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Fatigue Life Analysis of Pressure-resistant Shell of Underwater Vehicle Under Random Loads

Jian Wang, Kang Wang, Pei Zhang, and Chunbo Zhen

***Abstract*—This paper studies the fatigue life of underwater vehicle pressure-resistant shell structures under random diving load. Firstly, the random load sequence that conforms to the Gumbel distribution is given. Secondly, the mathematical relationship between diving depth and maximum stress is given based on the approximate model method. Then, 500 random load sequences based on the Monte Carlo sampling method are obtained. The calculation model of the stress intensity factor and fatigue crack propagation model is given, and the life of fatigue cracks under the random load sequence is calculated using the method of numerical integration calculation of cycle by cycle. Finally, the probability distribution of lives of fatigue crack propagation under random load sequences is analyzed. The results show that, under each initial crack size, the fatigue crack growth life of the pressure-resistant shell follows the normal distribution and has a high level of significance.**

***Index Terms*—Underwater vehicle; pressure-resistant shell; random load sequence; fatigue life**

I. Introduction

UNCREWED vehicles are essential in underwater detection, tracking, relay communication, and marine ecological environment investigation. Titanium alloy is usually used as a pressure shell material for underwater vehicles due to its high strength and strong corrosion resistance in seawater [1]. The pressure hull is the primary load-bearing device of the underwater vehicle. Each submergence and ascent process can be regarded as a pulsating cyclic load. The fatigue damage caused by several diving operations will significantly impact the structure's safety, so it is essential to study the fatigue life of the pressure shell structure.

Due to the importance of the pressure hull structure's fatigue life to the underwater vehicle's structural safety, scholars at home and abroad have conducted extensive research. Li et al. [2] used an improved formula based on McEvily 's crack propagation model to analyze the reliability of the fatigue life of pressure spherical shells, considering the randomness of titanium alloy material parameters, diving depth, and other influencing factors. Xue et al. [3] calculated the pressure spherical shell's fatigue reliability index using the response surface method based on the unified crack propagation prediction model, considering the crack closure effect and the three-dimensional constraint effect of the crack front. They analyzed the sensitivity of the relevant parameters. Yao et al. [4] introduced the calculation method of stress intensity factor and analyzed the fatigue life of pressure hulls based on fracture mechanics. Yu et al. [5] studied deep-sea pressure hull TC4 titanium alloy thick plate welding characteristics using the equivalent structural stress method. They estimated the fatigue life of the pressure hull under a certain pressure. It is concluded that the weld greatly influences the stress distribution of the whole spherical shell structure, and the stress concentration is easy to occur at the weld of the hatch covers. The modeling method of the weld area is proposed and its feasibility is proved. Using the probability density evolution method, Liu et al. [6] predicted the fatigue life of submarine pressure hull weld. Aiming at the complex characteristics of the stress intensity factor of the pressure hull weld, the probability density equation was established based on the crack growth rate model, and the probability density function of the crack size changing with the load cycle was obtained by numerical method, which was used to predict the reliability of fatigue life. Wang et al. [7] established a prediction model of constant amplitude cyclic load spectrum for estimating the life of pressure hulls for pressure materials of underwater vehicles. WANG et al. [8] proposed an improved short crack growth model for the influence of the abnormal growth rate of short cracks on the accurate prediction of the underwater vehicle's fatigue life of the pressure hull structure. Based on this model, the short crack growth rate and the corresponding fatigue life of the Ti-6Al-4V alloy under different stress ratios and stress levels were predicted, and the model's validity was verified. Li et al. [9] pointed out that low cycle fatigue failure may occur with the increase in the number of dive times of the pressure spherical shell. Therefore, based on the local stress-strain method, the fatigue analysis software MSC. Fatigue was used to calculate the fatigue life of the pressure spherical shell. The effects of the fatigue strength coefficient and effective stress concentration coefficient on the fatigue life of the pressure spherical shell were discussed. The results show that the design dive times of the spherical shell are safe. Zhang and Huang [10] used different numerical calculation methods to analyze the fatigue life of naval architecture and ocean structures under random fatigue loads. Luo et al. [11] studied and analyzed the influence of different numerical integration methods on the simulation results of fatigue crack propagation of marine structures. From the research status at home and abroad, more research on the current research has more calculation methods for the fatigue failure mode and life of the pressure structure, more research on the fatigue crack propagation life under constant amplitude load, and less on the fatigue crack propagation life under random load. However, the depth of each dive of the underwater vehicle is uncertain according to the needs of the mission, so it is essential to analyze the fatigue crack propagation life under random load.

