An Evolutionary Game Model for Enterprise Green Transformation with Stakeholder Win Strategies

Dandan Qi, Xinyue Li, Ganwei Zhang, Weicheng Zhang, Zhiyong Gao, and Shubo Jiang

Abstract— **The green transition of traditional enterprises not only plays a crucial role in reducing environmental pollution and addressing climate change but also enhances their innovation capabilities and helps them gain market advantages. This study, grounded in the core principles and methodologies of evolutionary game theory, develops a game model that examines the interactions of the government, enterprises, and consumers to examine the stability of enterprises' green transition strategies and analyze the effect of various parameters on the strategic choices of different stakeholders. A Stackelberg game is also introduced to simulate and analyze the evolving interactions between the government, as policymaker (leader), and enterprises (followers) during the green transition process. Corresponding countermeasures and recommendations are proposed. The findings reveal that under initial parameters, enterprises face challenges in adopting green transition strategies. However, optimizing certain parameters, such as reducing incremental costs for enterprises, lowering government regulatory costs, increasing the government's net additional benefits, and enhancing consumers' benefits from purchasing green products, can drive the game system toward an optimal solution. Government subsidies and consumers' willingness to buy green products contribute to market price stability and encourage enterprises to adopt more environmentally friendly and sustainable production methods.**

Index Terms—**evolutionary game model, green transition, matlab simulation, Stackelberg game**

I. INTRODUCTION

In recent years, the environment and climate have undergone dramatic changes, including severe pollution, excessive greenhouse gas emissions, and significant threats to biodiversity. Faced with this grim reality, individuals and societies are increasingly recognizing that the high-carbon growth model, primarily fueled by fossil fuels, has altered the atmospheric conditions in which humans live. Furthermore, the growing frequency of extreme climate events is beginning to have profound impacts on daily life and economic activities. From the United Nations Framework Convention on Climate Change adopted in the late 20th century to the Paris Agreement signed in 2016, we hope to protect the environment and enhance our capability to respond to climate change. Being the largest developing country, China is serious about addressing climate change, implementing the goals of saving energy and reducing emissions, and fulfilling its commitment to green development. In September 2020, General Secretary Xi Jinping solemnly announced at the 75th session of the United Nations General Assembly that "China aims to reach peak carbon dioxide emissions by 2030 and achieve carbon neutrality by 2060." Carbon peaking is a reasonable setting of a peak and time, reaching a peak within a specified time, and then not increasing, and gradually falling. Carbon neutrality is to reduce total greenhouse gases emitted directly or indirectly in a certain area and time, through the process of planting trees and expanding forested areas, saving energy and reducing emissions so as to achieve "near-zero emissions." The Central Economic Work Conference convened in Beijing in December 2020, which determined eight key tasks, one of which is to effectively achieve carbon peaking and carbon neutrality. In September 2021, China made an official declaration that it would halt the construction of new coal power projects abroad, fulfilling its obligations and commitments to environmental governance and protection. On the one hand, China faces significant challenges due to its high levels of traditional pollutant and carbon dioxide emissions, which severely impact green and low-carbon development and the construction of an ecological civilization and then affect modernization efforts aimed at improving people's well-being. On the one hand, if these energy sources are rushed to be one-size-fits-all, it will inevitably affect economic development. So, there is an urgent need for a green transition. The so-called green transformation is based on the circular economy; the development approach shifts towards sustainability, focusing on resource efficiency, environmental protection, and enabling green business operations, industrial green reconstruction, and government green supervision by changing the operation methods, industrial composition methods, and government supervision methods, so that the traditional black economy is transformed into a green economy. In this process,

Manuscript received May 17, 2024; revised December 31, 2024.

This work was supported by 2023 Harbin University of Commerce Jinghu Scholar Support Program, Project Number: JHQNRC02; Heilongjiang Province Philosophy and Social Science Planning Project: Research on Multi center Governance Strategies for Rural Non-point Source Pollution in Heilongjiang Province under the Background of "Dual Carbon", Project Number: 23JYA041.

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financial innovation support and government financial support are needed to finally transform the traditional manufacturing industry and achieve the purpose of alleviating environmental and climate pressure. This process is challenging and requires the active participation of all social actors.

This article constructs a game model based on consumers, enterprises, and governments, discusses the optimal decisions of all parties, and provides some suggestions for the green transformation of Chinese enterprises on this basis. This article makes the following contributions: 1. Perspective on the integration of dual carbon targets: this article takes China's proposed "carbon peak and carbon neutrality" (referred to as "dual carbon") target as the research background, and deeply explores the profound impact of this macro policy orientation on the enterprise green production. This research perspective that combines national strategic goals with micro-level corporate behavior provides a new theoretical framework for understanding the driving forces, paths, and strategies of corporate green transformation. 2. Application of Evolutionary Game Theory: this article innovatively applies evolutionary game theory for analysis, the strategic choices and the mechanisms of interaction among business entities in the process of green transformation. 3. Introduction of simulation technology: This paper utilizes numerical simulations to examine the strategic choices of stakeholders across various parameter combinations. Drawing on the research results, it offers specific policy recommendations, such as the government providing appropriate special support and subsidies, enterprises strengthening themselves, and attaching importance to the technology development process. 4. Application of the Integrated Game Model: This study builds on the existing evolutionary game framework involving the government, enterprises, and consumers by incorporating a Stackelberg game . This addition enables a more precise simulation and study of the evolving relationship between government, acting as policymaker (leader), and enterprises (followers) throughout the green transition process. This setup captures the government's leading role in driving the green transition and the enterprises' responsive behavior under government policy guidance.

II. LITERATURE REVIEW

The stakeholders involved in the green transformation of Chinese enterprises mainly include governments, enterprises, and consumers. Recently, academic discourse has focused on discussions around various stakeholders. Wang Wei et al. developed an evolutionary game framework examining the interaction between governments and enterprises within a joint reward and punishment framework, concluding that a substantial innovation subsidy rate can significantly accelerate the pace of green innovation adoption by enterprises [1]. Philipp Herzog and others argue that if governments can provide certain innovation subsidies and promote technological innovation within enterprises, they can help enterprises break through the threshold of technological innovation to a certain extent [2]. Using a sample of publicly traded companies in strategic emerging industries between 2009 and 2013, Lv Xiaojun empirically analyzed how government subsidies influenced corporate investment in technological innovation and the process of marketization [3]. Cao Xia et al. believe that low-intensity public environmental protection publicity, moderate innovation incentive compensation, and high-intensity pollution taxation can better promote the green transformation of enterprises [4]. Baocheng et al. developed a model based on evolutionary game theory to examine the interaction between consumers and enterprises concerning green consumption and technological innovation. They believe that reaching the ideal equilibrium state independently is challenging and that external forces, such as the government , are needed to promote it [5]. Xu Jianzhong developed an evolutionary game model to study the impact of government-led environmental technological advancements input subsidies and carbon tax influences manufacturing enterprises' decisions regarding green innovation strategies and analyzed the evolutionary path, stable equilibrium strategy, and impact mechanism theoretically and numerically. [6]. Viewed through the perspective of signal transmission, Wu Jian et al. believe that government subsidies can function as resource incentives, driving innovation investment within enterprises, and drive innovation output. On the other hand, they can also play the role of signal attributes to transmit information to society, help enterprises obtain investment and financing, and also drive innovation in enterprises [7]. Zhen Zhiyong et al. have developed a behavioral game model involving both the government and businesses and believe that the government will play a crucial part in shaping and advancing green innovation within the manufacturing sector [8]. Chen Hongmei et al. believe that there is also a game between enterprises; some enterprises will choose not to innovate, and then the government needs to reward or punish enterprises through transfer payments, tax credits, and various approaches, with the ultimate goal of realizing a green and low-carbon transition [9]. Krass et al. constructed a game model between enterprises and regulators, proving that environmental taxes and subsidies incentivize green technological innovation [10]. Li Cunfang et al. and Li Chunfa et al. believe that government regulatory decisions significantly influence corporate behavior [11]-[12]. Xia Dan argues that, with proper government supervision, consumer behavior can play a significant role in driving companies toward green transformation. By aligning consumer preferences with sustainable practices, businesses are incentivized to adopt environmentally friendly technologies and practices, thereby accelerating the transition to a greener economy [13]. Zhang Qian believes the stability of the system and the equilibrium strategy are influenced by the net profits of companies participating in environmentally friendly innovation efforts and enforcement of stringent environmental regulations by the government [14]. Deng et al. created a game model between local governments and businesses from a taxation perspective, suggesting that tax competition not only influences the development of environmentally-friendly technological advancements directly, but also exerts an indirect influence through the environmental governance efforts of local governments under environmental regulatory constraints

[15]. Lin et al. believe that for new energy vehicle companies, financial support from the government can act as the most effective driver for advancing environmentally sustainable technological development when environmental regulation is enforced at an appropriate level. [16]. Zhou Zehui et al. have developed a three-player game model that includes the government, businesses, and low-carbon service providers, arguing that the government's regulatory costs play a negative role in their choice of regulatory strategies and that enterprises are more sensitive to punishment than subsidies [17]. Sun Shupeng and Sun Xiaoyang established a tri-party game model featuring the government, enterprises, and environmental NGOs to study the evolution of law between subjects [18]. Xu Jianzhong and Lv Xichen constructed a trilateral game framework involving the government, businesses, and consumers, arguing that the current social environment does not have an evolutionary steady state of "the government allows enterprises to produce traditional production" or "consumers prefer high-carbon products of enterprises" [19]. Carlos et al., taking the US aviation industry as their research object, found that external policy influences play a vital role in shaping green technology innovation within the U.S. aviation sector [20]. Using an analysis of the issues encountered during low-carbon development, Guo Xiaohua examines the challenges associated with this process and proposes strategies for green transformation in logistics enterprises in the western region, within the "dual carbon" framework, to drive their development and transformation [21]. Niu Yiheng and Han Jieping analyzed the challenges faced by the power generation industry in transitioning to a low-carbon model and proposed an effective transformation direction that will help promote the scientific and efficient transformation of the power generation industry [22]. Wu Mouyuan et al. suggest that China's oil and gas enterprises should promote the swift expansion of industries related to renewable energy and associated initiatives, with state support to encourage green and low-carbon energy development [23].

