Portfolio Decision and Risk Analysis with Disappointment and Regret Emotions

Xue Deng, Zhi Li, Tiantian Zhang, Wen Zhou

Abstract—**The portfolio problem focuses on efficiently allocating securities to maximize profits and minimize risk. The classic mean-variance model is based on investors' rational assumption. However, psychological research indicates that emotions significantly influence investment behavior. This paper establishes a portfolio model considering disappointment and regret emotions. Firstly, based on investors' preferences for seeking rejoice or avoiding regret, we adopt linear and nonlinear (power-exponential) regret-rejoice functions to construct corresponding portfolio models. Secondly, considering investors' potential feelings of disappointment and aversion, we establish a portfolio model that incorporates both disappointment and regret emotions. By regarding disappointment aversion and regret avoidance as a dual-objective optimization problem, we employ an improved particle swarm optimization algorithm to enhance the efficiency of solving the model. Finally, by selecting stock data and applying our proposed model under emotional influence, we verify the impact of considering emotional factors on investor decisions, as well as the sensitivity of investors to emotions on portfolio composition. Additionally, a comparative analysis is presented on the effects of different regret-rejoice functions.**

Index Terms—**Portfolio, Disappointment theory, Regret theory, Utility function, Investors' preferences.**

I. Introduction

After the proposal of the mean-variance model, portfolio theory has been continuously evolving. However, investors' decisions are not constantly rational but are often influenced by emotions. By taking into account various anticipated emotions of investors in the process of risk investment, such as disappointment aversion, regret aversion, and pursuit of rejoice, not only can it help to establish investment portfolio models that are more in line with the actual investment process, but it can also better explain anomalies in the financial market. Therefore, the objective of this paper is to delve into the impact of emotional factors on investors' decisions and propose new portfolio models to guide investment behavior, thereby enhancing investment efficiency and risk management levels.

Since the introduction of the mean-variance model, portfolio theory has evolved to optimize the trade-off between risk and return. However, this classic approach assumes investor rationality, overlooking the significant impact of emotional factors on investment decisions. Psychological research indicates that emotions such as disappointment aversion and regret avoidance influence investor behavior, often leading to deviations from purely rational choices.

Incorporating these emotional factors into portfolio modeling can enhance the alignment between theoretical models and actual investment behaviors, and offer explanations for observed market anomalies. This paper aims to address the gap by introducing portfolio models that integrate disappointment and regret emotions. By developing both linear and nonlinear regret-rejoice functions and leveraging an enhanced particle swarm optimization algorithm, we propose a dual-objective framework that considers emotional sensitivities in portfolio selection. This approach not only enhances decision-making efficacy but also offers new insights into risk management and optimization in emotional contexts.

A. Literature review

Portfolio optimization plays a crucial role in investment, as it seeks to guide investment choices that fulfill investor objectives. Since Markowitz [1] introduced the mean-variance model in 1952, portfolio theory has continuously made new advancements. However, Ellsberg [2] revealed that investors do not always opt for the "correct" solutions. This is because investors, as imperfectly rational individuals, are susceptible to emotional influences in their investment behaviors, resulting in biases such as prediction errors, conservatism, and overconfidence. Expected emotions and immediate emotions play undeniable roles in the investment process. Expected emotions refer to investors' emotional responses to potential positive or negative outcomes in the future, while immediate emotions are the emotional states of decision-makers in the present decision-making activities. The main difference between the two lies in that immediate emotions directly influence decision-making behaviors without affecting investors' cognitive processing. Since Neumann and Morgenstern [3] proposed the "Expected Utility Theory" in the mid-20th century, many researchers have deeply studied expected emotions as important variables influencing decision-making. This research direction is not only used to explain anomalies in financial markets but also to optimize portfolio models. Significant progress has been made, particularly in the study of

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disappointment and regret emotions. In addition, there are some other scholars researched the application of portfolio decision. Deng and Geng [4] proposed an improved weighting method that combines interval-valued intuitionistic fuzzy AHP with entropy weight, along with a new score function, to overcome the limitations of unilateral weighting methods. Deng et al. [5] proposed a mean-variance-efficiency portfolio model that incorporates stock efficiency measured by a fuzzy DEA model and used a genetic algorithm for optimization, demonstrating its feasibility and highlighting the importance of considering portfolio efficiency in financial decision-making. Schober [6] proposed a dynamic programming approach using value function iteration on spatially adaptive sparse grids to solve Bellman equations in finance, particularly for modeling dynamic portfolio choice, and demonstrated how this method can mitigate the curse of dimensionality and compute optimal choices.

The origin of Disappointment Aversion Theory can be traced back to 1985 when Bell [7] proposed a definition of disappointment emotions in the investment process: decision-makers establish fixed reference points as standards, feeling satisfied when outcomes exceed these standards, and experiencing disappointment when outcomes fall below them. This definition successfully introduced psychological research into the field of finance. In financial markets, some phenomena (such as the "Allais paradox") cannot be explained by the expected utility hypothesis. Therefore, to reasonably explain these phenomena, scholars have developed Disappointment Aversion Theory models, where disappointment aversion describes investors' aversion to risk, demonstrating investors' asymmetric risk preferences. The introduction of disappointment aversion models prompted scholars to apply them to explain anomalies in financial markets and the field of risk investment. Ang et al. [8] successfully explained the phenomenon of stock returns exceeding risk-free asset returns in the U.S. market based on disappointment theory. However, a drawback of disappointment aversion models is the inability to calculate lifetime utility, leading to relatively slow development over a period of time. It was not until Delikouras [9] proposed the calculation of lifetime utility using investors' consumption data that Disappointment Aversion Theory was further developed. Cao et al. [10] combined disappointment aversion and the psychological aspect of seeking satisfaction during the newsboy problem, successfully applying disappointment theory to solve the newsboy problem and finding that the quantity of newspapers ordered by the newsboy decreases as the aversion to disappointment strengthens. Graves and Ringuest [11] tested the influence of overconfidence on portfolio based on disappointment theory. Wang et al. [12] combined prospect theory with disappointment theory to address the challenge of minimizing risk brought about by irrational investment decisions during wealth fluctuations. Zhang et al. [13] proposed a disappointment theory-based probabilistic hesitant fuzzy multi-attribute decision making method to solve the investment decision problem, which can integrate the psychological behavior of decision makers into the decision-making process and make the decision results more authentic and reliable.

Regret theory, also known as the theory of regret, originated in 1982 when Bell et al. [14] proposed the anticipated regret theory based on prospect theory. Regret emotions arise when investors' decision outcomes are not the optimal decisions among all available options. It was the first time regret emotions were incorporated into existing utility functions, revealing investors' motivation to seek rejoice and avoid regret during the investment process. Loomes and Sugden [15] suggested that regret emotions are anticipatory emotions generated by investors based on past investment experiences, which directly influence investors' decisions. Subsequently, regret theory gradually developed in the field of investment decision-making, leading to the proposition of "regret aversion theory", emphasizing that investors tend to minimize regret rather than minimize risk. Gao and Duan [16] established a mathematical model quantifying anticipated regret based on regret theory, demonstrating that introducing regret emotions can influence investors' decisions when constructing portfolio and encouraging investors to make alternative choices. Regret theory explains why investors sometimes choose conservative options while other times they lean towards aggressive options. Building upon this, regret theory has been extended to scenarios involving general choice sets and widely applied to solve various multi-option selection problems. For example, Muernann et al. [17] studied the impact of regret aversion on pension plans, while Dodonova [18] applied regret theory to capital asset pricing, successfully explaining the issue of excessive stock return volatility. Deng and Geng [19] proposed a novel two-parameter coherent fuzzy number that can flexibly capture investors' attitudes (pessimistic, optimistic, or neutral). Song et al. [20] proposed a new probabilistic hesitant fuzzy TOPSIS method based on the regret theory, the proposed method can consider both the probabilistic hesitant fuzzy information and regret aversion of experts at the same time in actual applications. Deng and Huang [21] proposed a novel mean-entropy portfolio model with risk curve and total mental accounts under multiple background risk is constructed.

B. Motivation

Psychological studies have shown that emotions play a significant role in investors' investment behavior. When investors are worried that their investment returns may not meet their expectations, they may feel disappointed. However, if the returns exceed their expectations, they may feel satisfied. Additionally, investors may experience regret emotions due to concerns that unchosen options might have performed better during the investment period, while feeling rejoice when the chosen options perform better. This paper integrates theoretical research findings on the impact of expected emotions on portfolio, taking into account investors' feelings of disappointment aversion, the pursuit of rejoice, and avoidance of regret. By seeking practical emotion utility functions to construct portfolio models based on knowledge from behavioral finance, statistics, and related fields, it aims to provide decision models that are meaningful and applicable for investors to refer to.

C. Organization

The structure of this paper is as follows. Section I provides a detailed exposition of the research background and current status of risk investment, portfolio management, expected emotions, Expected Utility Theory, as well as disappointment and regret emotions. Section II serves as the foundational theory, such as disappointment theory, regret theory, max-min regret criterion, and portfolio theory. Section III focuses on the portfolio models under disappointment and regret emotions. Section IV conducts empirical analysis on the portfolio models constructed earlier. Section V presents the conclusions and outlines prospects for future research.

