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A Numerical Simulation of the Kratom Plant Growth Model While Treated by a Specific Nutrient Using an Explicit Finite Difference Method

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***Abstract*—Kratom refers to both Mitragyna speciosa, a tree native to Southeast Asia, and products manufactured from its leaves sold as herbal supplements. Kratom leaves contain a range of chemical compounds known as bioactive alkaloids, which have physiological effects. A mathematical model of the Kratom plant under a particular nutritional treatment will be provided in this research. Also, the methods for setting the initial condition and boundary condition will be presented. Also, as the plant grows, the solution's domain shifts every time. Techniques for adjusting the specific nutrient's physical parameters are also provided. With the use of an explicit finite difference method, the solutions are approximated. The specific nutritional concentrations are calculated for each height level. As shown, the specific nutrient will spread from the root to the apex of the trunk. The nutrient has the capacity to stimulate the growth of the Kratom. The specific nutrient concentration along the trunk may be measured using the proposed mathematical model as the Kratom plant grows each day. A proposed numerical model with a specific nutrient can be used to develop a precise model, such as a one-dimensional model of branches and foliage. It would be more captivating if the plant nutrients indicated here were researched for their ability to accelerate the growth of large or medium-sized Kratom plants. In conclusion, the study shows that calcium dihydrogen phosphate monohydrate may be useful as a growth promoter for Kratom plants and suggests a way to measure its effects quantitatively.**

***Index Terms*—Kratom plant, Specific nutrient, Plant growth, Mathematical model, Finite difference method**

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I. INTRODUCTION

MITRAGYNA speciosa, a tree native to Southeast Asia, and items made from its leaves that are sold as herbal supplements are both referred to as Kratom. The leaves of kratom contain a variety of chemical substances called bioactive alkaloids that have physiological effects. Mitragynine and 7-hydroxymitragynine are the kratom-related substances that have received the most research [1]. Although the U.S. Food and Drug Administration has not declared kratom or its related chemicals safe and effective for any medical use, users claim that kratom products help them manage mental health issues, pain, and opioid withdrawal symptoms and cravings [3,4,11,12,13,14,15,16].

Kratom is commonly consumed by people as raw plant material in the form of capsules or powder, mixed into food or beverages, brewed as a tea, or taken orally as a liquid extract [1]. Both stimulant-like effects (increased energy, alertness, and high heart rate) and opioid- and sedative-like effects (relaxation, pain alleviation, and disorientation) are reported by kratom users [10,17]. Research and case studies have also revealed that kratom or certain kratom components may occasionally cause uncommon adverse effects [1,9]. According to anthropologists, kratom has been used for hundreds of years in traditional medicine in Southeast Asia as a multipurpose cure to boost energy and alertness while working and during social events [18]. Although estimates of the extent of kratom usage in the United States differ, [19] the growth of kratom suppliers and the rise in the number of case reports indicate that kratom use has increased over the past 20 years [20].

Kratom, or Mitragyna speciosa, is a tropical, small to medium-sized (4–16 m) tree that is native to Southeast Asian wetland forests. Kratom has a long history of usage as a moderate herbal stimulant, painkiller, and treatment for diarrhea and opium addiction in Thailand, Malaysia, and Indonesia [14]. Research on the cultivation and usage of kratom is necessary, given that it has historically been used as an analgesic and a medication to lessen opioid withdrawal symptoms. Kratom leaves are grown in Southeast Asia and are chewed or steeped in water to produce tea [14]. Because fresh kratom is not readily available in the Western Hemisphere, it is offered either as a dried and powdered powder or as a concentrated liquid extract [14].

Phyllotaxis frequently treats a plant as a predetermined geometrical entity without mentioning the underlying biological processes that cause the patterns to be seen. Plants may be viewed from a variety of perspectives as dynamic things that vary in size and shape over time as a result of various growth mechanisms. The reaction-diffusion systems and Turing structures are connected to the most well-known pattern creation method in mathematical biology [15,16,17,18]. There is no scientific proof that this process actually contributes to the development of biological patterns [15]. The optimization mechanism is used in some other methods. For instance, plants' branching patterns may be connected to maximizing light absorption [15]. [15] studies plant topology and design. One may get further information on plant modeling in [15]. Several experimental studies have demonstrated a connection between the expression of certain genes and the development of plant organs [15].

In [15], they describe the dynamics of developing plants, including how their size changes over time and how their morphologies arise. We must determine the key components of the growth process in order to propose a mathematical model of plant growth. Nutrients originating from the root through the xylem, metabolites created by the plant and dispersed throughout it by the phloem, and plant cells that multiply and absorb nutrients and metabolites are all included in the simplest and most schematic representation.