The research by scholars both domestically and internationally provides an important theoretical foundation for this study. However, in general terms, there are still certain shortcomings. On the one hand, the majority of studies focusing on the green transition of businesses is superficial and often does not deeply explore the intrinsic interest mechanisms among various stakeholders. On the other hand, while much of the literature emphasizes the role of government subsidies, it often overlooks the fact that such subsidies are temporary and will eventually cease. Most of the previous research stayed on a static and single perspective, and there were few multi-party game analyses based on a dynamic perspective, while there were certain shortcomings in the applicability and scientific nature of a static and single research perspective. Furthermore, many scholars focus on specific influencing factors but neglect to compare the relative impact of these factors." Based on this analysis, this study conducts a more comprehensive and in-depth investigation into the green transformation of Chinese enterprises.

III. Selection of Research Methods and Basic Assumption of Game Model

A.Selection of research methods

Evolutionary game theory provides a systematic approach to analyzing the strategy evolution of agents in repeated interactions through dynamic analysis and evolutionary stability, assuming limited rationality and learning behavior. In each round of the game, economically rational agents, constrained by bounded rationality, are unable to determine whether their strategy is optimal for that round. Instead, they optimize their strategies by imitating and learning from the strategies of similar agents, ultimately leading to an equilibrium state among all participants. This process of strategy optimization not only aids individuals in adapting to their environment but also fosters the co-evolution of the entire system. Furthermore, this approach can reveal the long-term impacts of different strategic choices on the environment and society. Utilizing evolutionary game theory to investigate the simulation and dynamic evolution of Chinese enterprises' green transition under the framework of the 'dual-carbon' policy provides several advantages, including a more thorough comprehension of the dynamic evolution process, the revelation of the underlying logic of strategy selection, the analysis of strategy stability, the provision of recommendations for strategy optimization, and the promotion of interdisciplinary research and application.

B.Basic Assumption

Enterprises: If a company continues to operate using the previous high-consumption, high-pollution model, the income is R_0, the cost is C_0, and the tax is T_0. Due to the high pollutant discharge, a fine A is paid. If the enterprise optimizes the industrial structure, actively develops and reforms, upgrades technology, and adds the creation of environmentally friendly and low-carbon products, the income is $R_0 + R_1$, the cost is C_0, and the transformation needs to invest in a new plant construction and technology research and development and other costs C_1, pay taxes T_1, but do not pay or pay very little penalty A, and at the same time if you get government subsidies with strong government support, etc., get additional benefits, as expressed in C_g. And if the government takes active measures, enterprises will get additional benefits D because of publicity and policy support, considering the sequential nature of the game, assuming that within the game of the government-enterprise, as long as the government takes positive measures, enterprises will get additional benefits D; In the game between consumers and enterprises, only companies that have adopted the green transition can obtain additional benefits.

Government: Amid the challenges posed by climate change, alongside efforts for energy efficiency and emissions mitigation, governments worldwide is actively promoting low-carbon policies. The decision to offer subsidies should be assessed according to the specific circumstances. This evaluation should take into account the economic impact of subsidies and their potential to promote sustainable practices. Concurrently, the influence of subsidy policies on the incentive mechanisms for corporate green transformation should be analyzed. This study assumes that the government has two strategies: subsidize or not subsidize.

Consumers: Consumers spend R_0 on traditional goods, and R_1 on additional spending on green and low-carbon products after the green transition. After the enterprise completes the green transformation, consumers have an additional benefit W , and the additional income of W means that after the completion of the transformation, consumers will be subsidized to purchase green goods, and the traditional commodity market will be impacted, resulting in the price reduction of traditional goods, thereby saving money, and the utility of higher living standards following environmental improvements. Furthermore, this transformation is likely to enhance consumers' awareness of green products, thereby further promoting the formation of green consumption habits. The gain that the consumer does not buy is B, that is, the gain obtained by keeping the money in his hand. Under traditional production, the income purchased by consumers is B_0 , that is, the income obtained when purchasing traditional products. Under the green transition, consumers can purchase B_1 benefits, and after the transformation, consumers can not only buy traditional products but also buy the benefits of green and low-carbon products.

The government's subsidy C_g for the transition enterprises, the enterprise according to the traditional mode of production to pay the government A fine and T_3 tax, if the enterprise carries out green and low-carbon transformation, the government receives tax T_4 , and at the same time, will bring certain incremental environmental and social benefits to the government, expressed in L_g . Under the active subsidy of the government, the transformation of enterprises is completed, and the government will obtain the credibility of the masses as a net benefit L_1 (including the cost of government supervision and accounting L_2) for active actions. However, if the enterprise still adopts the traditional mode of production, the government will pay the regulatory cost L_2 if it is found that the enterprise still produces according to the traditional model, and the subsidy C_g will not be paid and a fine A will be charged.

The tax burden T_0 and T_1 borne by the enterprise are the tax T_3 and T_4 obtained by the government; The subsidy Cg received by the enterprise is the subsidy C_g paid by the government; The pollution treatment fee A received by the government is the pollution treatment fee A paid by the enterprise; In the sales process, the income R_0 and R_1 obtained by the enterprise correspond to the consumer's expenditure R_3 and R_4 .

TABLE I

GAME SUBJECT PARAMETER SETTING AND SIGNIFICANCE

Game subject	Parameter	Indicator meaning
Enterprise	R_0	Benefits of traditional production methods
	R_1	Incremental benefits after green transition
	C_0	The cost of traditional production methods
	C_1	Incremental costs after the green transition
	T_0	The tax burden arising from traditional production methods active power
	T_1	Tax burden arising from the green transition
	D	The government takes active measures to gain additional benefits for enterprises
Government	C_g	Government subsidies
	A	Government penalties
	T_3	Taxes obtained under traditional production methods
	T_4	Taxes received after the green transition
	L_g	Incremental benefits under the green transformation of enterprises
	L_1	Additional net benefits such as public reputation
	L_2	Regulatory costs
Consumer	R_3	The cost of traditional products for consumers
	R_4	The cost of consumers buying green and low-carbon products
	W	Additional benefits for consumers after the green transition
	B	Basic earnings
	B_0	The benefits of consumer choice to buy under traditional production
	B_1	The benefits of consumer choice to buy under the green transition

IV. The Evolutionary Game between Enterprise and Government

A. Construction and Analysis of Evolutionary Models

Building upon the preceding analysis of the interaction involving the government and enterprise, along with the underlying assumptions, enterprise and government game payment matrix is presented in TABLE II:

TABLE II

Enterprise and government game payment matrix

		Government	
		Subsidy (x)	No subsidy (1-x)
Enterprise	Green transition (y)	(R_0+R_1-C_0-C_1-T_1+C_g+D, T_4+L_g-C_g+L_1)	(R_0+R_1-C_0-C_1-T_1, T_4+L_g)
	Traditional production (1-y)	(R_0-C_0-T_0-A+D, T_3 +A+L_2)	(R_0-C_0-T_0-A, T_3+A)

In the game between enterprises and governments, suppose the likelihood of the government opting for subsidies is x, while the likelihood of not providing subsidies is $1-x$. In a similar manner, the likelihood of a company embracing a green transition is y, and the likelihood of selecting not to transform and still operate the way it was before is $1-y$.

Then the government selects to "implement policies and subsidize" expected income

$$U_{g1} = y\left(T_4 + L_g - C_g + L_1\right) + (1-y)\left(T_3 + A + L_2\right) \tag{1}$$

Expected return

$$U_{g2} = y\left(T_4 + L_g\right) + (1-y)\left(T_3 + A\right) \tag{2}$$

Average return

$$U_g = xU_{g1} + (1-x)U_{g2} \tag{3}$$

During the course of the game, the equation for government's decision to offer subsidies in the dynamic replication model is:

$$\begin{aligned} F(x) &= x\left(U_{g1} - U_g\right) \\ &= x\left[U_{g1} - xU_{g1} - (1-x)U_{g2}\right] \\ &= x(1-x)\left(U_{g1} - U_{g2}\right) \\ &= x(1-x)\left[y\left(L_1 - C_g\right) + (1-y)L_2\right] \\ &= x(1-x)\left(yL_1 - yC_g - yL_2 + L_2\right) \end{aligned} \tag{4}$$

$$\begin{aligned} &i.e. F(x) = dx/dt \\ &= x(1-x)\left[y\left(L_1 - C_g - L_2\right) + L_2\right] \end{aligned} \tag{5}$$

At $F(x)=0$, the rate of adjustment in the government's strategy is 0, indicating that the government evolution has reached a stable state.

At this time, there is a solution $x_1 = 0,\ x_2 = 1$,

$$y^* = L_2 / \left(C_g + L_2 - L_1\right) \tag{6}$$

Similarly, in this game, the expected benefits of choosing a "green transition" are

$$\begin{aligned} U_{b1} &= x\left(R_0 + R_1 - C_0 - C_1 - T_1 + C_g + D\right) \\ &+ (1-x)(R_0 + R_1 - C_0 - C_1 - T_1) \end{aligned} \tag{7}$$

The expected benefit of enterprises choosing "traditional production" is $U_{b2} = R_0 - C_0 - T_0 - A + xD$

The average return is $U_b = y\ U_{b1} + (1-y)\ U_{b2}$

Then the replication dynamic equation for enterprises to choose green transformation is

$$\begin{aligned} F(y) &= y\left(U_{b1} - U_b\right) = y\left(U_{b1} - U_b\right) \\ &= y(1-y)\left(U_{b1} - U_{b2}\right) \\ &= y(1-y)\begin{pmatrix} R_0 + R_1 - C_0 - C_1 - T_1 \\ +xC_g - R_0 + C_0 + T_0 + A \end{pmatrix} \\ &= y(1-y)\left(R_1 - C_1 - T_1 + T_0 + A + xC_g\right) \end{aligned} \tag{8}$$

$$\begin{aligned} &i.e. F(y) = dy/dt \\ &= y(1-y)\left(xC_g + R_1 - C_1 - T_1 + T_0 + A\right) \end{aligned} \tag{9}$$

When $F(y)=0$, there is a solution $y_1 = 0,\ y_2 = 1$,

$$x^* = \left(C_1 + T_1 - R_1 - T_0 - A\right)/C_g \tag{10}$$

From this, five equilibrium points can be determined (1,0), (0,1), (0,0), (1,1), ($\frac{C_1 + T_1 - T_0 - R_1 - A}{C_g}$, $\frac{L_2}{C_g + L_2 - L_1}$).