II. PRELIMINARIES

A. Portfolio theory

Portfolio theory focuses on studying how investors diversify their investments to achieve the highest returns and minimize risk. It mainly comprises two aspects including the mean-variance model and the efficient frontier analysis approach.

a. Mean-variance model

In portfolio theory, risk and return are characterized using mean and variance, respectively. Specifically, the investor's utility function includes only mean and variance as objectives. The mean-variance model is a significant method for analyzing the risk-return situation using mean, variance, and covariance. This model is based on the following assumptions:

(1) The investor's utility functions including expected returns, variance, and covariance are known.
(2) Investors, driven by risk aversion, have two investment objectives. One is to maximize terminal returns under a specified level of risk, and the other is to minimize investment risk while achieving a predetermined level of return.
(3) Investors make portfolio based solely on the above inputs.

Portfolio theory aims to achieve investors' objectives by selecting available risk assets and continually adjusting their investment proportions accordingly.

b. Efficient frontier

Under the aforementioned measurements of expected returns and risk for portfolio, by randomly generating multiple sets of investment proportions W for investment, corresponding sets of results can be obtained. Representing the investment results in a scatter plot with volatility as the horizontal axis and expected returns as the vertical axis yields the feasible set, as shown in Fig. 1.

The boundary of the feasible set scatter plot is known as the efficient frontier. Portfolio outside this boundary cannot be constructed using the current risk assets. Portfolio lying on the efficient frontier represents the maximum returns achievable under certain levels of risk. Thus, the curve where points with higher returns for the same volatility lie is the efficient frontier. In the case of the minimum variance frontier, only the upper portion represents the efficient frontier, as illustrated in Fig. 2.

In the context, where the $x-$axis represents volatility and the $y-$axis represents expected returns, the convexity of the efficient frontier reflects investors' attitudes towards risk. Due to risk aversion, investors tend to prefer options with lower risk at the same level of expected returns. Therefore, the efficient frontier is convex to the right. The slope of the curve represents the degree of risk aversion among investors. A steeper curve indicates that investors require higher compensatory returns for the same increase in risk, implying

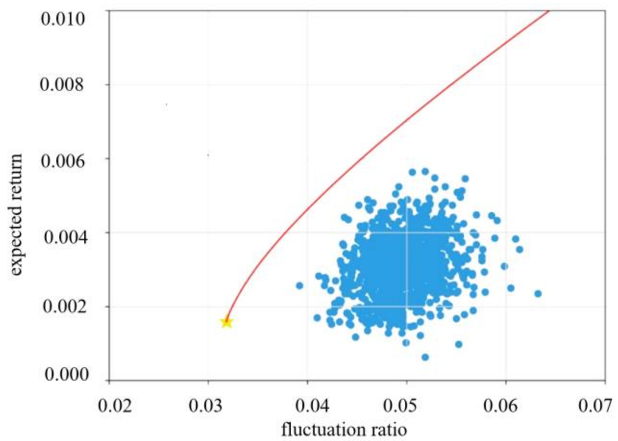


Fig.1. Scatter plot of feasible solutions

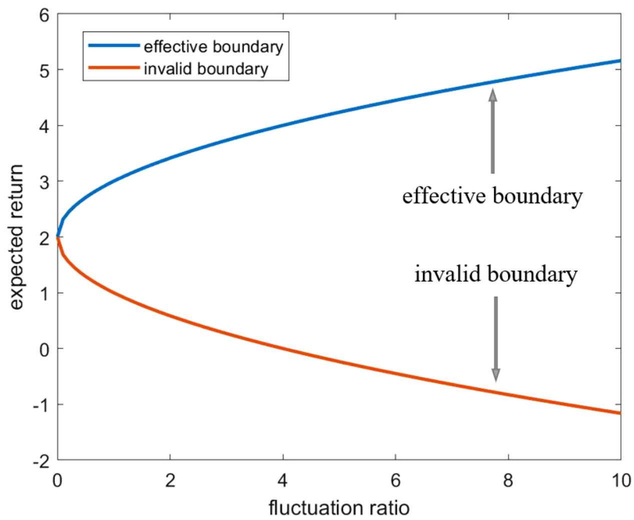


Fig. 2. Efficient and inefficient frontiers

a higher level of risk aversion. Conversely, a shallower curve suggests that investors require lower compensatory returns for the same increase in risk, indicating lower risk aversion.

B. Disappointment theory

a. Expected Utility Theory

Expected Utility Theory analyzes entirely rational investors under axiomatic assumptions, describing investors' references through utility functions. A utility function assigns a numerical value to a particular outcome. Since the proposal of Expected Utility Theory, experts and scholars have continuously researched and proposed a series of utility theories, expanding the depth and breadth of research. Tversky and Kahneman [22] introduced "Prospect Theory". Bell [7] and Loomes [23] proposed "Regret Theory". Gul [24] introduced "Disappointment Aversion Theory". These theories effectively address phenomena such as preference reversals in financial markets, helping to describe investors' asymmetric preferences.

b. Disappointment theory

Disappointment theory posits that when the outcome of a decision falls below the previously expected return target, investors experience feelings of disappointment. It is introduced to explain the "Allais Paradox". In Allais' designed experiments, investors' choices neither fully comply with the principle of certainty nor with the principle of independence, leading to contradictions.

Building upon the Allais Paradox, scholars such as Bell et al. [7] introduced the concept of "disappointment" and incorporated it into the portfolio process, offering a rational and intuitive explanation for the behavior of participants in the Allais Paradox. Delquié [25] expanded on disappointment theory by considering the impact of deviations between each outcome on the results. Denote the possible returns as follows: $X=(x_1,p_1;x_2,p_2;\cdots x_T,p_T)$.

Where x_i represents the returns, and p_i denotes the corresponding probabilities. Based on these parameters, a value-at-risk model can be constructed:

$$V(X)=\sum_{i=1}^{T}p_j v(x_j)-\frac{1}{2}\sum_{j,k=1}^{T}p_j v(x_j)H(|v(x_j)-v(x_k)|). \tag{1}$$

Consider a portfolio comprising N risky assets, where ω_i represents the investment proportion, and the historical returns for asset i over the past T periods are denoted by $\{x_{ij}, j=1,2,\cdots,T\}$. Using $\frac{1}{T}$ to represent p_j and p_k. The portfolio return is then given by:

$$M(\omega)=\sum_{i=1}^{N}\sum_{j=1}^{T}\frac{1}{T}\omega_i v(x_j). \tag{2}$$

The portfolio risk is denoted as:

$$\Delta(\omega)=\sum_{k=1}^{T}\sum_{j=1}^{T}\frac{1}{2T^2}H(|\sum_{i=1}^{N}\omega_i v(x_{ij})-v(x_{ik})|). \tag{3}$$

The function $H(\bullet)$ is a disappointment-satisfaction function defined on the non-negative interval, characterizing investors' emotional between potential outcomes. According to Cillo and Delquié [26], it can be defined as:

$$H(z)=z+e^{-mz}-1,\ 0\le m\le 1. \tag{4}$$

The function $v(\bullet)$ is an increasing function used to describe the subjective value of investors towards possible outcomes. According to Schneider's TAU function model [27], it can be defined as:

$$v(x_{ij})=\begin{cases}(x_{ij}-r)^a, & x_{ij}\ge r,\\ -(r-x_{ij})^b, & x_{ij}<r.\end{cases} \tag{5}$$

$$r=\max_{i=1,\cdots,n}(\min_{k=1,\cdots,n} x_{ik}). \tag{6}$$

Where the parameter $(a,b>1)$ represents the extent to which investors perceive deviations from expected targets. The larger the deviation, the greater this perception. Additionally, $a<b$ indicates that the perception of returns exceeding the target is less than the perception of returns falling below the target. In other words, investors are more inclined towards disappointment and aversion relative to pursuing satisfaction.

In this case, leveraging the disappointment theory to construct a portfolio model:

$$\begin{cases}\max\ f=M(\omega)-\Delta(\omega)\\ \text{s.t.}\ \ \sum_{i=1}^{N}\omega_i=1,\\ \quad l\varepsilon_i\le\omega_i\le u\varepsilon_i, i=1,2,\cdots,N,\\ \quad \sum_{i=1}^{N}\varepsilon_i\le K, \varepsilon_i\in\{0,1\}.\end{cases} \tag{7}$$

Where constraint $l\varepsilon_i\le\omega_i\le u\varepsilon_i$ defines the range of values for asset weights, the final equation $\sum_{i=1}^{N}\varepsilon_i\le K$ is used to constrain the selection of investment assets. Asset i is selected when $\varepsilon_i=1$, and not selected when $\varepsilon_i=0$.

C. Regret theory

a. Basic idea of regret theory

In regret theory, regret refers to the negative psychological emotion when the decision outcome is not the optimal choice, while rejoice is the positive psychological emotion when the decision outcome is the optimal choice among all available options. Therefore, when considering regret factors, investors' decisions are influenced by both the results obtained from choosing alternative options and the expectations of regret and rejoice.

When considering regret, the decision-makers' satisfaction with the decision is influenced by the difference between the current outcome and the optimal outcome. Suppose the decision-makers currently has two options A_1, A_2 to choose from, with potential outcomes X_1 and X_2 respectively. Then, the decision-makers' satisfaction with the current option depends not only on the current outcome X_i, but also on the difference between the outcomes X_1 and X_2. When the decision-makers' chosen option yields a lower outcome than the alternative option, regret is experienced, conversely, when the chosen option yields a higher outcome than the alternative option, rejoice is experienced.