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In this paper, aiming at the randomness of the diving depth during the service of the underwater vehicle, the fatigue crack propagation life of the pressure structure of the underwater vehicle under the random load spectrum is calculated and analyzed, and the fatigue crack propagation life is analyzed according to the uncertainty of the initial size of the crack caused by the welding defect.

II. THE BASIC IDEA OF RANDOM FATIGUE LIFE ANALYSIS FOR THE PRESSURE STRUCTURE OF THE UNDERWATER VEHICLE

In studying the fatigue crack propagation life of the pressure hull of the underwater vehicle, the diving depth of the underwater vehicle during service and the diving load spectrum are usually random. Studying the probability distribution characteristics of the diving load spectrum is the first problem to be solved for the random fatigue life of the pressure hull structure of the underwater vehicle. When the random load sequence acts on the pressure hull, the stress response of the pressure hull structure is also a random sequence. If the corresponding finite element software is used to calculate each time, the calculation amount will significantly increase. In the range of diving depth, a certain number of diving depth samples are extracted by the Latin hypercube sampling method. The maximum stress of the pressure shell is calculated by the finite element method, and the mathematical relationship between diving depth and maximum stress is constructed by the approximate model method. When calculating the random fatigue crack growth of the pressure hull, the cycle-by-cycle numerical integration method is used to calculate the fatigue crack growth of the pressure hull under random load. Considering the randomness of the random diving load spectrum and the randomness of the initial crack size, this paper makes a statistical analysis of the fatigue crack propagation life under different random load sequences, obtains the distribution law of the fatigue crack propagation life, and further obtains the statistical characteristics of the random fatigue life of the pressure structure of the underwater vehicle. The primary research idea of this paper is shown in Fig. 1.

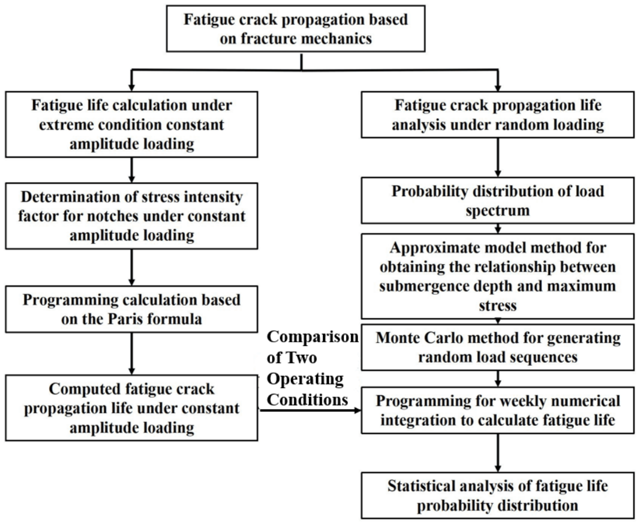


Fig. 1. Research flowchart

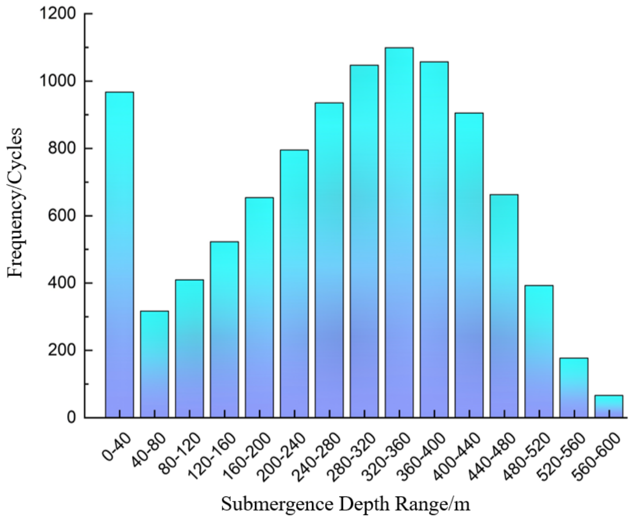


Fig. 2. Fatigue load spectrum of 600m deep underwater vehicle

III. STATISTICAL MODEL OF LOAD

The underwater vehicle needs to carry out the ascending-submerging operation repeatedly when it is working, and the seawater pressure changes with the change in the diving depth. To predict the fatigue crack life, the fatigue load spectrum of the pressure hull must be obtained. In this paper, the load spectrum distribution of the underwater vehicle is obtained by statistical analysis of the diving data recorded by the existing data.

