Subsidy Rules of Blockchain Technology for Low-carbon Supply Chain Considering Consumers' Perceived Distrust Level

Pan Liu, Zihou Liu, Haodong Tang, Jiamin Zhu, Ye Li, Muhammad Ahmad

***Abstract*—To investigate the subsidy strategy of blockchain technology for low-carbon supply chains considering consumers' perceived mistrust level, this paper first examines the potential applications of blockchain technology in the supply chain. Subsequently, the effects of government subsidy strategies on the supply chain benefits was analyzed, upon which a low-carbon supply chain game model involving a single producer and a single retailer was constructed. Four investment subsidy models were considered and analyzed. The following conclusions are reached: (1) CERD and CPDL are the main factors affecting producer and retailer profitability. (2) The cost subsidy coefficients of the producer and the retailer are positively correlated with their respective profits, while government revenue initially increases and then decreases as subsidies to both parties increase. (3) While formulating policies, the government should not consider the subsidy costs that reduce consumer perceived distrust levels.**

***Index Terms*—low-carbon supply chain; consumers' perceived distrust level; blockchain technology; subsidy rules**

I. Introduction

In response to the global climate anomalies caused by substantial greenhouse gas emissions, the Chinese government has formally pledged to strive to reach a peak in carbon dioxide (CO_2) emissions by 2030 and to achieve carbon neutrality by 2060, a goal known as the 'dual-carbon' target [1]. To achieve this long-term goal, it is essential to foster a collaborative relationship between the government and the market. According to the government's carbon emission policy, manufacturing enterprises will implement technologies for carbon emission reduction and engage in research and development of low-carbon products.

At the same time, along with the public's growing concern for green issues, consumers are also increasingly preferring low-carbon products [2], and this change in the consumer market has prompted enterprises to undertake low-carbon transition. However, for consumers, since they are far away from the production and retail links of products, they have relatively little knowledge of low-carbon information about products, which leads to their distrust of whether the products they buy are low-carbon products and the low-carbon level of the products. [3].

The disclosure of product carbon emissions reduction information can enhance consumer trust in low-carbon products; however, the unreliability of the information may, in turn, lead to a reduction in consumer trust. Most existing studies concur that the implementation of blockchain technology can effectively alleviate consumer distrust of product's carbon emission reduction information [4]. This study argues that consumers' perceived distrust of the product carbon reduction is the key to determining whether blockchain technology is applied. Accordingly, this study introduces the level of consumers' perceived distrust towards the degree of carbon emission reduction of products to explore the optimal subsidy strategy for the application of blockchain technology in low-carbon supply chains.

In China, the implementation of blockchain technology is characterized by the coordinated promotion of the market and the government. [5]. In order to encourage the integrated development of blockchain technology and low-carbon industries, the government has introduced incentives to guide enterprises to apply blockchain technology. In recent years, China and its local governments at all levels have introduced supportive policies to accelerate the integration of blockchain technology with various industries, as illustrated in Table 1. Consequently, the primary objective of this study is to investigate the optimal blockchain subsidy strategy for low-carbon supply chains, taking into account the level of consumers' perceived distrust. This investigation aims to address the following questions:

(1) Which subsidy input cost application is most effective?

(2) How can governments design subsidy strategies to maximize benefits for all parties?

To address the above issues, based on a low-carbon supply chain with a producer and a retailer, considering the changes of the CPDL, our study constructed and analyzed four

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investment subsidy models: applying and not applying blockchain technology, and subsidizing and not subsidizing blockchain technology.

TABLE I
SELECTED SUBSIDY POLICIES FOR BLOCKCHAIN TECHNOLOGY APPLICATIONS IN CHINA

Time	Policy Title and Related Content
2017	The Guidance on Actively Promoting Supply Chain Innovation and Application proposed to study the use of blockchain, artificial intelligence and other emerging technologies to establish a supply chain-based credit evaluation mechanism.
2017	The Guidance on Deepening the "Internet + Advanced Manufacturing" and Developing the Industrial Internet proposed to promote research and exploration of the application of blockchain and other emerging frontier technologies in the industrial Internet.
2019	The Outline for the Construction of a Strong Transportation Country proposed to promote the deep integration of new technologies such as blockchain with the transportation industry.
2019	The Guidance on Promoting High-Quality Development of Trade proposed to promote the organic integration of blockchain with trade and accelerate the cultivation of new dynamic energy.
2020	The Opinions on Leading the Accelerated Development of New Consumption with New Industries and New Models proposed to promote the development and integration of technologies such as blockchain and accelerate the application of blockchain in digital scenarios such as commodity traceability, cross-border remittances, supply chain finance and e-bills.
2021	The Opinions on Promoting the High-Quality Development of Public Hospitals proposed to promote the deep integration of new-generation information technologies such as blockchain with medical services.
2021	The Notice on the Issuance of the Fourteenth Five-Year Plan for the Development of Cold Chain Logistics proposed to accelerate the application of blockchain in the construction of intelligent monitoring and tracing systems for cold chain logistics and improve the authenticity, timeliness and credibility of tracing information.
2022	The Circular on the Issuance of Guidelines for the Construction of National Integrated Government Affairs Big Data System proposed to promote "blockchain + government services", "blockchain + government data sharing", "blockchain + social governance "and other scenarios of application innovation.
2023	The Guiding Opinions on Further Promoting Cross-sectoral Comprehensive Supervision proposes accelerating the application of technologies such as big data, artificial intelligence, IoT perception and blockchain, and actively carrying out off-site supervision featuring departmental collaborative remote supervision, mobile supervision, early warning, prevention and control, and so on.

Source from: http://www.gov.cn/

The main contributions of this study are (1) introducing the level of consumers' perceived distrust of the degree of carbon emission reduction of products, and revising the demand function of low-carbon products. (2) Considering the blockchain input cost subsidy and the consumer perceived distrust level reduction subsidy, the blockchain subsidy game model of low-carbon supply chain under three subsidy modes is constructed. (3) Compare the practical effects of the three subsidy modes with the control mode without applying blockchain technology in order to explore the optimal blockchain technology application subsidy mode for the government.

II. LITERATURE REVIEW

This study focused on the blockchain application in low-carbon supply chain and the impacts of government subsidy strategies on supply chain management, and this section collated and analyzed the relevant literature in the above areas.

A. The Application of Blockchain Technology in the Supply Chain

Over the past few years, blockchain technology has been widely used in various supply chains, and its distributed, transparent, and traceable characteristics played an important role in improving supply chain decision-making [6]-[8]. In the vaccine supply chain, the introduction of a blockchain-constructed vaccine traceability and anti-counterfeiting platform significantly improved the safety and economic efficiency of vaccines [3]. In the fresh food supply chain, the application of a blockchain-based anti-counterfeit traceability system had a positive impact on improving the unreliability of fresh product freshness information [9]-[11]. In the port and shipping supply chain, the investment in blockchain technology could also enhance the transparency and operational efficiency of the shipping business and improve the digitalization of the shipping industry [12]. In addition, blockchain technology was also widely used in the supply chain of pharmaceutical products, green agricultural products and luxury goods, significantly improving supply chain management [13]-[15].

Blockchain technology application in low-carbon supply chains is relevant to our paper. The blockchain application played a significant role in improving the synergy of low-carbon decision-making and the transparency of low-carbon information among the component companies of the supply chain [16]. In the low-carbon supply chain, the application of blockchain technology was mainly focused on low-carbon product information traceability and low-carbon enterprise carbon data monitoring, etc. For the blockchain investment study in different dual-channel supply chains, Jiang and Liu [17] argued that the application of this technology could lead to joint emission reduction among supply chain members. Ju et al. [18] affirmed the value of the carbon footprint traceability system built on blockchain technology in quickly tracing and tracking the carbon footprint data of enterprises and products. carbon footprint data. In addition, some low-carbon companies were working on blockchain applications to make carbon data transparent, Yang et al. [19] focused on the effects of the still opaque market information on the efficiency of such value-added services and explored the strategic choices between different information sharing modes and blockchain technology applications among low-carbon supply chain members.

B. The Impact of Government Subsidy Strategies on the Supply Chain

The choice of government investment subsidy strategies will have a huge impact on supply chain revenue. Many scholars have studied the government's subsidy decision-making problems. The government's subsidy strategies for supply chain enterprises mainly include output subsidies, technology subsidies and fixed subsidies. Of course, in some literature, these subsidy strategies also have other similar expressions, such as quantity subsidies [20], unit production subsidies, innovation effort subsidies [21], [22], etc. Brown et al. [23] found that regulators using output subsidies can reduce the cost burden of emission pricing and effectively alleviate rising electricity prices. Zhong et al. [20]

found in the study of dual-channel supply chain tracking product information that innovation subsidies can always obtain better social welfare than quantity subsidies. Yue et al. [24] argue that government subsidies can compensate for the lack of existence of technological innovation subjects and promote knowledge transformation and diffusion. Liu et al. [25] argue that producers and retailers investing in blockchain traceability services can increase revenues, but there is also an inflection point after which costs increase and revenues instead decrease, while increased retailer subsidies for blockchain traceability services can reduce mistrust of freshness information, which in turn increases sales. Nie et al. [26] 's research found that output subsidies can obtain a higher consumer surplus and a lower producer surplus than fixed subsidies. Li et al. [27] compared and analyzed the effects of three strategies adopted by the government: no subsidy, product subsidy and innovation subsidy, and obtained the government's optimal subsidy plan under different scenarios. Xie et al. [22] proposed the application environment of unit production subsidies and innovation subsidies when analyzing the choice of vaccine supply chain subsidies with risk-averse buyers. Chen et al. [28] 's research also involved these two types of subsidy strategies. In addition, the problems of supply chain subsidy strategies under centralized decision-making and decentralized decision-making [28]- [30] and the optimal decision-making problems of single and dual-channel supply chains [31], [32] have also received much discussion and exploration from many scholars. But government subsidies need to be controlled in terms of how much and to whom they are given, otherwise, they can lead to a (counter)tragedy of the commons [33].

The foregoing research indicated that studies on blockchain application in supply chain management and government investment subsidy strategies have been burgeoning. However, existing research exhibited some limitations as delineated below: (1) The existing literature on blockchain technology application in low-carbon supply chains did not take into account the role of consumer perceived unreliability of carbon reduction information about low-carbon products. (2) The existing literature focused on comparing the investment effectiveness of three single subsidy strategies, namely output subsidy, technology subsidy and fixed subsidy, and lacked comparative studies on the effectiveness of mixed subsidy strategies.

Therefore, our study considered the influence of the CPDL and constructed three models to address the aforementioned research lacuna: one subsidizing blockchain technology application cost, another subsidizing the CPDL reduction, and a hybrid model combining the two approaches.

III. Problem Description

In our study, a second-tier supply chain consisting of a retailer and a producer of low-carbon products was selected as the research object. Considering the change of the CPDL, we modified the demand function and constructed four investment subsidy models (Low-carbon product producer and retailer did not apply in blockchain technology, called PRN model; producer and retailer both applied in blockchain technology, and the government provided a certain percentage of the cost subsidy for blockchain technology application, called PRC model; the government subsidized the CPDL reduction, called PRR model; the government subsidized the blockchain technology application cost and the CPDL reduction simultaneously, called PRCR model). We derived the feasible interval of investment subsidy cost by analyzing the CPDL and the influence rules of the subsidy coefficient on the equilibrium price and the profits, as well as comparing and analyzing the revenues of the government, the producer and the retailer under different models.

A. Parameter Descriptions

The involved parameters of our study and their meaning are delineated in TABLE II:

TABLE II
PARAMETERS DESCRIPTION

Parameter	Meaning
c	Production cost of low-carbon products
w^i	Wholesale price of low-carbon products in the i model, $i=\{PRN, PRC, PRR, PRCR\}$
s_1	Cost subsidy factor for producer applying blockchain technology in the PRC model
s_2	Cost subsidy factor for retailers applying blockchain technology in the PRC model
s_3	Cost subsidy factor for producers applying blockchain technology in the PRCR model
s_4	Cost subsidy factor for retailer applying blockchain technology in the PRCR model
c_{op}	Blockchain technology application cost for producers
c_{or}	Blockchain technology application cost for retailers
F	Subsidy cost parameters for the CPDL reduction in the PRR model
F'	Subsidy cost parameters for the CPDL reduction in the PRCR model
γ^i	The CPDL in the i model
p^i	Retail prices of low-carbon products in the i model
D^i	Demand for low-carbon products in the i model
a	Potential market demand
e	The CERD
π_j^i	Profits of j in the i model, $j=\{r, p, g\}$

B. Research Hypothesis

The game-theoretic model developed in our study assumed the following premises:

Assumption 1: Before adopting blockchain technology, consumers face two products (one is a low-carbon product and the other is a non-low-carbon product) that differ only on the CERD, the number of consumers who choose the low-carbon product m. In the same situation, after adopting blockchain technology, the number of consumers who choose the low-carbon product is $(1+\gamma)m$. The percentage of consumers who did not trust in the CERD of the low-carbon product is $((1+\gamma)m-m)/m=\gamma$. In our study, we used γ to denote the CPDL. The demand is determined by the sales price, the CPDL and the CERD, thus the demand function is $D^i=a-p^i-\gamma^i e$.