Regret theory introduces regret and rejoice emotions into portfolio by adjusting the utility function. Let m be the number of alternative options $A_1, A_2,\cdots,A_m$, and let $x_1,x_2,\cdots,x_m$ denote the outcomes of these options. Then, the decision-makers' perceived utility for option A_i is given by:

$$U(A_i)=v(x_i)-kg[v(x^{\max})-v(x_i)] \tag{8}$$

Where $x^{\max}=\max\{x_i\mid i=1,2,\cdots,m\}$ represents the outcome obtained from the optimal decision; $v(x_i)$ denotes the utility brought by the outcome; the function $g(\bullet)$ is the regret-in-action function, taking the difference in outcomes as the independent variable, thus, the decision-makers' regret and rejoice emotions depend on the difference between the outcome and the optimal solution; parameter k represents the degree to which the investor perceives regret. The function $g(\bullet)$ satisfies the following 3 conditions.

(1) $g(0)=0$ indicating that when the outcome of the chosen option is the same as that of the optimal solution, the decision-makers do not experience regret.

(2) $g'(\bullet)>0$ implying that the regret function is

monotonically increasing; the greater the discrepancy between the actual decision and the optimal decision, the greater the regret.

(3) $g''(\bullet) > 0$ indicating that the regret function is concave.

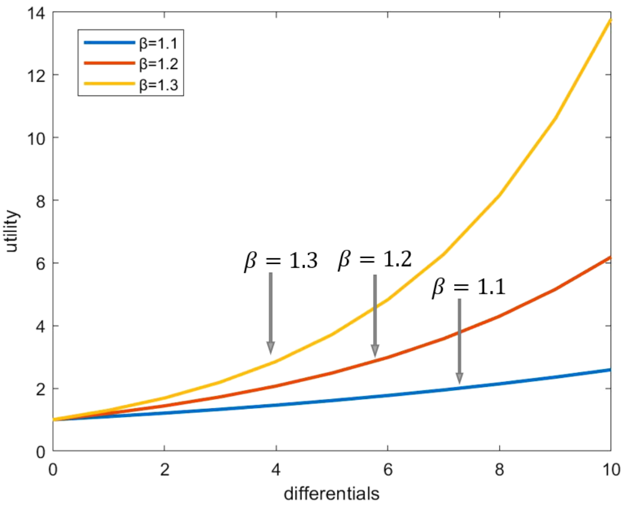


Fig. 3. Regret-rejoice function of different values of β

b. Minimax regret criterion

The minimax regret criterion is an important method for investors to minimize regret values. Its calculation process involves firstly computing the regret values of each option under different scenarios, identifying the maximum regret value for each option, and finally selecting the option with the smallest maximum regret value. The mathematical formulation of the minimax regret criterion can be expressed as:

$$\begin{cases} \theta_{RV} = \min\ z \\ \text{s.t.}\ \ y_s - \theta_s(1-z) \geq 0, \\ \qquad \theta_s = \max(y_s). \end{cases} \tag{9}$$

Where the variable z represents the degree of regret, y_s represents the objective function of the optimization problem under scenario s, and θ_s is the optimal value of the objective function under scenario s When faced with multiple scenarios, there will be multiple constraint conditions.

III. Portfolio Models under Disappointment and Regret Emotions

This section aims to establish portfolio models under different emotional states. Firstly, we adopt the classical Markowitz mean-variance model as Model 1, defining the risk of the portfolio as the volatility of returns. This model aims to balance investors' objectives, namely, seeking high returns and low risk. Secondly, Model 2 is based on regret theory, introducing investors' expected emotions during the investment process as an important factor. We use a simplified one-dimensional linear regret-rejoice function to characterize investors' emotional utility and construct a portfolio model considering regret emotions. Model 3 further selects a power form regret-rejoice function that conforms to regret theory and is more in line with actual utility. Through this adjustment, we construct a portfolio model that is closer to the actual investment process. Finally, Model 4 introduces the expected emotions of disappointment and satisfaction as another important influencing factor based on prospect theory. Using disappointment theory, we minimize disappointment emotions as another objective function, constructing a bi-objective optimization model. This model better fits investors' psychological tendencies of disappointment aversion, regret avoidance, and pursuit of rejoice in investment.

A. Markowitz classic mean-variance model

a. Mean-variance model

With the continuous development of risk investment, experts and scholars have been exploring methods to solve the portfolio investment problem, striving to achieve investors' decision-making goals: maximizing returns while minimizing risk. There are mainly two approaches to solving the portfolio investment problem: one is to establish models aimed at maximizing investors' expected returns, and the other is to construct risk investment models that minimize risk while ensuring a certain level of return. The most basic portfolio model is the mean-variance model proposed by Markowitz in 1952.

Suppose there are N risky assets, with the expected returns of each asset denoted as x_i, the covariance matrix denoted as V, the weight vector denoted as W, the identity matrix denoted as I, the maximum expected return of all securities in the portfolio denoted as $x_{\max}$, and the minimum expected return of all securities denoted as $x_{\min}$.

The portfolio risk at this time is expressed as:

$$\sigma_p^2 = W^T VW = Var(\sum_i \omega_i x_i) = \sum_{ij} \omega_i \omega_j \operatorname{cov}(x_i, x_j). \tag{10}$$

The portfolio return is expressed as:

$$R_p = W^T X = \sum_{i=1}^{N} \omega_i x_i. \tag{11}$$

The mean-variance model is essentially a nonlinear bi-objective optimization problem, with maximizing returns and minimizing risk as the two objectives. Its mathematical model is as follows:

$$\begin{cases} \min\ \sigma_p^2 = W^T VW \\ \max\ R_p = W^T X \\ \text{s.t.}\quad W^T I = 1. \end{cases} \tag{12}$$

b. Model solution

The Markowitz mean-variance model is a bi-objective optimization problem. Its solution first involves aggregating the dual objectives into a single objective problem, aiming to minimize portfolio risk while ensuring a certain level of return. In other words, keeping the goal of minimizing risk unchanged, returns are converted into constraints. The model at this stage is:

$$\begin{cases} \min\ \sigma_p^2 = W^T VW \\ \text{s.t.}\quad R_p = W^T X, \\ \qquad W^T I = 1. \end{cases} \tag{13}$$

The single-objective constrained problem can be solved using the method of Lagrange multipliers. We construct the Lagrange function as follows:

$$L = W^T VW - \lambda_1 (W^T X - R_p) - \lambda_2 (W^T I - 1). \quad (14)$$

The efficient frontier curve is obtained as:

$$\frac{\sigma_p^2}{\frac{1}{C}} - \frac{[R_p - \frac{A}{C}]^2}{\frac{D}{C^2}} = 1. \quad (15)$$

B. Linear regret-rejoice function model

The Markowitz mean-variance model is based on the assumption of investor complete rationality. However, in actual investment processes, investment decisions are influenced by investors' expected emotions, where avoiding regret and seeking rejoice both play important roles. According to the framework effect proposed by Tversky and Kahneman [22], the representation of a problem will affect people's decision-making. Additionally, according to the phenomenon of preference reversal discovered by Litchtenstein and Slovic [28], psychological cues also influence decision-makers' choices. Furthermore, group psychology also affects individual decisions. All these external factors will influence the final outcome of the portfolio through changes in expected emotional utility.

After considering the influence of expected emotions, the expected utility generated by risk investment is influenced by two factors: the function of expected returns and the function of expected emotional utility. It can be represented as:

$$V(X_i) = U(\overline{X_i}) + g(X). \quad (16)$$

Where $g(X)$ represents the expected emotional utility function, $U(\overline{X}_i)$ represents the utility of expected returns. Formula (16) satisfies the definition of regret theory. To simplify the application of regret theory in portfolio theory, the two expected emotions of regret avoidance and rejoice pursuit can be separately handled.

When the decision-makers' chosen option yields a lower outcome than the alternative option, regret emotions arise. Therefore, the utility of regret avoidance is:

$$V(X_i) = U(\overline{X_i}) + g(\overline{X_i} - X_{\max}). \quad (17)$$

When the decision-makers' chosen option yields a higher outcome than the alternative option, rejoice emotions arise. Therefore, the utility of rejoice pursuit is:

$$V(X_i) = U(\overline{X_i}) + g(\overline{X_i} - X_{\min}). \quad (18)$$

The regret-rejoice function should satisfy the three properties mentioned. For simplicity, we select one-dimensional linear regret and rejoice functions.

$$g(\overline{X_i} - X_{\max}) = \alpha(\overline{X_i} - X_{\max}). \quad (19)$$

$$g(\overline{X_i} - X_{\min}) = \beta(\overline{X_i} - X_{\min}). \quad (20)$$

Where parameters α and β respectively represent the degree of perception of regret and rejoice emotions by the investor.

