Optimizing Schedules of Feeder Bus Considering Passenger Flows Demand

Mingming Chen, Guanghui Zhou, Jilong Li, Zhengxiu Zhang, Sijia Wang, Zhen Wang

***Abstract* — In order to address the demand for transferring passengers between urban rail transit and feeder bus services, and effectively reduce passenger flow loss due to excessive waiting times. This paper uses transfer demand and urban rail transit arrival times to indicate the distribution of passenger flow. It describes the transfer time based on the arrival time of passengers and the departure time of the feeder bus. Under the limited number of feeder buses, the model considers bus headway, passenger loss, and transfer demand to achieve a multi-objective optimization that aims to minimize the number of feeder buses, passenger flow loss, and the total waiting time for transfer passengers. The model is solved using the Non-dominated Sorting Genetic Algorithm II (NSGA-II), and obtaining the Pareto solution set for the problem. A case study using an actual feeder bus line shows that the optimization model balances the operational cost of the bus and the transfer time cost of passengers, providing bus schedules that meet different needs. The maximum passenger loss in the solution set is 200 passengers, and the minimum is 105 passengers. The minimum number of buses used is 16, and the maximum is 22 that can meet the enterprise's requirements. By analyzing the optimized and uniform headway schedules, it is evident that the optimized schedule matches the arrival times of urban rail transit preferably. When the number of feeder buses is same, the total transfer waiting time of the optimized method is reduced by 8.0% compared with the uniform headway, and the average load factor of uneven headway is 59.3% which is better than the average load rate of 50.2% under the uniform departure interval. The calculation results validate the rationality of the model and algorithm.**

***Index Terms*—public transport; feeder bus; bus schedule; capacity matching; NSGA-II**

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Mingming Chen is a professor at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: lzcmm2000@mail.lzjtu.cn).

Guanghui Zhou is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (Corresponding author, phone: +86 17393180632, e-mail: 17393180632@163.com).

Jilong Li is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: 1440264642@qq. com).

Zhengxiu Zhang is an engineer at the Lanzhou depot, China Railway Lanzhou Group Co. Ltd. Lanzhou 730030, China. (e-mail: 3129335807@qq.com).

Sijia Wang is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: wangsijia0508@qq.com).

Zhen Wang is a postgraduate student at the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou 730070, China. (e-mail: 2423526876@qq. com).

I. INTRODUCTION

In recent years, the growing disparity between urban rail transit supply and demand has led to increasingly severe traffic congestion issues. Prioritizing public transport has been widely recognized as an effective strategy for mitigating urban traffic challenges, and some cities have established public transport with urban rail transit as the backbone [1]. This imperative calls for the operational coordination of urban rail transit and bus scheduling systems to reduce passenger transfer time costs, mitigate transfer flow attrition risks, and effectively meet passenger demand.

Existing research has extensively investigated coordinated optimization between rail transit and bus transfers, primarily addressing schedule synchronization and fleet management challenges. Among them, it primarily encompassed uniform headway scheduling and uneven departure interval [2],[3]. LIU et al. [4] aimed at the maximum number of buses arriving at the transfer station and the minimum number of buses, constructed a integer programming model to solve the time interval table of uneven departure. With the goal of minimizing number of buses and passenger transfer waiting time, PETERSEN et al. [5] used the local algorithm to solve the whole number programming model of the public transit time under the uneven interval.

Recent advancements have extended to multimodal coordination frameworks, where researchers have developed integrated optimization models accounting for rail transit and bus. Based on the temporal analyses of operational patterns that can reflect the characteristics of constrained optimization systems are established[6]. The average travel time of passengers including waiting time and onboard time was taken as the travel cost, and the average full rate of vehicles was taken as the passenger carrying cost of bus enterprises. Long [7] and Gkiotsalitis [8] established a scheduling model considering the impact of transfer waiting time and the number of cooperative transfers on schedule preparation. Yuan [9] and Guo [10] studied the optimization of feeder bus schedule in high speed railway stations, and established the model of feeder bus schedule under uneven departure interval. Xiong et al. [11] built a mixed integer programming model to solve the bus schedule, which minimized the total cost of public transportation. Tang [12] established a multi-objective model with the goal of minimizing passenger waiting time and number of flights, and evaluated parameters in the model.

