e^{MP-MR*}}{\partial\phi} - \frac{\partial e^{LP-MR*}}{\partial\phi} \le 0$ if $\gamma \ge (1-\alpha)c_m+\alpha t$

Proof of Corollary 2.

Taking the first-order partial derivatives of e^{LP-MR*} with respect to α and ϕ, we have

$$\frac{\partial e^{LP-MR*}}{\partial\alpha} = \frac{-4\Delta hk\phi}{(4h+(\alpha-1)k^2\phi)^2} < 0$$

$$\frac{\partial e^{LP-MR*}}{\partial\phi} = \frac{4(1-\alpha)\Delta hk}{(4h+(\alpha-1)k^2\phi)^2} > 0$$

Similarly, for the sensitivity of α and ϕ on p^{LP-MR*}, we can deduce that

$$\frac{\partial p^{LP-MR*}}{\partial\alpha} = \frac{-2\Delta hk^2\phi}{(4h+(\alpha-1)k^2\phi)^2} < 0$$

$$\frac{\partial p^{LP-MR*}}{\partial\phi} = \frac{2(1-\alpha)\Delta hk^2}{(4h+(\alpha-1)k^2\phi)^2} > 0$$

Proof of Proposition 3.

Through formula $CS = \int_{p_{min}}^{p_{max}} q_L dp = \frac{(\Delta-p+ke)^2}{2}$, we calculate the consumer surplus for both scenarios *MP-MR* and *LP-MR* as follows:

$$CS^{MP-MR*} = \frac{\begin{pmatrix}(\alpha-1)c_m(2h+(\alpha-1)k^2\phi)\\ +(\alpha-1)\phi(2\Delta h+k^2(\gamma-\alpha t))\\ +2h(\gamma+\Delta-\alpha(\Delta+t))\end{pmatrix}^2}{2(\alpha-1)^2(\phi-1)^2(4h+(\alpha-1)k^2\phi)^2}$$

$$CS^{LP-MR*} = \frac{2\Delta^2h^2}{(4h+(\alpha-1)k^2\phi)^2}$$

By deriving the difference of CS^{MP-MR*} and CS^{LP-MR*}, we have

$$CS^{MP-MR*} - CS^{LP-MR*} = -\frac{4\Delta^2h^2}{2(4h+(\alpha-1)k^2\phi)^2} + \frac{\begin{pmatrix}(\alpha-1)c_m(2h+(\alpha-1)k^2\phi)\\ +(\alpha-1)\phi(2\Delta h+k^2(\gamma-\alpha t))\\ +2h(\gamma+\Delta-\alpha(\Delta+t))\end{pmatrix}^2}{2(\alpha-1)^2(\phi-1)^2(4h+(\alpha-1)k^2\phi)^2}$$

Since $\frac{\partial^2(CS^{MP-MR*}-CS^{LP-MR*})}{\partial\gamma^2} = \frac{(2h+(\alpha-1)k^2\phi)^2}{(\alpha-1)^2(\phi-1)^2(4h+(\alpha-1)k^2\phi)^2}$ and this term is greater than 0. $CS^{MP-MR*} - CS^{LP-MR*}$ is convex on γ. Then, by solving the equation $CS^{MP-MR*} - CS^{LP-MR*} = 0$, we obtain two roots as follows:

$$\gamma_1 = (1-\alpha)c_m + \alpha t$$

$$\gamma_4 = \frac{\begin{gathered}4\Delta h(\alpha+\phi-1-\alpha\phi)+(\alpha-1)\alpha k^2t\phi\\ +(1-\alpha)c_m(2h+(\alpha-1)k^2\phi)+2\alpha ht\end{gathered}}{2h+(\alpha-1)k^2\phi}$$

Next, we discuss the magnitudes of two roots γ_1 and γ_4 and obtain

$$\gamma_1 - \gamma_4 = \frac{4\Delta h(\alpha-1)(\phi-1)}{2h+(\alpha-1)k^2\phi}$$

The sign of $\gamma_1 - \gamma_4$ is determined by the term $2h+(\alpha-1)k^2\phi$. Because $\frac{\partial(2h+(\alpha-1)k^2\phi)}{\partial\alpha} = k^2\phi > 0$, we further solve $2h+(\alpha-1)k^2\phi = 0$ with α as the threshold and obtain $\alpha = 1-\frac{2h}{k^2\phi}$. We can deduce that $\gamma_1 < \gamma_4$ if $0 < \alpha < 1-\frac{2h}{k^2\phi}$ and $\gamma_1 \ge \gamma_4$ if $1-\frac{2h}{k^2\phi} \le \alpha < 1$. Subsequently, we can summarize the conclusion stated in Proposition 3.

Proof of Proposition 4.