At this point, the portfolio model motivated by regret avoidance is:

$$\begin{cases} \min \ \sigma_p^2 = W^T VW \\ \text{s.t.} \ \ W^T I = 1, \\ \qquad R_p = W^T [X + \alpha(X - X_{\max} I)]. \end{cases} \quad (21)$$

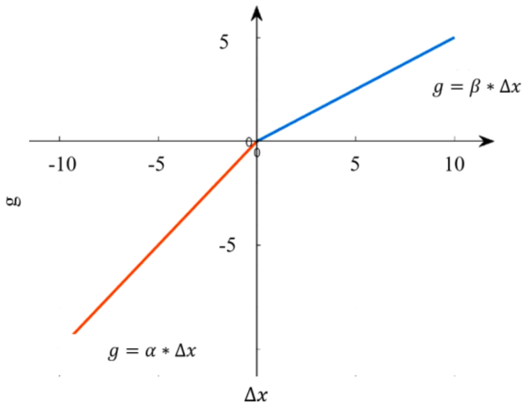


Fig. 4. Simplified regret-rejoice function

According to Formula (15) and coordinate transformation, we can obtain the efficient frontier equation for Formula (21) in the same coordinate system as:

$$\frac{\sigma_p^2}{\frac{1}{C}} - \frac{[\frac{R_p + \alpha X_{\max}}{1+\alpha} - \frac{A}{C}]^2}{\frac{D}{C^2}} = 1. \quad (22)$$

The portfolio model motivated by rejoice pursuit is:

$$\begin{cases} \min \ \sigma_p^2 = W^T VW \\ \text{s.t.} \ \ W^T I = 1, \\ \qquad R_p = W^T [X + \beta(X - X_{\min} I)]. \end{cases} \quad (23)$$

According to Formula (15) and coordinate transformation, we can obtain the efficient frontier equation for Formula (23) in the same coordinate system as:

$$\frac{\sigma_p^2}{\frac{1}{C}} - \frac{[\frac{R_p + \beta X_{\min}}{1+\beta} - \frac{A}{C}]^2}{\frac{D}{C^2}} = 1. \quad (24)$$

C. New power-exponential regret-rejoice function model

According to the conditions (1), (2), and (3) that Bell has proven the regret-rejoice function $g(\bullet)$ should satisfy, as mentioned in Part C of Section II:

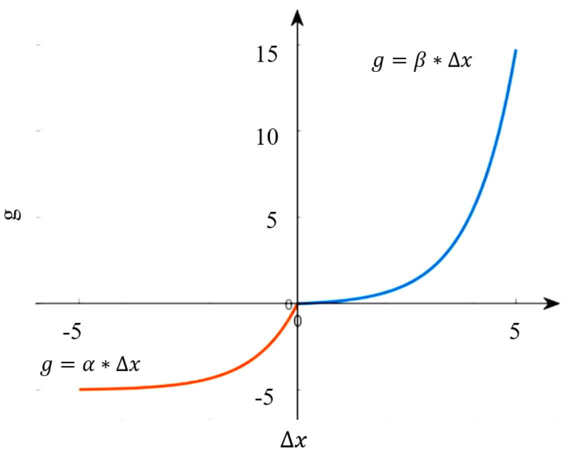


Fig. 5. Power form of regret-rejoice function

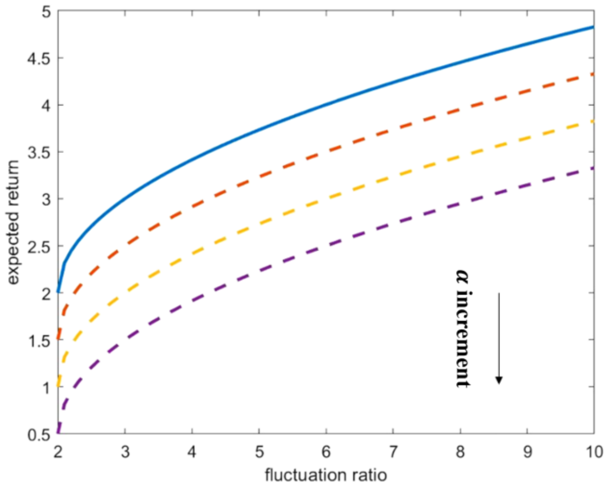
Therefore, to better align with the impact of rejoice pursuit and regret avoidance on portfolio outcomes in real investment processes, we select a power-form regret-rejoice function:

$$g(\overline{X_i} - X_{\max}) = \alpha(e^{\overline{X_i} - X_{\max}} - 1). \tag{25}$$

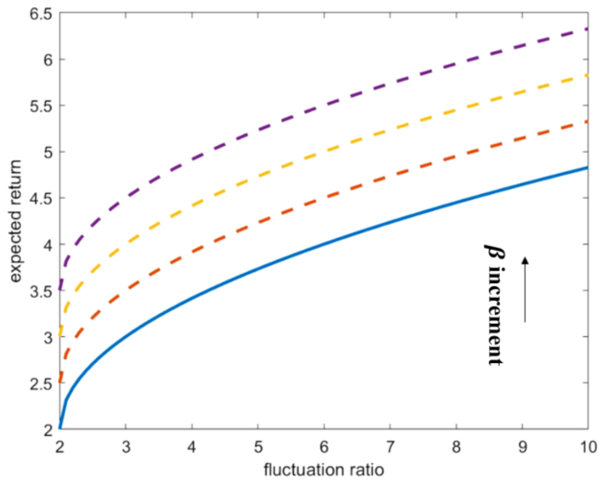
$$g(\overline{X_i} - X_{\min}) = \beta(e^{\overline{X_i} - X_{\min}} - 1). \tag{26}$$

Therefore, the portfolio model motivated by regret avoidance is:

$$\begin{cases} \min \ \sigma_p^2 = W^T V W \\ \text{s.t.} \ \ W^T I = 1, \\ \qquad R_p = W^T [X + \alpha(e^{\overline{X_i} - X_{\max} I} - I)]. \end{cases} \tag{27}$$



(a). Impact of α on results



(b). Impact of β on results

Fig.6. Impact of regret and rejoice on the efficient frontier

According to Formula (15) and coordinate transformation, we can obtain the efficient frontier of Formula (27) in the same coordinate system as:

$$\frac{\sigma_p^2}{\frac{1}{C}} - \frac{[R_p - \alpha W^T (e^{\overline{X_i} - X_{\max} I} - I) - \frac{A}{C}]^2}{\frac{D}{C^2}} = 1. \tag{28}$$

The portfolio model motivated by rejoice pursuit is:

$$\begin{cases} \min \ \sigma_p^2 = W^T V W \\ \text{s.t.} \ \ W^T I = 1, \\ \qquad R_p = W^T [X + \beta(e^{\overline{X_i} - X_{\min} I} - I)]. \end{cases} \tag{29}$$

According to Formula (15) and coordinate transformation, we can obtain the efficient frontier of Formula (29) in the same coordinate system as:

$$\frac{\sigma_p^2}{\frac{1}{C}} - \frac{[R_p - \beta W^T (e^{\overline{X_i} - X_{\min} I} - I) - \frac{A}{C}]^2}{\frac{D}{C^2}} = 1. \tag{30}$$

The parameters α and β respectively represent the degree of perception of regret and rejoice emotions by the investor. Their influence on the efficient frontier is depicted in Fig. 6.

D. Portfolio models under disappointment and regret emotions

a. Model construction

In addition to seeking rejoice and avoiding regret, disappointment aversion is also an important emotion that affects investors' investment decisions. According to Part B of Section II, disappointment theory suggests that when the selected outcome is greater than the investor's predetermined target, satisfaction emotions arise; conversely, when the outcome is lower than the predetermined target, investors experience disappointment emotions. Therefore, the characterization of disappointment and satisfaction emotions can be based on the deviation from the predetermined target value. Building on this foundation, Delquie [22] expanded disappointment theory to make each possible outcome a reference point, defining the deviation between each reference point as disappointment and satisfaction emotions.

Therefore, when considering the integration of disappointment aversion, regret avoidance, and rejoice pursuit as three types of expected emotions, a bi-objective programming model can be established as follows.

Motivated by regret avoidance:

$$\begin{cases} \min \ \sigma_p^2 = W^T V W \\ \min \ \Delta(\omega) = \sum_{k=1}^{T} \sum_{j=1}^{T} \frac{1}{2T^2} H(|\sum_{i=1}^{N} \omega_i v(x_{ij}) - v(x_{ik})|) \\ \text{s.t.} \ \ W^T I = 1, \\ \qquad R_p = W^T [X + \alpha(e^{\overline{X_i} - X_{\max} I} - I)]. \end{cases} \tag{31}$$

Motivated by rejoice pursuit:

$$\begin{cases} \min \ \sigma_p^2 = W^T V W \\ \min \ \Delta(\omega) = \sum_{k=1}^{T} \sum_{j=1}^{T} \frac{1}{2T^2} H(|\sum_{i=1}^{N} \omega_i v(x_{ij}) - v(x_{ik})|) \\ \text{s.t.} \ \ W^T I = 1, \\ \qquad R_p = W^T [X + \beta(e^{\overline{X_i} - X_{\min} I} - I)]. \end{cases} \tag{32}$$

The solution to the above bi-objective optimization problem can first be transformed into single-objective optimization problems by respectively converting Formulae (31) and (32) into the following single-objective optimization problems.

Motivated by regret avoidance:

$$\begin{cases} \min \ W^T V W + \sum_{k=1}^{T}\sum_{j=1}^{T}\frac{1}{2T^2}H(|\sum_{i=1}^{N}\omega_i v(x_{ij})-v(x_{ik})|) \\ \text{s.t.} \ \ W^T I = 1, \\ \quad R_p = W^T[X+\alpha(e^{\overline{X_i}-X_{\max}I}-I)]. \end{cases} \quad (33)$$

Motivated by rejoice pursuit:

$$\begin{cases} \min \ W^T V W + \sum_{k=1}^{T}\sum_{j=1}^{T}\frac{1}{2T^2}H(|\sum_{i=1}^{N}\omega_i v(x_{ij})-v(x_{ik})|) \\ \text{s.t.} \ \ W^T I = 1, \\ \quad R_p = W^T[X+\beta(e^{\overline{X_i}-X_{\min}I}-I)]. \end{cases} \quad (34)$$

b. Particle swarm optimization

In addition to seeking rejoice and avoiding regret, disappointment aversion is also an important emotion that affects investors' investment decisions. According to Part B of Section II, disappointment theory suggests that when the selected outcome is greater than the investor's predetermined target, satisfaction emotions arise; conversely, when the outcome is lower than the predetermined target, investors experience disappointment emotions. Therefore, the characterization of disappointment and satisfaction emotions can be based on the deviation from the predetermined target value. Building on this foundation, Delquie [22] expanded disappointment theory to make each possible outcome a reference point, defining the deviation between each reference point as disappointment and satisfaction emotions.

