Practical Predefined-time Synchronization for Multi-weighted Complex Networks Based on Event-triggered Control

Xin Zhou, Jie Gao*, and Xingyu Wu

***Abstract*—This study addresses the *practical predefined-time (PT)* synchronization problem for *multi-weighted complex networks (MWCNs)* using event-triggered control. We introduce two key time-varying functions: an exponentially decaying function to adjust the control input amplitude and a linearly increasing function to modulate the triggering threshold. This combination effectively balances response speed with control energy efficiency. Leveraging these functions, we propose a novel event-triggered control strategy and triggering condition, which utilize real-time network states to reduce communication overhead and computational complexity.We derive sufficient conditions for achieving PT synchronization within the predefined time and rigorously prove the system's stability over this interval. To prevent Zeno behavior, we incorporate a minimum triggering interval. Finally, numerical simulations validate the effectiveness of the proposed control strategy across various initial conditions and network topologies.**

***Index Terms*—PT synchronization, MWCNs, predefined-time control, time-varying functions, event-triggered control.**

I. Introduction

In recent years, complex networks have gained significant attention for their applications across diverse fields, including communication systems, biological networks, and economics. Many real-world networks involve multi-weighted couplings, such as transportation and social networks, which require accurate modeling through MWCNs) [1]–[3]. For instance, individuals in social networks interact through various channels (e.g., mobile phones, email, social media), making multi-weighted models ideal for capturing different interaction effects.

Synchronization is crucial for cooperative control in MWCNs, and various synchronization forms have been extensively studied. Initial research emphasized infinite-time synchronization (e.g., secure communication), but finite-time synchronization, which offers better robustness and anti-interference properties, is essential in certain cases [4]–[7]. Although finite-time convergence can be adjusted by control parameters, it is sensitive to initial conditions, which are often unknown due to the challenging environments of real-world networks.

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Fixed-time (FxT) control provides an advantage over finite-time control by making convergence time independent of initial conditions. FxT control strategies have been explored in [2], [4], [8], although FxT convergence relies on system parameters, such as control gain, making exact convergence time difficult to determine. Both finite-time and FxT convergence times are not freely adjustable, yet applications like robotic manipulators and motor systems often require specific predefined timings [9], [10].

To address the challenge of designing controllers that achieve predefined-time (PT) synchronization, the concept of PT synchronization and related control protocols were introduced in [11], enabling convergence time to be set in advance. Researchers have explored PT control to achieve pre-specified settling times based on task requirements. For example, [12], [13] introduced a distributed protocol with time-varying functions to ensure PT consensus for single-integrator agents, while [14] proposed controllers for leader-following consensus of double-integrator agents. Further studies on PT consensus [12], [15] and PT synchronization [16], [17] have also emerged. In particular, [16] developed an event-triggered controller using time-varying control gains, and [17] proposed a smooth controller for PT cluster synchronization.

Despite extensive research on PT consensus, most studies focus on single-weighted networks, and limited work has addressed practical PT synchronization in MWCNs [1], [4], [5], [18], [19]. Moreover, event-triggered mechanisms are rarely considered in MWCNs, though they are ideal for resource-constrained systems due to reduced communication requirements. Event-triggered control uses local triggering conditions to optimize communication, which can lead to better resource allocation compared to traditional periodic mechanisms. To prevent Zeno behavior, a positive inter-event interval is maintained between triggering events, ensuring practical implementability.

In [13], event-triggered consensus protocols with dynamic triggering conditions were developed using time-varying functions, establishing global PT stability criteria. Similarly, [20] proposed a distributed practical PT observer to estimate the reference trajectory in networked systems. Building on these findings, this work addresses PT consensus in MWCNs under event-triggered control, enhancing model alignment with real-world scenarios. The main contributions of this work are summarized as follows:

(I) This paper considers a *multi-weighted complex network* (MWCN), which represents a more realistic and comprehensive model of practical system.

(II) Based on the time-varying function proposed in [20], we construct a control law which is different from [13] and establish sufficient *Lyapunov conditions* for PT stability in MWCNs.

(III) The control scheme is fully distributed which is dependent on the neighbours' information without requiring global information. The controller includes time-varying control parameters which could enable the adjustment more flexible and efficient.