Based on the existing dive data of 'Alvin' and the relevant literature, this paper determines that the Gumbel distribution is more suitable for the distribution of the fatigue load spectrum of underwater vehicles [12]–[14]. For the different design depths, according to the function characteristics of the Gumbel distribution, the scale coefficient transformation of the diving depth is carried out without changing the shape of the function distribution so that the maximum depth can reach the design depth.

In this paper, the designed diving depth of the underwater vehicle is 600 m. The distribution function and probability density function of the diving depth obeying the Gumbel distribution are as follows:

$$\mathbf{F}(x) = 1 - \exp\left[-\exp\left(\frac{\frac{4.5}{0.6}x - 2585.5}{1000}\right)\right] \tag{1}$$

$$f(x) = \frac{1}{1000}\exp\left[\frac{\frac{4.5}{0.6}x - 2585.5}{1000} - \exp\left(\frac{\frac{4.5}{0.6}x - 2585.5}{1000}\right)\right] \tag{2}$$

Where x is submergence depth. It is assumed that the underwater vehicle is designed to dive 10,000 times. Through the above analysis, from the above distribution function characteristics of the underwater vehicle, the density function peak of the Gumbel distribution is about 350 m. The fatigue load spectrum of the underwater vehicle is shown in Fig. 2.

According to the force characteristics of the underwater vehicle, a diving process is a force cycle. In this paper, based on the fitted Gumbel distribution diving load, 500 groups of 10,000 diving random load sequences are generated by the

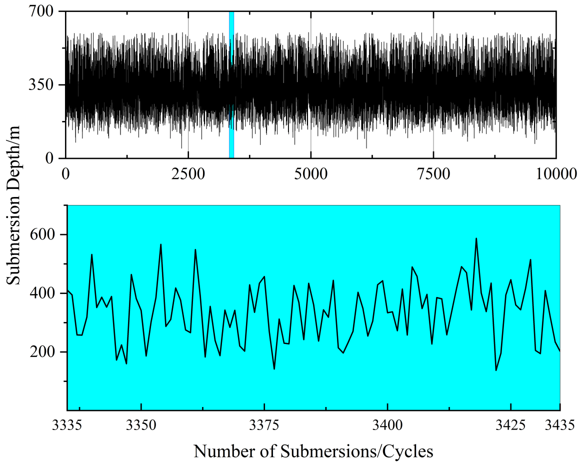


Fig. 3. Random Dive Depth Map

Monte Carlo sampling method. Fig. 3 is a set of diving depth maps generated by Gumbel distribution.

IV. CALCULATION MODEL OF FATIGUE CRACK PROPAGATION LIFE

A. Crack propagation mode

According to the characteristics of fatigue residual strength, the Paris crack propagation formula, which is widely used in the engineering field, is adopted in this paper.

$$\frac{da}{dN} = C(\Delta K)^n \tag{3}$$

Where C and N are material constants, which can be measured by test.

B. Calculation model of Stress intensity factor

According to the Paris formula, the stress intensity factor amplitude K is the primary determinant of crack growth rate. Therefore, the primary task of estimating the fatigue life of the pressure shell studied in this paper is to obtain the stress intensity factor K at the crack tip. The stress intensity factor K at the crack tip is a parameter used to represent the strength of the stress field near the crack tip in fracture mechanics, and its calculation formula is as follows [9,14]:

$$\Delta K = K_{eff} - K_{effth} \tag{4}$$

$$K_{eff} = K_{\max}\left(1 - f_{op}\right) \tag{5}$$

$$K_{effth} = K_{\max}\left(1 - f_{op}\left(a_{th}\right)\right) \tag{6}$$

$$K_{\max} = \sqrt{\pi r_e \left(\sec\frac{\pi}{2}\frac{\sigma_{\max}}{\sigma_v} + 1\right)}\left(1 + Y\left(a\right)\sqrt{\frac{a}{2r_e}}\right)\sigma_{\max} \tag{7}$$

$$f_{op} = \begin{cases} \max\left\{R, A_0 + A_1 R + A_2 R^2 + A_3 R^3\right\} & 0 \leqslant R \leqslant 1 \\ A_0 + A_1 R & -2 \leqslant R < 0 \end{cases} \tag{8}$$