Assumption 2: Consumers have environmentally sustainable preferences and are willing to consider the CERD. And they are risk-neutral and perfectly rational at the same time.

Assumption 3: In this model, the producer, as the leader in the supply chain game, determines the wholesale price and the CERD. The retailer is the follower of the game and sets the retail price. In addition, the producer in the supply chain has the ideal production capacity with sufficient production capacity to respond to consumer market demand.

Assumption 4: The government's profits consist of four components: the producer's profits, the retailer's profits, the consumer's surplus, and other government fiscal expenditures, where the consumer's surplus is $CS=\int_{p_i}^{p_{mas}} D^i dp=(D^i)^2/2$, and other fiscal expenditures include government subsidies to the retailer and the producer for their blockchain technology application cost and the CPDL reduction.

IV. Analyses of Subsidy Strategies

A. PRN Model

In the PRN model, the producer and the retailer do not choose to adopt the blockchain, and there is no other financial expenditure from the government. At this time, the CPDL is γ^{PRN}, and the revenues of the retailer, the producer, and the government are as follows:

$$\pi_r^{PRN}=(p^{PRN}-w^{PRN})D^{PRN} \tag{1}$$

$$\pi_p^{PRN}=(w^{PRN}-c)D^{PRN} \tag{2}$$

$$\pi_g^{PRN}=\pi_p^{PRN}+\pi_r^{PRN}+CS^{PRN} \tag{3}$$

Using the inverse solution method and based on Eq. (1), Eq. (2) and Eq. (3), the equilibrium prices, the quantity demand, and the revenues of the retailer, the producer, and the government are calculated as follows (the analytical procedure is shown in Appendix A):

$$\begin{cases} w^{PRN*}=\dfrac{a+c-e\gamma^{PRN}}{2} \\ p^{PRN*}=\dfrac{3a+c-3e\gamma^{PRN}}{4} \\ D^{PRN*}=\dfrac{a-c-e\gamma^{PRN}}{4} \end{cases} \tag{4}$$

$$\begin{cases} \pi_r^{PRN*}=\dfrac{(c-a+e\gamma^{PRN})^2}{16} \\ \pi_p^{PRN*}=\dfrac{(c-a+e\gamma^{PRN})^2}{8} \\ \pi_g^{PRN*}=\dfrac{7(c-a+e\gamma^{PRN})^2}{32} \end{cases} \tag{5}$$

Proposition 1:

(1) $\dfrac{\partial w^{PRN*}}{\partial e}<0$, $\dfrac{\partial p^{PRN*}}{\partial e}<0$, $\dfrac{\partial D^{PRN*}}{\partial e}<0$, $\dfrac{\partial \pi_r^{PRN*}}{\partial e}<0$, $\dfrac{\partial \pi_p^{PRN*}}{\partial e}<0$, $\dfrac{\partial \pi_g^{PRN*}}{\partial e}<0$;

(2) $\dfrac{\partial w^{PRN*}}{\partial \gamma^{PRN}}<0$, $\dfrac{\partial p^{PRN*}}{\partial \gamma^{PRN}}<0$, $\dfrac{\partial D^{PRN*}}{\partial \gamma^{PRN}}<0$, $\dfrac{\partial \pi_r^{PRN*}}{\partial \gamma^{PRN}}<0$, $\dfrac{\partial \pi_p^{PRN*}}{\partial \gamma^{PRN}}<0$, $\dfrac{\partial \pi_g^{PRN*}}{\partial \gamma^{PRN}}<0$.(The parsing process is shown in Appendix B-1).

Proposition 1 shows that the CERD and the CPDL, are negatively related to the wholesale price, the retail price, the quantity demand, and the profits of the retailer, the producer and the government. The CERD and the CPDL synergistically affect the wholesale price, the retail price, the demand, and the profits of chain members. When the CERD is maintained, the wholesale price, the retail price, the demand and the profits gained by chain members can increase by reducing the CPDL. Conversely, when the CPDL is maintained, the wholesale price, the retail price, the demand and the benefits of chain members will be decreased by increasing the CERD.

B. PRC Model

In the PRC model, to stimulate the motivation of the supply chain members to apply blockchain technology, the government provides them with a certain percentage of cost subsidies. The cost subsidy coefficients s_1 and s_2 are introduced for the producer and the retailer to apply in blockchain technology, and the CPDL is γ^{PRC}. At this time, the revenues of the retailer, the producer and the government are as shown below:

$$\pi_r^{PRC}=[p^{PRC}-w^{PRC}-(1-s_2)c_{or}]D^{PRC} \tag{6}$$

$$\pi_p^{PRC}=[w^{PRC}-c-(1-s_1)c_{op}]D^{PRC} \tag{7}$$

$$\pi_g^{PRC}=\pi_p^{PRC}+\pi_r^{PRC}+CS^{PRC}-(s_1c_{op}+s_2c_{or})D^{PRC} \tag{8}$$

Similarly with the PRN model, let $\partial\pi_r^{PRC}/\partial p^{PRC}=0$, $\partial\pi_p^{PRC}/\partial w^{PRC}=0$, and w^{PRC*}, p^{PRC*}, D^{PRC*}, π_r^{PRC*}, π_p^{PRC*}, π_g^{PRC*} are obtained as follows:

$$\begin{cases} w^{PRC*}=\dfrac{a+c+c_{op}+c_{or}s_2-e\gamma^{PRC}-c_{op}s_1-c_{or}}{2} \\ p^{PRC*}=\dfrac{3a+c+c_{op}+c_{or}-3e\gamma^{PRC}-c_{op}s_1-c_{or}s_2}{4} \\ D^{PRC*}=\dfrac{c_{op}s_1+c_{or}s_2-e\gamma^{PRC}-E}{4} \end{cases} \tag{9}$$

$$\begin{cases} \pi_r^{PRC*}=\dfrac{(c_{op}s_1+c_{or}s_2-e\gamma^{PRC}-E)^2}{16} \\ \pi_p^{PRC*}=\dfrac{(c_{op}s_1+c_{or}s_2-e\gamma^{PRC}-E)^2}{8} \\ \pi_g^{PRC*}=\dfrac{(E+e\gamma^{PRC}-c_{op}s_1-c_{or}s_2)(7E+7e\gamma^{PRC}+c_{op}s_1+c_{or}s_2)}{32} \end{cases} \tag{10}$$

Among them, $E=c+c_{op}+c_{or}-a$, the same below.

Proposition 2:

(1) $\dfrac{\partial w^{PRC*}}{\partial e}<0$, $\dfrac{\partial p^{PRC*}}{\partial e}<0$, $\dfrac{\partial D^{PRC*}}{\partial e}<0$, $\dfrac{\partial \pi_r^{PRC*}}{\partial e}<0$, $\dfrac{\partial \pi_p^{PRC*}}{\partial e}<0$;

(2) When $e < \frac{3c_{op}s_1 + 3c_{or}s_2 - 7E}{7\gamma^{PRC}}$, $\frac{\partial \pi_g^{PRC*}}{\partial e} < 0$; otherwise, $\frac{\partial \pi_g^{PRC*}}{\partial e} > 0$;

(3) $\frac{\partial w^{PRC*}}{\partial \gamma^{PRC}} < 0$, $\frac{\partial p^{PRC*}}{\partial \gamma^{PRC}} < 0$, $\frac{\partial D^{PRC*}}{\partial \gamma^{PRC}} < 0$, $\frac{\partial \pi_r^{PRC*}}{\partial \gamma^{PRC}} < 0$, $\frac{\partial \pi_p^{PRC*}}{\partial \gamma^{PRC}} < 0$;

(4) When $\gamma^{PRC} < \frac{3s_1c_{op} + 3s_2c_{or} - E}{7e}$, $\frac{\partial \pi_g^{PRC*}}{\partial \gamma^{PRC}} < 0$; otherwise, $\frac{\partial \pi_g^{PRC*}}{\partial \gamma^{PRC}} > 0$ (The parsing process is shown in Appendix B-2).

From Proposition 2, it can be seen that the effects of the CERD and the CPDL on the wholesale price, the retail price, the demand, and the revenues of the supply chain members are consistent with Proposition 1, which are also negatively correlated. The effects of the two parameters (i.e., the CERD and the CPDL) on the government's profits are divided into two stages. If the CERD is maintained at a certain level and the CPDL is less than $3s_1c_{op} + 3s_2c_{or} - E/7e$, the CPDL is negatively correlated with the government's profits, and higher government profits can be obtained by reducing the CPDL. Conversely, when the CPDL is greater than $3s_1c_{op} + 3s_2c_{or} - E/7e$, the CPDL is positively correlated with the government's profits, and the government's profits will decrease with the decrease of the CPDL. In contrast, when the CPDL is certain, the CERD has a similar effect on the government's profits.

Proposition 3:

(1) $\frac{\partial w^{PRC*}}{\partial s_1} < 0$, $\frac{\partial p^{PRC*}}{\partial s_1} < 0$, $\frac{\partial D^{PRC*}}{\partial s_1} > 0$, $\frac{\partial \pi_r^{PRC*}}{\partial s_1} > 0$, $\frac{\partial \pi_p^{PRC*}}{\partial s_1} > 0$;

(2) When $s_1 < \frac{-3E - 3e\gamma^{PRC} - c_{or}s_2}{c_{op}}$, $\frac{\partial \pi_g^{PRC*}}{\partial s_1} > 0$; otherwise, $\frac{\partial \pi_g^{PRC*}}{\partial s_1} < 0$;

(3) $\frac{\partial w^{PRC*}}{\partial s_2} > 0$, $\frac{\partial p^{PRC*}}{\partial s_2} < 0$, $\frac{\partial D^{PRC*}}{\partial s_2} > 0$, $\frac{\partial \pi_r^{PRC*}}{\partial s_2} > 0$, $\frac{\partial \pi_p^{PRC*}}{\partial s_2} > 0$;

(4) When $s_2 < -3E - 3e\gamma^{PRC} - c_{op}s_1/c_{or}$, $\partial \pi_g^{PRC*}/\partial s_2 > 0$, otherwise, $\partial \pi_g^{PRC*}/\partial s_2 < 0$ (The parsing process is shown in Appendix B-3).

From Proposition 3, it can be seen that the effects of the blockchain cost subsidy coefficients on the producer and the retailer on the supply chain under the PRC model remain the same. Both of them are negatively correlated with the retail price, and when one of them is maintained at a certain level, the retail price decreases with the increase of the other. Both of them are positively correlated with the demand, and profits of the retailer and the producer. When one of them is maintained at a certain level, the demand, and the profits of the retailer and the producer increase with the increase of the other variable. The effects of the cost subsidy coefficients s_1 s_2 on the government's profits are divided into two stages. Namely, if s_2 remains certain and s_1 is less than $(-3E - 3e\gamma^{PRC} - c_{or}s_2)/c_{op}$, s_1 is positively correlated with the government's profit. Therefore, the government's profits will increase with the increase of the subsidy coefficient s_1 . And when s_1 is greater than that $(-3E - 3e\gamma^{PRC} - c_{or}s_2)/c_{op}$, s_1 is negatively correlated with the government's profits. Conversely, if s_1 is held constant, s_2 is positively correlated with the government's profits when s_2 is less than $(-3E - 3e\gamma^{PRC} - c_{op}s_1)/c_{or}$, and negatively correlated with it when s_2 is greater than $(-3E - 3e\gamma^{PRC} - c_{op}s_1)/c_{or}$. In addition, the cost subsidy coefficients s_1、 s_2 have an opposite effect on the wholesale price. To be specific s_1 is negatively related to the wholesale price and s_2 is positively related to it.