II. Preliminaries

In this section, we consider the following *multi-weighted complex network system* (MWCNS):

$$\begin{aligned}\dot{y}_i(t) &= -Ey_i(t) + Gh(y_i(t)) \\ &\quad + \sum_{p=1}^{q}\sum_{k=1}^{M} b_p D_{ik}^p \Theta^p y_k(t) + J + v_i(t),\end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, M$ indexes the nodes; $y_i(t) \in R^n$ denotes the state vector of the i-th node; $E = \mathrm{diag}(e_1, e_2, \ldots, e_n) > 0$ represents the decay rates; G modulates the activation function $h(y_i(t))$; J is the external input; $v_i(t)$ is the control input; b_p and $\tilde{b}_p$ are the original and modified coupling strengths, respectively; D^p and $\tilde{D}_{ik}^p$ are the respective topology matrices describing the coupling interactions; Θ^p are the internal interaction matrices for each coupling form.

Throughout this paper, the function $h_l(\cdot)$ is assumed to satisfy the global Lipschitz condition. That is, there exists a constant $w_l > 0$ such that

$$|h_l(b) - h_l(a)| \le w_l |b - a|, \tag{2}$$

where

$$W = \mathrm{diag}(w_1^2, w_2^2, \cdots, w_n^2).$$

Definition 2.1. Consider the following system:

$$\dot{\eta}(t) = h(\eta(t)), \quad \eta(0) = \eta_0, \quad t \ge 0, \tag{3}$$

where $\eta \in R^n$ is the state vector, and $h : R^n \to R^n$ is a continuous function with $h(0) = 0$. The origin of the system in Eq. (3) is said to be *globally predefined-time practically stable* if, for a predefined time $T_c > 0$, there exists a constant Ψ such that:

$$\begin{cases} \lim_{t\to T_c} \|\eta(t)\| \le \Psi, \\ \|\eta(t)\| \le \Psi \quad \text{for} \quad t \ge T_c, \\ \lim_{t\to+\infty} \|\eta(t)\| = 0, \end{cases}$$

where Ψ is a designer-adjustable constant.

Definition 2.2. The practical predefined-time synchronization problem for the system in Eq. (1) is solved if the states of the nodes satisfy:

$$\begin{cases} \lim_{t\to T_c} \|y_i(t) - y_k(t)\| \le \tilde{\Psi}, \\ \|y_i(t) - y_k(t)\| \le \tilde{\Psi} \quad \text{for} \quad t \ge T_c, \\ \lim_{t\to+\infty} \|y_i(t) - y_k(t)\| = 0, \end{cases}$$

for all $i, k \in \{1, 2, \ldots, M\}$, where $\tilde{\Psi}$ is the predefined synchronization error bound.

III. Main Conclusion

This section presents a new class of time-varying functions, followed by the design of the event-triggering control scheme and a consistency analysis to validate the proposed approach.

A. Time-Varying Function Design

We define a smooth function as follows:

$$\omega_0(x) = \begin{cases} 0, & x \le 0, \\ e^{-1/x}, & x > 0, \end{cases} \quad x \in R, \tag{4}$$

and further define

$$\omega_b(t, T_c) = \frac{\omega_0(T_c - t)}{\omega_0(T_c - t) + \omega_0(t)}, \tag{5}$$

where $T_c > 0$ is a predefined time parameter chosen by the controller designer. We construct a time-dependent scaling function $\omega(t, T_c, \epsilon) \in R$ as follows:

$$\omega(t, T_c, \epsilon) = \frac{1}{\omega_b(t, T_c) + \epsilon}, \tag{6}$$

where $0 < \epsilon \ll 1$ is a small constant. The function $\omega(t, T_c, \epsilon)$ has the following properties:

1) At $t = 0$, $\omega(t, T_c, \epsilon) = \frac{1}{1+\epsilon}$, and as $t \ge T_c$, $\omega(t, T_c, \epsilon) = \frac{1}{\epsilon}$.
2) $\omega(t, T_c, \epsilon)$ is a C^∞ smooth function over $(0, +\infty)$.
3) $\omega(t, T_c, \epsilon)$ is monotonically increasing over $(0, T_c)$.