$$A_0 = (0.825 - 0.34\alpha' + 0.05\alpha'^2) \bullet [\cos(\pi\sigma_{max}/2\sigma_n)]^{1/\alpha'} \tag{9}$$

$$A_2 = 1 - A_0 - A_1 - A_3 \tag{10}$$

$$A_3 = 2A_0 + A_1 - 1 \tag{11}$$

$$\alpha' = \frac{1}{1-2v} + \frac{1 - \frac{1}{1-2v}}{\left[1 + 0.8861 \bullet \left(t/(K_{\max}/\sigma_y\ )^{\ 2}\right)^{3.2251}\right]^{0.75952}} \tag{12}$$

Where K_{eff} is the amplitude of the effective stress intensity factor; K_{effth} is the threshold value of stress intensity factor amplitude; f_{op} is the ratio of opening stress intensity factor; r_e is the size of hidden defect; $\sigma_{\max}$ is the peak stress of fatigue cyclic load; σ_V is the virtual strength (the strength of the material under ideal conditions); $Y(a)$ is the crack shape parameter. a is the crack length; R is the cyclic characteristic coefficient; v is the Poisson ratio of the material; t is the plate thickness; σ_y is the yield strength of the material.

C. Calculation of crack propagation life

Assuming that the peak value of the load spectrum under the pressure shell is constant,and the crack of the pressure shell extends from the initial crack size a_0 to the critical crack size a_c,the total number of crack propagation cycles N can be used as the fatigue life of the structure:

$$N = \int_{a_0}^{a_c} \frac{da}{C(\Delta K)^n} \tag{13}$$

When the peak value of the fatigue load spectrum changes continuously, the integral method is no longer applicable to calculate the fatigue crack growth life, and the final result must be calculated by the weekly numerical integration method. Once the calculation formula of stress intensity factor is determined, the crack propagation life and critical crack size can be calculated by programming. The calculation process of the weekly numerical integration method is shown in Fig. 4.

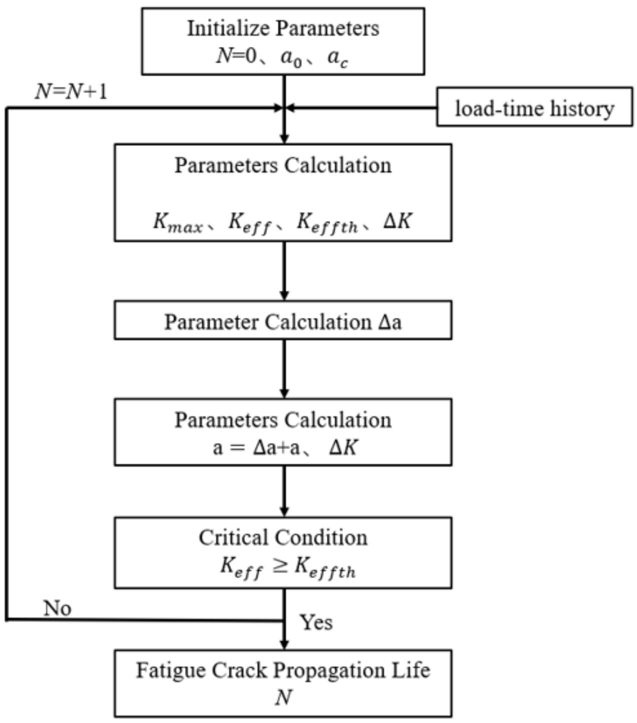


Fig. 4. Numerical integration method by cycle

In Fig.4, N is the fatigue crack propagation life; a_0 is the initial crack length; a_c is the critical crack size; Δa is the crack propagation size of a fatigue cycle.

V. EXAMPLE ANALYSIS

Taking an underwater vehicle pressure hull as the example, the structural parameters of the pressure shell of the underwater vehicle are designed as follows : the radius of pressure shell R = 0.23 m, the length of parallel section L = 5 m, the thickness of pressure shell plate t = 0.008 m, the spacing of ring ribs l = 0.3 m, the height of ring ribs h = 40 mm, the thickness of ring ribs m = 6 mm, the calculated water depth hj = 600 m, and the calculated load is 6 MPa. The finite element model of pressure shell is shown in Fig. 5.