The equilibrium analysis of the five points is carried out, and the bias derivation of $F(x)$ and $F(y)$ is obtained to obtain $\frac{\partial F(x)}{\partial x}$, $\frac{\partial F(x)}{\partial y}$, $\frac{\partial F(y)}{\partial x}$, $\frac{\partial F(y)}{\partial y}$. From which the Jacobian matrix of the game between government and enterprise is obtained

$$J = \begin{pmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} \\ \frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y} \end{pmatrix} \tag{11}$$

$$\frac{\partial F(x)}{\partial x} = (1-2x)\left[y\left(L_1 - C_g - L_2\right) + L_2\right] \tag{12}$$

$$\frac{\partial F(x)}{\partial y} = x(1-x)(L_1 - C_g - L_2) \tag{13}$$

$$\frac{\partial F(y)}{\partial y} = (1-2y)\left(R_1 - C_1 - T_1 + T_0 + A + xC_g\right) \tag{14}$$

$$\frac{\partial F(y)}{\partial x} = y(1-y)C_g \tag{15}$$

Further calculations of the determinant $det(j)$ and trace $tr(j)$ are employed to study the local stability of the equilibrium point.

$$\det(j) = \frac{\partial F(x)}{\partial x} \cdot \frac{\partial F(y)}{\partial y} - \frac{\partial F(x)}{\partial y} \cdot \frac{\partial F(y)}{\partial x} \tag{16}$$

$$tr(j) = \frac{\partial F(x)}{\partial c} + \frac{\partial F(y)}{\partial y} \tag{17}$$

TABLE III
TR(J) AND DET(J) OF THE JACOBIAN MATRIX OF THE ENTERPRISES -GOVERNMENT GAME

Equilibrium Point	det(j)	tr(j)
(0,0)	$L_2(R_1\text{-}C_1\text{-}T_1\text{+}T_0\text{+}A)$	$L_2\text{+}R_1\text{-}C_1\text{-}T_1\text{+}T_0\text{+}A$
(0,1)	$-(L_1\text{-}C_g)(R_1\text{-}C_1\text{-}T_1\text{+}T_0\text{+}A)$	$L_1\text{-}C_g\text{-}R_1\text{+}C_1\text{+}T_1\text{-}T_0\text{-}A$
(1,0)	$-L_2(R_1\text{-}C_1\text{-}T_1\text{+}T_0\text{+}A\text{+}C_g)$	$-L_2\text{+}R_1\text{-}C_1\text{-}T_1\text{+}T_0\text{+}A\text{+}C_g$
(1,1)	$(L_1\text{-}C_g)(R_1\text{-}C_1\text{-}T_1\text{+}T_0\text{+}A\text{+}C_g)$	$-(L_1\text{-}C_g)-(R_1\text{-}C_1\text{-}T_1\text{+}T_0\text{+}A\text{+}C_g)$
$\frac{C_1+T_1-T_0\text{-}R_1\text{-}A}{C_g}$, $\frac{L_2}{C_g+L_2\text{-}L_1}$	$(1\text{-}y)(1\text{-}x)L_2(C_1\text{+}T_1\text{-}R_1\text{-}T_0\text{-}A)$	0

When $y = y^* = L_2/\left(C_g + L_2 - L_1\right)$, $F(x)$ consistently equals 0, which means that all x always represents a steady state , and the government's policy choices make no difference in the outcome. If $y \neq y^*$, then $x_1 = 0,\ x_2 = 1$ are two stable states, when $C_g + L_2 - L_1 < 0$,if $y < y^*$, $y\left(L_1 - C_g - L_2\right) + L_2 < 0$, $F'(x_1) < 0$, then $x = 0$ is the evolutionary stabilization strategy, if $y > y^*$, $F'(x_2) < 0$, then $x = 1$ is the evolutionary stabilization strategy, if $C_g + L_2 - L_1 > 0$, for any $y \in [0,1]$,there is $y\left(L_1 - C_g - L_2\right) + L_2 < 0$, then $F'(x_1) < 0$, $i.e.\ x_1 = 0$ is the evolutionary stability strategy.

When $x = x^* = \left(C_1 + T_1 - R_1 - T_0 - A\right)/C_g$, $F(y)$ consistently equals 0, which means that all y is a steady state, and there is no difference in the outcome of the enterprise strategy choice. If $x \neq x^*$, then $y_1 = 0$ and $y_2 = 1$ are two stable states, when $C_1 + T_1 - T_0 - R_1 - A > 0$, if $x < x^*,\ F'(y_1) < 0,\ F'(y_2) > 0$, then $y = 0$ is the evolutionary stability strategy, if $x > x^*$, $F'(y_2) < 0$, then $y = 1$ is the evolutionary stability strategy. When $C_1 + T_1 - T_0 - R_1 - A < 0$,for any $x \in [0,1]$,there is $xC_g + R_1 - C_1 - T_1 + T_0 + A > 0$, then $F'(y_1) > 0, F'(y_2) < 0$, i.e., $y_2 = 1$ is the evolutionary stability strategy. Considering the current status quo of the green transition of conventional production enterprises in China, the assumption of $C_g + L_2 - L_1 < 0$, $C_1 + T_1 - T_0 - R_1 - A > 0$ is more in line with the status quo, at this time x* and y* are between (0,1), (0,0) and (1,1) are evolutionary stable equilibrium points, (0,1), (1,0), (x^*, y^*) three points are not stable, where $(x^*,\ y^*)$ represents the saddle point, which is an unstable point, in the saddle point, the strategy of both sides chooses itself and the other party's income, any combination of strategies is possible, but if there is a slight deviation from one side, it will evolve towards the equilibrium point.

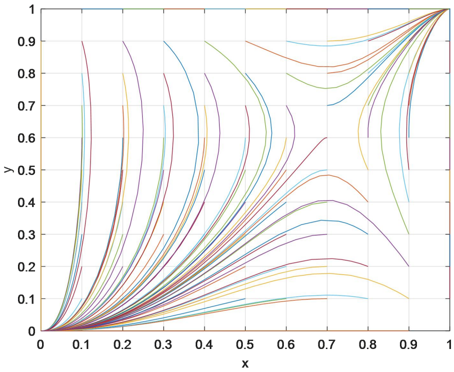
B. Simulation of government-enterprise game under initial parameters

Taking Chengdu Fulin Precision Industry Enterprise as an example, the main business of the enterprise was initially the production of auto parts and assembly and sales of automobiles, but with the change of market structure and the requirements and guidance of national and government policies, in recent years also began to transform and produce new energy vehicles, mainly new energy lithium battery cathode material research and development, manufacturing, and sales. From its 2020 annual report, it is evident that the relevant government subsidies are 6.75 million yuan, that is $C_g = 675$. $R_0 = 178800,\ C_0 = 112400$, $R_1 = 5630$, $C_1 = 6327$, $T_0 = T_3 = 16592.5$, $T_1 = T_4 = 16417.5$, $A = 50$, then $\frac{C_1 + T_1 - T_0\text{-}R_1\text{-}A}{C_g}$ =0.69. Here assume $L_1 = 800$ and $L_2 = -200$, $\frac{L_2}{C_g + L_2\text{-}L_1}$ =0.62. From the above data, it can be seen that $R_1 - C_1 - T_1 + T_0 + A < 0;\ R_1 - C_1 - T_1 + T_0 + A + C_g > 0$.

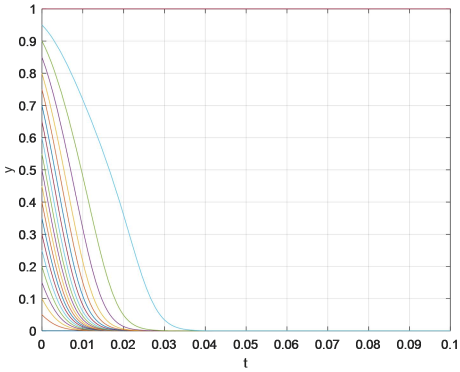
TABLE IV
STABILITY ANALYSIS OF LOCAL EQUILIBRIUM POINT UNDER INITIAL PARAMETERS

Equilibrium point	det(j)	tr(j)	Stability
(0,0)	+	-	ESS
(0,1)	+	+	Unstable
(1,0)	+	+	Unstable
(1,1)	+	-	ESS
(0.69,0.62)	-	0	Saddle point

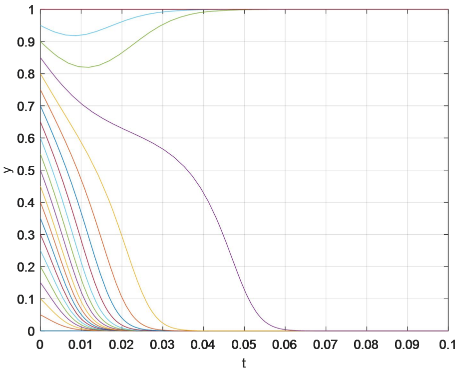
As stated above, government's replication dynamic equity $F(x) = x(1-x)\left(yL_1 - yC_g - yL_2 + L_2\right) = x(1-x)(325y - 200)$, and the replication dynamic equation for enterprises is $F(y) = y(1-y)\left(R_1 - C_1 - T_1 + T_0 + A + xC_g\right) = y(1-y)(675x - 472)$. The above parameters and equations were simulated to get Figure 1 a)-d). When the starting values of x and y are both 0, with a loop step of 0.1, the dynamic evolution trajectory of the interaction between the government and the enterprise group is illustrated in Figure 1a). At the same time, the cycle step is set to 0.05, and the probability x when the government chooses to implement policies and subsidies is set to be 0.3, 0.5 and 0.7, respectively, and the evolutionary equilibrium trend observed shows the evolution path as illustrated in Figure 1 b)-d).