C. PRR Model

In the PRR model, supply chain members also choose to apply blockchain, and the application costs for the producer and the retailer are the same as those in the aforementioned PRC model. The government provides economic subsidies based on the amount of reduction from the CPDL. Here, the subsidy cost parameter of the CPDL reduction F is introduced. At this time, the CPDL is γ^{PRR}. The revenues of the retailer, the producer, and the government are as below:

$$\pi_r^{PRR} = [p^{PRR} - w^{PRR} - c_{or}]D^{PRC} + F(\gamma^{PRN} - \gamma^{PRR}) \tag{13}$$

$$\pi_p^{PRR} = [w^{PRR} - c - c_{op}]D^{PRR} + F(\gamma^{PRN} - \gamma^{PRR}) \tag{14}$$

$$\pi_g^{PRR} = \pi_r^{PRR} + \pi_p^{PRR} + CS^{PRR} - 2F(\gamma^{PRN} - \gamma^{PRR}) \tag{15}$$

Similarly with the PRN model, let

$\partial \pi_r^{PRR}/\partial p^{PRR} = 0$, $\partial \pi_p^{PRR}/\partial w^{PRR} = 0$,

and w^{PRR*} , p^{PRR*} , D^{PRR*} , π_r^{PRR*} , π_p^{PRR*} , π_g^{PRR*} are obtained as follows:

$$\begin{cases} w^{PRR*} = \dfrac{a + c + c_{op} - c_{or} - e\gamma^{PRR}}{2} \\ p^{PRR*} = \dfrac{3a + c + c_{op} + c_{or} - 3e\gamma^{PRR}}{4} \\ D^{PRR*} = \dfrac{-E - e\gamma^{PRR}}{4} \end{cases} \tag{11}$$

$$\begin{cases} \pi_r^{PRR*} = \dfrac{(E + e\gamma^{PRR})^2}{16} + F(\gamma^{PRN} - \gamma^{PRR}) \\ \pi_p^{PRR*} = \dfrac{(E + e\gamma^{PRR})^2}{8} + F(\gamma^{PRN} - \gamma^{PRR}) \\ \pi_g^{PRR*} = \dfrac{7(E + e\gamma^{PRR})^2}{32} \end{cases} \tag{12}$$

Proposition 4:

(1) $\frac{\partial w^{PRR*}}{\partial e}<0$, $\frac{\partial p^{PRR*}}{\partial e}<0$, $\frac{\partial D^{PRR*}}{\partial e}<0$, $\frac{\partial \pi_r^{PRR*}}{\partial e}<0$, $\frac{\partial \pi_p^{PRR*}}{\partial e}<0$, $\frac{\partial \pi_g^{PRR*}}{\partial e}<0$;

(2) $\frac{\partial w^{PRR*}}{\partial \gamma^{PRR}}<0$, $\frac{\partial p^{PRR*}}{\partial \gamma^{PRR}}<0$, $\frac{\partial D^{PRR*}}{\partial \gamma^{PRR}}<0$, $\frac{\partial \pi_r^{PRR*}}{\partial \gamma^{PRR}}<0$, $\frac{\partial \pi_p^{PRR*}}{\partial \gamma^{PRR}}<0$, $\frac{\partial \pi_g^{PRR*}}{\partial \gamma^{PRR}}<0$;

(3) $\frac{\partial w^{PRR*}}{\partial F}=0$, $\frac{\partial p^{PRR*}}{\partial F}=0$, $\frac{\partial D^{PRR*}}{\partial F}=0$, $\frac{\partial \pi_r^{PRR*}}{\partial F}>0$, $\frac{\partial \pi_p^{PRR*}}{\partial F}>0$, $\frac{\partial \pi_g^{PRR*}}{\partial F}=0$ (The parsing process is shown in Appendix B-4).

In Proposition 4, the CERD and the CPDL have the same effects on the wholesale price, the retail price, the demand, or the profits of chain members with them in Proposition 1. The subsidy cost parameter of the CPDL reduction is not correlated with the wholesale price, the retail price, the demand and the government's profits, but is positively correlated with the profits of the retailer and the producer, so the profits of the retailer and the producer can be improved by increasing the subsidy cost parameter of the CPDL reduction.

D. PRCR Model

In the PRCR model, for the blockchain application by the producer and the retailer, the government provides the blockchain application cost subsidy and the CPDL reduction subsidy simultaneously. The revenues of the retailer, the producer, and the government are as below:

$$\pi_r^{PRCR}=[p^{PRCR}-w^{PRCR}-(1-s_4)c_{or}]D^{PRCR} +F^{'}(\gamma^{PRN}-\gamma^{PRCR}) \tag{16}$$

$$\pi_p^{PRCR}=[w^{PRCR}-c-(1-s_3)c_{op}]D^{PRCR}+F^{'}(\gamma^{PRN}-\gamma^{PRCR}) \tag{17}$$

$$\pi_g^{PRCR}=\pi_p^{PRCR}+\pi_r^{PRCR}+CS^{PRCR}-(s_3c_{op}+s_4c_{or})D^{PRCR} -2F^{'}(\gamma^{PRN}-\gamma^{PRCR}) \tag{18}$$

Similarly with the PRN model, let $\frac{\partial \pi_r^{PRCR}}{\partial p^{PRCR}}=0$, $\frac{\partial \pi_p^{PRCR}}{\partial w^{PRCR}}=0$,and w^{PRCR*} , p^{PRCR*} , D^{PRCR*} , π_r^{PRCR*} , π_p^{PRCR*} , π_g^{PRCR*} can be obtained as follows:

$$\begin{cases} w^{PRCR*}=\dfrac{a+c+c_{op}+s_4c_{or}-c_{or}-e\gamma^{PRCR}-s_3c_{op}}{2} \\ p^{PRCR*}=\dfrac{3a+c+c_{op}+c_{or}-3e\gamma^{PRCR}-s_3c_{op}-s_4c_{or}}{4} \\ D^{PRCR*}=\dfrac{s_3c_{op}+s_4c_{or}-E-e\gamma^{PRCR}}{4} \end{cases} \tag{19}$$

$$\begin{cases} \pi_r^{PRCR*}=\dfrac{(s_3c_{op}+s_4c_{or}-E-e\gamma^{PRCR})^2}{16}+F^{'}(\gamma^{PRN}-\gamma^{PRCR}) \\ \pi_p^{PRCR*}=\dfrac{(s_3c_{op}+s_4c_{or}-E-e\gamma^{PRCR})^2}{8}+F^{'}(\gamma^{PRN}-\gamma^{PRCR}) \\ \pi_g^{PRCR*}=\dfrac{(E+e\gamma^{PRCR}-s_3c_{op}-s_4c_{or})(7E+7e\gamma^{PRCR}+s_3c_{op}+s_4c_{or})}{32} \end{cases} \tag{20}$$

Proposition 5:

(1) $\frac{\partial w^{PRCR*}}{\partial e}<0$, $\frac{\partial p^{PRCR*}}{\partial e}<0$, $\frac{\partial D^{PRCR*}}{\partial e}<0$, $\frac{\partial \pi_r^{PRCR*}}{\partial e}<0$, $\frac{\partial \pi_p^{PRCR*}}{\partial e}<0$;

(2) When $e<\frac{3c_{op}s_3+3c_{or}s_4-7E}{7\gamma^{PRCR}}$, $\frac{\partial \pi_g^{PRCR*}}{\partial e}<0$; otherwise, $\frac{\partial \pi_g^{PRCR*}}{\partial e}>0$;

(3) $\frac{\partial w^{PRCR*}}{\partial \gamma^{PRCR}}<0$, $\frac{\partial p^{PRCR*}}{\partial \gamma^{PRCR}}<0$, $\frac{\partial D^{PRCR*}}{\partial \gamma^{PRCR}}<0$, $\frac{\partial \pi_r^{PRCR*}}{\partial \gamma^{PRCR}}<0$, $\frac{\partial \pi_p^{PRCR*}}{\partial \gamma^{PRCR}}<0$;

(4) When $\gamma^{PRCR}<\frac{3c_{op}s_3+3c_{or}s_4-7E}{7e}$, $\frac{\partial \pi_g^{PRCR*}}{\partial \gamma^{PRCR}}<0$; otherwise, $\frac{\partial \pi_g^{PRCR*}}{\partial \gamma^{PRCR}}>0$;

(5) $\frac{\partial w^{PRCR*}}{\partial F^{'}}=0$, $\frac{\partial p^{PRCR*}}{\partial F^{'}}=0$, $\frac{\partial D^{PRCR*}}{\partial F^{'}}=0$, $\frac{\partial \pi_r^{PRCR*}}{\partial F^{'}}>0$, $\frac{\partial \pi_p^{PRCR*}}{\partial F^{'}}>0$, $\frac{\partial \pi_g^{PRCR*}}{\partial F^{'}}=0$ (The parsing process is shown in Appendix B-5).

Proposition 5 indicates that the CERD and the CPDL are also negatively correlated with the wholesale price, the retail price, the demand, and the revenue of retailer and producer. Meanwhile, the effects of them on the government's profits are also divided into two stages. If the CERD is maintained at a certain level and $\gamma^{PRCR}<(3c_{op}s_3+3c_{or}s_4-7E)/7e$, γ^{PRCR} is negatively related to the government's profits. When $\gamma^{PRCR}>(3c_{op}s_3+3c_{or}s_4-7E)/7e$, γ^{PRCR} is positively related to the government's profits. Conversely, if the CPDL is maintained at a certain level, the CERD on the government's profits is also similar. The subsidy cost parameter $F^{'}$ is also not related to the wholesale price, the retail price, the demand and the government's profits, but is positively related to the profit of the retailer and the producer. Similarly, we can increase the value of the subsidy cost parameter $F^{'}$ to increase the revenues of the supply chain members.

Proposition 6:

(1) $\frac{\partial w^{PRCR*}}{\partial s_3}<0$, $\frac{\partial p^{PRCR*}}{\partial s_3}<0$, $\frac{\partial D^{PRCR*}}{\partial s_3}>0$,

$\frac{\partial \pi_r^{PRCR*}}{\partial s_3} > 0$, $\frac{\partial \pi_p^{PRCR*}}{\partial s_3} > 0$;

(2) When $s_3 < \frac{-3E - 3e\gamma^{PRCR} - c_{or}s_4}{c_{op}}$, $\frac{\partial \pi_g^{PRCR*}}{\partial s_3} > 0$; otherwise, $\frac{\partial \pi_g^{PRCR*}}{\partial s_3} < 0$;

(3) $\frac{\partial w^{PRCR*}}{\partial s_4} > 0$, $\frac{\partial p^{PRCR*}}{\partial s_4} < 0$, $\frac{\partial D^{PRCR*}}{\partial s_4} > 0$, $\frac{\partial \pi_r^{PRCR*}}{\partial s_4} > 0$, $\frac{\partial \pi_p^{PRCR*}}{\partial s_4} > 0$;

(4) When $s_4 < \frac{-3E - 3e\gamma^{PRCR} - c_{op}s_3}{c_{or}}$, $\frac{\partial \pi_g^{PRCR*}}{\partial s_4} > 0$; otherwise, $\frac{\partial \pi_g^{PRCR*}}{\partial s_4} < 0$ (The parsing process is shown in Appendix B-6).

In Proposition 6, the effects of the cost subsidy coefficient for the producer blockchain application s_3 and the cost subsidy coefficient for the retailer blockchain application s_4 on the supply chain are similar to Proposition 3. s_3 is negatively correlated with the wholesale price and s_4 is positively correlated with the wholesale price. s_3 and s_4 are negative with the retail prices, while positive with the demand, the retailer's revenue and the producer's revenue. In a perfectly competitive market, producer often adopt a low-price strategy to gain a competitive edge; hence, producer subsidies tend to reduce product prices. Similarly, when retailers provide subsidies, such as through coupons or discounts, this is typically aimed at increasing sales volume or market share. Such subsidies usually increase the retailer's costs, and in order to maintain profitability, the retailer may demand that the producer lower the wholesale price. However, if the retailer's subsidy strategy is highly successful and leads to an increase in product demand, the retailer might raise the retail price of the product, representing a dynamic game-theoretic process. The effects of the cost subsidy coefficient s_3 and s_4 on the government's profits are divided into two stages. If s_4 is maintained, when $s_3 < (-3E - 3e\gamma^{PRCR} - c_{or}s_4)/c_{op}$, s_3 is positively correlated with the government's profits, then by raising the value of the cost subsidy coefficient s_3, the government can get greater profits. And when $s_3 > (-3E - 3e\gamma^{PRCR} - c_{or}s_4)/c_{op}$, s_3 is negatively correlated with the government's profits, then raising the cost subsidy coefficient s_3 will cause a decline in the government's profits. Conversely, if s_3 is maintained, s_4 has a similar effect on the government's profits.