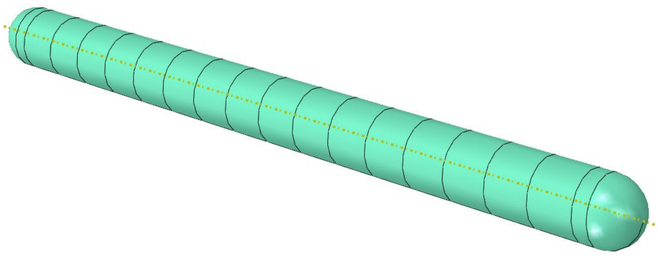


Fig. 5. Pressure-resistant shell finite element model

A. Approximate model of stress and diving depth

It is found by calculation that the maximum stress of the pressure-resistant structure is located at the intersection of the rib and the pressure-resistant shell plate and the root area of the rib web. In this paper, these parts are selected as the stress of fatigue analysis, and the calculated stress cloud diagram is shown in Fig. 6.

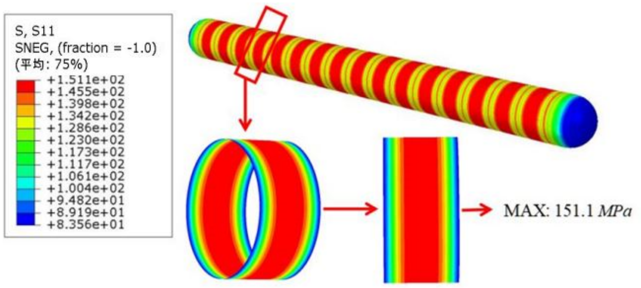


Fig. 6. Stress contour plot

In this paper, 20 sets of the depth data samples were uniformly extracted from the range of 0 to 600 meters. The corresponding maximum stress was calculated by ABAQUS, and the maximum stress of the pressure shell in full depth was obtained by linear fitting using approximate model technology. The fitting analysis shows that there is a linear relationship between the depth of diving and the maximum stress of the pressure shell, and the expression is $y = 0.251x + 0.045$, where is the depth of diving. Fig. 7 is the fitting curve of diving depth and maximum stress.

B. Fatigue life calculation under random load

In this paper, when calculating the fatigue crack propagation life under random load, it is assumed that the initial

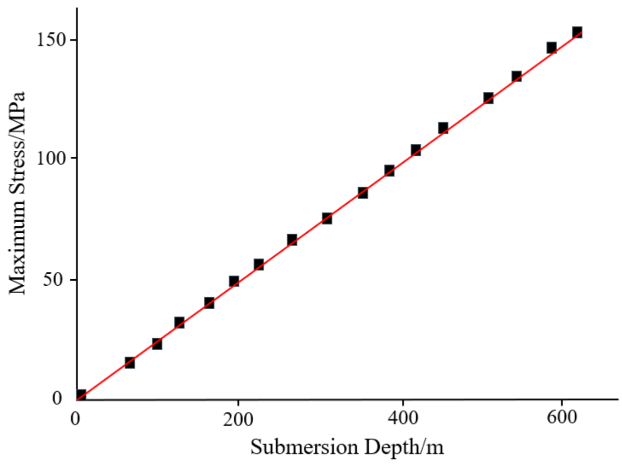


Fig. 7. Fitting curve of diving depth and maximum stress

crack size is 0.2 mm, 0.3 mm, 0.4 mm, 0.5 mm, 0.6 mm, and the critical crack size is 2 mm. Referring to the design life of foreign underwater vehicles, the number of dive fatigue cycles of the pressure hull of the underwater vehicle is designed to be 10,000 times. As a comparative analysis, this paper also gives the fatigue life results under the design water depth, as shown in Fig. 8.