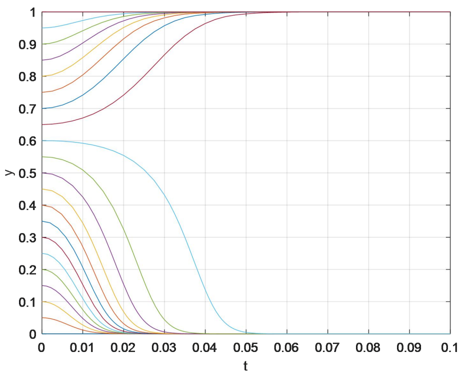
a) Evolutionary path



b) Enterprise evolutionary(x=0.3)



c) Enterprise ~ (x=0.5)



d) Enterprise ~(x=0.7)

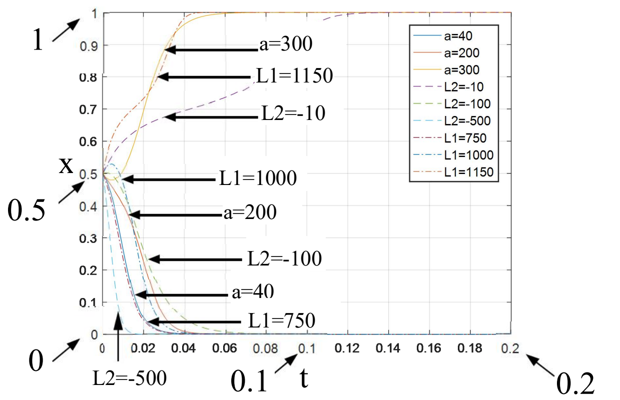
Fig.1. Game simulation diagram of government and enterprise using initial parameters

As illustrated in Figure 1a), it is evident that in the upper-right area of the saddle point (0.69, 0.62), all paths evolve towards (1,1) (subsidy, green transformation), and on the lower left side of the saddle point, all paths evolve towards (0,0) (unsubsidized, traditional production). Under the current parameter settings, the lower-left region is significantly larger than the upper-right region, and the game between the two is more inclined to evolve in the direction of (0,0). When the likelihood of the government selecting subsidies is 0.3, most enterprises opt for high-pollution, high-energy-consumption production. However, when the probability of subsidies increases to 0.7, the likelihood of enterprises choosing green transformation becomes greater." In this scenario, the effectiveness of subsidy policies is crucial in encouraging businesses to adopt environmentally friendly actions.

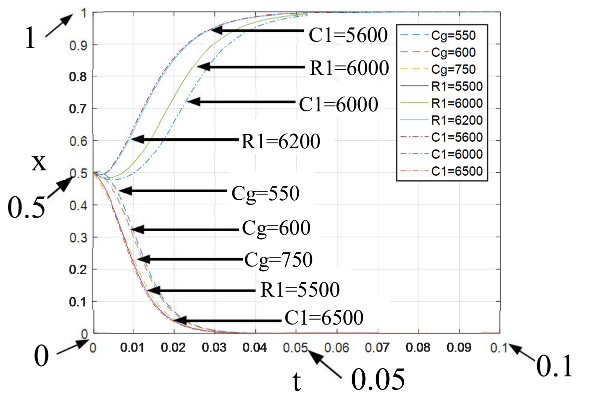
C. Dynamic Simulation under Parameter Optimization of Government and Enterprise

In the game process under initial parameters involving the government and enterprise, the game between the two is likely to evolve into an inferior equilibrium solution of (0,0). In the evolution path diagram of the government and enterprises illustrates, position of the saddle point $(\frac{C_1+T_1-T_0-R_1-A}{C_g}, \frac{L_2}{C_g+L_2-L_1})$ will affect the final effect of evolution, and the parameters will affect the location of the saddle point, and by adjusting the size of the parameters to make the saddle point closer to (0,0), the optimal equilibrium solution of the model can be achieved with a greater probability (1,1). The parameters include the incremental cost of green transformation C_1, the tax burden of enterprises after green transformation T_1, the tax burden of enterprises under the traditional mode T_0, the incremental benefit of green transformation R_1, pollution control fee A, government subsidy C_g, additional net income L_1 of public reputation obtained by the government after green transformation, and government supervision cost L_2. Among them, the key parameters related to the government are C_g, L_1, L_2, T_0, T_1 and A, and the key parameters related to enterprises are C_g, T_1, T_0, R_1 and A.

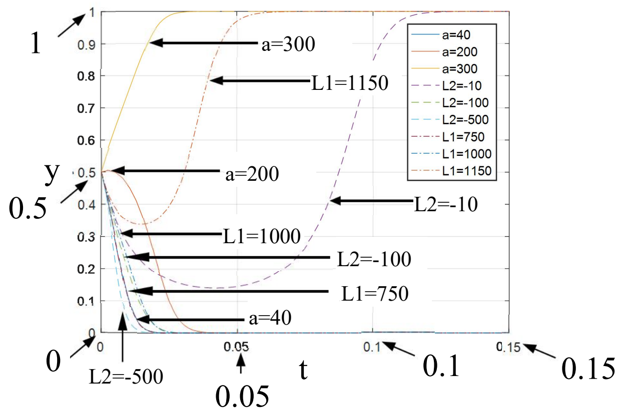
To foster the progress of government and enterprises toward an ideal equilibrium point (1,1), and further analyze how parameter changes influence the game will be conducted, because the tax burden is primarily associated with the production and operation of enterprises, T_1 and T_0 are mainly affected by R_1, R_0, C_1, C_0, and because the technology and market of the traditional mode of production are quite mature and gradually tend to be stable, R_0, C_0 and T_0 are assumed to be constant. The relevant key parameters are optimized separately, and the control variable method is adopted, and a certain degree of change is taken for C_g, L_1, L_2, R_1, C_1 and A respectively according to the actual situation under the initial parameters. The initial parameters were $C_g = 675$, $L_1 = 800$, $L_2 = -200$, $R_0 = 178800$, $C_0 = 112400$, $R_1 = 5630$, $C_1 = 6327$, $A = 50$, $T_0 = 16592.5$, $T_1 = 16417.5$. In the optimization process, A takes 40, 200, and 300 respectively. L_2 takes -10, -100, -500. L_1 takes 750, 1000, 1150. C_g takes 550, 600, 750. R_1 takes 5500, 6000, 6200. C_1 takes 5600, 6000, 6500. And assume that the proportion of initial government selecting subsidies $x = 0.5$, and the proportion of enterprises selecting green transformation $y = 0.5$. Enter the above parameters into MATLAB to get the trend of the proportions of x and y over time, as shown in the figure below.



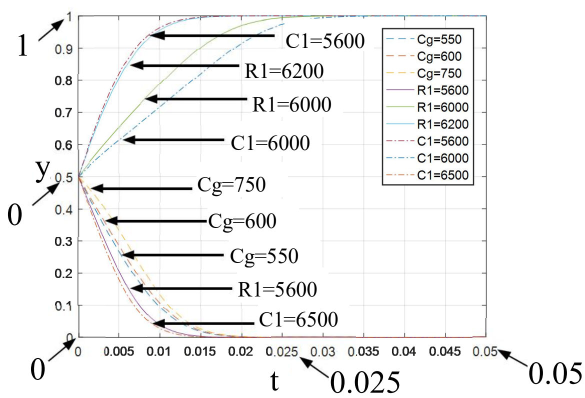
a) Evolution trend of government strategy (A, L_1, L_2)



b) Evolution trend of government strategy (C_g, R_1, C_1)



c) Evolution trend of enterprise strategy (A, L_1, L_2)



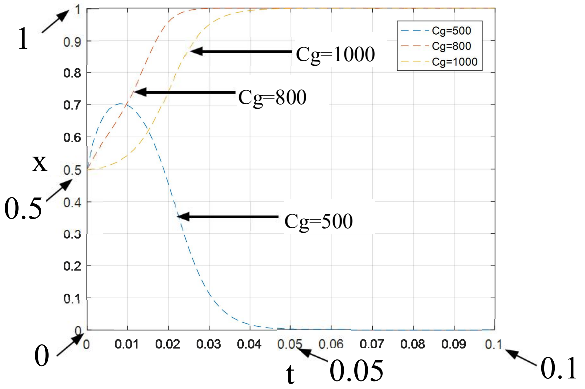
d) Evolution trend of enterprise strategy (C_g, R_1, C_1)

Fig.2. Strategy evolution diagram after government parameter optimization

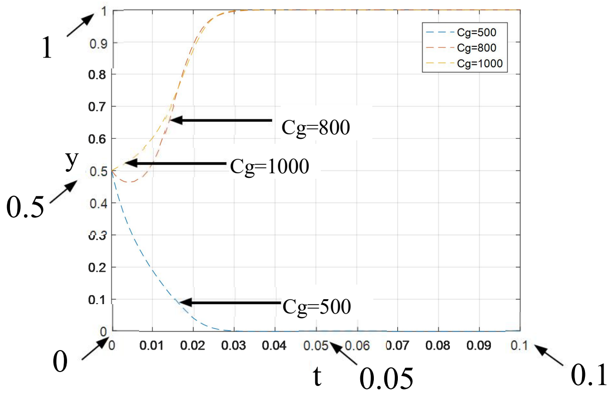
From the a) and b) of Figure 2), it is evident that the government parameter optimization analysis based on the government's perspective shows that when the government charges for the pollution control fee A, public reputation and other additional benefits L_1 charged by enterprises for pollutants discharged by enterprises, the income R_1 of enterprises on new products after green transformation increases, or the regulatory cost L_2 and the cost of new products C_1 after green transition in enterprises decrease, the change trend of strategy that originally tended to 0 gradually tends to 1, and with the gradual increase of A, L_1, R_1 values or gradual decrease of C_1 and L_2 . The time for the strategy to tend to 1 is gradually shortened. Government policies(such as increasing pollution control fees), market factors(such as consumer demand for eco-friendly products), and corporate cost control measures(such as reducing transition costs) work together to alter the cost-benefit structure for businesses, thereby prompting them to make green transformation decisions more swiftly. This integrated effect enhances businesses' awareness of environmental responsibilities, making green transformation not just a compliance requirement but also a strategy for enhancing competitiveness. As time progresses, the more pronounced the changes in these factors, the stronger the trend towards corporate green transformation and the shorter the decision-making time becomes.

In the same way, c), d) in Figure 2, it is clear that the government parameter optimization analysis based on the perspective of the enterprise, when the pollution control fee A , the incremental income of the enterprise R_1 , the expected return of the government increase or the incremental cost C_1 and the regulatory expenses incurred by the company decrease, the strategy of v the enterprise originally tending to 0 gradually tends to 1, and with the increase or decrease of the value, the time for the strategy to tend to 1 is gradually shortened, which shows that the parameter adjustment of both parties in the game contributes to reaching the system's optimal equilibrium solution.