E. Comparative Analyses of Subsidy Strategies

In this section, the conditions for blockchain technology application in a low-carbon supply chain are explored by comparing the government's profits with and without using blockchain technology.

Proposition 7: The blockchain technology application can be considered only when $\Delta\gamma \geq \min(\delta_1, \delta_2, \delta_3)$ ($\Delta\gamma$ means the reduction of the CPDL) (The parsing process is shown in Appendix C-1).

Due to the blockchain application in a low-carbon supply chain, the CPDL is reduced. However, the CPDL reduction should meet a certain amount so that the government's profits with applying blockchain technology is greater than the profits without applying blockchain technology. Otherwise, it means that the blockchain technology application cannot achieve the expected profits and will cause the investment to fall short of income. Proposition 7 obtains the threshold values of applying blockchain technology about the three applied blockchain technology models. Only when the CPDL reduction $\Delta\gamma$ is greater than δ_1, δ_2 and δ_3 respectively, the blockchain application in the models of PRC, PRR and PRNR can recover the input cost and the choice of applying blockchain technology is economically feasible.

Proposition 8: Among the three models of applying blockchain in a low-carbon supply chain, if the CPDL remains consistent and $s_m c_{op} + s_n c_{or}$ is less than $-3E - 3e\gamma^i$, the government's profits is positively correlated with the per unit cost subsidy value of blockchain application for the producer and the retailer $s_m c_{op} + s_n c_{or}$. Conversely, they are negatively correlated (The parsing process is shown in Appendix C-2).

By comparing the government profit in the models of PRC, PRR, and PRCR, it can be found that the government's profits are not related to the subsidy value of the CPDL reduction. When the CPDL is at the same level for the three models after applying blockchain, there is a certain function relationship between the government's profits and the per unit cost subsidy value of the blockchain application for the producer and the retailer $s_m c_{op} + s_n c_{or}$. When the per unit cost subsidy value $s_m c_{op} + s_n c_{or}$ is smaller $-3E - 3e\gamma^i$, higher government profits can be obtained by increasing the per unit cost subsidy value. When the per unit cost subsidy value $s_m c_{op} + s_n c_{or}$ is larger than $-3E - 3e\gamma^i$, the subsidy will not be sufficient if the per unit cost subsidy value continues to increase. It is also clear that when the per unit cost subsidy value of government subsidy per unit of blockchain application cost for the producer and the retailer is $-3E - 3e\gamma^i$, the government's profits will reach the maximum.

V. Numerical Analyses

To verify the validity of the above conclusions, this section uses MATLAB to perform a numerical simulation of the relevant parameters. Based on the basic assumptions, our study selected the business information of Sunshine Rose Grapes produced by China Jiaxing Green River Grape Professional Cooperative as the data source. The relevant information showed that the carbon label applied by the Green River Grape Professional Cooperative can record the greenhouse gas emissions throughout the life cycle of the grapes, and according to the project team's tracking, measurement, and calculation, each kilogram of Sunshine Rose grapes produces 0.824 kg of carbon dioxide.

Based on the information of data related to Sunshine Rose grapes, the parameters of this paper were set as follows: $a=4$, $c=2$, $c_{op}=0.3$, $c_{or}=0.2$, $F=0.2$, $F^{'}=0.1$. Take γ, e, s_1, s_2, s_3, s_4 as the independent variable parameters and satisfy $\gamma\in[0,1]$, $e\in[0,1]$, $s_1\in[0,1]$, $s_2\in[0,1]$, $s_3\in[0,1]$, $s_4\in[0,1]$.

A. The Effect of the CPDL on the Supply Chain in Different Models

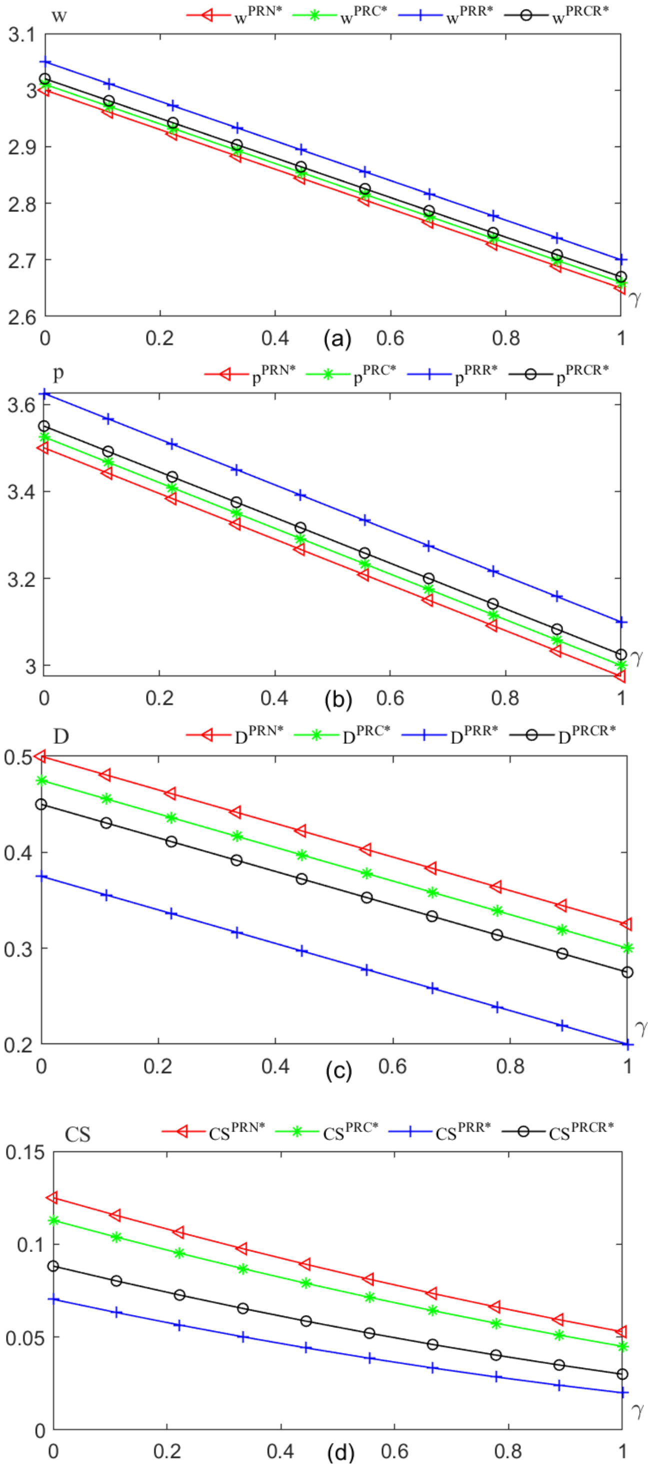


Fig .1. The impacts of the CPDL on the supply chain

From Fig .1 (a) to Fig. (d), we can see that the CPDL is negatively correlated with the wholesale price, the retail price, the demand and the consumer surplus of low-carbon products in the proposed four models.

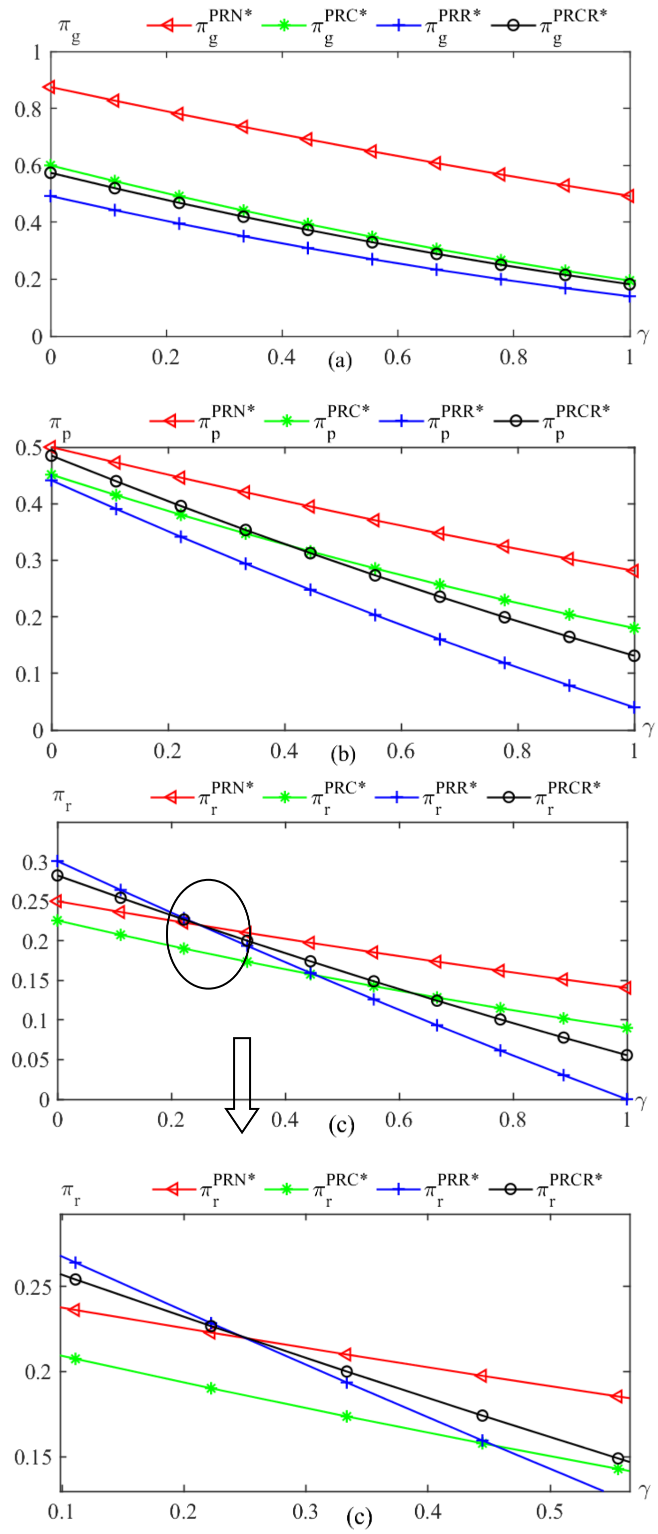
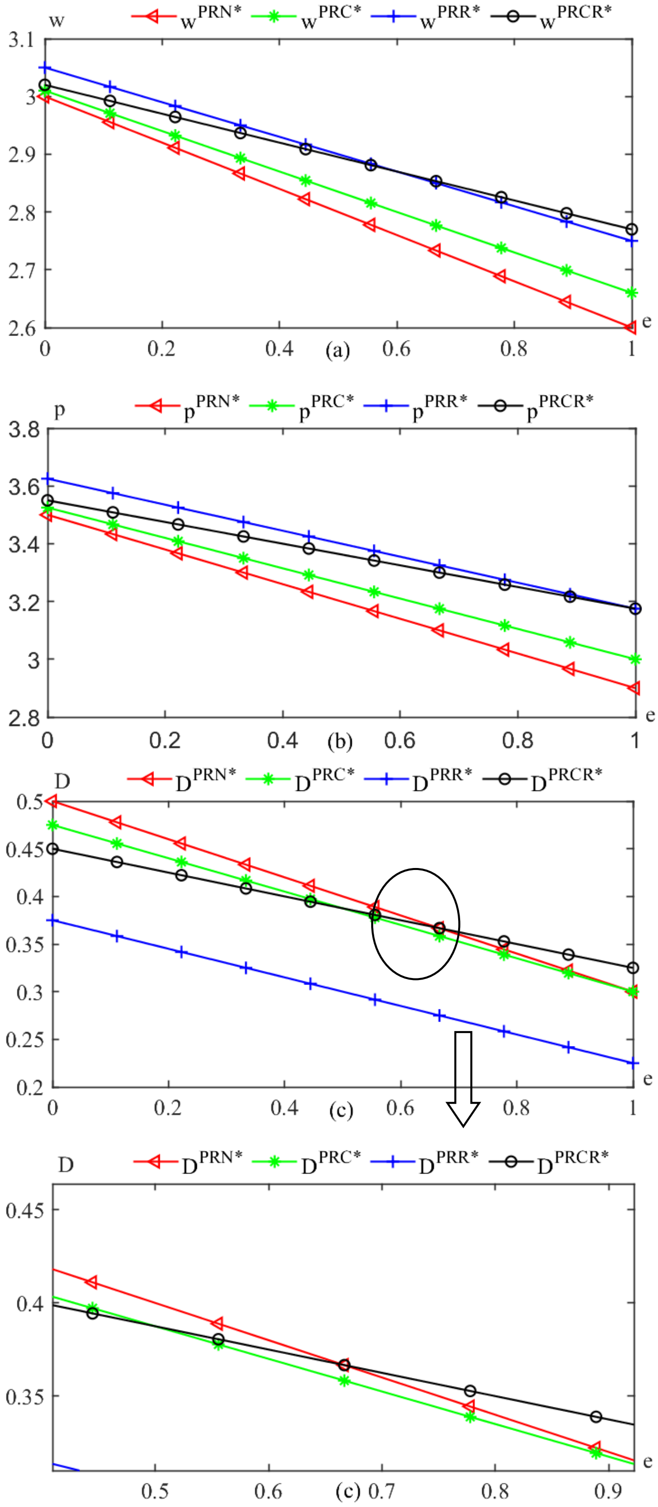


Fig. 2. The impact of the CPDL on the supply chain profits

From Fig. 2 (a) to Fig. (c), it can be seen that the profits of the retailer, the producer and the government all show decreasing results as the CPDL increases, in which the relationships between the profits of the retailer and the producer and the CPDL are consistent with the conclusion of the previous proposition, but the relationships between the government's profits and the CPDL are limited by the range of variables.