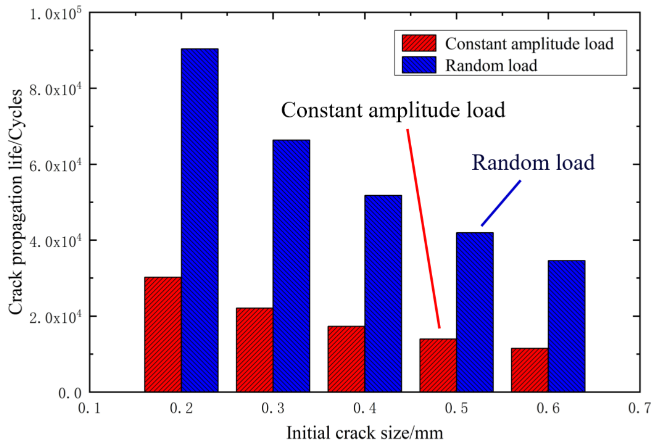


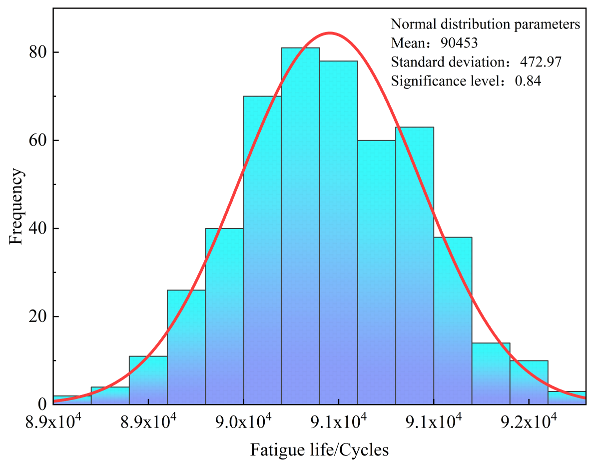
Fig. 8. Fatigue life of different initial crack depths

The calculation results of Fig. 8 show that under the premise of the above initial crack size assumption, the fatigue life of the pressure shell studied in this paper is greater than the design life, and the structural fatigue has a high safety margin, whether it is constant amplitude load or random load. The fatigue crack propagation life under a constant amplitude load is much smaller than that under a random load, and the fatigue crack propagation life under a random load is about 3 times that under a constant amplitude load.

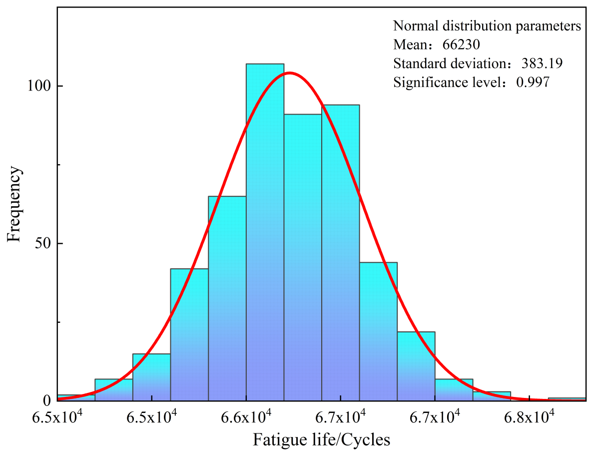
C. Statistical analysis of fatigue life under random load sequence

Considering the randomness of underwater vehicles' random dives, this paper analyzes fatigue life under a random load sequence based on statistical ideas. Firstly, the Monte Carlo sampling method is used to randomly generate 500 sets of random load sequences with 10,000 dive times. Then, the fatigue crack propagation life under different initial crack sizes is calculated for each random load sequence according to the cycle-by-cycle numerical integration method. Finally, the fatigue life under 500 sets of random load sequences is statistically analyzed.

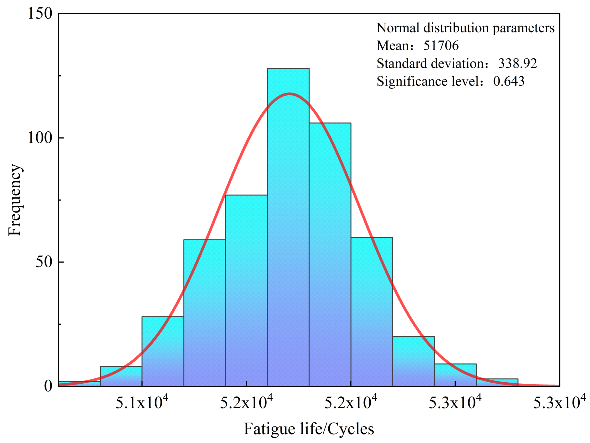
Fig. 9 shows the distribution of fatigue crack propagation life of the pressure shell when the initial crack size is 0.2 mm-0.6 mm. The figure shows that the fatigue life distribution of the pressure shell under each initial crack size obeys normal distribution, and the significance level is more significant than 0.05. Under each initial crack size, the standard deviation of fatigue life is insignificant, and the discreteness of fatigue life is not large, concentrated near to the mean value.