The above analysis does not discuss the impact of the specific value change of government subsidy C_g on the game, and From b), it is evident that an increase in the subsidy C_g value, the faster the government's strategy tends to 0, this means that the government's decision to reduce or stop providing subsidies will happen more quickly; It can be seen from d) that as the subsidy C_g value decreases, the faster the company's strategy tends to 0. However, under the existing parameters, no C_g can accelerate the evolution of the strategies of the two subjects to 1, from the setting of the initial parameters, L_1 and L_2 limit the range of C_g, so assume that L_1 takes 1200 and C_g takes 500, 800, 1000 to analyze how variations in subsidy values affect the strategic evolution of government and enterprise interactions.And assume that the initial proportion of the government selecting subsidies is $x = 0.5$, proportion of enterprises selecting green transformation $y = 0.5$. Enter the above parameters into MATLAB to get the trend of the proportions of x and y over time, as shown in the figure below.



a) Evolution trend of government strategy (L_1=1200, C_g)



b) Evolution trend of enterprise strategy (L_1=1200, C_g)

Fig.3. Strategy evolution diagram after government parameter optimization

As illustrated in Figure 3, when the government's income L_1 increases, the government not only can carry out more subsidies, but as the amount of subsidies increases, the original strategy of 0 gradually tends to 1, but not as the subsidy amount increases, the trend accelerates, Figure 3 a) $C_g = 800$ than $C_g = 1000$ can trend to strategy 1 faster, indicating that the government should consider its own the financial situation in the process of subsidy. This is consistent with the curve in Figure 2 b), in the case of initial L_1, the government's increase in the amount of subsidies will lead to a faster choice for the government not to subsidize, and the government's blind subsidy without considering the fiscal situation is not conducive to the trend strategy 1. If fiscal permitting, such as an increase in L_1, the government's finances are in good shape. At this time, increasing the value of the subsidy C_g can change the original strategy towards 0 and gradually tend to 1. Figure 3 b) similarly, under the condition that the government can subsidize more, the government subsidy C_g increases, the original strategy of enterprises that tends to 0 gradually tends to 1, which is similar to the conclusion in Figure 2 d), under the initial L_1, as government subsidies increase, the pace at which businesses reach strategy 0 diminishes, indicating that subsidies from the government have an impact on encouraging enterprises' green transformation. If the government fails to offer adequate subsidies, it will hinder the shift towards sustainability in businesses. However, indiscriminate increases in subsidies could lead to a reduction due to fiscal constraints, ultimately causing enterprises to abandon their transformation efforts. Since the key parameters of the enterprise align with those of the government, and the image of the optimized relevant parameters of the enterprise is consistent with Figure 2, the strategy evolution diagram after the optimization of the enterprise parameters is not additionally analyzed in the "government-enterprise" game.

V. The Evolutionary Game between Enterprise and Consumers

A. *Construction and Analysis of Evolutionary Models*

TABLE V
Enterprise and Consumers game payment matrix

		Consumers	
		Buy (m)	Do not buy (1-m)
Enterprise	Green transition (n)	(R_0+R_1-C_0-C_1-T_1+C_g+D, B_1-R_3-R_4+W)	(Cg-C0-C1+D,B+W)
	Traditional production (1-n)	(R_0-C_0-T_0-A, B_0-R_3)	(-C_0-A, B)

After experiencing the interaction between enterprises and the government in the game, the government will only subsidize the enterprises that adopt the green transition, and the enterprises with the green transition will receive additional market benefits. In the game between consumers and enterprises, it is assumed that the odds of consumers selecting to "buy" is m, and the odds of not buying is (1-m). At this time, the odds of enterprises selecting "green transformation" is n and the odds of selecting "traditional production" is (1-n). So, the expected return of consumers choosing to "buy" is

$U_{p1} = n\left(B_1 - R_3 - R_4 + W\right) + \left(1-n\right)\left(B_0 - R_3\right)$.

The expected payoff of selecting "Do Not Buy"

$U_{p2} = n\left(B + W\right) + \left(1-n\right)B = nB + nW + B - nB = nW + B$

The consumers' average anticipated return is $U_p = m\, U_{p1} + \left(1-m\right)U_{p2}$

Then the replication dynamic equation that the consumer chooses to buy is:

$$\begin{aligned} F(m) &= m(U_{p1} - U_p) \\ &= m(1-m)(U_{p1} - U_{p2}) \\ &= (1-m)[n(B_1 - R_4 - B_0) + (B_0 - R_3 - B)] \end{aligned} \tag{18}$$

$$\begin{aligned} &i.e.F(m) = dm/dt \\ &= m(1-m)[\begin{matrix} n(B_1 - R_4 - B_0) \\ +(B_0 - R_3 - B) \end{matrix}] \end{aligned} \tag{19}$$

When F(m)=0, there are solutions m_1=0, m_2=1,

$$n^* = \frac{B - B_0 + R_3}{B_1 - R_4 - B_0} \tag{20}$$

Similarly, in this game, the expected return of enterprises choosing "green transformation" is

$$U_{b1} = m\left(R_0 + R_1 - C_0 - C_1 - T_1 + C_g + D\right) + \left(1-m\right)\left(C_g - C_0 - C_1 + D\right) = mR_0 + mR_1 - mT_1 + C_g - C_0 - C_1 + D \quad (21)$$

The anticipated benefit of enterprises opting for "traditional production" is

$$U_{b2} = m\left(R_0 - C_0 - T_0 - A\right) + (1-m)(-C_0 - A) = mR_0 - mT_0 - C_0 - A \quad (22)$$

At this time, enterprise's average anticipated return is $U_b = n\ U_{b1} + \left(1-n\right)\ U_{b2}$

The replication dynamic equation for enterprises to choose "green transformation" is:

$$F(n) = n(U_{b1} - U_b) = n\left(1-n\right)\left(U_{b1} - U_{b2}\right) = n\left(1-n\right)\begin{pmatrix} mR_1 - mT_1 + mT_0 \\ +C_g - C_1 + A + D \end{pmatrix} \quad (23)$$

$$i.e. F(n) = dn/dt = n\left(1-n\right)\begin{bmatrix} m(R_1 - T_1 + T_0) \\ +\left(C_g - C_1 + A + D\right) \end{bmatrix} \quad (24)$$

When F(n)=0, there are solutions $n_1=0$, $n_2=1$,

$$m^* = \frac{C_1 - C_g - A - D}{R_1 - T_1 + T_0} \quad (25)$$

From this, five equilibrium points can be determined (0,0), (0,1), (1,0), (1,1), ($\frac{C_1-C_g-A-D}{R_1-T_1+T_0}$, $\frac{B-B_0+R_3}{B_1-R_4-B_0}$)

The equilibrium analysis of the five points is carried out, and the bias of F(m) and F(n) is obtained to obtain $\frac{\partial F(m)}{\partial m}$, $\frac{\partial F(m)}{\partial n}$, $\frac{\partial F(n)}{\partial m}$, $\frac{\partial F(n)}{\partial n}$,and obtain the Jacobian matrix of the game between consumers and enterprise.

$$\frac{\partial F(m)}{\partial(m)} = (1-2m)[n(B_1 - R_4 - B_0) + (B_0 - R_3 - B)] \quad (26)$$

$$\frac{\partial F(m)}{\partial n} = m(1-m)(B_1 - B_4 - B_0) \quad (27)$$

$$\frac{\partial F(n)}{\partial(m)} = (1-2n)[m(R_1 - T_1 + T_0) + (C_g - C_1 + A + D)] \quad (28)$$

$$\frac{aF(n)}{an} = n(1-n)(R_1 - T_1 + T_0) \quad (29)$$

$$J = \begin{bmatrix} \frac{\partial F(m)}{\partial m} & \frac{\partial F(m)}{\partial n} \\ \frac{\partial F(n)}{\partial m} & \frac{\partial F(n)}{\partial n} \end{bmatrix} \quad (30)$$

Further calculations of the determinant det(j) and trace tr(j) are employed to assess the local stability of the equilibrium point.

$$\det\left(j\right) = \frac{\partial F(m)}{\partial m} \cdot \frac{\partial F(n)}{\partial n} - \frac{\partial F(m)}{\partial n} \cdot \frac{\partial F(n)}{\partial m} \quad (31)$$

$$tr\left(j\right) = \frac{\partial F(m)}{\partial m} + \frac{\partial F(n)}{\partial n} \quad (32)$$

TABLE VI
TR(J) AND DET(J) OF THE JACOBIAN MATRIX OF THE ENTERPRISES - CONSUMERS GAME

Equilibrium Point	det(j)	tr(j)
(0,0)	$(B_0-R_3-B)(C_g-C_1+A+D)$	$B_0-R_3-B+C_g-C_1+A+D$
(0,1)	$-(B_1-R_4-R_3-B)(C_g-C_1+A+D)$	$B_1-R_4-R_3-B-C_g+C_1-A-D$
(1,0)	$-(B_0-R_3-B)(R_1-T_1+T_0+C_g-C_1+A+D)$	$R_3+B-B_0+R_1-T_1+T_0+C_g-C_1+A+D$
(1,1)	$(B_1-R_4-R_3-B)(R_1-T_1+T_0+C_g-C_1+A+D)$	$R_3+B-B_1+T_1+C_1-T_0-C_g-A-D$
($\frac{C_1-C_g-A-D}{R_1-T_1+T_0}$, $\frac{B-B_0+R_3}{B_1-R_4-B_0}$)	$-(1-m)(1-n)(C_1-C_g-A-D)(B-B_0+R_3)$	0

B. Simulation diagram of consumer-enterprise game under initial parameters

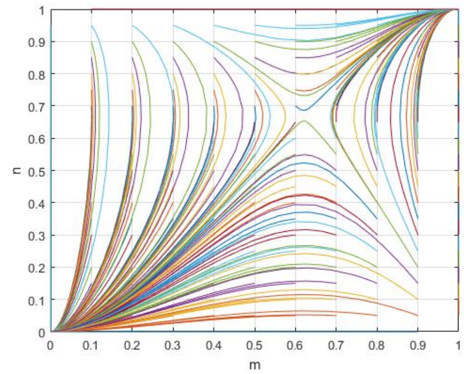
From the above, $R_0 = R_3 = 178800$, $C_g = 675$, $C_0 = 112400$, $R_1 = R_4 = 5630$, $C_1 = 6327$, $T_0 = 16592.5$, $T1 = 16417.5$, $A = 50$, set $D = 2000$. So there are $C_g - C_1 + A + D < 0$, $R_1 - T_1 + T_0 + C_g - C_1 + A + D > 0$.