There is no inflection point in the image, but there are two influence intervals of the CPDL on the government's profits. The calculation results show that the coordinates of the inflection point are still to the right CPDL definition domain interval. Therefore, the image exhibits monotonously decreasing characteristics left of the inflection point, within the definition domain.

B. The Effect of the CERD on the Supply Chain in Different Models



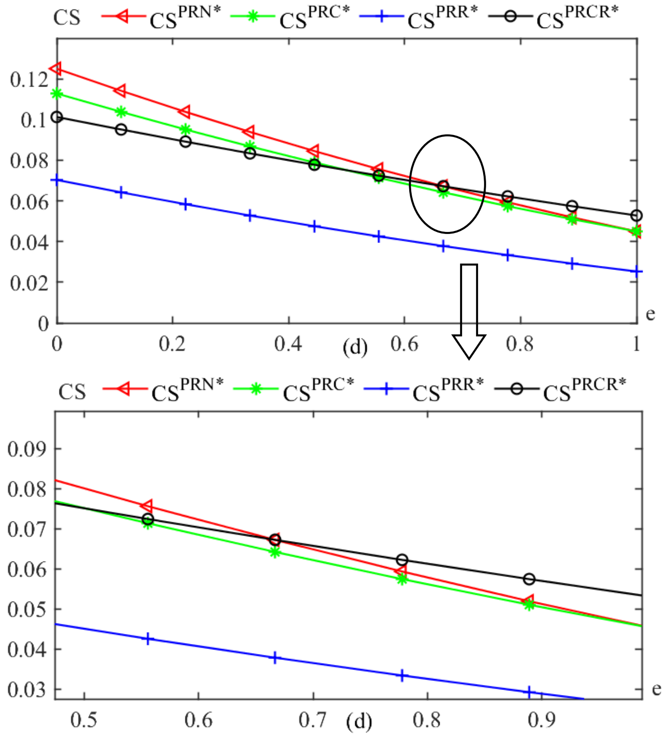


Fig. 3. The impact of the CERD on the supply chain

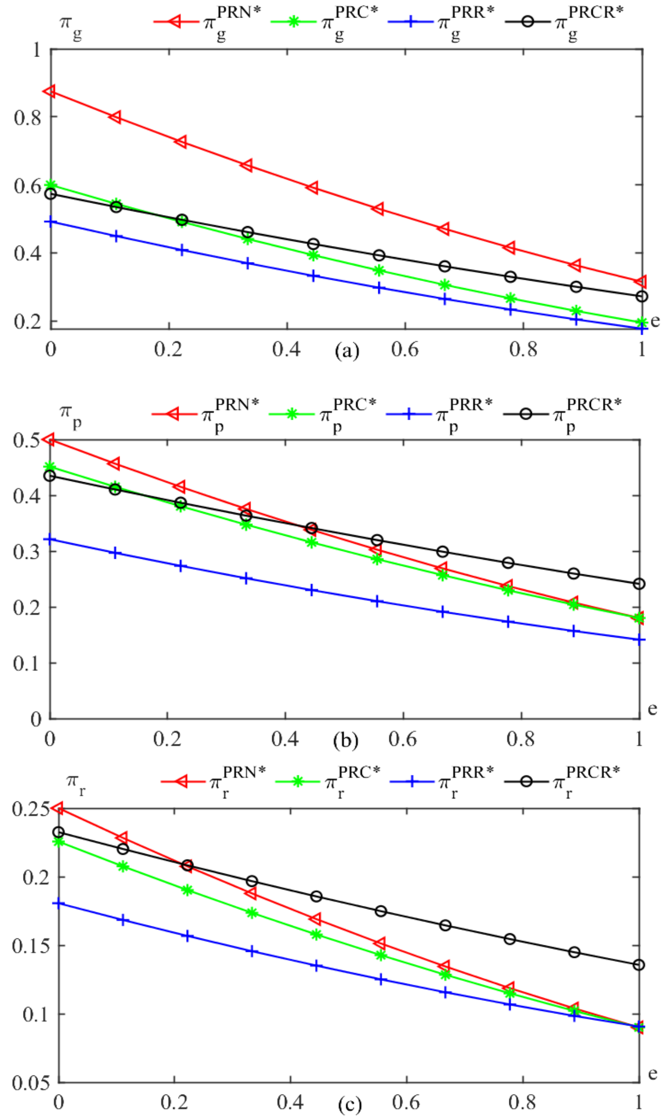


Fig. 4. The impact of the CERD on the supply chain profits

From Fig. 3 (a) to Fig. (d), we can find that the wholesale price, the retail price, the demand and the consumer surplus will decrease with the increase of the product carbon emission reduction in the proposed four models. They show the similar effect of the CPDL on the supply chain in the previous section.

From Fig. 4 (a) to Fig.4 (c), we can see that the revenues of the supply chain members and the government in the four models also show a decreasing trend as the CERD increases. The relationships between the profits of the retailer the producer and the CERD are consistent with the previous proposition.

There is no inflection point in this image due to the restriction of the definition domain of the independent variable. Besides, from this image, we can get that the government's profits are more sensitive to changes in the CERD.

C. The Effect of Blockchain Application Cost Subsidy Coefficient on the Supply Chain

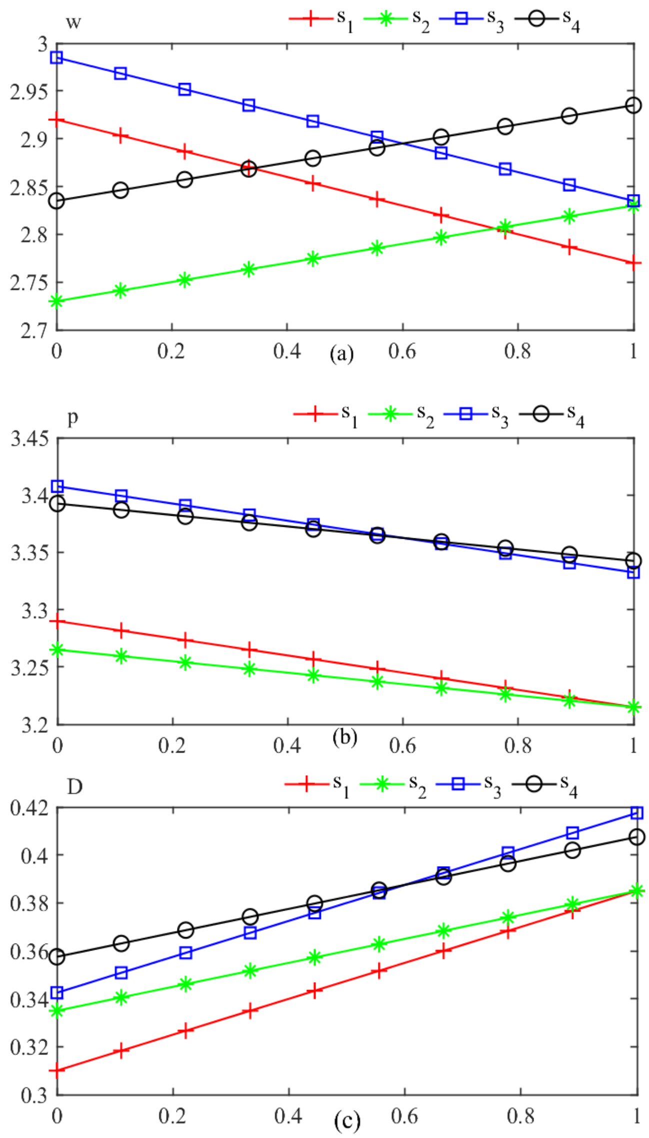


Fig. 5. The impact of blockchain application cost subsidy coefficients on the supply chain ($\gamma^{PRN}=0.8$, $\gamma^{PRC}=0.7$, $\gamma^{PRCR}=0.5$, $e=0.7$)

In the PRC model and PRCR model, the government offers a certain subsidy for the blockchain technology application of the retailer and the producer in the low-carbon supply chain. In this section, the effects of the subsidy coefficient s_1, s_2, s_3 and s_4 on the optimal prices and the profits of the chain members are discussed in the two models. Combining Proposition 3 and Proposition 6, the changes about the wholesale prices, the retail prices, and the demand concerning the subsidy coefficient s_1, s_2, s_3 and s_4 are obtained under the two models, which are in Fig. 5.

From Fig. 5 (a), we get that as the cost subsidy coefficient rises, the wholesale price drops for the producer applying blockchain technology, while increasing in the PRC model and the PRCR model. This maybe because as the percentage increase of government subsidies related to the producer behavior of investing in blockchain technology, the producer cost for producing low-carbon products relatively decreases, and the producer has more incentives to reduce their wholesale costs to further increase the wholesale quantities. On the contrary, if the cost subsidy coefficients for the retailer's blockchain applications are only increased unilaterally, it will cause dissatisfaction among the producer, who tend to increase the wholesale prices and make them gain more profits from the retailer. As can be seen from Fig. 5 (b), the retail price of low-carbon products will decrease with the increase of the cost subsidy coefficient. From Fig. 5 (c), we obtain that the increases in the cost subsidy coefficient will lead to a decrease of the producer's production cost, and the producer may offer a low wholesale price to the retailer, and then the demand is boosted as a result.

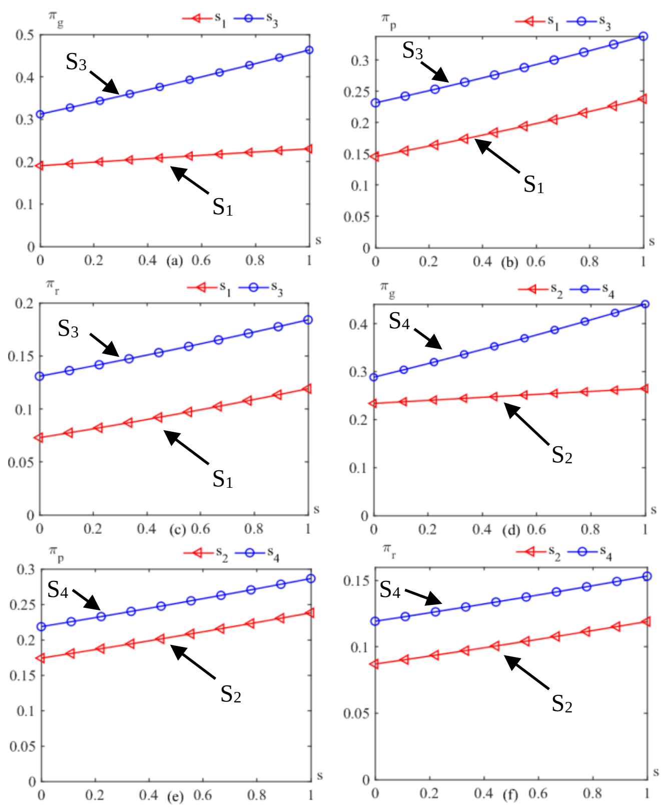


Fig. 6. The impact of blockchain application cost subsidy coefficient on the supply chain profits

As shown from Fig. 6 (a) to Fig. 6 (f), we get that the cost subsidy coefficients s_1, s_2, s_3 and s_4 show positive impacts on the profits of the producer and the retailer. In addition, these cost subsidy coefficients show a positive impact on the government's profits. Due to the limitation of the definition domain of the independent variable, the actual simulation results are difficult to show the characteristics of the theoretical results, thus showing the monotonically increasing function characteristics in the left interval in Proposition 3 and Proposition 6.