There are several methods to solve the single-objective optimization problems represented by Formulae (33) and (34), such as genetic algorithms, Jumping Spider Optimization Algorithm, etc. Particle Swarm Optimization (PSO) is a method that seeks the optimal solution to a problem by simulating the foraging behavior of birds. In PSO, particles represent candidate solutions to the optimization problem and have two important attributes: velocity and position. The basic steps of the Particle Swarm Optimization algorithm are as follows. PSO algorithm proceeds through the following basic steps:

Step 1: Initialize the parameters of the particle swarm, such as the dimensionality of the search space *D*, inertia factor, learning factor, etc.

Step 2: Randomly initialize a group of particles. Record the position and velocity of the *i*-th particle as follows:

$$x_{id} = (x_{i1}, x_{i2}, \cdots, x_{iD}). \quad (35)$$

$$v_{id} = (v_{i1}, v_{i2}, \cdots, v_{iD}). \quad (36)$$

Step 3: Save the current best solution p_{best} for each particle and the global best solution g_{best} for the entire population. Then, update the position and velocity of each particle based on these best solutions:

$$v_{id} = w \cdot v_{id-1} + c_1 r_1 (p_{besti} - x_{id}) + c_2 r_2 (g_{besti} - x_{id}). \quad (37)$$

$$x_{id+1} = x_{id} + v_{id}. \quad (38)$$

Where w is the inertia factor, c_1 and c_2 are the learning factors, and r_1 and r_2 are random numbers in the range [0,1]. The next movement direction of a particle is influenced by three factors: its own inertia, its individual best direction, and the global best direction.

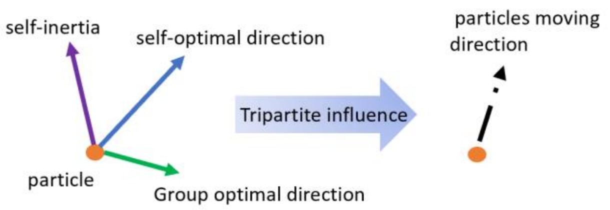


Fig. 7. Particle update schematic

Step 4: Repeat Step 3 continuously until reaching the maximum number of iterations or until the minimum difference between fitness values of two consecutive iterations is achieved.

IV. Empirical Analysis

This section selects data for the portfolio models with disappointment and regret emotions as proposed in Section III, applying them for a detailed comparative analysis.

A. Data selection

This paper selected 9 stocks from the constituents of the Shanghai Stock Exchange 50 Index as the risk assets available for investors to choose from, labeled as 1 to 9 respectively. The relevant stock codes and information can be found in Table I. We extracted monthly return rate data from January 2018 to December 2022, sourced from the RESSET database (https://www.resset.com/).

Table I Stock code, expected return, and standard deviation

i	Stock Code	Expected Return	Standard Deviation
1	600887	0.00514	0.0869
2	600958	0.00108	0.1164
3	601006	0.00149	0.0430
4	601088	0.01202	0.0786
5	601166	0.00672	0.0710
6	601857	-0.00263	0.0731
7	601988	0.00131	0.0758
8	601985	0.00144	0.0326
9	601998	0.00177	0.0524

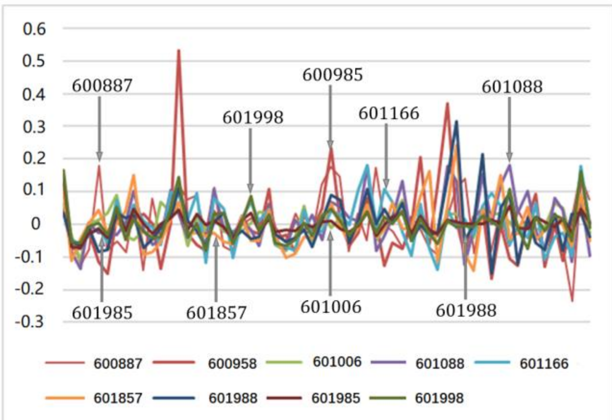


Fig. 8. Monthly stock return

Based on the monthly return rate data of the sample stocks from 2018 to 2022, the expected return and standard deviation of the sample stocks are calculated as shown in Table I. From the expected return matrix in the Table I, it can be observed that insert values: $x_{\max} = 1.2\%$, $x_{\min} = -0.26\%$. Using statistical software to analyze the above stock returns and standard deviations, the results are shown in Table II.

Through correlation analysis, it is indicated that the correlation between the return sequence and the risk sequence is weak, satisfying the condition of "risk and return independently influencing utility."

Fig. 8 shows the monthly return volatility of multiple stocks, reflecting their performance differences and similarities over a specific period. Although the volatility levels vary among stocks, the overall trend exhibits synchronized fluctuations, indicating that they may be influenced by common market factors. Certain stocks exhibit significant peaks or troughs in specific months, suggesting that these fluctuations could be related to market events or company-specific factors. Overall, the average return rates of these stocks hover around zero, indicating market stability, yet with notable short-term fluctuations, providing valuable insights for portfolio optimization.

Table II Correlation test result

Index	Correlation coefficient	Two-tailed test probability
Spearman	0.170	0.662
Kendal	0.029	0.916

Fig. 9 presents the covariance matrix of the return rates for the aforementioned stocks over various periods, illustrating the covariance relationships between different stocks. The matrix element values range from 0.001 to 0.014, with varying shades indicating different return rates. Specifically, the color gradient from light blue to dark blue represents return rates from low to high, with darker blue areas indicating higher return rates, and lighter blue areas representing lower returns.

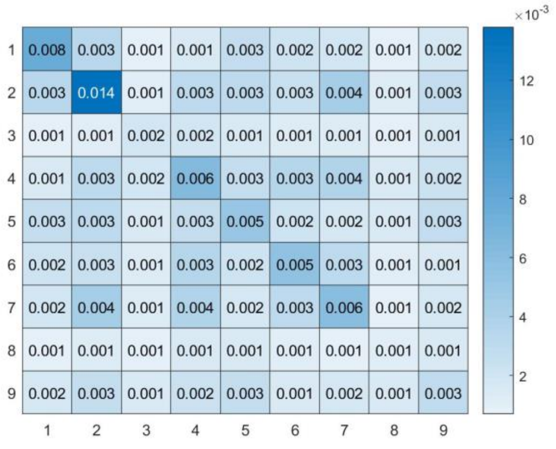


Fig. 9. Covariance matrix V

In this matrix, the darkest blue area in the second row and second column represents the highest return rate of 0.014 for the corresponding stock during that period. The lighter blue areas indicate lower return rates, reflecting relatively weaker performance of these stocks in certain periods. This color gradient allows for quick identification of each stock's performance over different periods.

Additionally, the heatmap provides insights into the covariance of return rates between stocks. For example, adjacent stocks (such as the first and second columns) with relatively high covariance values may indicate strong return correlations between them. Conversely, lighter areas away from the diagonal generally show lower covariance values, suggesting weaker return correlations. Overall, this visualization serves as a valuable tool for investors to quickly identify similarities and differences in stock return rates, aiding in the recognition of correlations and the diversification of risk in portfolio optimization.

By using this covariance heatmap, investors can assess which stocks have higher correlations, enabling better risk management and balanced portfolio allocation.

B. Model application

Bring the data processed in Part A of Section IV into the portfolio models under disappointment and regret emotions proposed in Section III, solve the feasible set and efficient frontier of portfolio under different models, and conduct comparative analysis.

a. Application of mean-variance model

According to the Lagrange Multiplier Method for solving the classic variance model by Markowitz in Part A of Section III, we can obtain its efficient frontier as follows:

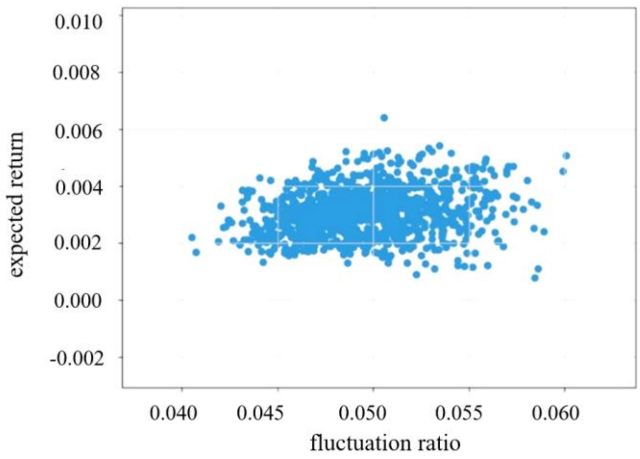
$$\frac{\sigma_p^2}{\frac{1}{C}} - \frac{[R_p - \frac{A}{C}]^2}{\frac{D}{C^2}} = 1. \qquad (39)$$

Using the historical return data of the nine stocks mentioned above in the Markowitz classic variance model, we obtained their feasible set and efficient frontier as shown in Fig. 10(a) and Fig. 10(b).

