A Comparative Study on the Performance of MARCOS Method and Weighted Sum Method

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Abstract—**Recently, a multi-criteria decision-making method known as the Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS) was proposed by Stević, et al. In this paper, we first provide a theoretical proof demonstrating that the MARCOS method is equivalent to the weighted sum method. To further illustrate the equivalence of these two methods, we present several cases, including simulation experiments and practical applications, to enhance readers' understanding.**

Index Terms—**Decision Making, MARCOS, Weighted sum method, Alternative Ranking**

I. Introduction

The Multi-Criteria Decision-Making (MCDM) approach is a method used to evaluate and select alternatives based on multiple criteria or objectives [9], [11], [14]. In MCDM, decision-makers assess various factors or criteria that are pertinent to the decision at hand, assigning weights or levels of importance to each criterion in order to guide the decision-making process. The objective is to systematically analyze and evaluate various alternatives in order to make informed decisions that are aligned with the desired goals and objectives. The Multi-Criteria Decision Making (MCDM) generally involves the following steps:

Step 1. Identification of Criteria: Define the criteria or objectives relevant to the decision-making process. These criteria represent the various dimensions or aspects that must be taken into account during the decision-making process.

Step 2. Weight Assignment: Assign weights to each criterion based on its relative importance or priority. This reflects the decision-maker's preferences and the relative significance of each criterion in achieving the overall objective.

Step 3. Alternative Evaluation: Evaluate each alternative with respect to each criterion. This evaluation may involve data collection, performance assessment, or the use of expert opinions.

Step 4. Aggregation: Combine the evaluations of alternatives across all criteria, taking into account the assigned weights. Various aggregation methods, such as the weighted sum or weighted product, may be employed.

Step 5. Ranking and Selection: Rank the alternatives based on the aggregated scores. The alternative with the highest score is regarded as the preferred choice.

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MCDM is applicable across various fields, including business, engineering, environmental management, and public policy, where decisions frequently involve multiple conflicting objectives. It offers a structured and transparent framework for decision-making, considering the complexity and diversity of decision criteria.

The classical weighted sum method is a simple yet powerful tool for aggregating multiple criteria into a single score for decision-making. Its simplicity and flexibility make it suitable for a wide range of applications. The basic steps involved are as follows:

Step 1. Define the Criteria: First, identify the criteria or objectives that will be used to evaluate the alternatives. These could include factors such as cost, quality, time, or other relevant performance measures.

Step 2. Quantify the Alternatives Performance: Assess how each alternative performs with respect to each criterion. The performance is typically quantified on a scale (e.g., 0 to 10, 1 to 100%, etc.). Denote the performance of the i-th alternative on the j-th criterion by x_{ij}:

$$X = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}.$$

Step 3. Assign Weights to the Criteria: Assign a weight w_j to each criterion to reflect its relative importance. The sum of all weights must be equal to 1:

$$\sum_{j=1}^{m} w_j = 1.$$

Step 4. Calculate the Weighted Sum for Each Alternative: For each alternative i, calculate the weighted sum by multiplying each score x_{ij} by the weight w_j for the corresponding criterion and summing the results:

$$S_i = \sum_{j=1}^{m} w_j \cdot x_{ij},$$

where:

- S_i is the weighted sum for the i-th alternative,
- w_j is the weight of the j-th criterion,
- x_{ij} is the score of the i-th alternative on the j-th criterion.

Step 5. Rank the Alternatives: After calculating the weighted sum for each alternative, compare the results. The alternative with the highest weighted sum is typically chosen as the best option.

Step 6. Make the Decision: After performing the ranking, select the alternative with the highest weighted sum as the final decision.

In 2020, an MCMD method called Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS) was presented by Stević, Pamučar, Puška and et al in [10]. Subsequently, numerous scholars have applied the MARCOS method to various fields, such as influenza forecasting [4], failure mode and effect analysis [7], site evaluation of subsea tunnels [15], supplier evaluation and selection [1], regional evaluation study of VFTO interference [12]. Meanwhile, several generalizations of the MARCOS method have been proposed by numerous scholars, including MARCOS technique under intuitionistic fuzzy environment [3], MARCOS method under integrated fuzzy FUCOM [2], MARCOS method under fuzzy ZE-numbers [5], [6], a hybrid method for maximizing the reliability of drug supply chain based on machine learning and MARCOS [8]. In summary, the MARCOS evaluation approach has been widely applied and further developed.

In this paper, we demonstrate the equivalence of the MARCOS method and the weighted sum method through theoretical analysis and illustrative cases.