From Fig. 7 (a) to Fig. 7 (f), we can see that the subsidy cost about the CPDL reduction are independent of the wholesale price, the retail price, the demand, and the government's profits, while they are positively related to the revenues of the supply chain members.

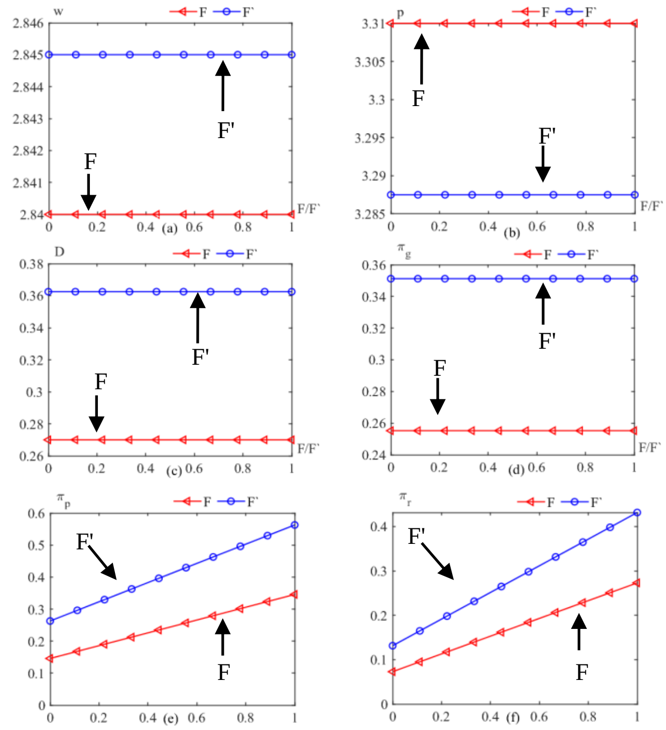


Fig. 7. The impact of the subsidy cost about the CPDL reduction on the supply chain benefits
（$s_3 = 0.6$ ， $s_4 = 0.6$ ， $\gamma^{PRN} = 0.8$ ， $\gamma^{PRR} = 0.6$ ， $\gamma^{PRCR} = 0.5$ ）

D. Comparative Analyses of the Profits of the Four Models

According to the conclusions related to Proposition 7 and Proposition 8, we obtain the images of Fig. 8 (a) and Fig. 8 (b). From Fig. 8(a) and Fig. 8 (b), it can be seen that there is a certain threshold value for investing in blockchain technology, and when the CPDL reduction reaches the threshold value, more profits can be achieved by applying blockchain technology. When the CPDL reduction reaches 0.50, the supply chain members choose to invest in blockchain technology to achieve higher profits. The subsidy about the CPDL reduction is not related to the government's profits. In this case, the unit subsidy value is larger in the PRC model, therefore, the PRC model is the best model.

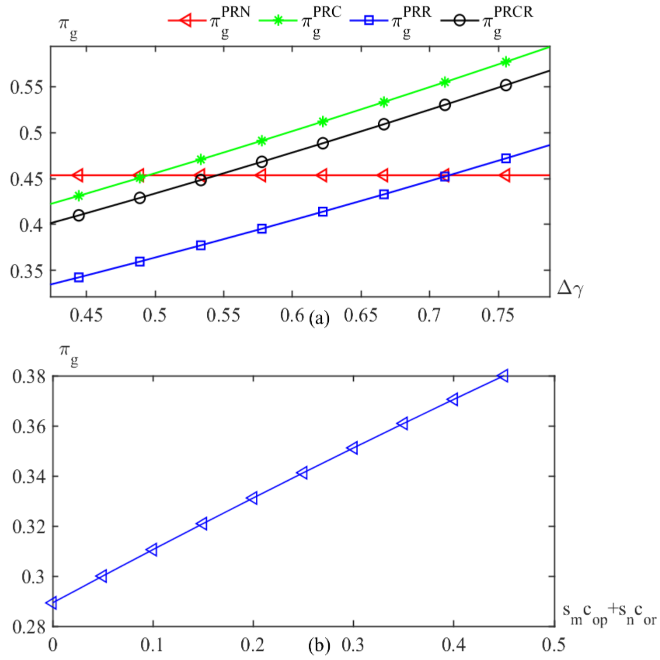


Fig. 8. Comparison of the government's profits under different subsidy models（ $s_1 = 0.8$, $s_2 = 0.8$, $s_3 = 0.6$, $s_4 = 0.6$, $\gamma^{PRN} = 0.8$ ）

VI. Robustness Analysis

To analyze the robustness of our findings, we will extend the three subsidy models proposed in this paper (models of PRC, PRR and PRCR). These three modes belong to the variable subsidy mode and the existing subsidy modes mainly include fixed subsidy mode and variable subsidy mode. Therefore, this section expanded the existing three subsidy models, including the FPRC model, FPRR model and FPRCR model, and introduced the fixed subsidy parameter of C_1 , C_2 and C_3 (C_1 is the fixed subsidy parameter of producer and retailer blockchain application cost subsidy model, C_2 is the fixed subsidy parameter of CPDL reduction subsidy model, and C_3 is the fixed subsidy parameter of the producer and the retailer blockchain application cost and CPDL reduction subsidy model at the same time). The detailed analysis process can be seen in Appendix D. The results show that whether the government provides fixed or variable subsidies, the equilibrium price of the low-carbon supply chain and the government's profits are consistent with the results in the modes of PRC, PRR and PRCR. Therefore, the models constructed in our study have good robustness.

VII. Discussion

In this section, the similarities and differences between our study and the related studies are explored. Distinguishing from the existing literature, the differences in the research contents are as follows:

1) Biswas et al. [4] concluded that the application of blockchain technology had significant advantages in reducing the CPDL of product carbon information. We also found that the blockchain technology investment behavior in low-carbon supply chains depended on the CPDL towards their low-carbon products.
2) Some related studies investigated the impacts of per-unit production subsidies and innovation effort subsidies on supply chains' sustainability innovation [24] and the differences in the effect of output subsidies versus fixed subsidies [26]. Our study constructed three different subsidy models for the application of blockchain technology (including a subsidy model for the cost of blockchain technology application, a subsidy model for the CPDL reduction, and a hybrid subsidy model that subsidizes both the cost of blockchain technology application and the CPDL reduction) and aimed to investigate the optimal government subsidy model for the blockchain technology application.

In addition, the differences in the results are shown below.

Our study proposed the existence of certain thresholds for the blockchain technology application in a low-carbon supply chain, which was similar to the findings of Jiang and Liu [17], and they concluded that investing in blockchain technology had specific thresholds for different models. However, the threshold in the conclusion of our study was related to the CPDL. Our study argued that the choice to apply blockchain technology was reasonable only when the amount of the CPDL reduction reached a certain threshold.

VIII. Conclusions

This study focused on the optimal subsidy strategy for the implementation of blockchain technology in a low-carbon supply chain. We introduced CPDL as a key influence factor and presented a control model without the application of blockchain technology. Additionally, we proposed three potential subsidy modes for the implementation of blockchain technology. Then, a comparative analysis of these four modes had been conducted. The findings are as follows.

1) In the four models, CERD and CPDL are negatively correlated with the profits of the retailer and the producer. In the models of PRC and PRCR, the change trends of the government benefit are more complex following the change of CERD and CPDL, exhibiting an inflection point effect. Namely, producers and retailers should improve the transparency and accuracy of low-carbon product information to reduce the CPDL, and thereby increase their own profits. The government should comprehensively consider the impact of the CERD and the CPDL on its revenue under different models, and then formulate targeted policies such as tax reductions, financial incentives, or promotion of emerging technologies. In the models of PRC and PRCR, The government should also pay more attention to the balance between profits and losses, and then set reasonable policy objectives to promote the carbon emission reduction of the whole supply chain.
2) In the models of PRC and PRCR, the cost subsidy coefficients for the producer and the retailer are positively correlated with their profits. Conversely, government revenue initially increases but then decreases as the increase of government subsidies. That is to say, producers and retailers should actively adopt blockchain technology and seek government subsidies, aiming for sustained profits growth. The government must consider the inflection point effect, and then set a reasonable subsidy cap to enhance the profits of producers and retailers. Once the inflection point is exceeded, a meticulous assessment of the subsidy methods and their intensity become necessary.
3) In the models of PRR and PRCR, the subsidy costs for reducing CPDL are positively correlated with the profits of the producer and the retailer, but not related to government revenue.
4) There is a certain threshold value for the investment cost of blockchain technology in low-carbon supply chains, and the threshold value is related to CPDL. When the reduction in CPDL reaches a certain range, investing in blockchain technology can obtain more profits. Within a certain interval, the government's profits are positively correlated with the per unit subsidy value of blockchain application for the producer and the retailer, and beyond this interval, the government's profits and the subsidy value are negatively correlated. Namely, when companies make investment decisions on blockchain technology, they should consider the CPDL reduction that can be achieved, so as to ensure that the investment covers the cost. When the government makes decisions on subsidies, it is also necessary to consider the per unit subsidy value, otherwise, a high subsidy may not yield the expected profits.

IX. Limitations and Future Research

Our study has some academic and applied value. Firstly, we introduced the CPDL as a decision variable to analyze subsidy strategies for the application of blockchain technology in a low-carbon supply chain. This will enrich the subsidy theory research on the blockchain technology application in low-carbon supply chains. At the same time, our findings are also practical guidance for the decision-making of blockchain technology applications and the selection of government subsidy strategy in a low-carbon supply chain. For instance, in the aspect of blockchain technology application decision, the CPDL reduction that can be achieved should be carefully considered, otherwise, the high investment may not bring the expected profits. Regarding the decision on government subsidy strategies, within a certain range, increasing the cost subsidy proportions of blockchain technology applications for the producer and the retailer can improve the overall profits of the supply chain, while beyond this range, it may result in the government's financial subsidies being unable to cover these expenses. Therefore, this subsidy approach should be avoided when making subsidy decisions for blockchain technology.

At the same time, there are some limitations in this study: We only considered a simple low-carbon supply chain model. In actual application, the operation of low-carbon supply chains may encounter more complex situations. Additionally, our analysis focused on scenarios where both manufacturers and retailers apply blockchain technology and neither applies blockchain technology In cases where blockchain technology is adopted, low-carbon information is fully shared between the manufacturer and the retailer without any instances of information fraud. Therefore, in future research, we can optimize and expand the above limitations, further refine the supply chain operation process, build a more realistic and credible low-carbon supply chain model, consider a more complex and changing market environment, and conduct a more in-depth and comprehensive discussion.

APPENDIX

Appendix A

Firstly, substitute D^{PRN} into π_r^{PRN} to obtain the retailer's profits function, let $\frac{\partial \pi_r^{PRN}}{\partial p^{PRN}}=0$, then obtain: $p^{PRN}=\frac{a+w^{PRN}-e\gamma^{PRN}}{2}$. Substituting p^{PRN} into π_p^{PRN} yields the producer's profits function $\pi_p^{PRN}(w^{PRN},e)$, calculate the Hessian matrix of $\pi_p^{PRN}(w^{PRN},e)$ with respect to w^{PRN} and e, then obtain:

$$H_1=\begin{pmatrix}\frac{\partial^2\pi_p^{PRN}}{\partial(w^{PRN})^2} & \frac{\partial^2\pi_p^{PRN}}{\partial w^{PRN}\partial e}\\ \frac{\partial^2\pi_p^{PRN}}{\partial e\partial w^{PRN}} & \frac{\partial^2\pi_p^{PRN}}{\partial e^2}\end{pmatrix}=\begin{pmatrix}-1 & -\frac{\gamma^{PRN}}{2}\\ -\frac{\gamma^{PRN}}{2} & 0\end{pmatrix}.$$

$\frac{\partial^2\pi_p^{PRN}}{\partial(w^{PRN})^2}\frac{\partial^2\pi_p^{PRN}}{\partial e^2}-(\frac{\partial^2\pi_p^{PRN}}{\partial w^{PRN}\partial e})^2=-\frac{(\gamma^{PRN})^2}{4}<0$ is obtained for general $\gamma^{PRN}>0$, therefore, $\pi_p^{PRN}(w^{PRN},e)$ is not a jointly concave function of w^{PRN} with e . At this point, assuming that e is determined, solve for the manufacturer's optimal wholesale price: $w^{PRN*}=\frac{a+c-e\gamma^{PRN}}{2}$. Substitute equation w^{PRN*} into p^{PRN} that is the optimal retail price of the product in the PRN model: $p^{PRN*}=\frac{3a+c-3e\gamma^{PRN}}{4}$. Substitute p^{PRN*} into D^{PRN} to obtain the optimal demand: $D^{PRN*}=\frac{a-c-e\gamma^{PRN}}{4}$, and thus

$\pi_r^{PRN*}=\frac{(c-a+e\gamma^{PRN})^2}{16}$, $\pi_p^{PRN*}=\frac{(c-a+e\gamma^{PRN})^2}{8}$, $\pi_g^{PRN*}=\frac{7(c-a+e\gamma^{PRN})^2}{32}$.