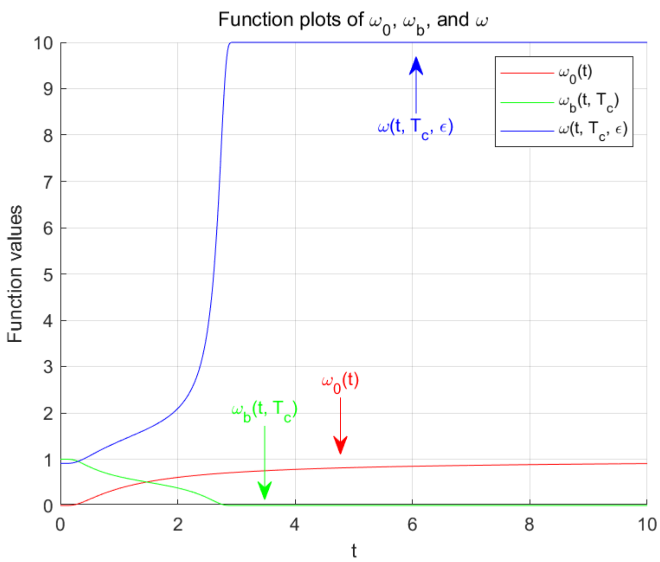

Fig. 1: Trajectories of $\omega_0(x)$, $\omega_b(t, T_c)$, and $\omega(t, T_c, \epsilon)$.

Lemma 1: Suppose there exists a Lyapunov function $V(t)$ satisfying

$$\dot{V}(t) \le -\left(\zeta_1 + \zeta_2 \frac{\dot{\omega}(t, T_c, \epsilon)}{\omega(t, T_c, \epsilon)}\right) V(t), \quad V(0) = V_0, \tag{7}$$

where $\zeta_1 > 0$ is a constant, and $\omega(t, T_c, \epsilon)$ is defined in (6). Then, $V(t)$ converges within the region $V \le \epsilon V_0$ within the predefined time T_c and asymptotically approaches zero as $t \to \infty$.

Proof: Rewrite inequality (7) as

$$\dot{V}(t) \le -\zeta_2 \frac{\dot{\omega}(t)}{\omega(t)} V(t) - \zeta_1 V(t). \tag{8}$$

Multiplying both sides by $\omega^{\zeta_2}(t)$ yields

$$\begin{aligned}\omega^{\zeta_2}(t)\dot{V}(t) &\le -\zeta_2 \dot{\omega}(t)\omega^{\zeta_2 - 1}(t)V(t) - \zeta_1 \omega^{\zeta_2}(t)V(t) \\ &= -\frac{d}{dt}\left(\omega^{\zeta_2}(t)V(t)\right) - \zeta_1 \omega^{\zeta_2}(t)V(t).\end{aligned} \tag{9}$$

Integrating both sides, we have

$$\omega^{\zeta_2}(t)V(t) \leq \omega^{\zeta_2}(0)V(0)e^{-\zeta_1 t}. \tag{10}$$

Thus, $V(t)$ reaches $\left(\frac{\epsilon}{1+\epsilon}\right)^{\zeta_2} V_0 e^{-\zeta_1 T_c} \leq V_0$ as $t \to T_c$ and converges to zero as $t \to \infty$.

Remark 1: The introduced time-varying function class offers flexible tunability by adjusting parameters like ϵ and T_c, enabling the system to meet diverse application requirements. This controller design achieves flexibility, practicality, and stability through appropriate selection of time-varying functions and control parameters.

B. Controller Design

Utilizing the time-varying function defined, we propose an event-triggered control input:

$$\begin{aligned} v_i(t) = &-\left(b(t) + \frac{\dot{\omega}(t,T_c,\epsilon)}{\omega(t,T_c,\epsilon)}\right) \\ &\times \sum_{p=1}^{q}\sum_{k\in\mathcal{N}_i} \tilde{b}_p \tilde{D}_{ik}^{p}\left(y_i(t_r^i) - y_k(t_r^k)\right), \end{aligned} \tag{11}$$

where $b(t)$ is an exponential decay function designed to satisfy $b(t) > \tilde{b}$ for all $t \geq 0$;t_r^i denotes the most recent event-triggered time of node i; $y_i(t_r^i)$ represents the state of node i at time t_r^i.