With the rise in environmental awareness among consumers and increased government support, new energy vehicles are gaining a foothold in the automotive market. Compared to conventional cars, new energy vehicles have lower purchase costs and, with improvements in public transportation, make conventional cars less cost-effective, leading consumers to prefer new energy vehicles. In summary, it is evident that the utility of people buying new energy vehicles > the utility of not purchasing > the utility of purchasing traditional cars, so $B_1 - R_1 - R_0 > B > B_0 - R_0$. Set $B = 3000, B_1 = 188430, B_0 = 179800$ $\frac{C_1-C_g-A-D}{R_1-T_1+T_0}$ =0.62, $\frac{B-B_0+R_3}{B_1-R_4-B_0}$ =0.66, additional analysis of the stability at the equilibrium point.

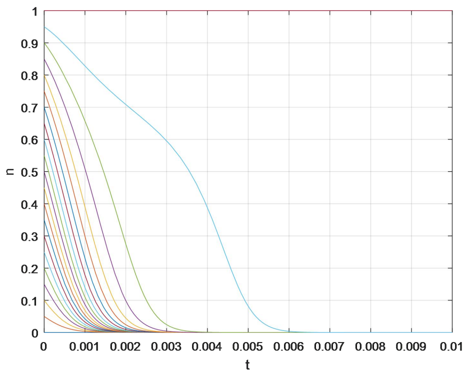
TABLE VII
STABILITY ANALYSIS OF GAME EQUILIBRIUM POINT BETWEEN ENTERPRISES AND CONSUMERS

Equilibrium point	det (j)	tr(j)	Stability
(0,0)	+	-	ESS
(0,1)	+	+	Unstable
(1,0)	+	+	Unstable
(1,1)	+	-	ESS
(0.62,0.66)	-	0	Saddle point

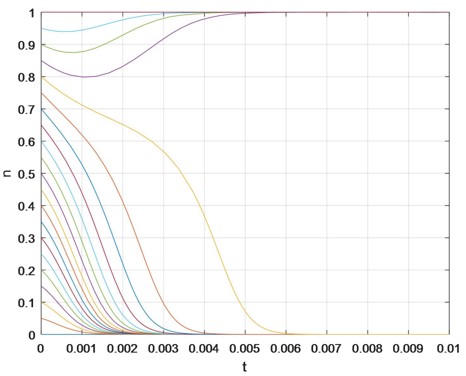
The consumer's replication dynamic equation is $F\left(m\right) = dm/dt = m\left(1-m\right)\left(300n - 200\right)$; The replication dynamic program of the enterprise is $F\left(n\right) = dn/dt = n\left(1-n\right)\ \left(5805m - 3602\right)$. The above parameters and equations are simulated in MATLAB to obtain Figure 4 a)-d), and when the initial value of m,n is 0 and the loop step is set to 0.05, the dynamic progression of the game between the enterprise group and consumers is illustrated in Figure 4a). At the same time, the cycle step is set to 0.05, and the probability m when consumers choose to purchase is set to be 0.3, 0.5 and 0.7, respectively, the evolution path is illustrated in Figure 4 b)-d).



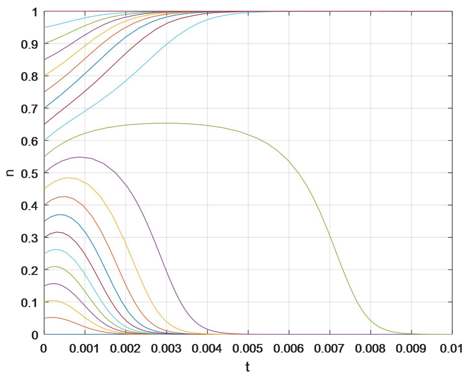
a) evolutionary path



b) enterprise evolutionary(m=0.3)



c) enterprise ~ (m=0.5)



d) enterprise ~(m=0.7)

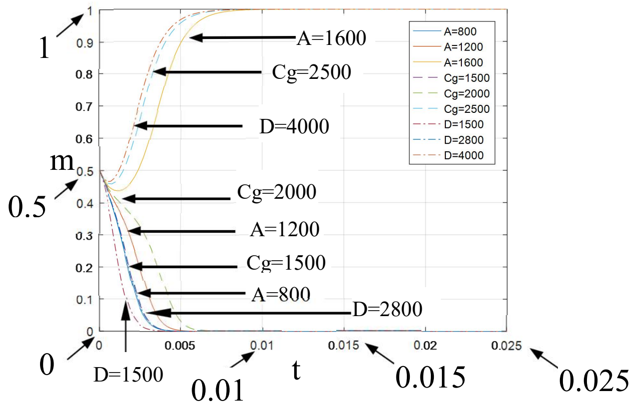
Fig.4. Game simulation diagram of consumers and enterprises under initial parameters

It can be seen that in the top-right area of the saddle point (0.62, 0.66), all paths evolve towards (1,1) (purchase, green transformation), and on the bottom-left region of the saddle point, all paths evolve towards (0,0), that is, (no purchase, traditional production). And at the level of existing parameters, the game between the two tends to evolve in the direction of (0,0). When the probability of consumers choosing to buy is 0.3, enterprises will choose traditional production approaches characterized by high pollution and significant energy consumption; when the probability of consumers choosing to buy increases to 0.7, a large number of enterprises will choose green transformation. This dynamic indicates that changes in consumer purchasing behavior have a significant impact on the choice of production methods by businesses.

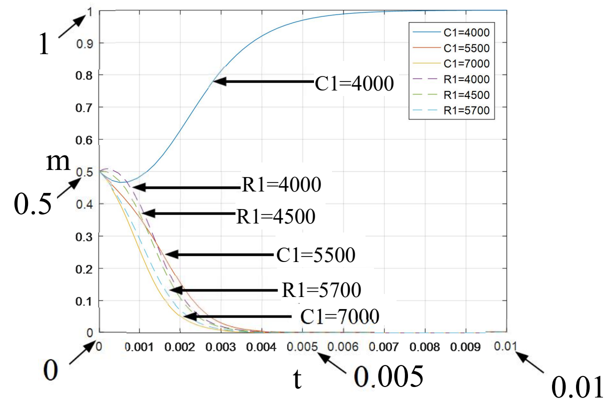
C. Dynamic Simulation under Parameter Optimization for Enterprises and Consumers

In the evolutionary path diagram of consumers and enterprises, the position of the saddle point $(\frac{C_1\text{-}C_g\text{-}A\text{-}D}{R_1\text{-}T_1\text{+}T_0}, \frac{B\text{-}B_0+R_3}{B_1\text{-}R_4\text{-}B_0})$ will affect the final effect of evolution, and by adjusting the size of the parameters to bring the saddle point closer to (0,0), the optimal equilibrium solution of the model can be achieved with a greater probability (1,1). The parameters include the incremental cost of green transformation C_1, the tax burden of enterprises after green transformation T_1, the tax burden of enterprises under the traditional mode T_0, the incremental income of green transformation R_1, pollution control fee A, government subsidy C_g, additional income D of enterprises after the government takes active measures, and benefits B, B_0 and B_1 purchased by consumers under different circumstances. Among them, the key parameters related to enterprises are C_g, C_1, D, R_1, T_0, T_1 and A, and the key parameters related to consumers are R_3, R_4, B, B_0 and B_1.

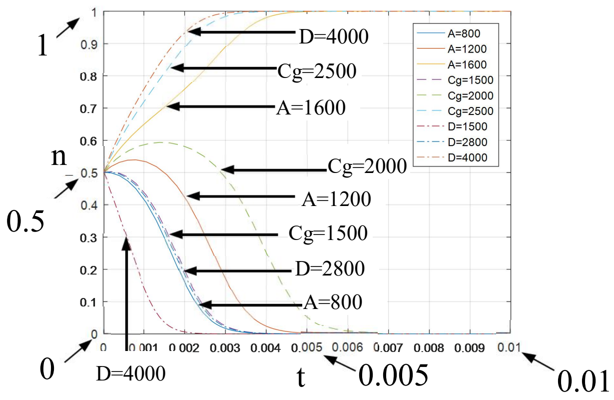
To promote the evolution of consumers and enterprises to the optimal equilibrium point (1,1), and further analyze of how variations in enterprise parameters affect the game, R_0, C_0 and T_0 remain unchanged, and other related key parameters are optimized separately, and the control variable method is adopted to change C_g, L_1, L_2, R_1, C_1, and A by certain magnitude under the initial parameters and take into account the actual circumstances. The initial parameters were $C_g = 675,\ R_0 = R_3 = 178800,\ C_0 = 112400,\ R_1 = R_4 = 5630$, $C_1 = 6327,\ A = 50,\ T_0 = 16592.5,\ T_1 = 16417.5,\ B = 0,\ B_1 = 184530$, $B_0 = 178600$. In the optimization process, A takes 800, 1200, and 1600 respectively. C_g take 1500, 2000, 2500. D take 1500, 2800, 4000. $R_1 = R_4$ takes 4000, 4500, 5700. C_1 take 4000, 5500, 7000. And assume that the odds of consumers who initially select to buy is $m = 0.5$, and the odds of enterprises choosing green transformation is $n = 0.5$. Enter the above parameters into MATLAB to obtain the trend of the ratio of m and n with time, as shown in the following figure.