Appendix B-1

Proof of Proposition 1: Taking the partial derivative of w^{PRN*} , p^{PRN*} , D^{PRN*} , π_r^{PRN*} , π_p^{PRN*} and π_g^{PRN*} with respect to e, it is easy to know $\frac{\partial w^{PRN*}}{\partial e}<0$, $\frac{\partial p^{PRN*}}{\partial e}<0$, $\frac{\partial D^{PRN*}}{\partial e}<0$, $\frac{\partial \pi_r^{PRN*}}{\partial e}=\frac{\gamma^{PRN}(c-a+e\gamma^{PRN})}{8}$, $\frac{\partial \pi_p^{PRN*}}{\partial e}=\frac{\gamma^{PRN}(c-a+e\gamma^{PRN})}{4}$ and $\frac{\partial \pi_g^{PRN*}}{\partial e}=\frac{7\gamma^{PRN}(c-a+e\gamma^{PRN})}{16}$. To ensure that the model is meaningful, therefore $D^{PRN*}>0$, it can be obtained $a-c-e\gamma^{PRN}>0$, from which $\frac{\partial \pi_r^{PRN*}}{\partial e}<0$, $\frac{\partial \pi_p^{PRN*}}{\partial e}<0$, and $\frac{\partial \pi_g^{PRN*}}{\partial e}<0$ can be obtained. Similarly, taking the partial derivative of w^{PRN*} , p^{PRN*} , D^{PRN*} , π_r^{PRN*} , π_p^{PRN*} , π_g^{PRN*} with respect to γ , $\frac{\partial w^{PRN*}}{\partial \gamma^{PRN}}<0$, $\frac{\partial p^{PRN*}}{\partial \gamma^{PRN}}<0$, $\frac{\partial D^{PRN*}}{\partial \gamma^{PRN}}<0$, $\frac{\partial \pi_r^{PRN*}}{\partial \gamma^{PRN}}<0$, $\frac{\partial \pi_p^{PRN*}}{\partial \gamma^{PRN}}<0$, $\frac{\partial \pi_g^{PRN*}}{\partial \gamma^{PRN}}<0$ can be obtained. Therefore, Proposition 1 is proved.

Appendix B-2

Proof of Proposition 2: Taking the partial derivative of w^{PRC*} , p^{PRC*} , D^{PRC*} , π_r^{PRC*} , π_p^{PRC*} with respect to e, it is easy to know $\frac{\partial w^{PRC*}}{\partial e}<0$, $\frac{\partial p^{PRC*}}{\partial e}<0$, $\frac{\partial D^{PRC*}}{\partial e}<0$, $\frac{\partial \pi_r^{PRC*}}{\partial e}=\frac{\gamma^{PRC}(c+c_{op}+c_{or}+e\gamma^{PRC}-a-s_1c_{op}-s_2c_{or})}{8}$, $\frac{\partial \pi_p^{PRC*}}{\partial e}=\frac{\gamma^{PRN}(c+c_{op}+c_{or}+e\gamma^{PRC}-a-s_1c_{op}-s_2c_{or})}{4}$. To ensure that the model is meaningful, therefore $D^{PRC*}>0$, it can be obtained $a+c_{op}s_1+c_{or}s_2-c-e\gamma^{PRC}-c_{op}-c_{or}>0$, from which $\frac{\partial \pi_r^{PRC*}}{\partial e}<0$, $\frac{\partial \pi_p^{PRC*}}{\partial e}<0$ can be obtained. Taking the partial derivative of π_g^{PRC*} with respect to e, we can obtain that if $e<\frac{7a+3c_{op}s_1+3c_{or}s_2-7c-7c_{op}-7c_{or}}{7\gamma^{PRC}}$, then $\frac{\partial \pi_g^{PRC*}}{\partial e}<0$,and if

$e>\frac{7a+3c_{op}s_1+3c_{or}s_2-7c-7c_{op}-7c_{or}}{7\gamma^{PRC}}$,

then $\frac{\partial \pi_g^{PRC*}}{\partial e}>0$ .Similarly, taking the partial derivative of w^{PRC*}, p^{PRC*}, D^{PRC*}, π_r^{PRC*}, π_p^{PRC*}, π_g^{PRC*} with respect to γ , $\frac{\partial w^{PRC*}}{\partial \gamma^{PRC}}<0$, $\frac{\partial p^{PRC*}}{\partial \gamma^{PRC}}<0$, $\frac{\partial D^{PRC*}}{\partial \gamma^{PRC}}<0$, $\frac{\partial \pi_r^{PRC*}}{\partial \gamma^{PRC}}<0$, $\frac{\partial \pi_p^{PRC*}}{\partial \gamma^{PRC}}<0$ if $\gamma^{PRC}<\frac{7a+(3s_1-7)c_{op}+(3s_2-7)c_{or}-7c}{7e}$, then $\frac{\partial \pi_g^{PRC*}}{\partial \gamma^{PRC}}<0$, and if $\gamma^{PRC} > \frac{7a+(3s_1-7)c_{op}+(3s_2-7)c_{or}-7c}{7e}$, then $\frac{\partial \pi_g^{PRC*}}{\partial \gamma^{PRC}} > 0$ can be obtained. Therefore, Proposition 2 is proved.

Appendix B-3

Proof of Proposition 3: Taking the partial derivative of w^{PRC*}, p^{PRC*}, D^{PRC*}, π_r^{PRC*}, π_p^{PRC*}, π_g^{PRC*} with respect to s_1, it is easy to know $\frac{\partial w^{PRC*}}{\partial s_1} < 0$, $\frac{\partial p^{PRC*}}{\partial s_1} < 0$, $\frac{\partial D^{PRC*}}{\partial s_1} > 0$,

$$\frac{\partial \pi_r^{PRC*}}{\partial s_1} = \frac{c_{op}(a+c_{op}s_1+c_{or}s_2-c-c_{op}-c_{or}-e\gamma^{PRC})}{8},$$

$$\frac{\partial \pi_p^{PRC*}}{\partial s_1} = \frac{c_{op}(a+c_{op}s_1+c_{or}s_2-c-c_{op}-c_{or}-e\gamma^{PRC})}{4},$$

$$\frac{\partial \pi_g^{PRC*}}{\partial s_1} = \frac{c_{op}(3a-3c-3c_{or}-3c_{op}-3e\gamma^{PRC}-c_{op}s_1-c_{or}s_2)}{16}.$$

To ensure that the model is meaningful, therefore $D^{PRC*} > 0$, it can be obtained $a+c_{op}s_1+c_{or}s_2-c-e\gamma^{PRC}-c_{op}-c_{or} > 0$, from which $\frac{\partial \pi_r^{PRC*}}{\partial s_1} > 0$, $\frac{\partial \pi_p^{PRC*}}{\partial s_1} > 0$ can be obtained. We can also obtain that if $s_1 < \frac{3a-3c-3c_{op}-3c_{or}-3e\gamma^{PRC}-c_{or}s_2}{c_{op}}$, then $\frac{\partial \pi_g^{PRC*}}{\partial s_1} > 0$, and if $s_1 > \frac{3a-3c-3c_{op}-3c_{or}-3e\gamma^{PRC}-c_{or}s_2}{c_{op}}$, then $\frac{\partial \pi_g^{PRC*}}{\partial s_1} < 0$. Similarly, taking the partial derivative of w^{PRC*}, p^{PRC*}, D^{PRC*}, π_r^{PRC*}, π_p^{PRC*}, π_g^{PRC*} with respect to s_2, $\frac{\partial w^{PRC*}}{\partial s_2} > 0$, $\frac{\partial p^{PRC*}}{\partial s_2} < 0$, $\frac{\partial D^{PRC*}}{\partial s_2} > 0$, $\frac{\partial \pi_r^{PRC*}}{\partial s_2} > 0$, $\frac{\partial \pi_p^{PRC*}}{\partial s_2} > 0$, if $s_2 < \frac{3a-3c-3c_{op}-3c_{or}-3e\gamma^{PRC}-c_{op}s_1}{c_{or}}$, then $\frac{\partial \pi_g^{PRC*}}{\partial s_2} > 0$, and if $s_2 > \frac{3a-3c-3c_{op}-3c_{or}-3e\gamma^{PRC}-c_{op}s_1}{c_{or}}$, then $\frac{\partial \pi_g^{PRC*}}{\partial s_2} < 0$ can be obtained. Therefore, Proposition 3 is proved.

Appendix B-4

Proof of Proposition 4: Taking the partial derivative of w^{PRR*}, p^{PRR*}, D^{PRR*}, π_r^{PRR*}, π_p^{PRR*}, π_g^{PRR*} with respect to e, it is easy to know $\frac{\partial w^{PRR*}}{\partial e} < 0$, $\frac{\partial p^{PRR*}}{\partial e} < 0$, $\frac{\partial D^{PRR*}}{\partial e} < 0$,

$$\frac{\partial \pi_r^{PRR*}}{\partial \gamma^{PRR}} = \frac{e(c+c_{op}+c_{or}+e\gamma^{PRR}-a)}{8} - F,$$

$$\frac{\partial \pi_p^{PRR*}}{\partial \gamma^{PRR}} = \frac{e(c+c_{op}+c_{or}+e\gamma^{PRR}-a)}{4} - F,$$

$$\frac{\partial \pi_r^{PRR*}}{\partial \gamma^{PRR}} = \frac{7e(c+c_{op}+c_{or}+e\gamma^{PRR}-a)}{16}.$$

To ensure that the model is meaningful, therefore $D^{PRR*} > 0$, it can be obtained $a-c-c_{op}-c_{or}-e\gamma^{PRR} > 0$, from which $\frac{\partial \pi_r^{PRR*}}{\partial \gamma^{PRR}} < 0$, $\frac{\partial \pi_p^{PRR*}}{\partial \gamma^{PRR}} < 0$, $\frac{\partial \pi_g^{PRR*}}{\partial \gamma^{PRR}} < 0$ can be obtained. Similarly, taking the partial derivative of w^{PRR*}, p^{PRR*}, D^{PRR*}, π_r^{PRR*}, π_p^{PRR*}, π_g^{PRR*} with respect to γ and F respectively, $\frac{\partial w^{PRR*}}{\partial \gamma^{PRR}} < 0$, $\frac{\partial p^{PRR*}}{\partial \gamma^{PRR}} < 0$, $\frac{\partial D^{PRR*}}{\partial \gamma^{PRR}} < 0$, $\frac{\partial \pi_r^{PRR*}}{\partial \gamma^{PRR}} < 0$, $\frac{\partial \pi_p^{PRR*}}{\partial \gamma^{PRR}} < 0$, $\frac{\partial \pi_g^{PRR*}}{\partial \gamma^{PRR}} < 0$. $\frac{\partial w^{PRR*}}{\partial F} = 0$, $\frac{\partial p^{PRR*}}{\partial F} = 0$, $\frac{\partial D^{PRR*}}{\partial F} = 0$, $\frac{\partial \pi_r^{PRR*}}{\partial F} > 0$, $\frac{\partial \pi_p^{PRR*}}{\partial F} > 0$, $\frac{\partial \pi_g^{PRR*}}{\partial F} = 0$ can be obtained. Therefore, Proposition 4 is proved.