Remark 2: In this control law, $b(t)$ and $\omega(t, T_c, \epsilon)$ are time-varying functions, allowing dynamic control strategy adjustments based on time. This enables enhanced stability and robustness while achieving predefined-time synchronization in complex and dynamic network environments.

Define the event-triggered error $\phi_i(t) = y_i(t_r^i) - y_i(t)$. The event-triggering condition is defined as

$$\begin{aligned} t_{r+1}^i = \inf\Bigg\{ t > t_r^i : \|\phi_i(t)\| > \rho \Bigg\| \sum_{p=1}^{q}\sum_{k\in\mathcal{N}_i} \tilde{D}_{ik}^{p} \\ \times \left(y_i(t_r^i) - y_k(t_r^k)\right) \Bigg\| \Bigg\}, \end{aligned} \tag{12}$$

where $t \in [t_r^i, t_{r+1}^i)$ and $\rho > 0$ is a constant.

C. Synchronous Analysis

Let $z^* = (z_1^*, z_2^*, \cdots, z_n^*)$ denote an equilibrium solution vector of the network (1):

$$0 = -Ez^* + Gh(z^*) + \sum_{p=1}^{q} b_p D^p \Theta^p z^*.$$

Define the error $e_i(t) = y_i(t) - z^*$, yielding the error system:

$$\begin{aligned} \dot{e}_i(t) = &-Ee_i(t) + G\hat{h}(e_i(t)) + \sum_{p=1}^{q}\sum_{k=1}^{M} b_p D_{ik}^{p} \Theta^p e_k(t) \\ &-\left(b(t) + \frac{\dot{\omega}(t,T_c,\epsilon)}{\omega(t,T_c,\epsilon)}\right) \\ &\times \sum_{p=1}^{q}\sum_{k\in\mathcal{N}_i} \tilde{b}_p \tilde{D}_{ik}^{p}\left(e_i(t_r^i) - e_k(t_r^k)\right), \end{aligned} \tag{13}$$

where $\hat{h}(e_i(t)) = h(y_i(t)) - h(z^*)$ represents the nonlinear error term.

Based on the event-triggering condition, we derive

$$\begin{aligned} \|\phi_i(t)\| &\leq \rho \left\| \sum_{p=1}^{q}\sum_{k\in\mathcal{N}_i} \tilde{D}_{ik}^{p}\left(e_i(t_r^i) - e_k(t_r^k)\right) \right\| \\ &\leq 2\rho \sum_{p=1}^{q} \tilde{c}_p \left(\|\phi(t)\| + \|e(t)\|\right), \end{aligned} \tag{14}$$

where $0 < \rho < \frac{1}{2M\sum_{p=1}^{q}\tilde{c}_p}$ and $\tilde{\rho} = \frac{1}{2M\sum_{p=1}^{q}\tilde{c}_p}$.

Theorem 3.1: The MWCNS (1) achieves practical predefined-time synchronization under the event-triggered condition if a function $b(t) > \tilde{b} = \frac{\lambda_1}{\lambda_2}$ exists, where

$$\lambda_1 = -E + (G \otimes W) + \sum_{p=1}^{q} b_p (D^p \otimes \Theta^p),$$

$$\lambda_2 = \sum_{p=1}^{q} \tilde{b}_p + 2M \sum_{p=1}^{q} \tilde{b}_p (\tilde{c}^p)^2 \left(\frac{\rho^2}{(\tilde{\rho} - \rho)^2} + 1\right).$$

Proof: We construct a Lyapunov function $V(t)$ as follows:

$$V(t) = \frac{1}{2}\sum_{i=1}^{M} e_i^T(t) e_i(t). \tag{15}$$

Taking the derivative of $V(t)$ with respect to time, we get:

$$\begin{aligned} \dot{V}(t) = &\sum_{i=1}^{M} e_i^T(t) \Bigg[\sum_{p=1}^{q}\sum_{k=1}^{M} b_p D_{ik}^{p} \Theta^p e_k(t) + G\hat{h}(e_i(t)) \\ &-\left(b(t) + \frac{\dot{\omega}(t,T_c,\epsilon)}{\omega(t,T_c,\epsilon)}\right) \\ &\times \sum_{p=1}^{q}\sum_{k\in\mathcal{N}_i} \tilde{b}_p \tilde{D}_{ik}^{p}\left(e_i(t) + \phi_i(t) - e_k(t) - \phi_k(t)\right) \\ &- Ee_i(t) \Bigg]. \end{aligned} \tag{16}$$