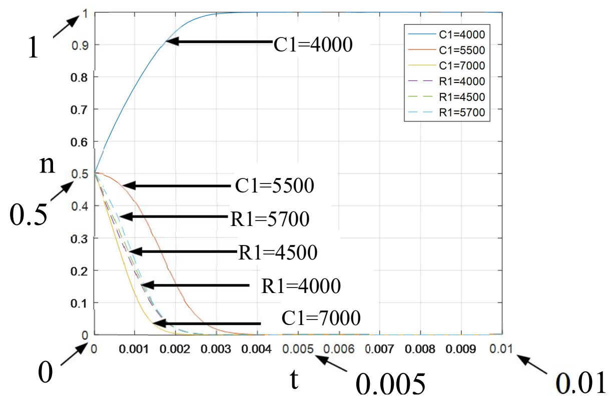
a) Evolution Trend of Consumer Strategy (A, C_g, D)



b) Evolution Trend of Consumer Strategy (C_1, R_1)



c) Evolution Trend of Enterprise Strategy (A, C_g, D)



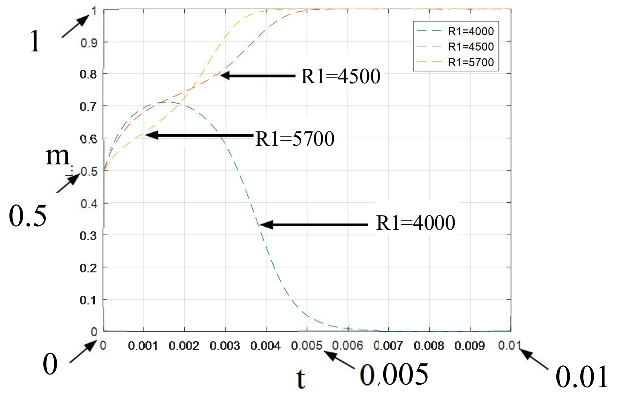
d) Evolution Trend of Enterprise Strategy (C_1, R_1)

Fig.5. Strategy evolution diagram after enterprise parameter optimization

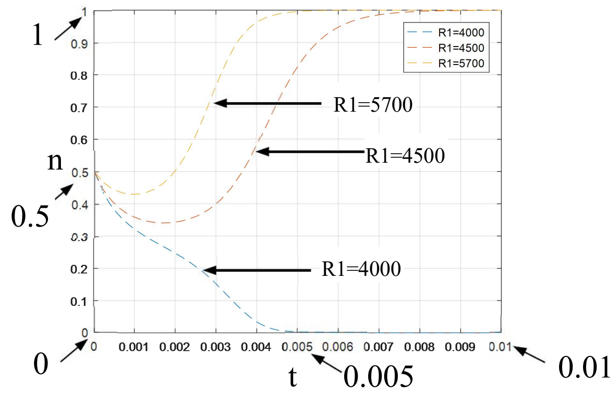
Figure 5 shows the strategy evolution diagram between two subjects after partial parameter optimization in the process of consumer-enterprise game. Figures 5 a) and b) are based on the optimization analysis of enterprise parameters from the perspective of consumers, when the fine A, the government subsidy C_g, and the additional income D of the enterprise after the government adopts an active strategy increase, or the incremental cost of the green transformation of the enterprise C_1 decreases, the consumer's original strategy of 0 gradually tends to 1, and with the gradual increase of A, C_g, D values or the gradual decrease of C_1, the time for the strategy to tend to 1 is gradually shortened. The government, by increasing fines for polluting activities, offering financial subsidies, and incentivizing businesses to earn additional revenues, along with businesses' own efforts to reduce the costs of green transformation, has collectively altered market supply and demand dynamics and the cost-benefit balance for consumers. These policy adjustments aim to use economic incentives to encourage businesses to reduce pollution and invest in green technologies, thereby influencing market trends. These changes not only boost the market demand for environmentally friendly products but also accelerate the shift of businesses towards greener production methods. As a result, consumers are more inclined to choose green products, which in turn promotes overall societal sustainability. Reflecting a shift in consumer preferences towards environmentally friendly attributes, the greater the increase in fines, subsidies, and additional revenues, and the more significant the reduction in green transition costs, the stronger the trend of consumers choosing green products and the faster the decision-making process becomes.

Similarly, Figures 5 c) and d) are based on the optimization analysis of enterprise parameters from the perspective of enterprises, when the penalty A, government subsidy C_g, the additional income D of the enterprise after the government adopts an active strategy increases, or the incremental cost of the green transformation of the enterprise C_1 decreases, the strategy of the enterprise that originally tends to 0 gradually tends to 1, and with the gradual increase of A, C_g, and D values or the gradual decrease of C_1, the time for the strategy to tend to 1 is gradually shortened.

Figure 5 b) causes consumers to tend to strategy 0 more slowly as the R_1 value decreases, but eventually does not tend to strategy 1. Figure 5 d) shows that the R_1 value decreases, the evolution to strategy 0 accelerates at the beginning of the period, but because of the decrease in R_1 increases the probability of consumers choosing to buy, the speed of enterprises tending to strategy 0 after a certain period slows, but eventually, it will not tend to strategy 1. Here R_1 is mainly affected by the size of B_1, in order to explore the influence of R_1 optimization step by step, assume that $B_1 = 191000$, R_1 takes 4000, 4500, 5700. And assume that the likelihood of consumers initially opting to purchase is $m = 0.5$, and the likelihood of enterprises choosing green transformation is $n = 0.5$. Enter the above parameters into MATLAB to obtain the trend of the ratio of m and n with time, as shown in the following figure.



a) Evolution trend of consumer strategy (B_1=191000, R_1)

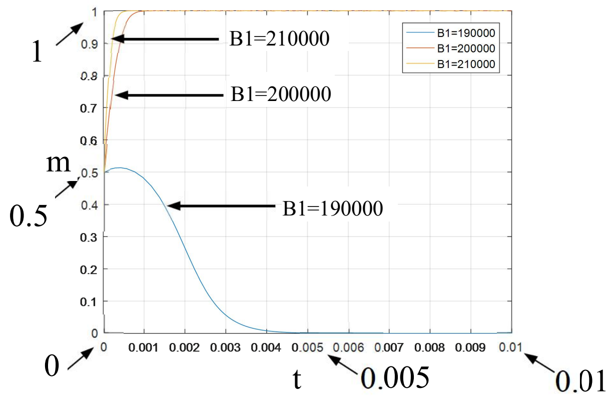


b) Evolution trend of enterprise strategy (B_1=191000, R_1)

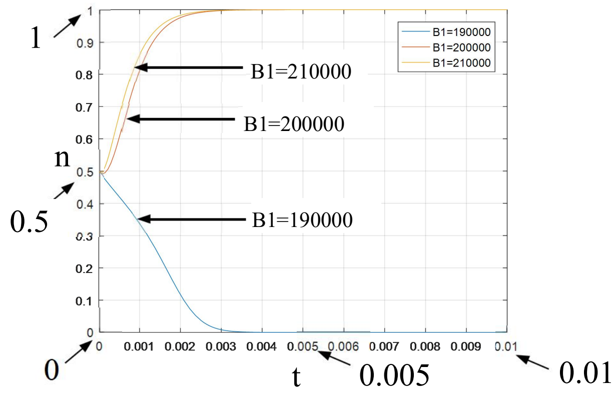
Fig.6. Strategy evolution diagram after enterprise parameter optimization

By comparing Figure 5 with Figure 6, it is evident that under the initial parameters, changing the incremental income R_1 of enterprise green transformation within a suitable range has little impact on both consumers and enterprises, and eventually evolves to strategy 0. In the case of an increase in the yield B_1 purchased under the green transition, the increase in R_1 within a certain range will prompt consumers to choose strategy 1, and the probability of enterprises choosing strategy 1 due to the increase in incremental income R_1 will increase. This is because the increase in income B_1 has led to an increase in consumer payment R_1 , but consumers will still choose to buy. This choice reflects consumers' desire for higher quality, better experiences, and more sustainable lifestyles. Over time, if these benefits continue to increase, the trend of consumers choosing green products will become more pronounced.so in this case, as long as the enterprise chooses green transformation, consumers will definitely buy the green and low-carbon products they produce to obtain higher utility, and ultimately promote the evolution of strategy 1 on both sides.

Further optimization of key consumer-relevant parameters. The optimization process, B_1 takes 190000, 200000, and 210000. The traditional production model is relatively mature, and its benefits to consumers are relatively stable, assuming that B and B_0 are fixed. R_1 has been analyzed above and is not discussed here. Assume that the likelihood of consumers initially opting to purchase is $m = 0.5$, and the proportion of enterprises choosing green transformation is $n = 0.5$. Enter the above parameters into MATLAB to obtain the trend of the ratio of m and n with time, as shown in the following figure.



a) Evolution Trend of Consumer Strategy (B_1)



b) Evolution Trend of Enterprise Strategy (B_1)

Fig.7. Strategy evolution diagram after consumer parameter optimization

Figure 7 shows the strategy evolution diagram between two subjects after the optimization of consumer partial parameters in the process of the consumer-enterprise game. Figure 7 a) is based on the optimization analysis of consumer parameters from the perspective of consumers, when the benefit B_1 of consumer purchase increases under the green transition, the strategy of consumers originally tending to 0 gradually tends to 1 and with the gradual increase of B_1 value, consumers perceive a higher net benefit and make decisions more quickly. This change reflects consumers' pursuit of high-quality, healthy and sustainable lifestyles, the time for the strategy to tend to 1 gradually shortens. Similarly, Figure 7 b) is based on the optimization analysis of consumer parameters from the perspective of enterprises, when the benefit of consumer purchase B_1 increases under the green transition, the strategy of enterprises that originally tended to 0 gradually tends to 1, and with the gradual increase of B_1 value, the time for the strategy to tend to 1 is gradually shortened.

VI. Stackelberg Game Analysis

A. Research Method

The Stackelberg game is a dynamic game model involving a leader and one or more followers. In this framework, the leader makes decisions first, and the followers respond based on the leader's actions. When enterprises invest in innovation, they often face high upfront costs. In such cases, financial subsidies from the government become a critical factor for enterprises. Thus, the interaction between the government and enterprises can be viewed as a Stackelberg game, with government taking on the role of leader, taking initiative to formulate supportive policies for enterprise innovation. Specifically, the government determines the innovation subsidy rate ω, which is founded on the goal of maximizing social welfare . In other words, the government sets subsidy policies with the broader societal interest in mind, while enterprises make their innovation investment decisions within this policy framework to optimize their behavior and maximize their benefits. The Stackelberg game provides an effective framework for analyzing the interactions among the government, enterprises, and consumers. This analysis helps to better understand how government subsidies can assist enterprises in bearing the substantial upfront expenses associated with green technology research, and mitigate the product price increases caused by adopting new technologies. These subsidies not only alleviate the economic pressure on enterprises during the green transition but also help maintain relative price stability, preventing market disruptions due to price fluctuations. Ultimately, such measures promote a more sustainable market trajectory.

B. Basic Model

This study assumes an industry with two firms, Firm 1 and Firm 2, both undertaking green transition initiatives.

1. It is assumed that green transition efforts by the firms increase their respective product demand. Let Δq_1 Δq_2 denote the demand increments for Firm 1 and Firm 2, respectively, resulting from their green transitions. Assuming a linear relationship between green transition investments and the increase in product demand, we have the following:

$$\Delta q_1 = \lambda M_1 \tag{33}$$

$$\Delta q_2 = \lambda M_2 \tag{34}$$

In the equations: λ represents the revenue parameter of innovation activities, $\lambda \geq 0$. M_1 , M_2 denote the innovation investment of the two firms, respectively. $M1, M2 \geq 0$.