(1) By solving $\Pi_m^{MP-LR*} - \Pi_m^{MP-MR*} = 0$, we have

$$\gamma_5 = \frac{4\Delta h(\alpha+\phi-\alpha\phi-1)+\alpha k^2 t(\alpha-1)(2\phi-1) + 2\alpha h t - 2(\alpha-1)c_m(2h+(\alpha-1)k^2\phi)}{4h+2(\alpha-1)k^2\phi}$$

Because $\frac{\partial(\Pi_m^{MP-LR*}-\Pi_m^{MP-MR*})}{\partial\gamma} = \frac{2\alpha t(2h+(\alpha-1)k^2)(2h+(\alpha-1)k^2\phi)}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)^2}$, it can be observed that the term $A=(2h+(\alpha-1)k^2)(2h+(\alpha-1)k^2\phi)$ determines the sign of $\frac{\partial(\Pi_m^{MP-LR*}-\Pi_m^{MP-MR*})}{\partial\gamma}$. Further, the sign of A is determined by the following two thresholds

$$\alpha_1 = 1-\frac{2h}{k^2\phi}; \alpha_2 = 1-\frac{2h}{k^2}$$

Because $\frac{\partial^2(2h+(\alpha-1)k^2)(2h+(\alpha-1)k^2\phi)}{\partial\gamma^2} = 2k^4\phi > 0$, A is a parabola opening upward with respect to α. It can be concluded that $\frac{\partial(\Pi_m^{MP-LR*}-\Pi_m^{MP-MR*})}{\partial\gamma} > 0$ if $\alpha < 1-\frac{2h}{k^2\phi}$ or $\alpha > 1-\frac{2h}{k^2}$ and $\frac{\partial(\Pi_m^{MP-LR*}-\Pi_m^{MP-MR*})}{\partial\gamma} \le 0$ if $1-\frac{2h}{k^2\phi} \le \alpha \le 1-\frac{2h}{k^2}$. Based on the monotonicity in different scenarios mentioned above, we obtain the findings of Proposition 4 (1).

(2) With γ as the threshold, we solve for the root of $\Pi_l^{MP-LR*} - \Pi_l^{MP-MR*}$ as follows

$$\gamma_6 = (\alpha-1)\Delta(\phi-1)^2 - (\alpha-1)c_m + \frac{\alpha t\phi}{2}$$

Because $\frac{\partial(\Pi_l^{MP-LR*}-\Pi_l^{MP-MR*})}{\partial\gamma} = \frac{2\alpha h t}{(\alpha-1)(\phi-1)^2(4h+(\alpha-1)k^2\phi)} < 0$, it can be proved that $\Pi_l^{MP-LR*} - \Pi_l^{MP-MR*}$ is a monotonically decreasing function with respect to γ. We can conclude that $\Pi_l^{MP-LR*} > \Pi_l^{MP-MR*}$ if $\gamma_1 < \gamma_6$; otherwise, we have $\Pi_l^{MP-LR*} \le \Pi_l^{MP-MR*}$.

Proof of Proposition 5.

With the condition of $0<\alpha<1$, $0<\phi<1$ and $4h+(\alpha-1)k^2\phi>0$, we compare the selling efforts and selling prices between scenarios *MP-LR* and *MP-MR*.

$$e^{MP-LR*} - e^{MP-MR*} = \frac{\alpha k t(2-\phi)}{(\phi-1)(4h+(\alpha-1)k^2\phi)} < 0$$

$$e^{MP-LR*} - e^{MP-MR*} = -\frac{\alpha t(2h+(\alpha-1)k^2(\phi-1))}{(\alpha-1)(\phi-1)(4h+(\alpha-1)k^2\phi)} < 0$$

Proof of Corollary 3.

(1) We compute the first-order partial derivatives of e^{MP-LR*} and e^{MP-MR*} with respect to α and subtract them to obtain

$$\frac{\partial e^{MP-LR*}}{\partial\alpha} - \frac{\partial e^{MP-MR*}}{\partial\alpha} = \frac{kt(\phi-2)(k^2\phi-4h)}{(\phi-1)(4h+(\alpha-1)k^2\phi)^2}$$

The term $k^2\phi-4h$ determines the sign of the above expression. Then, we can deduce that $\frac{\partial e^{MP-LR*}}{\partial\alpha} - \frac{\partial e^{MP-MR*}}{\partial\alpha} < 0$ if $0<\phi<\frac{4h}{k^2}$ and $\frac{\partial e^{MP-LR*}}{\partial\alpha} - \frac{\partial e^{MP-MR*}}{\partial\alpha} \ge 0$ if $\frac{4h}{k^2} \le \phi < 1$.

(2) Similarly, Corollary 3 (2) can be proved.

Proof of Proposition 6.