It's easy to deduce

$$\sum_{i=1}^{M} e_i^T(t) G\hat{h}(e_i(t)) \leq e^T(t)(G \otimes W)e(t). \tag{17}$$

$$\sum_{i=1}^{M} e_i^T(t) \sum_{p=1}^{q} b_p D_{ik}^{p} \Theta^p e_k(t) = \sum_{p=1}^{q} b_p e^T(t)(D^p \otimes \Theta^p)e(t). \tag{18}$$

$$\sum_{i=1}^{M} e_i^T(t)(-E)e_i(t) = e^T(t)(I_M \otimes (-E))e(t). \tag{19}$$

$$\begin{aligned} &\sum_{i=1}^{M} e_i^T(t)\left(b(t) + \frac{\dot{\omega}(t,T_c,\epsilon)}{\omega(t,T_c,\epsilon)}\right) \\ &\quad\times \sum_{p=1}^{q}\sum_{k\in\mathcal{N}_i} \tilde{b}_p \tilde{D}_{ik}^{p}\left(e_i(t) - e_k(t)\right) \\ &\leq \frac{1}{2}\left(b(t) + \frac{\dot{\omega}(t,T_c,\epsilon)}{\omega(t,T_c,\epsilon)}\right) \sum_{p=1}^{q} \tilde{b}_p e^T(t) e(t) \\ &\quad + 2M\left(b(t) + \frac{\dot{\omega}(t,T_c,\epsilon)}{\omega(t,T_c,\epsilon)}\right) \\ &\quad\times \sum_{p=1}^{q} \tilde{b}_p (\tilde{c}^p)^2 e^T(t) e(t). \end{aligned} \tag{20}$$

Combining the above results, we have:

$$\begin{aligned}\dot{V}(t) &\leq -\left[\lambda_1 - b(t)\lambda_2\right] V(t) - \lambda_2 \frac{\dot{\omega}(t,T_c,\epsilon)}{\omega(t,T_c,\epsilon)} V(t) \\ &= -\left(z_1 + \lambda_2 \frac{\dot{\omega}(t,T_c,\epsilon)}{\omega(t,T_c,\epsilon)}\right) V(t),\end{aligned} \tag{21}$$

where $z_1 = \lambda_1 - b(t)\lambda_2 > 0$ and $\lambda_2 > 0$ as defined in the theorem.

Referring to Lemma 1, the practical predefined-time synchronization of the system (1) is achieved. ■

Theorem 3.2: Under the event-triggering condition with a fixed positive lower bound τ, Zeno behavior is excluded, ensuring that the inter-event intervals $t^i_{r+1} - t^i_r$ are uniformly bounded below by $\tau > 0$ for all $r > 0$ and $i = 1, 2, \ldots, M$.

Proof: From the definition of the event-triggered error $\phi_i(t) = y_i(t^i_r) - y_i(t)$, we have:

$$\|\dot{\phi}_i(t)\|_2 \leq \alpha \|\phi_i(t)\|_2 + \beta, \tag{22}$$

where

$$\begin{aligned}\alpha &= \lambda_{\max}(E) + |\lambda_{\max}(G)|w + \sum_{p=1}^{q} b_p \|D^p\|_2 \lambda_{\max}(\Theta^p), \\ \beta &= \left(\lambda_{\max}(E) + |\lambda_{\max}(G)|w\right) \|e_i(t^i_r)\|_2 \\ &\quad + \sum_{p=1}^{q} b_p \|D^p\|_2 \lambda_{\max}(\Theta^p) \|e(t^i_r)\|_2 \\ &\quad + \hat{U}_i(t^i_r, t^k_r),\end{aligned} \tag{23}$$

with $w = \max_l\{w_l\}$ and

$$\hat{U}_i(t^i_r, t^k_r) = \sum_{p=1}^{q} \sum_{k\in\mathcal{N}_i} \tilde{b}_p \tilde{D}^p_{ik} \|e_i(t^i_r) - e_k(t^k_r)\|_2. \tag{24}$$