2. Assuming the product demand and price exhibit a linear relationship, the demand functions for the two firms are given by:

$$q_1 = a - b * p_1 + \Delta q_1 \tag{35}$$

$$q_2 = a - b * p_2 + \Delta q_2 \tag{36}$$

In the equations: a represents the market capacity, and b denotes price elasticity. $q_1 + q_2 = 1$.

3. Firms engaging in green production must invest a certain amount of funds. The government supports firms' green transition through fiscal subsidies, with a subsidy rate of ω. The government subsidy to the firm is ωM_i, where consumers' readiness to spend extra on eco-friendly products is denoted as k. Thus, the portion of the green production cost borne by the firm is $(1-\omega-k)M_i$.

4. The production costs for the two firms are c_1 and c_2, respectively. Consequently, the profit functions for the two firms can be expressed as:

$$\pi_1 = (p_1 - c_1)(a - b * p_1 + \lambda M_1) - (1-\omega-k)M_1 \tag{37}$$

$$\pi_2 = (p_1 - c_1)(a - b * p_1 + \lambda M_2) - (1-\omega-k)M_2 \tag{38}$$

5. The government determines the innovation subsidy rate by maximizing social welfare L, which is described as the total of both consumer surplus and producer surplus:

$$L = CS + (\pi_1 + \pi_2) \tag{39}$$

where consumer utility CS follows the DIXIT-STIGLITZ model, consumer surplus CS is assumed to have a quadratic relationship with market demand.

$$CS = \frac{1}{2}(q_1 + q_2) \tag{40}$$

$$L = \frac{1}{2}(q_1 + q_2) + (\pi_1 + \pi_2) \tag{41}$$

C. Model Solution

Using the profit functions for the two firms, equations (37) and (38), and the condition of profit maximization, set $\frac{\partial \pi_1}{\partial p_1} = 0; \frac{\partial \pi_2}{\partial p_2} = 0$, the equilibrium prices for the products of the two firms can be obtained as:

$$p_1 * = \frac{(a - bc_1 + \lambda M_1)}{2b} \tag{42}$$

$$p_2 * = \frac{(\mathrm{a} + \mathrm{bc}_2 + \lambda \mathrm{M}_2)}{2\mathrm{b}} \tag{43}$$

Substituting the equilibrium prices p_1^*, p_2^* into equations (37) and (38), the equilibrium profits are derived as:

$$\pi_1^* = \frac{(a - bc_1 + \lambda M_1)^2}{4b} - (1-w-k)M_1 \tag{44}$$

$$\pi_2^* = \frac{(\mathrm{a} - \mathrm{bc}_2 + \lambda M_2)^2}{4b} - (1-w-k)M_2 \tag{45}$$

Based on equations (44) and (45), let $\frac{\partial \pi_1 *}{\partial \mathrm{M}_1} = 0; \frac{\partial \pi_2 *}{\partial \mathrm{M}_2} = 0$, we have the following:

$$M1^* = \frac{2b(1-\omega-k) - \lambda a + \lambda bc_1}{\lambda^2} \tag{46}$$

$$M_2^* = \frac{2b(1-\omega-k) - \lambda a + \lambda bc_2}{\lambda^2} \tag{47}$$

Substituting M_1^*, M_2^* back into equations (44) and (45) yields:

$$p_1^* = c_1 + \frac{1-w-k}{\lambda} \tag{48}$$

$$p_2^* = c_2 + \frac{1-w-k}{\lambda} \tag{49}$$

The government determines the innovation subsidy rate ω by maximizing the social welfare function. Substituting equations (35), (36), (37), and (38) into equation (41):

$$L = \frac{1}{2}[(a - b * p_1 + \lambda M_1) + (a - b * p_2 + \lambda M_2)]^2 + (p_1 - c_1)(a - b * p_1 + \lambda M_1) - (1 - \omega - k) M_1 + (p_2 - c_2)(a - b * p_2 + \lambda M_1) - (1 - \omega - k) M_2 \tag{50}$$

By substituting $p_1^*,\ p_2^*, M_1^*, M_2^*$ into Equation (50) and set $\frac{\partial L}{\partial w} = 0$, we have the following:

$$w^* = \frac{bk + 1 - k - b}{1 - b}$$

Conclusion 1: A higher government innovation subsidy rate leads to lower product prices for firms.

Demonstrate：Solve each one separately $\frac{\partial p_1^*}{\partial w}, \frac{\partial p_2^*}{\partial w}$, we have the following:

$$\frac{\partial p_1 *}{\partial w} = \frac{\partial p_1 *}{\partial w} = -\frac{1}{\lambda} \langle 0$$

Thus, Conclusion 1 holds. By providing financial support, the government helps firms offset the high initial expenses related to the development and innovation of environmentally friendly technologies. This helps to mitigate the rise in product prices caused by the adoption of new technologies. Such subsidies not only alleviate the economic pressure on firms during their green transition but also help maintain relatively stable product prices, avoiding market disruptions due to price volatility.

A stable pricing environment enhances consumer confidence, promotes the sale of green products, and incentivizes firms to further invest in green technologies and improvements in production processes. Therefore, government subsidies not only directly reduce the costs of green transition for firms but also indirectly encourage a shift toward more environmentally friendly and sustainable production methods by stabilizing market mechanisms.

Conclusion 2: A higher consumer willingness to purchase green products leads to lower product prices for firms.

Demonstrate: From Equations (48) and (49), solve each one separately $\frac{\partial p_1^*}{\partial k}, \frac{\partial p_2^*}{\partial k}$, we have the following:

$$\frac{\partial p_1 *}{\partial \mathrm{k}} = \frac{\partial p_1 *}{\partial \mathrm{k}} = -\frac{1}{\lambda} \langle 0$$

Thus, Conclusion 2 holds. The desire of consumers to buy green products is essential in driving sustainability efforts in stabilizing the market. When consumers show a stronger preference for environmentally friendly products, the resulting demand helps mitigate price volatility that may arise during the innovation process. Increased demand for green products provides firms with a more predictable revenue stream, fostering investment in green technologies and production methods.

In summary, consumer green purchasing behavior not only reduces the market's sensitivity to price fluctuations but also provides market-driven incentives for firms to undergo green transformation. This enables firms to maintain competitiveness while meeting the requirements of sustainable development.

VII. Conclusions and Suggestion

At present, many large manufacturing enterprises in China are facing the problem of green transformation and upgrading. This study takes a listed company as an example, quantifies the incremental benefits and incremental costs of green transformation, combines the behavior of various subjects, establishes the profit and loss variables of government, enterprises, and consumers, develops a game model, and founded on the actual data of specific cases, carries out simulation analysis under initial parameters and optimization parameters. A Stackelberg game is further introduced to simulate and analyze the evolving interactions involving the government, as the policymaker (leader), and enterprises (followers) during the green transition process. The findings indicate that traditional manufacturing enterprises struggle to complete green transformation rapidly due to their large scale. Although the company began its new energy business in 2016, its 2020 annual report shows that new energy business revenue accounted for less than 10% of total revenue. Despite promising prospects, the early stages of transformation are plagued by challenges such as low market share, minimal government subsidies, technological hurdles, and high production costs. Coupled with the low government subsidies and insufficient willingness of consumers to buy, it is impossible to maximize the overall benefits in the current state, hindering enterprises from carrying out green transformation. Firstly, the size of government subsidies plays a crucial role in promoting enterprises' green transition. Larger subsidies strengthen enterprises' willingness to transition; however, excessive subsidies may negatively impact government revenues. If the government ultimately opts not to provide subsidies, it could hinder enterprises from completing the transition. Secondly, government subsidies play a vital role in stabilizing market prices and mitigating price fluctuations caused by enterprise innovation. Subsidies help maintain relative price stability, preventing market disruption due to price volatility. Lastly, if consumers demonstrate a higher willingness to purchase green products, enterprises can achieve green transformation even when initial incremental benefits are lower than incremental costs, suggesting enterprises can strategically forego short-term profits to capture market share, facilitating long-term development. The additional net benefits of the government, the increase in the income consumers choose to purchase within the context of enterprises' green transformation, the reduction of incremental costs for enterprises and government regulatory costs can all encourage enterprises to choose green transformation strategies.

Building on above conclusions, we propose the following recommendations: Firstly, as both a policymaker and market regulator, the government is crucial in fostering and advancing the green transformation of enterprises in their initial stages. This includes appropriately investing human, material, and financial resources, improving laws and regulations, establishing suitable industry standards and norms, and increasing fines for excessive pollutant emissions by enterprises. Additionally, the government should provide targeted support and subsidies without applying a one-size-fits-all approach to avoid excessive financial pressure and avoid indiscriminate subsidies. Promoting the sustainability concept of products, enhancing public awareness and acceptance, and creating a favorable environment for enterprises' green transformation are also crucial. The government should comprehensively consider economic, social, and environmental benefits and pay attention to implied benefits, including the development of the industrial chain, the improvement of residents' satisfaction, and the reduction of unemployment. Furthermore, enhancing the efficiency of relevant personnel in discharging their official duties and minimizing supervision costs is imperative. Secondly, enterprises should consolidate themselves, attach importance to technological development, achieve technological breakthroughs as soon as possible, reduce costs, form scale effects, strengthen cooperation with universities and research institutes, comprehensively enhance soft power and hard power, and turn losses into profits as soon as possible. Enterprises should reasonably set product prices, and in the early stage, they cannot pay too much attention to profits but focus on occupying part of the market to achieve the purpose of publicizing products. At the same time, enterprises should conduct a comprehensive assessment, not only paying attention to the actual income and expenditure but also considering the expected benefits brought by the green transformation, such as brand enhancement benefits, government-enterprise cooperation benefits, etc., and then making reasonable decisions. Thirdly, as the demand source and users of green transition products, consumers significantly influence the market. They should embrace new products with an open mind and a long-term perspective, fostering green transformation under government guidance. The products after the green transformation have the advantages of energy savings, emission reduction, green environmental protection, etc.; consumers should change their concepts and accept new things so as to improve the benefits of purchasing under the green transformation, leverage the strengths of users and actively drive the promotion of new products.

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