A mathematical model of the Kratom plant under a particular nutritional treatment will be provided in this research. Also, the methods for obtaining the initial condition and boundary condition will be discussed. Also, as the plant grows, the solution's domain varies. Techniques for setting the specific nutrient's physical parameters are also presented. With the use of an explicit finite difference method, the solutions are calculated. At each height level, the specific nutritional concentrations will be determined. It will be shown how a certain nutrient may boost the development of the Kratom.

This study uses calcium dihydrogen phosphate monohydrate (Ca(H2PO4)2H2O) [19] as a fertilizer as the specific nutrient to treat the Kratom. It was successfully created utilizing prepared triple superphosphate (TSP), which is made from oyster shell waste, as the starting material for a recrystallization process. This bio-green, eco-friendly method of making a crucial fertilizer can support the development of a sustainable civilization. The distilled water-dissolved TSP made from shell waste was maintained at 30, 50, and 80 °C.

This study will introduce a mathematical model of the Kratom plant under a specific nutritional treatment. Methods for setting the initial and boundary conditions will also be given.

II. GOVERNING EQUATION

A. Kratom plant growth model

Another approach to plant modelling is based on attempts to describe the kinetics of plant growth. If $L(t)$ is the plant size that depends on time t, then we can consider the empirical equation [15]:

$$\frac{dL}{dt} = F(L), \qquad (1)$$

where F can be proportional to L as an autocatalytic growth.

To simplify the modeling, we do not include photosynthesis. Assuming that there is enough light and that it is distributed equally, photosynthesis will not be a limiting stage for metabolite creation. As a result, it will be implicitly considered in the model's parameters. Another simplification is that we do not consider root growth. We shall assume that the plant originates at ground level.

Fig. 1. Kratom plants in a field experiment.

Given that plants have a very basic structure and that their developing portion is firmly confined, fairly intuitive mathematical models to describe their growth are suggested. In free-boundary problems, plant development is described by the motion of the interface, which is equivalent to the displacement of the apical meristem. The meristem's self-accelerating production of plant growth factors and the plant's internal nutrient diffusion and convection fluxes regulate the interface's rate of development or mobility [15].

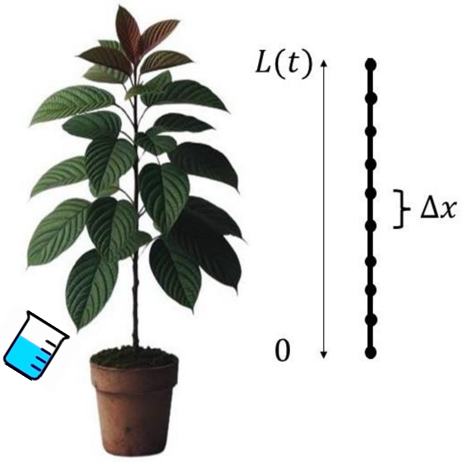

Fig. 2. Kratom plant height is transformed into a varied domain. In a plant container, 100 cm^3 of calcium dihydrogen phosphate monohydrate is applied to the soil twice a day, at 6:00 a.m. and 6:00 p.m.

If the plant's length L (or height) is substantially more than the diameter of its trunk, then a one-dimensional example is justified. As a result, we take into account the interval $0 \leq x \leq L(t)$, whose length varies with time. The root is represented by the left endpoint $x = 0$. Its function is to supply the nutrient flux that is considered by the boundary condition. The root development is not modeled here. The left boundary is therefore established. As seen in Fig. 2, the apex is represented by the right endpoint, $x = L(t)$. Compared to the plant, it is substantially narrower. In the model, we assume that it's a mathematical point. Over time, the value $L(t)$ rises. The concentration of metabolites at , which we represent by R, determines the growth rate based on the previous assumption. Thus [15]

$$\frac{dL}{dt} = f(R), \tag{2}$$

where the function $f(R)$ will be specified by the rate of change of plant length over time t.

Recall that differentiated cells that transport nutrients from the root to the apex are represented by interval $0 \le x \le L(t)$. They are in a liquid solution, we assume. Their concentration, which depends on x and t, is indicated by C. Supposing the fluid is incompressible and fills the xylem (the portion of plant tissue inside the cambium layer that conducts nutrients from below to above) less equally. The diffusion equation describes its evolution,

$$\frac{\partial C}{\partial t} = d\frac{\partial^2 C}{\partial x^2}, \tag{3}$$

for all $(x,t) \in [0, L(t)] \times [0, \Gamma]$, where d is the diffusion coefficient.