Appendix B-5

Proof of Proposition 5: Taking the partial derivative of w^{PRCR*}, p^{PRCR*}, D^{PRCR*}, π_r^{PRCR*}, π_p^{PRCR*} with respect to e, it is easy to know

$\frac{\partial w^{PRCR*}}{\partial e} < 0$, $\frac{\partial p^{PRCR*}}{\partial e} < 0$, $\frac{\partial D^{PRCR*}}{\partial e} < 0$,

$$\frac{\partial \pi_r^{PRCR*}}{\partial e} = \frac{\gamma^{PRCR}(c+c_{op}+c_{or}+e\gamma^{PRCR}-a-s_3c_{op}-s_4c_{or})}{8},$$

$$\frac{\partial \pi_p^{PRCR*}}{\partial e} = \frac{\gamma^{PRCR}(c+c_{op}+c_{or}+e\gamma^{PRCR}-a-s_3c_{op}-s_4c_{or})}{4},$$

$$\frac{\partial \pi_g^{PRCR*}}{\partial e} = \frac{\gamma^{PRCR}(7c+7c_{op}+7c_{or}+7e\gamma^{PRC}-7a-3s_3c_{op}-3s_4c_{or})}{16}$$

To ensure that the model is meaningful, therefore $D^{PRCR*} > 0$, it can be obtained

$a+s_3c_{op}+s_4c_{or}-c-c_{op}-c_{or}-e\gamma^{PRCR} > 0$,

from which $\frac{\partial \pi_r^{PRCR*}}{\partial e} < 0$, $\frac{\partial \pi_p^{PRCR*}}{\partial e} < 0$ can be obtained. We can also obtain that if

$e < \frac{7a+3c_{op}s_3+3c_{or}s_4-7c-7c_{op}-7c_{or}}{7\gamma^{PRCR}}$,

then $\frac{\partial \pi_g^{PRCR*}}{\partial e} < 0$,

and if $e > \frac{7a+3c_{op}s_3+3c_{or}s_4-7c-7c_{op}-7c_{or}}{7\gamma^{PRCR}}$, then $\frac{\partial \pi_g^{PRCR*}}{\partial e} > 0$. Similarly, taking the partial derivative of w^{PRCR*}, p^{PRCR*}, D^{PRCR*}, π_r^{PRCR*}, π_p^{PRCR*}, π_g^{PRCR*} with respect to γ

and $F^{'}$ respectively, w^{PRCR*}, p^{PRCR*}, D^{PRCR*}, π_r^{PRCR*}, π_p^{PRCR*}, π_g^{PRCR*}, if $\gamma^{PRCR} < \frac{7a+3c_{op}s_3+3c_{or}s_4-7c-7c_{op}-7c_{or}}{7e}$,

then $\frac{\partial \pi_g^{PRCR*}}{\partial \gamma^{PRCR}} < 0$,

and if $\gamma^{PRCR} > \frac{7a+3c_{op}s_3+3c_{or}s_4-7c-7c_{op}-7c_{or}}{7e}$, then $\frac{\partial \pi_g^{PRCR*}}{\partial \gamma^{PRCR}} > 0$. $\frac{\partial w^{PRCR*}}{\partial F^{'}} = 0$, $\frac{\partial p^{PRCR*}}{\partial F^{'}} = 0$, $\frac{\partial D^{PRCR*}}{\partial F^{'}} = 0$, $\frac{\partial \pi_r^{PRCR*}}{\partial F^{'}} > 0$, $\frac{\partial \pi_p^{PRCR*}}{\partial F^{'}} > 0$, $\frac{\partial \pi_g^{PRCR*}}{\partial F^{'}} = 0$ can be obtained. Therefore, Proposition 5 is proved.

Appendix B-6

Proof of Proposition 6: Taking the partial derivative of w^{PRCR*}, p^{PRCR*}, D^{PRCR*}, π_r^{PRCR*}, π_p^{PRCR*}, π_g^{PRCR*} with respect to s_3, we can know

$\frac{\partial w^{PRCR*}}{\partial s_3} < 0$, $\frac{\partial p^{PRCR*}}{\partial s_3} < 0$, $\frac{\partial D^{PRCR*}}{\partial s_3} > 0$,

$$\frac{\partial \pi_r^{PRCR*}}{\partial s_3} = \frac{c_{op}(a+c_{op}s_3+c_{or}s_4-c-c_{op}-c_{or}-e\gamma^{PRCR})}{8},$$

$$\frac{\partial \pi_p^{PRCR*}}{\partial s_3} = \frac{c_{op}(a+c_{op}s_3+c_{or}s_4-c-c_{op}-c_{or}-e\gamma^{PRCR})}{4},$$

$$\frac{\partial \pi_g^{PRCR*}}{\partial s_3} = \frac{c_{op}(3a-3c-3c_{or}-3c_{op}-3e\gamma^{PRC}-c_{op}s_3-c_{or}s_4)}{16}.$$

To ensure that the model is meaningful, therefore $D^{PRCR*} > 0$, it can be obtained $a+c_{op}s_3+c_{or}s_4-c-e\gamma^{PRC}-c_{op}-c_{or} > 0$, from which $\frac{\partial \pi_r^{PRCR*}}{\partial s_3} > 0$, $\frac{\partial \pi_p^{PRCR*}}{\partial s_3} > 0$ can be obtained. We can also obtain that if $s_3 < \frac{3a-3c-3c_{op}-3c_{or}-3e\gamma^{PRCR}-c_{or}s_4}{c_{op}}$,

then $\frac{\partial \pi_g^{PRCR*}}{\partial s_3} > 0$,

and if $s_3 > \frac{3a-3c-3c_{op}-3c_{or}-3e\gamma^{PRCR}-c_{or}s_4}{c_{op}}$, then $\frac{\partial \pi_g^{PRCR*}}{\partial s_3} < 0$ .Similarly, taking the partial derivative of w^{PRCR*}, p^{PRCR*}, D^{PRCR*} π_r^{PRCR*}, π_p^{PRCR*}, π_g^{PRCR*} with respect to s_4, $\frac{\partial w^{PRCR*}}{\partial s_4} > 0$, $\frac{\partial p^{PRCR*}}{\partial s_4} < 0$, $\frac{\partial D^{PRCR*}}{\partial s_4} > 0$, $\frac{\partial \pi_r^{PRCR*}}{\partial s_4} > 0$, $\frac{\partial \pi_p^{PRCR*}}{\partial s_4} > 0$, if $s_4 < \frac{3a-3c-3c_{op}-3c_{or}-3e\gamma^{PRCR}-c_{op}s_3}{c_{or}}$,

then $\frac{\partial \pi_g^{PRCR*}}{\partial s_4} > 0$,

and if $s_4 > \frac{3a-3c-3c_{op}-3c_{or}-3e\gamma^{PRCR}-c_{op}s_3}{c_{or}}$, then $\frac{\partial \pi_g^{PRCR*}}{\partial s_4} < 0$ can be obtained. Therefore, Proposition 6 is proved.

Appendix C-1

Proof of Proposition 7:

$$\pi_g^{PRC*}-\pi_g^{PRN*} = \frac{(E+e\gamma^{PRC}-c_{op}s_1-c_{or}s_2)(7E+7e\gamma^{PRC}+c_{op}s_1+c_{or}s_2)-7(c-a+e\gamma^{PRN})^2}{32},$$

when $\Delta\gamma > \frac{7E+7e\gamma^{PRN}+E_3-3c_{op}s_1-3c_{or}s_2}{7e}$, $\pi_g^{PRC*}-\pi_g^{PRN*} > 0$, this situation is called δ_1; $\pi_g^{PRR*}-\pi_g^{PRN*} = \frac{7(E+e\gamma^{PRR})^2}{32} - \frac{7(c-a+e\gamma^{PRN})^2}{32}$,

when $\Delta\gamma > \frac{c_{op}+c_{or}}{e}$, $\pi_g^{PRR*}-\pi_g^{PRN*} > 0$, this situation is called δ_2; besides,

$$\pi_g^{PRCR*}-\pi_g^{PRN*} = \frac{(E+e\gamma^{PRCR}-s_3c_{op}-s_4c_{or})(7E+7e\gamma^{PRCR}+s_3c_{op}+s_4c_{or})-7(c-a+e\gamma^{PRN})^2}{32},$$

when $\Delta\gamma > \frac{7E+7e\gamma^{PRN}+E_6-3c_{op}s_3-3c_{or}s_4}{7e}$, $\pi_g^{PRCR*}-\pi_g^{PRN*} > 0$, this situation is called δ_3, where $E_3 = \sqrt{49(a-c)^2+49e\gamma^{PRN}(2c+e\gamma^{PRN}-2a)+16(c_{op}s_1+c_{or}s_2)^2}$.

Appendix C-2

Proof of Proposition 8:

$$\pi_g^{i*} = \frac{(E+e\gamma^i-C)(7E+7e\gamma^i+C)}{32},$$

where $i=\{PRC, PRR, PRCR\}$, $C = s_mc_{or}+s_nc_{op}$, $\frac{\partial \pi_g^{i*}}{\partial C} = \frac{-C-3E-3e\gamma^i}{16}$. Therefore, when $C < -3E-3e\gamma^i$, $\frac{\partial \pi_g^{i*}}{\partial C} > o$, and when $C > -3E-3e\gamma^i$, $\frac{\partial \pi_g^{i*}}{\partial C} < o$.

Appendix D:

Variable Subsidies in the Extended Model

In the FPRC model, the profits functions of low-carbon supply chain producers, retailers and the government are as follows: $\pi_r^{FPRC} = [p^{FPRC}-w^{FPRC}-(1-s_2)c_{or}]D^{FPRC}+C_1$, $\pi_p^{FPRC} = [w^{FPRC}-c-(1-s_1)c_{op}]D^{FPRC}+C_1$, $\pi_g^{FPRC} = \pi_p^{FPRC}+\pi_r^{FPRC}+CS^{FPRC}-(s_1c_{op}+s_2c_{or})D^{FPRC}-2C_1$.

Based on this, it can be solved that:

$$\begin{cases} w^{FPRC*}=\dfrac{a+c+c_{op}+c_{or}s_2-e\gamma^{FPRC}-c_{op}s_1-c_{or}}{2} \\ p^{FPRC*}=\dfrac{3a+c+c_{op}+c_{or}-3e\gamma^{FPRC}-c_{op}s_1-c_{or}s_2}{4} \\ D^{FPRC*}=\dfrac{c_{op}s_1+c_{or}s_2-e\gamma^{FPRC}-E}{4} \end{cases}$$

$$\begin{cases} \pi_r^{FPRC*}=\dfrac{(c_{op}s_1+c_{or}s_2-e\gamma^{FPRC}-E)^2}{16}+C_1 \\ \pi_p^{FPRC*}=\dfrac{(c_{op}s_1+c_{or}s_2-e\gamma^{FPRC}-E)^2}{8}+C_1 \\ \pi_g^{FPRC*}=\dfrac{(E+e\gamma^{FPRC}-c_{op}s_1-c_{or}s_2)(7E+7e\gamma^{FPRC}+c_{op}s_1+c_{or}s_2)}{32} \end{cases}$$

According to the above results, it can be known that: fixed subsidy C_1 has no effect on the equilibrium price and the government's profits, and is positively correlated with the producer's and the retailer's profits. The calculation process of the FPRR and FPRCR models is similar to this and will not be elaborated here.

Fixed subsidies in the Extended Model

In the FPRC model, the profits functions of low-carbon supply chain producers, retailers and the government are as follows:

$\pi_r^{FPRC}=[p^{FPRC}-w^{FPRC}-c_{or}]D^{FPRC}+C_1$,

$\pi_p^{FPRC}=[w^{FPRC}-c-c_{op}]D^{FPRC}+C_1$,

$\pi_g^{FPRC}=\pi_p^{FPRC}+\pi_r^{FPRC}+CS^{FPRC}-2C_1$.

Based on this, it can be solved that:

$$\begin{cases} w^{FPRC*}=\dfrac{a+c+c_{op}-e\gamma^{FPRC}-c_{or}}{2} \\ p^{FPRC*}=\dfrac{3a+c+c_{op}+c_{or}-3e\gamma^{FPRC}}{4} \\ D^{FPRC*}=\dfrac{-e\gamma^{FPRC}-E}{4} \end{cases}$$

$$\begin{cases} \pi_r^{FPRC*}=\dfrac{(-e\gamma^{FPRC}-E)^2}{16}+C_1 \\ \pi_p^{FPRC*}=\dfrac{(-e\gamma^{FPRC}-E)^2}{8}+C_1 \\ \pi_g^{FPRC*}=\dfrac{7(E+e\gamma^{FPRC})}{32} \end{cases}$$

According to the above results, it can be known that: fixed subsidy A has no effect on the equilibrium price and the government's profits, and is positively correlated with the producer's and the retailer's profits. The calculation process of the FPRR and FPRCR models is similar to this and will not be elaborated here.

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