Solving the differential inequality $\|\dot{\phi}_i(t)\|_2 \leq \alpha\|\phi_i(t)\|_2 + \beta$ yields:

$$\|\phi_i(t)\|_2 \leq \frac{\beta}{\alpha}\left(e^{\alpha(t-t^i_r)} - 1\right). \tag{25}$$

At the event-triggering time t^i_{r+1}, the triggering condition is satisfied:

$$\|\phi_i(t^i_{r+1})\|_2 = \rho \left\| \sum_{p=1}^{q} \sum_{k\in\mathcal{N}_i} \tilde{D}^p_{ik} \left(e_i(t^i_r) - e_k(t^k_r)\right) \right\|_2. \tag{26}$$

Substituting the bound of $\|\phi_i(t)\|_2$ into the triggering condition, we get:

$$\frac{\beta}{\alpha}\left(e^{\alpha(t^i_{r+1}-t^i_r)} - 1\right) = \rho \left\|\cdot\right\|, \tag{27}$$

where $\|\cdot\|$ represents the norm from the triggering condition.

Rearranging terms, we have:

$$e^{\alpha(t^i_{r+1}-t^i_r)} = 1 + \frac{\alpha\rho}{\beta}\left\|\cdot\right\|. \tag{28}$$

To ensure a positive lower bound τ for $t^i_{r+1} - t^i_r$, we utilize the inequality:

$$\ln(1+x) \geq \frac{x}{1+x}, \quad x > 0. \tag{29}$$

Applying this inequality, we obtain:

$$\alpha(t^i_{r+1} - t^i_r) \geq \frac{\alpha\rho\delta}{\beta + \alpha\rho\delta}, \tag{30}$$

where $\delta = \min_{i,r} \left\|\cdot\right\| > 0$.

Therefore, the inter-event time interval satisfies:

$$t^i_{r+1} - t^i_r \geq \frac{\rho\delta}{\beta + \alpha\rho\delta} = \tau > 0, \tag{31}$$

which provides a positive lower bound on the inter-event intervals, effectively excluding Zeno behavior. ■

Remark 3: The inequality used in the proof is crucial for establishing the positive lower bound on the inter-event intervals. By ensuring that the logarithmic expression is bounded below, we confirm that the intervals $t^i_{r+1} - t^i_r$ cannot approach zero, thus eliminating the possibility of Zeno behavior in the event-triggered control system.

IV. Simulations

Consider the following MWCNNs with event-triggered communication:

$$\begin{aligned}\dot{y}_i(t) =& -Ey_i(t) + Gh(y_i(t)) + \sum_{p=1}^{3}\sum_{k=1}^{6} b_p D^p_{ik} \Theta^p y_k(t) \\ &+ J - \left(b(t) + \frac{\dot{\omega}(t,T_c,\epsilon)}{\omega(t,T_c,\epsilon)}\right) \\ &\times \sum_{p=1}^{3}\sum_{k\in\mathcal{N}_i} \tilde{b}_p \tilde{D}^p_{ik} (y_i(t^i_r) - y_k(t^k_r)),\end{aligned} \tag{32}$$

where $i = 1, 2, \cdots, 6$; $h(y_i(t)) = \frac{|y_i(t)+1| - |y_i(t)-1|}{4}$, $p = 1, 2, 3$; $J = (J_1, J_2, J_3)^T = (0, 0, 0)^T$; $b_1 = 0.1$, $b_2 = 0.08$, $b_3 = 0.06$; $\widetilde{b}_1 = 0.1$, $\widetilde{b}_2 = 0.05$, $\widetilde{b}_3 = 0.03$; $E = \mathrm{diag}(2, 2, 2, 2, 2, 2)$; $\Theta^1 = \mathrm{diag}(0.3, 0.3, 0.4, 0.3, 0.2, 0.2)$, $\Theta^2 = \mathrm{diag}(0.4, 0.2, 0.2, 0.3, 0.2, 0.3)$, $\Theta^3 = \mathrm{diag}(0.2, 0.4, 0.3, 0.2, 0.2, 0.3)$; $G = 0.1I_6$.