B. The Initial and Boundary Conditions

We complete equation (3) by setting the boundary conditions

$$C(0,t) = C_0, \tag{4}$$

and

$$\frac{\partial C}{\partial x}(L(t),t) = C_A, \tag{5}$$

where C_0 is the concentration of the specified nutrient on root and C_A is the rate of change of concentration around the apex. The second boundary condition demonstrates that the concentration $C(L(t),t)$ is proportionate to the nutrient flux from the plant's main body to the meristem.

The plant has potential nutrient. As a result, the initial conditions become,

$$C(x,0) = g(x), \tag{6}$$

for all $x \in [0, L(t)]$.

III. NUMERICAL TECHNIQUES FOR SOLUTION OF GOVERNING EQUATION

We now discretize the domain of Eq.(3) by dividing the interval $[0, L(t)]$ into M subintervals such that $M\Delta x = L(t)$ and the time interval $[0, \Gamma]$ into N subintervals such that $N\Delta t = \Gamma$. We then approximate $C(x_m, t_n)$ by C_m^n , at the point $x_m = m\Delta x$ and $t_n = n\Delta t$, where $0 \le m \le M$ and $0 \le n \le N$ in which M and N are positive integers.

The forward time-centered space finite difference method is used to approximate their solutions. Consequently, the finite difference approximation becomes [20],

$$C(x_m, t_n) \cong C_m^n, \tag{7}$$

$$\frac{\partial C}{\partial t} \cong \frac{C_m^{n+1} - C_m^n}{\Delta t}, \tag{8}$$

$$\frac{\partial^2 C}{\partial x^2} \cong \frac{C_{m+1}^n - 2C_m^n + C_{m-1}^n}{(\Delta x)^2}, \tag{9}$$

Substituting Eqs.(7)-(9) into Eq.(3), we obtain,

$$\frac{C_m^{n+1} - C_m^n}{\Delta t} \cong \frac{C_{m+1}^n - 2C_m^n + C_{m-1}^n}{(\Delta x)^2}, \tag{10}$$

for $1 \le m \le M-1$and $0 \le n \le N-1$. Equation (10) can be written in an explicit form of finite difference as follows,

$$C_m^{n+1} \cong \mu C_{m+1}^n + (1-2\mu)C_m^n + \mu C_{m-1}^n, \tag{11}$$

for $1 \le m \le M-1$ and $0 \le n \le N-1$. where $\mu = \frac{\Delta t}{(\Delta x)^2}$.

IV. NUMERICAL SIMULATION

A. Numerical simulation of the Kratom plant growth model while treated by a specified nutrient

As a specific nutrient, calcium dihydrogen phosphate monohydrate (Ca(H2PO4)2H2O) [19] is applied to a Kratom plant at a concentration of 5 mg in 1000 cm^3 of clean water. In a plant container, calcium dihydrogen phosphate monohydrate with 100 cm^3 is added to the soil twice a day, at 6:00 a.m. and 6:00 p.m., as shown in Fig. 2.

The Kratom plant has an initial height of 50 cm. The specific nutrient has physical parameters such as a diffusion coefficient of 1 cm^2 per day and a concentration around the root of 0.1 mg/l.

Assuming that there is no rate of change in the nutrient concentration around the apex and that there is no initial nutrient along the Kratom trunk. Their parameters are assumed as shown in Table I.

The Kratom plant is treated with the nutrient for 120 days at a static concentration as show in Fig. 1. The growth of the plant is measured by its height, as shown in Table II and Fig. 3.

We will approximate their solutions to the governing equations Eqs. (3–6) by employing Eqs. (7–10).

The specific nutrient concentration (mg/l) over three height levels (25, 50, and 75 cm) is predicted by the mathematical model. The predicted specific nutrient concentration is shown in Fig. 4 and Table III.

TABLE I
PARAMETER SETTING

Parameters	Values
d	1 cm^2/day
C_0	0.10 mg/l
C_A	0
g	0 mg/l
Γ	120 days

TABLE II
THE HEIGHT OF TREATED AND UNTREATED KRATOM PLANTS 0-120 DAYS.