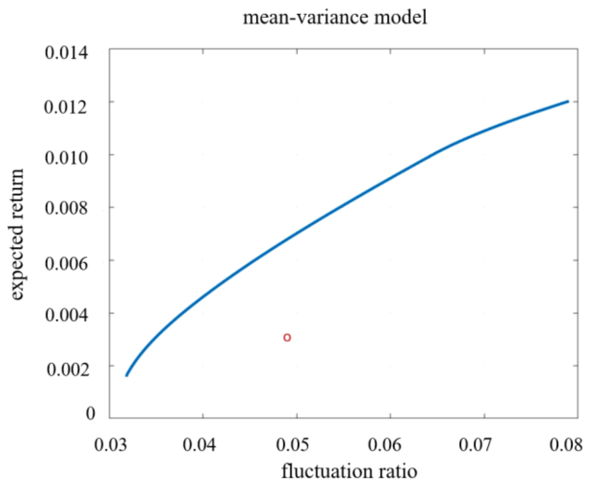
Fig. 10(a) presents a scatter plot illustrating the relationship between expected returns and volatilities for various portfolio combinations. Each blue dot represents a distinct portfolio configuration, capturing how different weight allocations across the stocks affect the overall return and risk profile. This scatter plot helps to visualize the range of possible portfolios, revealing how expected returns vary with increasing or decreasing volatility. The spread of blue dots highlights the diversity of portfolio choices available, with some achieving higher returns with greater volatility, while others offer lower returns but also lower risk.

Fig. 10(b) shows the efficient frontier, a critical concept in modern portfolio theory, represented by the blue curve. The efficient frontier demonstrates the set of optimal portfolios that maximize expected return for a given level of risk. The shape of the curve reflects the stock correlations and how diversification can mitigate risk. By following this curve, investors can identify portfolios that offer the best trade-off between risk and return, making it a valuable tool for investment decision-making.

Red dots in both figures denote the equal weight portfolio of the nine stocks, where each stock contributes equally to the portfolio. In Figure 10(b), if these red dots lie below the efficient frontier, it indicates that the equal weight portfolio is suboptimal. This positioning implies that better returns or lower risk could potentially be achieved by adjusting the portfolio weights to align closer with the efficient frontier.



(a). Feasible set of the mean-variance model



(b). Efficient frontier of the mean-variance model

Fig. 10. Feasible set and efficient frontier of the Markowitz model

Essentially, the comparison suggests that more sophisticated weighting strategies might yield higher returns for the same risk level, highlighting the benefits of optimization in portfolio construction. By juxtaposing these points with the efficient frontier, investors can readily gauge the performance gap and consider rebalancing their portfolios to enhance efficiency.

Together, Figures 10(a) and 10(b) offer comprehensive insights into the feasible set of portfolio outcomes and the potential advantages of aligning with the efficient frontier, emphasizing the importance of portfolio optimization in achieving desired risk-return outcomes.

b. Application of the linear regret-rejoice function model

According to Part B of Section III, the efficient frontier of the portfolio model motivated by regret avoidance when selecting a linear regret-rejoice function is obtained as follows:

$$\frac{\sigma_p^2}{\frac{1}{C}}-\frac{[\frac{R_p+\alpha X_{\max}}{1+\alpha}-\frac{A}{C}]^2}{\frac{D}{C^2}}=1. \tag{40}$$

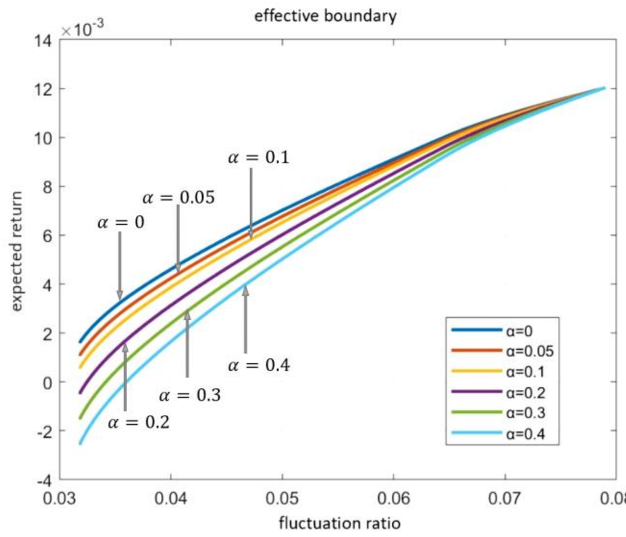


Fig. 11. Efficient frontier with regret avoidance

Since $X_{\max}$ represents the maximum return rate and $R_p < X_{\max}$, the efficient frontier of regret-considering portfolio, with the same volatility σ_p^2, should be located below the mean-variance model, and the deviation increases as α becomes larger. Solving the model with the data yields Fig. 11, where the efficient frontier curves under different regret parameters align with theoretical analysis results. The curves demonstrate how the expected return increases with higher fluctuation ratios and varies with different α values. As α increases, the curves shift upwards, indicating higher expected returns for the same level of risk.

Similarly, according to Part B of Section III, the efficient frontier of the portfolio model motivated by seeking rejoice when selecting a linear regret-rejoice function can be obtained as follows:

$$\frac{\sigma_p^2}{\frac{1}{C}}-\frac{[\frac{R_p+\beta X_{\min}}{1+\beta}-\frac{A}{C}]^2}{\frac{D}{C^2}}=1. \tag{41}$$

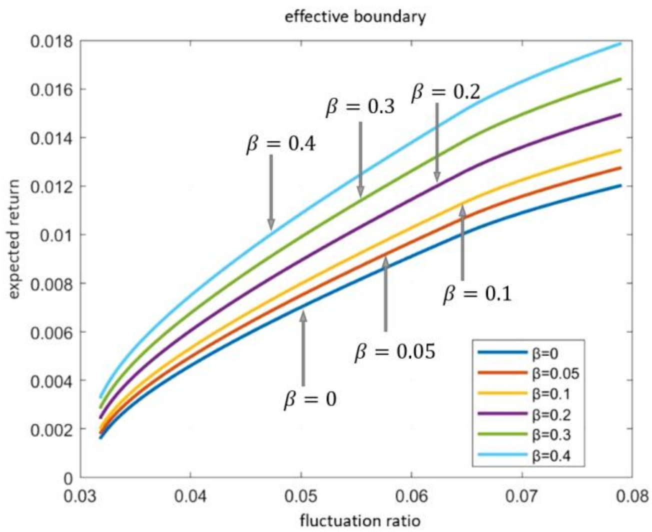


Fig. 12. Efficient frontier with rejoice pursuit

Since $X_{\max}$ represents the maximum return rate and $R_p > X_{\min}$, the efficient frontier of regret with the same volatility σ_p^2 should be located above the mean-variance model, and the deviation increases as β becomes larger.

Table III Comparison of volatility at the same return rate

α	Expected Return	volatility	β	Expected Return	volatility
0	0.0016	0.0319	0	0.004	0.0378
0.05	0.0016	0.0326	0.05	0.004	0.0368
0.1	0.0016	0.0336	0.1	0.004	0.0359
0.2	0.0016	0.0357	0.2	0.004	0.0345
0.3	0.0016	0.0378	0.3	0.004	0.0334
0.4	0.0016	0.0399	0.4	0.004	0.0327

Solving the model with the data yields the above Fig. 12, where the efficient frontier curves under different regret parameters align with the theoretical analysis results. The comparison of the volatility of feasible sets of portfolio with the same return rate is as follows:

c. Application of the novel power exponential regret-rejoice function model

According to Part C of Section III, the efficient frontier of the portfolio model motivated by regret avoidance when selecting the power exponential regret-rejoice function is obtained as follows:

$$\frac{\sigma_p^2}{\frac{1}{C}} - \frac{[R_p - \alpha W^T(e^{\overline{X}_i - X_{\max}I} - I) - \frac{A}{C}]^2}{\frac{D}{C^2}} = 1. \tag{42}$$

Since $X_{\max}$ represents the maximum return rate, and $X < X_{\max}$, the efficient frontier of the portfolio with regret emotion should lie below the mean-variance model when having the same volatility σ_p^2. Moreover, the greater the α, the greater the deviation. By plugging in the data into the model for solving, we obtain Fig. 13, where the efficient frontier curves under different regret parameters align with the theoretical analysis results.