II. Equivalence analysis

Firstly, the execution steps of the MARCOS method involve constructing an initial evaluation matrix. Subsequently, the negative ideal solution (AAI) and the positive ideal solution (AI) are introduced into the initial evaluation matrix, resulting in an extended matrix X:

$$X = \begin{array}{c} AAI \\ A_1 \\ A_2 \\ \vdots \\ A_m \\ AI \end{array} \begin{bmatrix} x_{aai1} & x_{aai2} & \cdots & x_{aain} \\ x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \\ x_{ai1} & x_{ai2} & \cdots & x_{ain} \end{bmatrix}.$$

The negative ideal solution and the positive ideal solution are calculated using equalities (2.1) and (2.2):

$$AAI = \begin{cases} \min\limits_i x_{ij}, j \in B \\ \max\limits_i x_{ij}, j \in C \end{cases}, \tag{2.1}$$

$$AI = \begin{cases} \max\limits_i x_{ij}, j \in B \\ \min\limits_i x_{ij}, j \in C \end{cases}, \tag{2.2}$$

where B and C represent the maximization indicator and the minimization indicator, respectively.

Subsequently, the extended matri x undergoes dimensionless processing as follows:

$$n_{ij} = \frac{x_{ij}}{x_{ai}}, j \in B, \tag{2.3}$$

$$n_{ij} = \frac{x_{ai}}{x_{ij}}, j \in C, \tag{2.4}$$

where n_{ij} represents the element in the matrix N after dimensionless processing.

The elements of the dimensionless matrix N are multiplied by the weights of the indicators to obtain the weighted matrix V, as defined by equality (2.5):

$$v_{ij} = n_{ij} \times w_j, \tag{2.5}$$

where w_j represents the weight coefficients of each indicator.

Calculate K_i^+ and K_i^- , which represent the utility of the alternative solutions relative to the positive and negative ideal solutions, respectively:

$$K_i^+ = \frac{S_i}{S_{ai}}, \tag{2.6}$$

$$K_i^- = \frac{S_i}{S_{aai}}, \tag{2.7}$$

where S_i represents the sum of the elements in the i-th row of the weighted matrix V, as defined by equality (2.8):

$$S_i = \sum_{j=1}^{n} v_{ij} \tag{2.8}$$

and S_{ai}, S_{aai} are the sum of positive ideal and negative ideal solutions, respectively.

Utilizing equalities (2.6) and (2.7) to determine the utility functions associated with the positive ideal and negative ideal solutions:

$$f(K_i^+) = \frac{K_i^-}{K_i^- + K_i^+}, \tag{2.9}$$

$$f(K_i^-) = \frac{K_i^+}{K_i^- + K_i^+}. \tag{2.10}$$

Finally, determine the utility function $f(K_i)$ of the alternative solution, as defined by equality (2.11):

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \dfrac{1 - f(K_i^+)}{f(K_i^+)} + \dfrac{1 - f(K_i^-)}{f(K_i^-)}} \tag{2.11}$$

and rank each alternative based on its utility function value $f(K_i)$.

Next, we will demonstrate that the utility function value $f(K_i)$ of the MARCOS method is equal to $C \times S_i$, where C is a constant greater than zero, which implies that the MARCOS method is equivalent to the weighted sum method.

Note that, if we substitute equalities (2.6) and (2.7) into equalities (2.9) and (2.10) respectively, we have

$$f(K_i^+) = \frac{S_i S_{ai}}{S_i S_{aai} + S_i S_{ai}} = \frac{S_{ai}}{S_{aai} + S_{ai}}, \tag{2.12}$$

$$f(K_i^-) = \frac{S_i S_{aai}}{S_i S_{ai} + S_i S_{aai}} = \frac{S_{aai}}{S_{ai} + S_{aai}}. \tag{2.13}$$

Furthermore, by equalities (2.12) and (2.13), we obtain

$$\begin{aligned} \frac{1 - f\left(K_i^+\right)}{f\left(K_i^+\right)} &= \frac{1 - \dfrac{S_{ai}}{S_{aai} + S_{ai}}}{\dfrac{S_{ai}}{S_{aai} + S_{ai}}} \\ &= \frac{\dfrac{S_{aai}}{S_{aai} + S_{ai}}}{\dfrac{S_{ai}}{S_{aai} + S_{ai}}} \\ &= \frac{S_{aai}}{S_{ai}}, \end{aligned} \tag{2.14}$$

$$\frac{1-f\left(K_i^-\right)}{f\left(K_i^-\right)}=\frac{1-\dfrac{S_{aai}}{S_{ai}+S_{aai}}}{\dfrac{S_{aai}}{S_{ai}+S_{aai}}}=\frac{\dfrac{S_{ai}}{S_{ai}+S_{aai}}}{\dfrac{S_{aai}}{S_{ai}+S_{aai}}}=\frac{S_{ai}}{S_{aai}}. \quad (2.15)$$