The overall demand of passengers in the process of travel was studied, and the optimization model of feeder bus route was established. However, the influence of bus schedule on passenger demand was not considered [13]. Most of the existing studies focused on the single vehicle and electric buses, the optimization method of bus schedule and vehicle configuration could be proposed considering the size of the bus[14]. Fan [15] proposed an adaptive departure strategy and built an optimization model aiming at the expected generalized system cost. At the same time, the author addressed issues such as oversaturation or undersaturation in public transportation systems. Chang [16] incorporated the heterogeneous information based on the passenger demand. The interval speed was used to represent the actual road elastic time, and the speed guidance strategy was used to coordinate the planning of bus routes to expand the quality solution. Cao [17] examined the transportation mode of rail transit passenger and addressed the feeder bus network's optimization within a three dimensional framework.

Prevailing studies predominantly focus on single-vehicle operational paradigms that prove inadequate in addressing asymmetric passenger flow patterns, with inherent capacity limitations failing to accommodate volumetric fluctuations during peak demand cycles. In response, researchers have formulated constrained optimization through single-line case studies that integrally incorporate vehicle capacity and bus number.[18],[19],[20]. Sun [21] considered the difference of passenger demand, and studied the problem of optimization in bus route, vehicle model adaptation and selection.

To sum up, there is a certain research foundation for the operation schedules and network optimization of feeder bus services. The research focused the realm of bus schedules and route optimization issues. When studying the schedule of feeder buses, scholars assumed that all passenger transfered to buses, rarely considering the phenomenon of passenger flow loss due to long waiting time and the arrival time of urban rail transit trains. Therefore, this paper fully considers the arrival time and the tolerance time of rail transit, and establishes a multi-objective model with the objectives of the number of feeder buses, the loss of interchange passengers, and the total waiting time of passengers. The model includes constraints such as departure time intervals and passenger transfer demands. At the same time, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is designed to solve the model and obtain the Pareto solution set, providing decision makers with different bus schedules.

II. Problem Description

In urban public transport system, urban rail transit and conventional bus form a comprehensive transportation network in space, where feeder bus primarily enhance the coverage and accessibility of urban rail transit. The urban rail transit network exhibits characteristics of a zonal layout, where the feeder bus network complements the urban rail transit network, forming a stratified layout pattern together. As depicted in Fig.1, urban rail transit operates across different regions, serving the purpose of long distance and interregional travel. Feeder Bus Route A establishes critical multimodal connectivity between destination clusters and urban rail transit nodes, specifically engineered to bridge last-mile mobility gaps through optimized short-distance trip supplementation. Beyond conventional feeder bus routes terminating at metro stations, advanced feeder systems feature intermediate stops traversing rail nodes, creating multimodal transfer hierarchies that integrate short-distance shuttle services with trunk line connectivity—as exemplified by Feeder Bus Route B's dual functionality in last-mile distribution and arterial network integration.

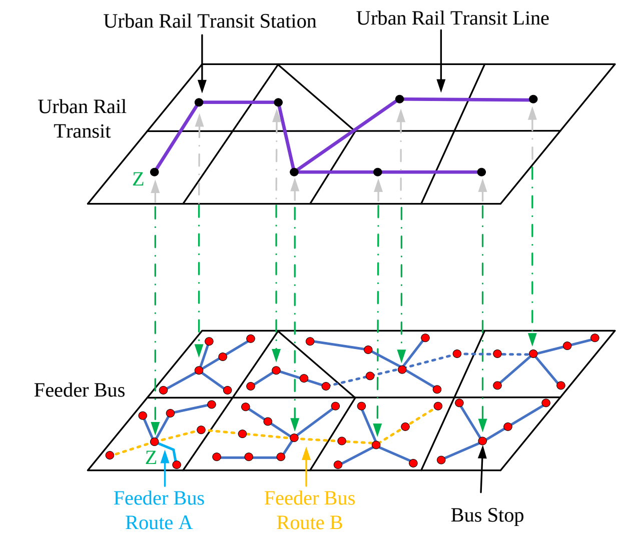


Fig. 1. Schematic diagram of urban rail transit and feeder bus

In actual operations, the transfer between urban rail transit and feeder buses primarily encompasses two scenarios. Firstly, the passenger transfers from feeder bus to urban rail transit, where feeder buses mainly serve to aggregate passenger flow, transporting passengers from various neighborhoods to urban rail transit, as depicted in Fig.2.

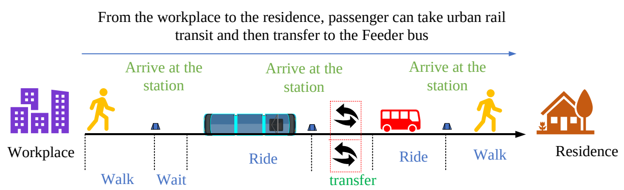


Fig. 2. The integrated transfer process of urban rail transit and feeder bus

Under normal conditions, the interchange between urban rail transit and feeder bus is influenced by passenger flow. Due to differences in service intensity and driving speed, the departure interval of vehicles is different. During a specific time period, the operating hours of urban rail transit are fixed. Therefore, it is essential to leverage the flexible scheduling advantages of feeder buses, compiling their timetables based on the departure times of urban rail transit and passenger demand, ensuring the continuity and efficiency of passengers' entire travel process.