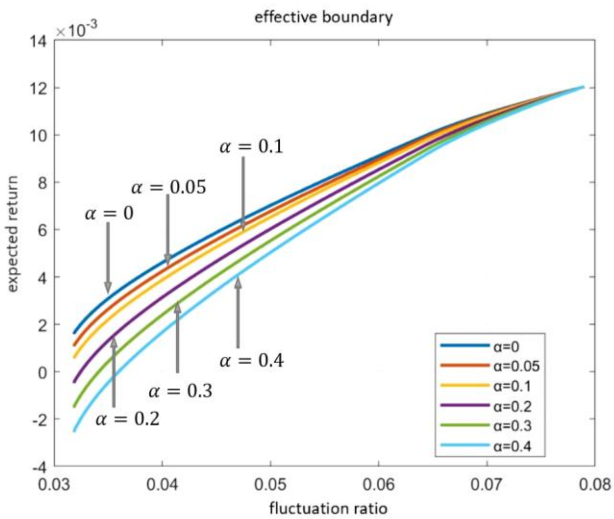


Fig. 13. Efficient frontier with regret avoidance

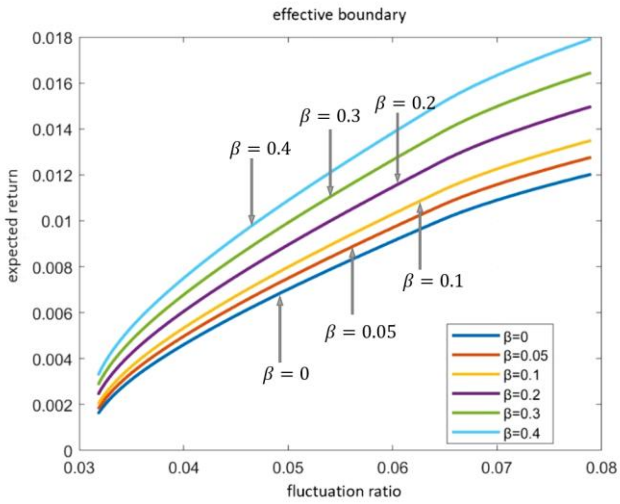


Fig. 14. Efficient frontier with rejoice pursuit

Similarly, according to Part C of Section III, the efficient frontier for the portfolio model motivated by pursuing rejoice when selecting the power form regret-rejoice function is obtained:

$$\frac{\sigma_p^2}{\frac{1}{C}} - \frac{[R_p - \beta W^T(e^{\overline{X}_i - X_{\min}I} - I) - \frac{A}{C}]^2}{\frac{D}{C^2}} = 1. \tag{43}$$

Table IV Comparison of volatility for portfolio feasible sets with the same returns

α	Expected Return	Volatility	β	Expected Return	Volatility
0	0.002	0.0325	0	0.005	0.0415
0.05	0.002	0.0334	0.05	0.005	0.0401
0.1	0.002	0.0345	0.1	0.005	0.0389
0.2	0.002	0.0367	0.2	0.005	0.0369
0.3	0.002	0.0389	0.3	0.005	0.0354
0.4	0.002	0.0409	0.4	0.005	0.0343

Since $X_{\max}$ represents the maximum return rate, and $X > X_{\min}$, when considering regret emotion in the portfolio, the efficient frontier should be located above the mean-variance model, and the greater the β, the greater the deviation. Solving the model with the data yields Fig. 14, where the efficient frontier curves under different regret parameters align with theoretical analysis results.

Comparison of portfolio feasible sets' volatility when selecting the same return rate is as Table IV.

d. Application of portfolio models under disappointment and regret emotions

Based on the portfolio model under disappointment and regret constructed in Part D of Section III, we obtain the single-objective optimization problems represented by Formulae (33) and (34). In the function $v(\bullet)$, the parameter a, b signify the investor's perception of disappointment and satisfaction emotions and should satisfy $a, b > 1$. Since investors tend to be more inclined towards disappointment aversion relative to seeking satisfaction, this paper selects $a = 1.3, b = 1.7$. By applying the data to the portfolio model under disappointment and regret, the effective frontier when

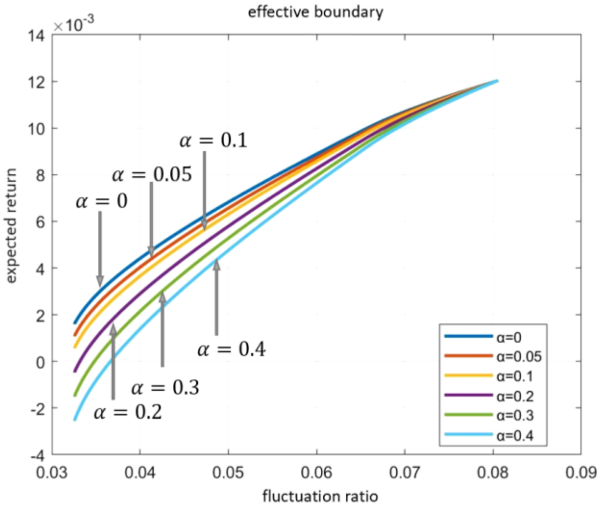


Fig. 15. Efficient frontier with regret avoidance

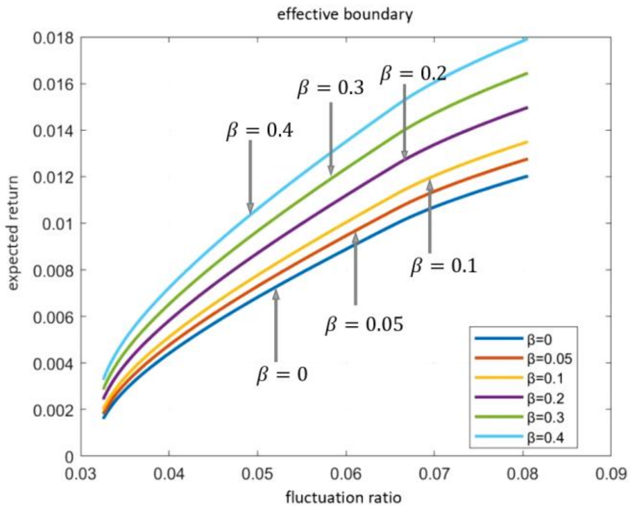


Fig. 16. Efficient frontier with rejoice pursuit

avoiding regret is obtained as Fig. 15.

It illustrates the relationship between fluctuation ratio and expected return under different α values (ranging from 0 to 0.4). It shows that as the α value decreases, the expected return at the same fluctuation ratio also increases. This indicates that lower α values correspond to higher expected returns, although they come with increased volatility.

Table V Comparison of volatility for the same returns

α	Expected Return	Volatility	β	Expected Return	Volatility
0	0.002	0.0331	0	0.005	0.0422
0.05	0.002	0.0341	0.05	0.005	0.0408
0.1	0.002	0.0352	0.1	0.005	0.0396
0.2	0.002	0.0374	0.2	0.005	0.0376
0.3	0.002	0.0396	0.3	0.005	0.0361
0.4	0.002	0.0416	0.4	0.005	0.0350

When pursuing rejoice as a motivation, we obtain Fig. 16, it illustrates the relationship between fluctuation ratio and expected return under different β values (ranging from 0 to 0.4). It shows that as the β value increases, the expected return at the same fluctuation ratio also increases. This indicates that higher β values correspond to higher expected returns, although they come with increased volatility.

C. *Comparison analysis*

a. Comparison of different regret-rejoice functions

In the model established in Part B of Section III, a linear regret-rejoice function was employed, while in Part C of Section III, a power form of the regret-rejoice function was used. When selecting the same expected return of 0.004, a comparison of the volatility under different regret-rejoice functions for the motivation of avoiding regret shown in Fig. 17.

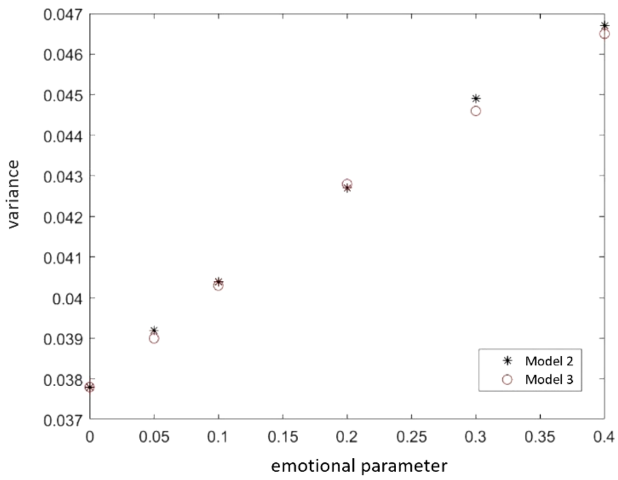


Fig. 17. Volatility comparison with regret avoidance

When pursuing rejoice as the motivation and selecting the same expected return of 0.01, a comparison of volatility under different regret-rejoice functions is illustrated in Fig. 18.

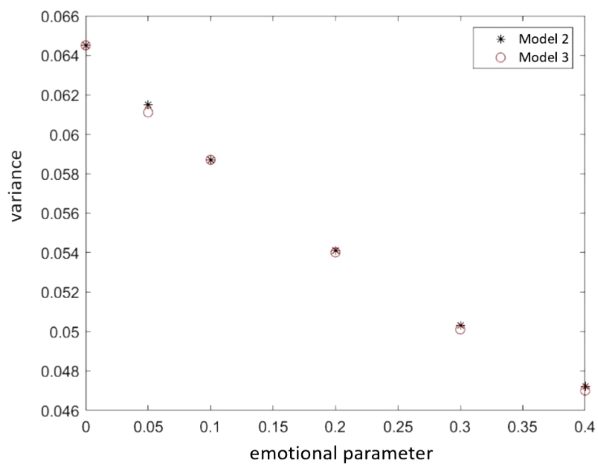


Fig. 18. Volatility comparison with rejoice pursuit

Fig.17 and Fig.18 illustrate the relationship between the variance of portfolio returns and the emotional parameter for two different regret-rejoice functions: the linear function (Model 2) and the power function (Model 3). In both figures, it can be observed that for a given level of return, portfolios constructed using the power regret-rejoice function generally exhibit lower volatility compared to those using the linear regret-rejoice function. This trend suggests that the power function may be more effective in mitigating the effects of emotional volatility on investment decisions.