Substitute equalities (2.6), (2.7), (2.14), (2.15) into (2.11) yields

$$f\left(K_i\right)=\frac{\dfrac{S_i}{S_{ai}}+\dfrac{S_i}{S_{aai}}}{1+\dfrac{S_{aai}}{S_{ai}}+\dfrac{S_{ai}}{S_{aai}}}=\frac{\dfrac{\left(S_{ai}+S_{aai}\right)S_i}{S_{ai}S_{aai}}}{\dfrac{S_{ai}S_{aai}+S_{aai}^2+S_{ai}^2}{S_{ai}S_{aai}}}=C\times S_i, \quad (2.16)$$

where

$$C=\frac{S_{ai}+S_{aai}}{S_{ai}^2+S_{aai}^2+S_{ai}S_{aai}}. \quad (2.17)$$

Therefore, by equalities (2.8) and (2.16), we know that the MARCOS method is equivalent to the weighted sum method.

III. Comparative analysis

This section presents several cases, including simulation experiments and practical applications, that illustrate the equivalence between the MARCOS method and the weighted sum method.

Case 1. In this case, we evaluate five alternatives $A_1, A_2, \cdots, A_5$, each of which is assessed based on six criteria $X_1, X_2, \cdots, X_6$, we assume that all criteria are positive criteria, which means that all criteria are "benefit" criteria, hence the extended matrix X is as follows:

$$X=\begin{array}{c} AAI \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ AI \end{array}\begin{bmatrix} 2.00 & 2.00 & 3.00 & 1.00 & 7.71 & 2.00 \\ 9.00 & 2.00 & 3.00 & 4.50 & 19.28 & 2.00 \\ 2.00 & 3.00 & 6.00 & 5.00 & 13.82 & 9.00 \\ 3.00 & 4.00 & 5.00 & 6.00 & 9.68 & 13.00 \\ 4.00 & 10.00 & 4.30 & 1.00 & 34.85 & 4.00 \\ 5.50 & 17.00 & 3.30 & 7.00 & 7.71 & 8.00 \\ 9.00 & 17.00 & 6.00 & 7.00 & 34.85 & 13.00 \end{bmatrix}.$$

By equality (2.3), the following dimensionless matrix N can be obtained:

$$N=\begin{array}{c} AAI \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ AI \end{array}\begin{bmatrix} 0.2222 & 0.1176 & 0.5000 & 0.1429 & 0.2212 & 0.1538 \\ 1.0000 & 0.1176 & 0.5000 & 0.6429 & 0.5532 & 0.1538 \\ 0.2222 & 0.1765 & 1.0000 & 0.7143 & 0.3966 & 0.6923 \\ 0.3333 & 0.2353 & 0.8333 & 0.8571 & 0.2778 & 1.0000 \\ 0.4444 & 0.5882 & 0.7167 & 0.1429 & 1.0000 & 0.3077 \\ 0.6111 & 1.0000 & 0.5500 & 1.0000 & 0.2212 & 0.6154 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \end{bmatrix}.$$

In this case, we assumes that the weights of each indicator are

$$w_j=(0.1636, 0.2384, 0.1561, 0.1578, 0.1613, 0.1227)^T,$$

respectively. By using equality (2.5) to weight the normalization matrix N, the weighting matrix is as follows:

$$V=\begin{array}{c} AAI \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ AI \end{array}\begin{bmatrix} 0.0364 & 0.0280 & 0.0781 & 0.0225 & 0.0357 & 0.0189 \\ 0.1636 & 0.0280 & 0.0781 & 0.1014 & 0.0893 & 0.0189 \\ 0.0364 & 0.0421 & 0.1561 & 0.1127 & 0.0640 & 0.0850 \\ 0.0545 & 0.0561 & 0.1301 & 0.1353 & 0.0448 & 0.1227 \\ 0.0727 & 0.1402 & 0.1119 & 0.0225 & 0.1613 & 0.0378 \\ 0.1000 & 0.2384 & 0.0859 & 0.1578 & 0.0357 & 0.0755 \\ 0.1636 & 0.2384 & 0.1561 & 0.1578 & 0.1613 & 0.1227 \end{bmatrix}.$$

Simple calculations show that

$$S_i=(0.4793, 0.4962, 0.5435, 0.5465, 0.6933)^T$$

and

$$S_{ai}=1.0000, S_{aai}=0.2196.$$

Next, calculate the utility of five alternatives relative to the positive ideal solution and negative ideal solution, respectively, the following results can be obtained:

$$K_i^+=(0.4793, 0.4962, 0.5435, 0.5465, 0.69331)^T,$$
$$K_i^-=(2.1828, 2.2598, 2.4754, 2.4887, 3.1573)^T.$$

So the utility function related to positive ideal and negative ideal solutions can be determined:

$$f(K_i^+)=(0.8200, 0.8200, 0.8200, 0.8200, 0.8200)^T,$$
$$f(K_i^-)=(0.1800, 0.1800, 0.1800, 0.1800, 0.1800)^T.$$

Finally, calculate the utility function of each alternative based on the above formula equality (2.11) and so

$$f(K_i)=(0.4611, 0.4773, 0.5229, 0.5257, 0.6669)^T.$$

On the other hand, by using equality (2.17) and S_i, we can obtain $C=0.9620$, therefore, by equality (2.16), we have

$$f(K_i)=(0.4611, 0.4773, 0.5229, 0.5257, 0.6669)^T.$$

So it can be seen that equalities (2.11) and (2.16) are equivalent.