Day	Treated Height (cm)	Untreated Height (cm)
0	50.00	50.00
20	54.00	53.20
40	70.40	53.50
60	77.80	53.80
80	80.60	54.30
100	83.20	56.20
120	87.00	58.10

TABLE III
NUTRIENT CONCENTRATION OVER 3 HEIGHT LEVELS: 25, 50, AND 75 CM

	Nutrient concentration (mg/l)		
Day, Height	25(cm)	50(cm)	75(cm)
0	0	-	-
30	0.01331	0.00265	-
60	0.02788	0.00498	0.00498
90	0.03755	0.01003	0.00687
120	0.04440	0.01466	0.00436

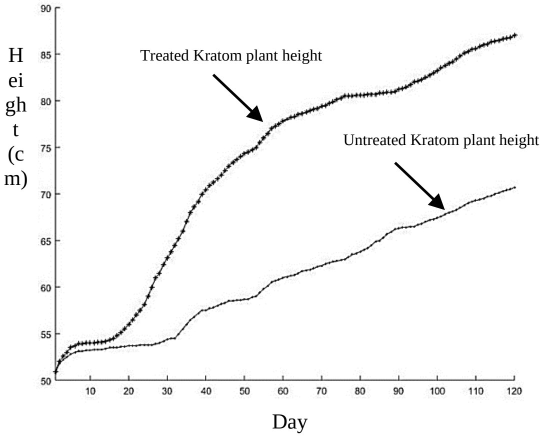

Fig. 3. The growth of the plant is measured by its height along 120 days.

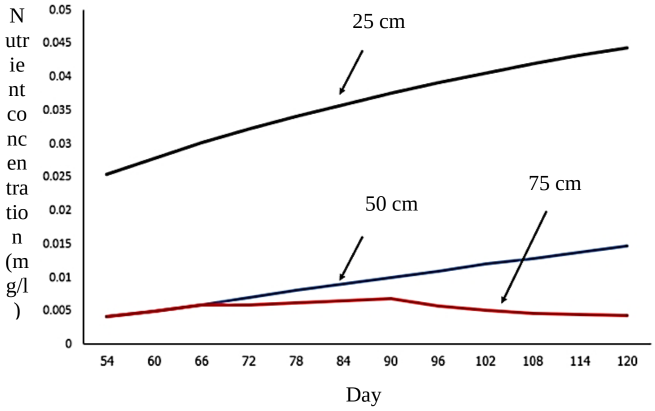

Fig. 4. The specific nutrient concentration (mg/l) over three height levels: 25, 50, and 75 cm between 54-120 days.

V. DISCUSSION

The specific nutrient will diffuse along the trunk from the root to the apex, as can be seen. The Kratom plant that has been filled with the specific nutrient grows faster than the typical control Kratom plants, as seen in Fig. 4 and Table III.

From Table II and Fig. 3, we can see that the treated Kratom plant by the calcium dihydrogen phosphate monohydrate (Ca(H2PO4)2H2O) grows faster than the untreated Kratom plant. This indicates that the specific nutrient, calcium dihydrogen phosphate monohydrate, has the ability to increase Kratom growth.

Table II and Figure 3 indicate that Kratom plants treated with calcium dihydrogen phosphate monohydrate exhibit faster growth compared to untreated plants. This suggests that the specific nutrient, Ca(H2PO4)2H2O, can positively influence Kratom plant growth.

VI. CONCLUSION

Calcium dihydrogen phosphate monohydrate (Ca(H2PO4)2H2O) is used as a particular nutrition in Kratom plants at a dosage of 5 mg per 1000 cm^3 of pure water. Calcium dihydrogen phosphate monohydrate (100 cm^3) is given to the soil twice a day, at 6:00 a.m. and 6:00 p.m. A mathematical model of the Kratom plant under a specific nutritional treatment is suggested. The strategies for setting the initial condition and the boundary condition are introduced. Due to the plant's growth, the solution's domain changes. Techniques for adjusting the particular nutrient's physical parameters are also presented. With the use of an explicit finite difference method, the solutions are approximated. At each height level, the specific nutrient concentrations are determined. As can be observed, the particular nutrient will spread up the trunk from the root to the apex. The nutrient has the ability to accelerate Kratom's growth. The proposed mathematical model can be used to measure the specific nutrient concentration along the trunk while the plant is growing each day. It is achievable to develop a precise numerical model with specific nutrients, such as a one-dimensional model with branches and foliage. The plant nutrients presented here would be interesting if they could be tested to accelerate the growth of medium-sized and large Kratom plants. The proposed mathematical model shows that calcium dihydrogen phosphate monohydrate (Ca(H2PO4)2H2O) used as a particular nutrition can help the Kratom plant grow faster than the untreated Kratom plant.

In summary, the research proposes a mathematical method for analyzing the impacts of calcium dihydrogen phosphate monohydrate and shows its potential as a growth stimulant for Kratom plants.

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