In Fig.17, which focuses on a lower range of emotional parameters, there is a noticeable decrease in variance as the emotional parameter increases. The decline in volatility is more pronounced for Model 3, indicating that as investors assign greater weight to emotional considerations, the power function better accommodates these fluctuations, leading to more stable returns. This reduction in variance demonstrates the ability of the power function to provide a smoother response to changes in investor sentiment, which can be particularly valuable in volatile markets where emotional responses can greatly impact decision-making.

Similarly, Fig.18, which expands the analysis to a higher range of emotional parameters, reinforces this observation. As the emotional parameter continues to increase, the variance for Model 2 portfolios tends to fluctuate more than for Model 3, particularly at higher levels. This highlights the robustness of the power regret-rejoice function, suggesting that it offers greater stability and resilience against emotional influences on risk. Investors utilizing the power function can potentially achieve comparable returns with reduced volatility, optimizing their portfolios to better manage emotional risk.

Overall, these figures emphasize the advantage of incorporating a power regret-rejoice function over a linear one in investment models. By leveraging the power function, investors may benefit from reduced variance, implying that this approach is more effective in moderating the impact of emotional parameters on portfolio risk. This insight is critical for developing more resilient investment strategies that can better withstand the emotional fluctuations often encountered in financial decision-making.

b. Contrast before and after introducing disappointment emotion

Compared to the portfolio model established in Part C of Section III using a regret-rejoice function, the model in Part D of Section III introduces disappointment emotion. When avoiding regret is the motivation and choosing the same expected return 0.004, the volatility contrast before and after introducing disappointment emotion is shown in Fig. 19.

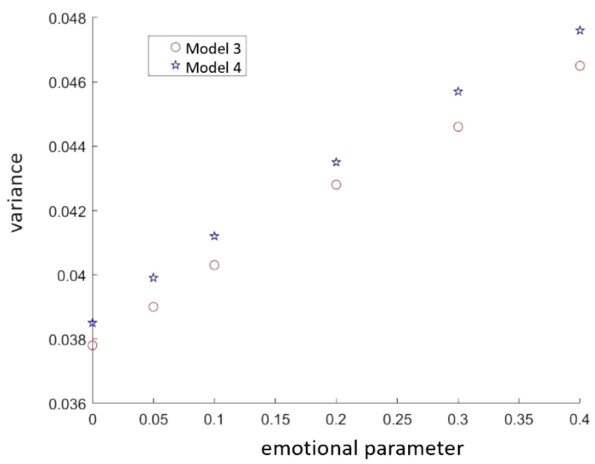


Fig. 19. Comparison of volatility with regret avoidance

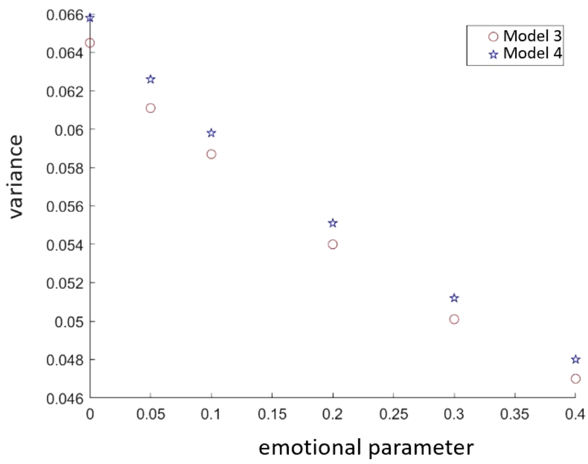


Fig. 20. Comparison of volatility with rejoice pursuit

When pursuing rejoice as motivation and selecting the same expected return of 0.01, the contrast of volatility before and after introducing disappointment emotion is shown in Fig. 20.

Fig. 19 and Fig. 20 provide insights into the influence of the emotional parameter on portfolio volatility, comparing two models: Model 3, which considers regret, and Model 4, which additionally incorporates disappointment. The figures demonstrate that for the same level of return, Model 4 consistently exhibits higher volatility than Model 3, especially as the emotional parameter increases. This trend suggests that when disappointment is considered alongside regret, investors' sensitivity to risk is heightened, leading to greater fluctuations in expected returns.

In Fig. 19, we see that Model 4's variance increases more sharply with the emotional parameter than Model 3, which reflects a linear regret function. This increase in volatility signifies that investors who account for both regret and disappointment are likely more affected by adverse outcomes, which amplifies their risk perception. As the emotional parameter escalates, portfolios under Model 4 experience a pronounced rise in variance, suggesting that incorporating disappointment into the model shifts the risk-return curve to the left of the original curve without disappointment. This shift implies that portfolios accounting for disappointment require a higher tolerance for risk to achieve similar returns as those with only regret.

Fig. 20 further illustrates the effect of adding disappointment to the emotional parameters. Across various levels of the emotional parameter, Model 4 consistently presents higher variances, emphasizing the significant impact that complex emotions can have on investment volatility. The contrast between Model 3 and Model 4 across different emotional intensities underscores the importance of considering multiple emotional factors when optimizing a portfolio. In practical terms, this indicates that investors who weigh disappointment more heavily may face increased volatility, which could lead to more conservative portfolio choices if they seek to avoid high fluctuations.

Overall, these figures highlight how integrating emotions like disappointment and regret can considerably alter a portfolio's risk profile. By acknowledging these emotional influences, investors and portfolio managers can make more informed decisions, adjusting their strategies to better manage the heightened risks associated with emotional volatility. This analysis underscores the role of psychological factors in financial decision-making, revealing how emotions can shape not only the selection of assets but also the overall investment strategy, ultimately impacting the risk-return trade-offs that define portfolio performance.

V. Conclusion

The classical mean-variance model assumes investors are fully rational, but considering a psychological perspective, investors' behavior is often driven by emotions. Based on this viewpoint, this paper integrates theoretical research findings on the impact of expected emotions on portfolio from domestic and foreign sources, fully considering investors' emotions such as disappointment, aversion, pursuit of rejoice, and avoidance of regret. The aim is to seek emotion utility functions that better reflect real-world scenarios and to construct portfolio models based on these findings, providing beneficial insights for investors.

Firstly, to capture investors' psychological preferences for rejoice and regret avoidance, we adopt a research approach ranging from simple to complex and from basic to specialized. Specifically, we quantify investors' regret and rejoice emotions using both linear regret-rejoice functions and novel power exponential regret-rejoice functions. Building upon the mean-variance model, we establish portfolio models motivated by rejoice pursuit or regret avoidance.

Secondly, we expand the model to incorporate disappointment aversion, employing disappointment theory to account for the emotional cost associated with outcomes falling below expectations. By integrating insights from behavioral finance, statistics, and related fields, we construct comprehensive portfolio models that simultaneously consider both disappointment and regret. Treating disappointment aversion and regret avoidance as dual objectives, we transform the problem of "minimizing risk while pursuing predetermined returns" into a bi-objective optimization to enhance efficiency by an improved particle swarm algorithm.

Finally, we analyze data from nine stocks of the Shanghai Stock Exchange 50 Index constituents from January 2018 to December 2022, deriving posterior expected returns and covariance through data processing. Our empirical results confirm the significant influence of emotional factors on investor decisions, demonstrating that models which account for emotions yield portfolio configurations that better align with investor preferences in risk-taking and emotional tolerance. Applying the constructed portfolio models under disappointment and regret emotions, we verify the impact of considering emotional factors on investor decisions and the sensitivity of investors to emotions on portfolio construction. Additionally, we conduct comparative analysis on the influence of different regret-rejoice functions and the introduction of disappointment emotions on the results.

The study underscores the significant role of emotions in investment decision-making. By moving beyond the rational assumptions of classical models and incorporating emotional factors, we provide a richer and more accurate depiction of investor behavior. The constructed models offer practical insights into how investors can better manage their portfolios by acknowledging and strategically responding to their emotional inclinations. Our analysis indicates that the power regret-rejoice function yields portfolios with lower volatility than the linear function, while incorporating disappointment increases overall risk, particularly at higher emotional intensities. These findings highlight the critical need to integrate nuanced emotional factors for a more realistic and resilient portfolio strategy. This approach not only aligns more closely with real-world investor experiences but also offers potential for improving portfolio performance by leveraging a deeper understanding of emotional impacts.

For the proposed portfolio models under disappointment and regret emotions, we will refine further to enhance their practical applicability. This refinement can involve selecting emotion utility functions that not only capture the essence of investor emotions but also align more closely with real investment processes. Additionally, incorporating cardinality constraints into the models are verified to be beneficial, as it can help mitigate the complexity and costs associated with managing large portfolios in real-world scenarios. By limiting the number of assets in the portfolio, cardinality constraints can improve the feasibility and efficiency of implementation.

Furthermore, in terms of solving portfolio models, we will explore the use of intelligent algorithms to expedite and optimize the solution process. Leveraging advanced computational techniques, such as machine learning algorithms and optimization methods, can enable quicker and more efficient portfolio optimization, especially when dealing with large datasets and complex constraints. Future research will also explore algorithmic improvements to address the computational challenges posed by multi-objective optimization in emotionally driven portfolio models, aiming to deliver solutions that are both effective and scalable for institutional investors. Research in this area focuses on developing algorithms that not only consider traditional risk-return trade-offs but also incorporate emotional factors to provide more comprehensive and robust portfolio solutions.

In summary, refining the proposed portfolio models can improve their practical utility to better reflect real-world investment dynamics and integrating cardinality constraints. Additionally, exploring the use of intelligent algorithms can enhance the efficiency and effectiveness of solving these models, ultimately contributing to more informed and successful investment decision-making processes.

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