(1) By deriving the difference of the profits of manufacturer's profits in scenarios *MP-MR* and *LP-MR* and solving the equation $\Pi_m^{LP-LR*} - \Pi_m^{LP-MR*} = 0$, we obtain the threshold

$$\gamma_7 = \frac{2h(\alpha t(\phi-1)) + (\alpha-1)k^2\phi((1-\alpha)\Delta\phi + \alpha t(\phi-1)) + ((1-\alpha)\phi c_m(4h+(\alpha-1)k^2\phi)) - 4(\alpha-1)\Delta h\phi^2}{\phi(4h+(\alpha-1)k^2\phi)}$$

Next, we will examine the monotonicity of the term $\Pi_m^{LP-LR*} - \Pi_m^{LP-MR*}$ and obtain $\frac{\partial(\Pi_m^{LP-LR*}-\Pi_m^{LP-MR*})}{\partial\gamma} = \frac{2\alpha h t}{(\alpha-1)\phi(4h+(\alpha-1)k^2\phi)} < 0$. In other words, $\Pi_m^{LP-LR*} - \Pi_m^{LP-MR*}$ decreases as γ increases. Then, we can conclude that $\Pi_m^{LP-LR*} > \Pi_m^{LP-MR*}$ if $\gamma < \gamma_7$ and $\Pi_m^{LP-LR*} \le \Pi_m^{LP-MR*}$ if $\gamma \ge \gamma_7$.

(2) By deriving the difference of the profits of the livestreamer in scenarios *LP-LR* and *LP-MR*, we have

$$\Pi_l^{LP-LR*} - \Pi_l^{LP-MR*} = \frac{\alpha h t(\alpha t - 2(1-\alpha)\Delta\phi)}{(1-\alpha)\phi(4h+(\alpha-1)k^2\phi)}$$

It can be observed that the term $\alpha t - 2(1-\alpha)\Delta\phi < 0$ in the numerator determines the sign of $\Pi_l^{LP-LR*} - \Pi_l^{LP-MR*}$. From the non-negative demand expression $q_L^{LP-LR*} = \frac{2h((\alpha-1)\Delta\phi+\alpha t)}{(\alpha-1)\phi(4h+(\alpha-1)k^2\phi)} > 0$ in scenario *LP-LR*, we can obtain the condition $\alpha t < (1-\alpha)\Delta\phi$, and further deduce that $\alpha t - 2(1-\alpha)\Delta\phi < 0$. Therefore, it is straightforward to have $\Pi_l^{LP-LR*} - \Pi_l^{LP-MR*} < 0$ all the time.

Proof of Proposition 7.

(1) By comparing the selling efforts and demand in scenario *LP-LR* and *LP-MR*, we obtain

$$e^{LP-LR*} - e^{LP-MR*} = \frac{-\alpha k t}{4h+(\alpha-1)k^2\phi} < 0$$

$$q_L^{LP-LR*} - q_L^{LP-MR*} = \frac{2\alpha h t}{\phi(\alpha-1)(4h+(\alpha-1)k^2\phi)} < 0$$

(2) Next, we derive the difference of p^{LP-LR*} and p^{LP-MR*}

$$p^{LP-LR*} - p^{LP-MR*} = \frac{-\alpha t(2h+(\alpha-1)k^2\phi)}{\phi(\alpha-1)(4h+(\alpha-1)k^2\phi)}$$

Taking ϕ as the threshold and solving for $p^{LP-LR*} - p^{LP-MR*} = 0$, we obtain $\phi = \frac{2h}{(1-\alpha)k^2}$. Because $\frac{\partial(p^{LP-LR*}-p^{LP-MR*})}{\partial\phi} = \frac{\alpha t}{2(\alpha-1)}(\frac{1}{\phi^2} + \frac{(\alpha-1)^2k^4}{(4h+(\alpha-1)k^2\phi)^2}) < 0$, we can find that $p^{LP-LR*} - p^{LP-MR*}$ decreases with the increase of ϕ. Next, we can conclude that $p^{LP-LR*} > p^{LP-MR*}$ if $0 < \phi < \frac{2h}{(1-\alpha)k^2}$ and $p^{LP-LR*} \le p^{LP-MR*}$ if $\frac{2h}{(1-\alpha)k^2} \le \phi < 1$.

Proof of Proposition 8.

The consumer surplus in scenarios *LP-LR* and *LP-MR* is obtained as follows:

$$CS^{LP-LR*} = \frac{2((\alpha-1)\Delta h\phi + \alpha h t)^2}{(\alpha-1)^2\phi^2(4h+(\alpha-1)k^2\phi)^2}$$

$$CS^{LP-MR*} = \frac{2\Delta^2 h^2}{(4h+(\alpha-1)k^2\phi)^2}$$

By deriving the difference of CS^{LP-LR*} and CS^{LP-MR*}, we have

$$CS^{LP-LR*} - CS^{LP-MR*} = \frac{2\alpha h^2 t(2(\alpha-1)\Delta\phi + \alpha t)}{(\alpha-1)^2\phi^2(4h+(\alpha-1)k^2\phi)^2}$$

Similar to the proof of Proposition 6 (2), we can obtain $\alpha t - 2(1-\alpha)\Delta\phi < 0$. Therefore, we have $CS^{LP-LR*} - CS^{LP-MR*} = \frac{2\alpha h^2 t(2(\alpha-1)\Delta\phi+\alpha t)}{(\alpha-1)^2\phi^2(4h+(\alpha-1)k^2\phi)^2} < 0$.

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