The matrices D^p are chosen as follows:

$$D^1 = \begin{pmatrix} -1 & 0.01 & 0.03 & 0.02 & 0 & 0.04 \\ 0.01 & -1 & 0.03 & 0.01 & 0.02 & 0 \\ 0.03 & 0.03 & -1 & 0.01 & 0.01 & 0.03 \\ 0.02 & 0.01 & 0.01 & -1 & 0 & 0.02 \\ 0 & 0.02 & 0.01 & 0 & -1 & 0.03 \\ 0.04 & 0 & 0.03 & 0.02 & 0.03 & -1 \end{pmatrix},$$

$$D^2 = \begin{pmatrix} -1 & 0.01 & 0.02 & 0.02 & 0.01 & 0.05 \\ 0.01 & -1 & 0.01 & 0.03 & 0.02 & 0 \\ 0.02 & 0.01 & -1 & 0.02 & 0.03 & 0.03 \\ 0.02 & 0.03 & 0.02 & -1 & 0 & 0.02 \\ 0.01 & 0.02 & 0.03 & 0 & -1 & 0.01 \\ 0.05 & 0 & 0.03 & 0.02 & 0.01 & -1 \end{pmatrix},$$

$$D^3 = \begin{pmatrix} -1 & 0.02 & 0.01 & 0.03 & 0.04 & 0.06 \\ 0.02 & -1 & 0.02 & 0.01 & 0.03 & 0 \\ 0.01 & 0.02 & -1 & 0.04 & 0.02 & 0.1 \\ 0.03 & 0.01 & 0.04 & -1 & 0 & 0.01 \\ 0.04 & 0.03 & 0.02 & 0 & -1 & 0.02 \\ 0.06 & 0 & 0.01 & 0.01 & 0.02 & -1 \end{pmatrix}.$$

The variable $b(t)$ is designed as $b(t) = 0.1 \cdot e^{-0.5t} + \tilde{b}$. Set $T_c = 3$s and $\epsilon = 0.1$. For the triggering condition in Eq. (12), we choose $\rho = 0.5$, $\widetilde{\rho} = 1$, and $N = 3$. Under the initial states $y(:,1) = [1; 2; 3; 4; 5; 6]$, the state trajectories of $\|y_i(t)\|$ are plotted in Fig. 3, illustrating that the states of $\|y_i(t)\|$ converge to a small neighborhood of 0 as time approaches 3s.

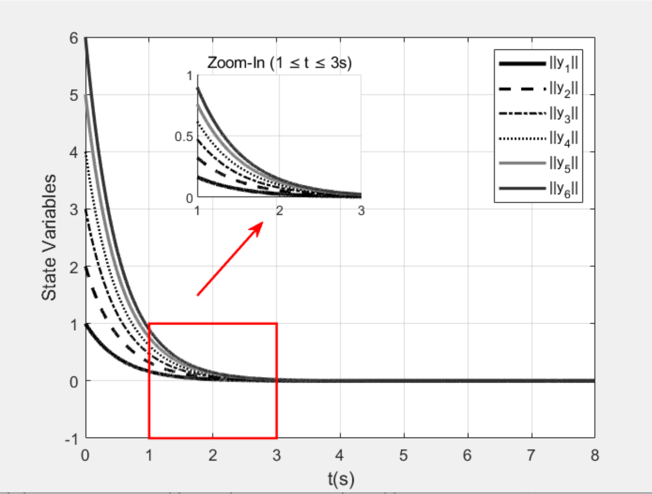

Fig. 2: The evolution of the state vectors $\|y_i(t)\|$, $(i = 1, 2, \ldots, 6)$.

V. Conclusion

This paper addresses the challenge of achieving practical predefined-time (PT) consensus in multi-weighted complex networks (MWCNs) using event-triggered control. For the first time, we introduce the concepts of practical PT consensus and associated time-varying functions into this context. We propose a fully distributed, event-triggered controller that enables each node to rely solely on local information from its neighborhood and its own state, thereby achieving practical PT synchronization without the need for global knowledge. Moreover, this approach effectively eliminates Zeno behavior by enforcing a positive minimum inter-event interval.

Future work will extend these results to the practical implementation of PT synchronization in directed graphs and networks with switching topologies, further broadening the applicability of the proposed method.

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