A. Transfer Waiting Time

Assuming that this paper does not account for variations in pedestrian travel times, the walking time is considered a constant value. The transfer process between urban rail transit and feeder bus is depicted in Fig.3. In the Fig.3, g_u indicates the arrival time of the rail transit train u, e_{ul} indicates the walking time of the rail transit transferring to the feeder bus m, b_l^m indicates the first departure time of the mth vehicle of the feeder bus route l, and pull into the 0-1 variable $\delta_u^{l,m}$. In order to address the loss of passengers due to long waiting times, the Maximum Waiting Time Tolerance is introduced, denoted by $R_{\max}$[13]. As can be seen from Fig.3, passengers walk to the feeder bus line l after getting off the rail transit train u to transfer, due to the time discontinuity, there will be two cases of successful transfer ($\delta_u^{l,m}=1$) and failed transfer ($\delta_u^{l,m}=0$), which are explained as follows:

(1) When the passenger arrives at the bus stop, the (m-1)th bus of the feeder bus route l is already running, and the passenger have to wait for the next bus, at which time $\delta_u^{l,m}=0$ and $g_u+e_{ul}>b_l^{m-1}$.

(2) When the passenger arrives at the bus stop earlier than the departure time of the (m+1)th bus of the feeder bus route, and the passenger has a chance to transfer successfully, then $\delta_u^{l,m}=1$ and $g_u+e_{ul}\le b_l^m\le g_u+e_{ul}+R_{\max}$. If the interchange flow exceeds the bus capacity limit, some passengers can not interchange and will continue to wait for the next bus.

(3) When the waiting time for interchange has exceeded the maximum waiting tolerance time, and some passengers will not interchange to the (m+1)th vehicle and choose to interchange to other modes of transport, then the interchange is considered to have failed. In this case, $\delta_u^{l,m}=0$ and $g_u+e_{ul}+R_{\max}<b_l^{m+1}$.

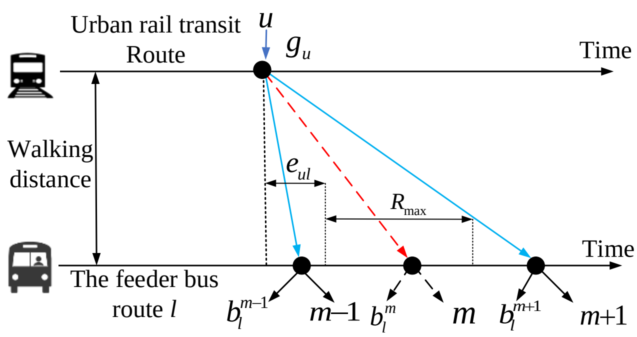


Fig. 3. Time and space diagram of transfer between urban rail transit and feeder buses

B. Matching of Transportation Capacity

In order to embody the degree of coordination between rail and feeder bus capacity, the capacity matching degree that can be expressed by ρ. It can be defined as the ratio of the passenger flow transferring from rail transit to feeder bus to the transportation capacity of the feeder bus within the study period.[22]. When ρ<1, it means a better matching of capacity. A small value of ρ indicates that the capacity of the feeder bus services significantly exceeds the transfer demand, which will result in a waste of bus. The classification of capacity bottlenecks based on the matching degree is presented Table I[22].

TABLE I
CLASSIFICATION OF MATCHING VALUES FOR OPERATIONAL CAPACITY

Transport capacity matching degree	Type	Matching degree value
Capacity surplus	Wastage	[0,0.40)
Capacity surplus	Surplus	[0.40,0.75)
Capacity coordination	Fine	[0.75,0.80)
Capacity coordination	Better	[0.80,0.90)
Capacity bottleneck	Mild	[0.90,1.00)
Capacity bottleneck	Moderate	[1.00,1.10)
Capacity bottleneck	Severe	[1.10,+∞)

Several scholars have verified through the evaluation system that the energy matching grading is coordinated at 0.7~0.95 [23]. They considered the change of interchange passenger flow in the operation time period, and the ideal interval of capacity matching degree in this paper is taken as [0.75,0.90), such as formula (1) and equation (2). When the capacity matching degree falls outside the specified range, the schedule can be adjusted in accordance with the capacity.