As shown in Table 1, the final rankings derived from both methods are identical.

TABLE 1
MARCOS method VS weighted sum method

A_i	$f\left(K_i\right)$	rank	S_i	rank	$\dfrac{f\left(K_i\right)}{S_i}$
A_1	0.4611	5	0.4793	5	0.9620
A_2	0.4773	4	0.4962	4	0.9620
A_3	0.5229	3	0.5435	3	0.9620
A_4	0.5257	2	0.5465	2	0.9620
A_5	0.6669	1	0.6933	1	0.9620

Case 2. Now, we aim to illustrate the similarity between the two methods by presenting a case involving non-beneficial criteria. Consider a decision-making problem with five alternatives $A_1, A_2, \cdots, A_5$, each evaluated against six criteria $X_1, X_2, \cdots, X_6$. We assume that criteria X_1 and X_2 are non-beneficial criteria, which are "cost" criteria, while the remaining four criteria are beneficial, which are "benefit" criteria. Based on this assumption, the corresponding extended matrix X is presented below:

$$X=\begin{array}{c} AAI \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ AI \end{array}\begin{bmatrix} 9.00 & 17.00 & 3.00 & 1.00 & 7.71 & 2.00 \\ 9.00 & 2.00 & 3.00 & 4.50 & 19.28 & 2.00 \\ 2.00 & 3.00 & 6.00 & 5.00 & 13.82 & 9.00 \\ 3.00 & 4.00 & 5.00 & 6.00 & 9.68 & 13.00 \\ 4.00 & 10.00 & 4.30 & 1.00 & 34.85 & 4.00 \\ 5.50 & 17.00 & 3.30 & 7.00 & 7.71 & 8.00 \\ 2.00 & 2.00 & 6.00 & 7.00 & 34.85 & 13.00 \end{bmatrix}$$

By equalities (2.3) and (2.4), the following dimensionless matrix N can be obtained:

$$N = \begin{array}{c} AAI \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ AI \end{array} \begin{bmatrix} 0.2222 & 0.1176 & 0.5000 & 0.1429 & 0.2212 & 0.1538 \\ 0.2222 & 1.0000 & 0.5000 & 0.6429 & 0.5532 & 0.1538 \\ 1.0000 & 0.6667 & 1.0000 & 0.7143 & 0.3966 & 0.6923 \\ 0.6667 & 0.5000 & 0.8333 & 0.8571 & 0.2778 & 1.0000 \\ 0.5000 & 0.2000 & 0.7167 & 0.1429 & 1.0000 & 0.3077 \\ 0.3636 & 0.1176 & 0.5500 & 1.0000 & 0.2212 & 0.6154 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \end{bmatrix}.$$

In this case, we assumes that the weights of each indicator are

$$w_j = (0.1836, 0.2184, 0.1761, 0.1578, 0.1613, 0.1027)^T,$$

respectively. By using equality (2.5) to weight the normalization matrix N, the weighting matrix is as follows:

$$V = \begin{array}{c} AAI \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ AI \end{array} \begin{bmatrix} 0.0408 & 0.0257 & 0.0881 & 0.0225 & 0.0357 & 0.0158 \\ 0.0408 & 0.2184 & 0.0881 & 0.1014 & 0.0892 & 0.0158 \\ 0.1836 & 0.1456 & 0.1761 & 0.1127 & 0.0640 & 0.0711 \\ 0.1224 & 0.1092 & 0.1468 & 0.1353 & 0.0448 & 0.1027 \\ 0.0918 & 0.0437 & 0.1262 & 0.0225 & 0.1613 & 0.0316 \\ 0.0668 & 0.0257 & 0.0969 & 0.1578 & 0.0357 & 0.0632 \\ 0.1836 & 0.2184 & 0.1761 & 0.1578 & 0.1613 & 0.1027 \end{bmatrix}.$$