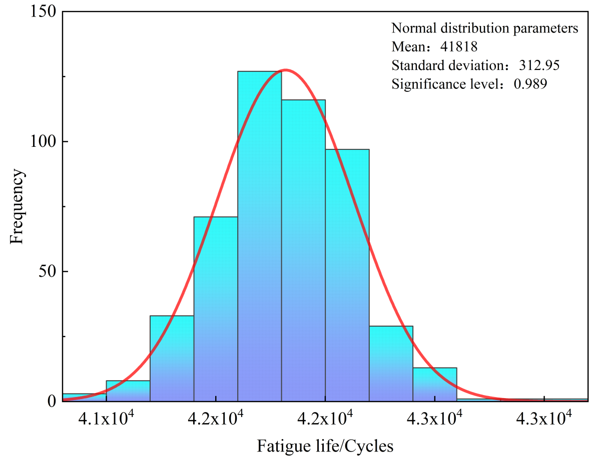
(a) 0.2 mm



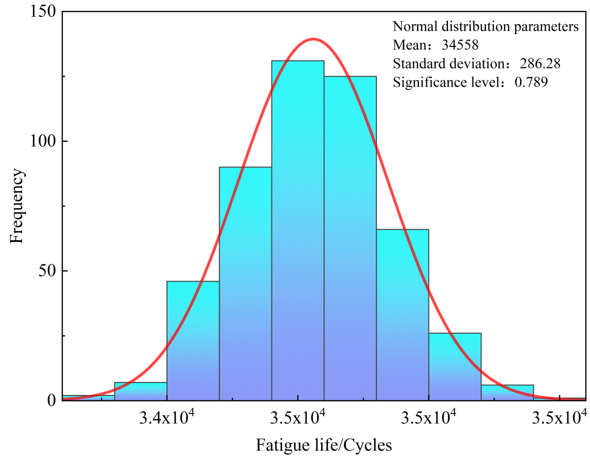
(b) 0.3 mm



(c) 0.4 mm



(d) 0.5 mm



(e) 0.6 mm

Fig. 9. Fatigue life distribution under different

The statistical analysis results of the mean and standard deviation of fatigue life under each initial crack size are shown in Fig. 10. It can be seen from the results of Fig.10 that the mean and standard deviation of fatigue life decrease with the increase of initial crack size. When the initial crack size changes from 0.2 mm to 0.6 mm, the fatigue life will be reduced by 61.3%, and the initial crack size dramatically influences the fatigue life.

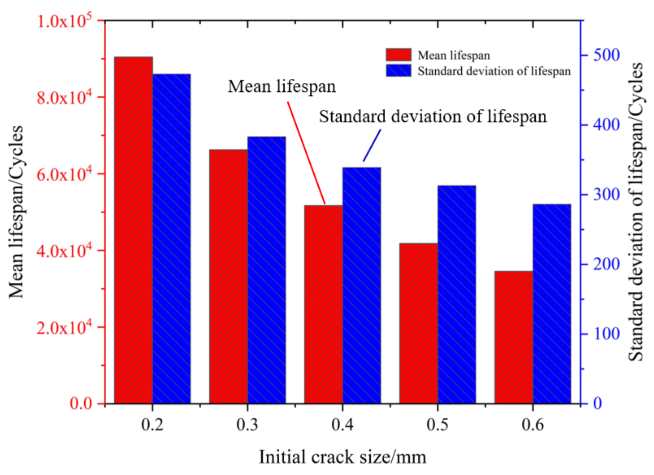


Fig. 10. The mean value and standard deviation of fatigue life under different initial crack sizes

VI. CONCLUSION

Based on the fatigue crack propagation theory of fracture mechanics, this paper analyzes the fatigue crack propagation life of underwater vehicle pressure structures under random load. Through this study, the following conclusions can be drawn :

1) The fatigue crack growth life of the pressure structure of underwater vehicle under random load is significantly higher than that of the constant amplitude load under the design water depth. If the fatigue analysis of the pressure structure of underwater vehicle is carried out according to the constant amplitude load, the result is conservative.

2) Under the premise that the random load obeys the Gumbel distribution, the statistical analysis shows that the fatigue crack growth life of the pressure structure of underwater vehicle obeys the normal distribution, and the different initial crack sizes have a significant influence on the mean value of the fatigue crack growth life.

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