$$0.75\le\rho=\frac{Q_{Transfer}^{u\to l}}{Q_{Total}^{l}}=\frac{\sum_{u=1}^{U}C_u\beta_{ru}\varphi_{ul}}{\sum_{m=1}^{M}C_l p_{\max}}<0.9 \quad (1)$$

$$\varphi_{ul}=\frac{D_u^{l,m}}{D_u} \quad (2)$$

In formula (1) and (2), D_u is the number of passengers getting off at the rail transit station. $D_u^{l,m}$ represents the total number of passengers that rail transit train u transfers to the feeder bus line l. C_l and C_u represent the rated passenger capacities of the feeder bus and rail transit respectively. β_{ru} denotes the full load rate of the rail transit train u. The transfer ratio φ_{ul} is influenced by the volume of transferring passengers.

This study develops a capacity-coordinated scheduling framework for feeder bus operations, integrating critical operational parameters including passenger demand and fleet allocation strategies. If the transportation capacity matching evaluation level is rated as "capacity bottleneck", it can be considered that the station is experiencing a surge in passenger flow. In this case, the buses depart frequently and the waiting time is shorter. In order to study the impact of waiting time on vehicle scheduling, this paper establishes a feeder bus schedule model coordinated with the arrival time of rail transit, and studies the optimization of feeder bus schedule under the coordination of transport capacity.

III. MODEL CONSTRUCTION

A. Parameter Setting

To facilitate model development, the operating hours of the urban rail and feeder bus routes are specified. The sets, parameters and decision variables involved in the model are defined below.

L — Collection feeder bus routes, $l\in L$.

U — Urban rail transit arriving train assembly, $u,v\in U$.

M — Gathering of feeder buses, $m,i,j,k\in \mathrm{M}$.

$d_u^{l,m}$ — The number of passengers of train u transfers to the mth vehicle of bus line l, passenger.

d_u^{l} — The number of passengers who failed to transfer from rail transit train u to bus line l, passenger.

$r_u^{l,m}$ — Rail transit train u transfer passengers are still waiting for the number of passengers of the mth bus of line l, passenger.

$w_u^{l,m}$ — The number of passengers who have left the queue among the transfer passengers of train u before the departure of the *m*th bus of the feeder bus line l, passenger.

q_l^t — The number of buses sent by the feeder bus line l in the study period t, vehicle.

$N_{\max}$ — The maximum number of buses that can be dispatched in the garage, vehicle.

$h_{\min}$ — The minimum departure time interval of the feeder bus line l, min.

$h_{\max}$ — The maximum departure time interval of the feeder bus line l, min.

b_l^{mz} — The *m*th bus of the feeder bus line l departs at station *z*.

p_l — Full capacity rate of feeder bus line l.

$p_{\max}$ — The maximum full capacity rate of feeder bus.

$R_{\min}$ —The minimum tolerance time for passengers to transfer , min.

$R_{\max}$ — The maximum tolerance time for passengers to transfer , min.

ϕ — The probability that a passenger will leave the queue when the waiting time exceeds the maximum waiting tolerance time.

Q_l — The maximum loss of passengers of feeder bus line l, passenger.

$TS_{\min}$ — Start time of the research period.

$\delta_u^{l,m}$ — A binary 0-1 variable that indicates the passengers of urban rail transit train u transfer to the *m*th vehicle of the feeder bus line l. $\delta_u^{l,m}$ =1 if the passengers of train u transfer to the *m*th vehicle of the feeder bus line l, and $\delta_u^{l,m} = 0$ otherwise.

$a_u^{l,m}$ — A binary 0-1 variable that indicates the waiting time of passengers on urban rail transit train u transfer to the *m*th vehicle of the feeder bus line l exceeds the maximum waiting time. $a_u^{l,m}$ =1 if the waiting time exceeds the maximum waiting time, and $a_u^{l,m} = 0$ otherwise.

$H_{l,z}$ — A binary 0-1 variable that indicates the feeder bus line l pass through stop z. $H_{l,z}$ =1 if the feeder bus line l pass through stop z, and $H_{l,z} = 0$ otherwise.

$\xi_{u,m}^{l,z}$ — A binary 0-1 variable that indicates the passengers of rail transit train u transfer to the *m*th vehicle of the feeder bus line l, and the bus destination is the bus stop z. $\xi_{u,m}^{l,z}$ =1 if the passengers of train u transfer to the *m*th vehicle of the feeder bus line l and arrive at stop z ,and $\xi_{u,m}^{l,z} = 0$ otherwise.