Simple calculations show that

$$S_i = (0.5537, 0.7531, 0.6611, 0.4771, 0.4460)^T,$$

and

$$S_{ai} = 1.0000, S_{aai} = 0.2286.$$

Next, calculate the utility of five alternatives relative to the positive ideal solution and negative ideal solution, respectively, the following results can be obtained:

$$K_i^+ = (0.5538, 0.7532, 0.6612, 0.4772, 0.4460)^T,$$

$$K_i^- = (2.4226, 3.2947, 2.8923, 2.0874, 1.9512)^T.$$

So the utility function related to positive ideal and negative ideal solutions can be determined:

$$f(K_i^+) = (0.8139, 0.8139, 0.8139, 0.8139, 0.8139)^T,$$

$$f(K_i^-) = (0.1861, 0.1861, 0.1861, 0.1861, 0.1861)^T.$$

Finally, calculate the utility function of each alternative based on the above formula equality (2.11) and so

$$f(K_i) = (0.5312, 0.7224, 0.6342, 0.4577, 0.4278)^T.$$

On the other hand, by using equality (2.17) and S_i, we can obtain $C = 0.9593$, therefore, by equality (2.16), we have

$$f(K_i) = (0.5312, 0.7224, 0.6342, 0.4577, 0.4278)^T.$$

So it can be seen that equalities (2.11) and (2.16) are equivalent.

Table 2 presents a comparative analysis of the results obtained using the MARCOS method and the weighted sum method. As shown in Table 2, the final rankings derived from both methods are identical.

TABLE 2
MARCOS METHOD VS WEIGHTED SUM METHOD

A_i	$f(K_i)$	rank	S_i	rank	$\frac{f(K_i)}{S_i}$
A_1	0.5312	3	0.5537	3	0.9593
A_2	0.7224	1	0.7531	1	0.9593
A_3	0.6342	2	0.6611	2	0.9593
A_4	0.4577	4	0.4771	4	0.9593
A_5	0.4278	5	0.4460	5	0.9593

Case 3. In this case, we consider a practical application involving the risk assessment of distribution network equipment. The risk assessment of distribution network equipment is of great significance for reducing distribution network failures and improving the reliability of distribution network power supply.

Regarding the risk assessment of distribution network equipment, Wang, et al [13] identified six evaluation indicators, denoted as $X_1, X_2, \cdots, X_6$, which are as follows: new load, equipment load rate, degree of aging, importance of load, exceeding the power supply radius limit, exceeding the voltage drop limit. Additionally, relevant data were collected for 51 alternative lines, as detailed in Table 3:

TABLE 3
RISK ASSESSMENT VALUES OF DISTRIBUTION NETWORK EQUIPMENT

A_i	X_1	X_2	X_3	X_4	X_5	X_6
A_1	8	8	4	6	0	0
A_2	8	8	4	6	0	0
A_3	6	6	2	6	0	0
A_4	4	2	4	4	0	0
A_5	4	2	4	4	2	0
A_6	2	2	4	2	0	0
A_7	4	4	4	4	0	0
A_8	4	6	6	4	0	0
A_9	2	2	6	2	2	0
A_{10}	4	4	6	2	0	0
A_{11}	2	2	4	2	2	0
A_{12}	2	2	6	2	0	0
A_{13}	2	2	6	2	2	0
A_{14}	0	0	8	2	0	0
A_{15}	0	0	8	2	0	0
A_{16}	10	10	2	6	0	0
A_{17}	10	10	6	6	4	4
A_{18}	10	10	2	6	0	0
A_{19}	8	8	6	8	0	2
A_{20}	8	8	6	6	0	2
A_{21}	6	8	6	4	0	0
A_{22}	6	6	8	2	0	0
A_{23}	8	6	6	6	0	2
A_{24}	4	6	6	6	0	0
A_{25}	6	6	8	8	0	0
A_{26}	4	4	6	4	4	2
A_{27}	4	4	6	4	0	0
A_{28}	2	4	8	2	4	4
A_{29}	2	2	6	4	0	0
A_{30}	0	2	6	2	0	0
A_{31}	0	2	4	2	0	0
A_{32}	0	2	4	2	0	0
A_{33}	0	2	6	2	0	0
A_{34}	2	2	6	4	0	0
A_{35}	2	2	8	8	0	0
A_{36}	0	0	6	2	4	2
A_{37}	0	0	6	2	2	2
A_{38}	0	0	6	4	8	6
A_{39}	0	0	6	4	4	2
A_{40}	0	0	6	2	4	2
A_{41}	0	0	6	2	2	4
A_{42}	0	0	6	4	4	2
A_{43}	0	0	6	2	6	6
A_{44}	0	0	8	2	10	6
A_{45}	0	0	4	8	4	2
A_{46}	0	0	6	2	2	2
A_{47}	0	0	4	2	8	4
A_{48}	0	0	6	2	10	6
A_{49}	0	0	6	2	2	2
A_{50}	0	0	6	2	2	2
A_{51}	0	0	4	2	10	6

In [13], the weights of each criterion are obtained through the combined weighting method as follows:

$$w_j = (0.3684, 0.2105, 0.1287, 0.1237, 0.0958, 0.0729)^T.$$

Finally, based on the MARCOS method and combined weighting, Wang, et al [13] calculated the risk coefficients of the aforementioned 51 alternative lines.

Based on the aforementioned alternatives and criteria, the extended matrix X is as follows:

$$X = \begin{array}{c} AAI \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \\ \vdots \\ A_{42} \\ A_{43} \\ A_{44} \\ A_{45} \\ A_{46} \\ A_{47} \\ A_{48} \\ A_{49} \\ A_{50} \\ A_{51} \\ AI \end{array} \begin{bmatrix} 0 & 0 & 2 & 2 & 0 & 0 \\ 8 & 8 & 4 & 6 & 0 & 0 \\ 8 & 8 & 4 & 6 & 0 & 0 \\ 6 & 6 & 2 & 6 & 0 & 0 \\ 4 & 2 & 4 & 4 & 0 & 0 \\ 4 & 2 & 4 & 4 & 2 & 0 \\ 2 & 2 & 4 & 2 & 0 & 0 \\ 4 & 4 & 4 & 4 & 0 & 0 \\ 4 & 6 & 6 & 4 & 0 & 0 \\ 2 & 2 & 6 & 2 & 2 & 0 \\ 4 & 4 & 6 & 2 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 6 & 4 & 4 & 2 \\ 0 & 0 & 6 & 2 & 6 & 6 \\ 0 & 0 & 8 & 2 & 10 & 6 \\ 0 & 0 & 4 & 8 & 4 & 2 \\ 0 & 0 & 6 & 2 & 2 & 2 \\ 0 & 0 & 4 & 2 & 8 & 4 \\ 0 & 0 & 6 & 2 & 10 & 6 \\ 0 & 0 & 6 & 2 & 2 & 2 \\ 0 & 0 & 6 & 2 & 2 & 2 \\ 0 & 0 & 4 & 2 & 10 & 6 \\ 10 & 10 & 8 & 8 & 10 & 6 \end{bmatrix}.$$

By equalities (2.3) and (2.4), the following dimensionless matrix N can be obtained:

$$N = \begin{array}{c} AAI \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \\ \vdots \\ A_{42} \\ A_{43} \\ A_{44} \\ A_{45} \\ A_{46} \\ A_{47} \\ A_{48} \\ A_{49} \\ A_{50} \\ A_{51} \\ AI \end{array} \begin{bmatrix} 0 & 0 & 0.25 & 0.25 & 0 & 0 \\ 0.8 & 0.8 & 0.5 & 0.75 & 0 & 0 \\ 0.8 & 0.8 & 0.5 & 0.75 & 0 & 0 \\ 0.6 & 0.6 & 0.25 & 0.75 & 0 & 0 \\ 0.4 & 0.2 & 0.5 & 0.5 & 0 & 0 \\ 0.4 & 0.2 & 0.5 & 0.5 & 0.2 & 0 \\ 0.2 & 0.2 & 0.5 & 0.25 & 0 & 0 \\ 0.4 & 0.4 & 0.5 & 0.5 & 0 & 0 \\ 0.4 & 0.6 & 0.75 & 0.5 & 0 & 0 \\ 0.2 & 0.2 & 0.75 & 0.25 & 0.2 & 0 \\ 0.4 & 0.4 & 0.75 & 0.25 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0.75 & 0.5 & 0.4 & 0.33 \\ 0 & 0 & 0.75 & 0.25 & 0.6 & 1 \\ 0 & 0 & 1 & 0.25 & 1 & 1 \\ 0 & 0 & 0.5 & 1 & 0.4 & 0.33 \\ 0 & 0 & 0.75 & 0.25 & 0.2 & 0.33 \\ 0 & 0 & 0.5 & 0.25 & 0.8 & 0.67 \\ 0 & 0 & 0.75 & 0.25 & 1 & 1 \\ 0 & 0 & 0.75 & 0.25 & 0.2 & 0.33 \\ 0 & 0 & 0.75 & 0.25 & 0.2 & 0.33 \\ 0 & 0 & 0.5 & 0.25 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

In this example, we know that the weights of each indicator are

$$w_j = (0.3684, 0.2105, 0.1287, 0.1237, 0.0958, 0.0729)^T,$$

respectively. By using equality (2.5) to weight the normalization matrix N, the weighting matrix is as follows:

$$V = \begin{array}{c} AAI \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \\ A_{10} \\ \vdots \\ A_{42} \\ A_{43} \\ A_{44} \\ A_{45} \\ A_{46} \\ A_{47} \\ A_{48} \\ A_{49} \\ A_{50} \\ A_{51} \\ AI \end{array} \begin{bmatrix} 0 & 0 & 0.0322 & 0.0309 & 0 & 0 \\ 0.2947 & 0.1684 & 0.0644 & 0.0928 & 0 & 0 \\ 0.2947 & 0.1684 & 0.0644 & 0.0928 & 0 & 0 \\ 0.1474 & 0.0421 & 0.0644 & 0.0619 & 0 & 0 \\ 0.1474 & 0.0421 & 0.0644 & 0.0619 & 0.0192 & 0 \\ 0.0737 & 0.0421 & 0.0644 & 0.0309 & 0 & 0 \\ 0.1474 & 0.0842 & 0.0644 & 0.0619 & 0 & 0 \\ 0.1474 & 0.1263 & 0.0965 & 0.0619 & 0 & 0 \\ 0.0737 & 0.0421 & 0.0965 & 0.0309 & 0.0192 & 0 \\ 0.1474 & 0.0842 & 0.0965 & 0.0309 & 0 & 0 \\ 0.0737 & 0.0421 & 0.0644 & 0.0309 & 0.0192 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0.0965 & 0.0619 & 0.0383 & 0.0243 \\ 0 & 0 & 0.0965 & 0.0309 & 0.0575 & 0.0729 \\ 0 & 0 & 0.1287 & 0.0309 & 0.0958 & 0.0729 \\ 0 & 0 & 0.0644 & 0.1237 & 0.0383 & 0.0243 \\ 0 & 0 & 0.0965 & 0.0309 & 0.0192 & 0.0243 \\ 0 & 0 & 0.0644 & 0.0309 & 0.0766 & 0.0486 \\ 0 & 0 & 0.0965 & 0.0309 & 0.0958 & 0.0729 \\ 0 & 0 & 0.0965 & 0.0309 & 0.0192 & 0.0243 \\ 0 & 0 & 0.0965 & 0.0309 & 0.0192 & 0.0243 \\ 0 & 0 & 0.0644 & 0.0309 & 0.0958 & 0.0729 \\ 0.3684 & 0.2105 & 0.1287 & 0.1237 & 0.0958 & 0.0729 \end{bmatrix}$$

Simple calculations show that

$$S_i = (0.6202, 0.6202, 0.4723, \cdots, 0.1709, 0.2640)^T,$$

and

$$S_{ai} = 1.0000, S_{aai} = 0.0631.$$

Next, calculate the utility of five alternatives relative to the positive ideal solution and negative ideal solution, respectively, the following results can be obtained:

$$K_i^+ = (0.6202, 0.6202, 0.4723, \cdots, 0.1709, 0.2640)^T,$$

$$K_i^- = (9.8296, 9.8296, 7.4848, \cdots, 2.7086, 4.1834)^T.$$

So the utility function related to positive ideal and negative ideal solutions can be determined:

$$f(K_i^+) = (0.9406, 0.9406, 0.9406, \cdots, 0.9406, 0.9406)^T,$$

$$f(K_i^-) = (0.0594, 0.0594, 0.0594, \cdots, 0.0594, 0.0594)^T.$$

Finally, calculate the utility function of each alternative based on the above formula equality (2.11) and so

$$f(K_i) = (0.6179, 0.6179, 0.4705, \cdots, 0.1703, 0.2630)^T.$$

On the other hand, by using equality (2.17) and S_i, we can obtain $C = 0.9593$, therefore, by equality (2.16), we have

$$f(K_i) = (0.6179, 0.6179, 0.4705, \cdots, 0.1703, 0.2630)^T.$$

So it can be seen that equalities (2.11) and (2.16) are equivalent.

In this practical application, we present the risk coefficients for each alternative line, which were obtained using both the MARCOS method and the weighted sum method. Table 4 presents a comparison of the results between the MARCOS method $f(K_i)$ and the weighted sum method S_i.

TABLE 4
MARCOS METHOD VS WEIGHTED SUM METHOD

A_i	$f(K_i)$	rank	S_i	rank	$\frac{f(K_i)}{S_i}$
A_1	0.6179	7	0.6202	7	0.9963
A_2	0.6179	7	0.6202	7	0.9963
A_3	0.4705	12	0.4723	12	0.9963
A_4	0.3145	23	0.3157	23	0.9963
A_5	0.3336	21	0.3348	21	0.9963
A_6	0.2103	38	0.2111	38	0.9963
A_7	0.3564	20	0.3578	20	0.9963
A_8	0.4304	15	0.4320	15	0.9963
A_9	0.2614	29	0.2624	29	0.9963
A_{10}	0.3577	19	0.3590	19	0.9963
A_{11}	0.2294	34	0.2302	34	0.9963
A_{12}	0.2423	33	0.2432	33	0.9963
A_{13}	0.2614	29	0.2624	29	0.9963
A_{14}	0.1590	48	0.1596	48	0.9963
A_{15}	0.1590	48	0.1596	48	0.9963
A_{16}	0.7012	3	0.7039	3	0.9963
A_{17}	0.8519	1	0.8551	1	0.9963
A_{18}	0.7012	3	0.7039	3	0.9963
A_{19}	0.7050	2	0.7076	2	0.9963
A_{20}	0.6742	5	0.6767	5	0.9963
A_{21}	0.5458	10	0.5478	10	0.9963
A_{22}	0.5051	11	0.5070	11	0.9963
A_{23}	0.6323	6	0.6346	6	0.9963
A_{24}	0.4612	13	0.4630	13	0.9963
A_{25}	0.5975	9	0.5997	9	0.9963
A_{26}	0.4509	14	0.4526	14	0.9963
A_{27}	0.3885	17	0.3899	17	0.9963
A_{28}	0.4029	16	0.4044	16	0.9963
A_{29}	0.2731	26	0.2742	26	0.9963
A_{30}	0.1689	46	0.1696	46	0.9963
A_{31}	0.1369	50	0.1374	50	0.9963
A_{32}	0.1369	50	0.1374	50	0.9963
A_{33}	0.1689	46	0.1696	46	0.9963
A_{34}	0.2731	26	0.2742	26	0.9963
A_{35}	0.3668	18	0.3682	18	0.9963
A_{36}	0.1894	40	0.1901	40	0.9963
A_{37}	0.1703	42	0.1709	42	0.9963
A_{38}	0.3068	24	0.3079	24	0.9963
A_{39}	0.2202	35	0.2210	35	0.9963
A_{40}	0.1894	40	0.1901	40	0.9963
A_{41}	0.1945	39	0.1952	39	0.9963
A_{42}	0.2202	35	0.2210	35	0.9963
A_{43}	0.2569	31	0.2578	31	0.9963
A_{44}	0.3271	22	0.3283	22	0.9963
A_{45}	0.2497	32	0.2507	32	0.9963
A_{46}	0.1703	42	0.1709	42	0.9963
A_{47}	0.2197	37	0.2205	37	0.9963
A_{48}	0.2950	25	0.2962	25	0.9963
A_{49}	0.1703	42	0.1709	42	0.9963
A_{50}	0.1703	42	0.1709	42	0.9963
A_{51}	0.2630	28	0.2640	28	0.9963

From Table 4, it is evident that the final results obtained by the MARCOS method and the weighted sum method are equivalent. This equivalence suggests that, despite the differences in the computational procedures and underlying assumptions of these two methods, they lead to the same ranking of alternatives.

In this paper, the weighted sum method is dimensionlessized using equations (2.3) and (2.4). These equations are specifically designed to standardize the decision matrix by eliminating the influence of differing units and scales across criteria. However, the classical weighted sum method employs the following formula for dimensionlessization:

$$y_{ij} = \frac{x_{ij} - x_{j\min}}{x_{j\max} - x_{j\min}}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n.$$

This difference in dimensionlessization techniques leads to variations in the final ranking results. Table 5 presents the results obtained from the classical weighted sum method, illustrating the rankings of the alternatives under this approach.

TABLE 5
THE CLASSICAL WEIGHTED SORTING RESULTS

A_i	rank	A_i	rank	A_i	rank
A_1	8	A_{18}	3	A_{35}	17
A_2	8	A_{19}	2	A_{36}	40
A_3	13	A_{20}	5	A_{37}	42
A_4	24	A_{21}	10	A_{38}	23
A_5	22	A_{22}	11	A_{39}	34
A_6	38	A_{23}	6	A_{40}	40
A_7	20	A_{24}	12	A_{41}	39
A_8	15	A_{25}	7	A_{42}	34
A_9	29	A_{26}	14	A_{43}	31
A_{10}	19	A_{27}	18	A_{44}	21
A_{11}	36	A_{28}	16	A_{45}	28
A_{12}	33	A_{29}	26	A_{46}	42
A_{13}	29	A_{30}	48	A_{47}	37
A_{14}	46	A_{31}	50	A_{48}	25
A_{15}	46	A_{32}	50	A_{49}	42
A_{16}	3	A_{33}	48	A_{50}	42
A_{17}	1	A_{34}	26	A_{51}	32

To facilitate a better comparison between the MARCOS method and the classical weighted sum method, we calculated the correlation coefficient between the ranking results of both methods, which yielded a value of 0.9968. This indicates that the results from these two methods are highly correlated.

IV. Conclusion

In this short paper, we demonstrate through algebraic calculations that the MARCOS method and the weighted sum method are equivalent. Furthermore, to provide a more intuitive comparison between these two methods, we use examples to illustrate our findings. In future research, greater attention should be paid to the inherent relationships among different multi-attribute decision-making methods, in order to avoid misinterpreting methods that are equivalent to existing ones as entirely new approaches.

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