B. Model Assumption

To simplify the issue, the following assumptions are made.

Assumption 1: Assuming that all buses travel at a fixed time throughout the journey and that bus schedules run continuously, there is no situation where the first bus is unable to return after all vehicles have travelled.

Assumption 2: The number of passengers interchanging from urban rail to feeder buses is given and can be obtained from statistical analysis of historical passenger flow data.

Assumption 3: Passengers know the time of the next bus departure, if the waiting time exceeds the maximum waiting tolerance time can choose other options.

Assumption 4: The first and last stops of feeder bus route are in the vicinity of the rail stations, and the buses are a uniform type, taking into account only the flow of passengers transferring from the rail to the bus.

C. Objective Function

During the study period, when the waiting time exceeds the maximum waiting tolerance time, some passengers will choose other modes to travel. In order to reduce the loss of passengers and attract more passengers, the loss of passenger is used as an optimization objective, which can be calculated by equation (3).

$$Z_1 = \sum_{u\in U}\sum_{l\in L}\sum_{m\in \mathrm{M}}\left(D_u^{l,m} - d_u^{l,m}\right) \tag{3}$$

The company has a limited number of buses to purchase, and transporting passengers with fewer vehicles is a key consideration for companies. Therefore, a number of feeder buses are chosen as an optimization objective and can be calculated by equation (4).

$$Z_2 = \sum_{l\in L}\sum_{t} q_l^t \tag{4}$$

In the whole journey service, the transfer waiting time is a key consideration for passengers when transferring, and the total passenger transfer waiting time is an optimization objective, which can be calculated by equation (5).

$$Z_3 = \sum_{l\in L}\sum_{u\in U}\sum_{m\in \mathrm{M}}\left(b_l^m - g_u - e_{ul}\right)D_u^l \delta_u^{l,m} \tag{5}$$

According to the above analysis, the optimization model is constructed with three objectives minimum respectively, and the optimization objective can be expressed as equation (6) ~(8).

$$\min Z_1 = \min \sum_{u\in U}\sum_{l\in L}\sum_{m\in \mathrm{M}}\left(D_u^{l,m} - d_u^{l,m}\right) \tag{6}$$

$$\min Z_2 = \min \sum_{l\in L}\sum_{t} q_l^t \tag{7}$$

$$\min Z_3 = \min \sum_{l\in L}\sum_{u\in U}\sum_{m\in \mathrm{M}}\left(b_l^m - g_u - e_{ul}\right)D_u^l \delta_u^{l,m} \tag{8}$$

D. Constraint Condition

Departure time interval constraint

In order to meet the transfer demand and ensure that the feeder bus operation is in line with the actual situation, the interval of the feeder bus departure should be controlled in an interval, as in formula (9). At the same time, each trip of feeder bus departs in sequence, and in order to avoid crosstown, the moment of departure should be in order, as in formula (10). During the study period, the first feeder bus must depart within the study time frame, and the last feeder bus must be successfully connected to the last urban rail transit, as in formula (11)~(12). The feeder bus departure moments and departure intervals are integer variables $\mathbb{Z}$, ensuring that the feeder bus operation scheduling is easy to execute in practice, as in formula (13).

$$h_{\min} \le h_l = \frac{60 C_l p_l}{D_u^l} = b_l^{m+1} - b_l^m \le h_{\max} \tag{9}$$

$$b_l^m < b_l^{(m+1)}, b_l^{mz} < b_l^{(m+1)z} \quad (10)$$

$$TS_{\min} \le b_l^1 \le TS_{\min} + h_1 \quad (11)$$

$$b_l^{m(last)} \ge g_{u(last)} + e_{ul} \quad (12)$$

$$h_l, h_1, b_l^1, b_l^m \in \mathbb{Z}, \qquad \forall l \in L, m \in \mathrm{M} \quad (13)$$

Vehicle number and passenger flow loss constraint

In actual operation, the total number of buses can not exceed the maximum number of buses, as in formula (14). Each vehicle can connect up to one subsequent bus and one preceding bus at most, and x_{ij} indicates the feeder bus trips i and j are connected, as in formula (15). If the departing trips i and j can be connected, take the value of 1, otherwise it is 0.

$$\sum_{l\in L}\sum_{t} \frac{t\max\left(D_u^l\right)}{C_l p_l} \le N_{\max} \quad (14)$$

$$\sum_j x_{ij} \le 1, \qquad \forall i \in \mathrm{M}$$
$$\sum_i x_{ij} \le 1, \qquad \forall j \in \mathrm{M} \quad (15)$$

In order to control the amount of passenger loss, the maximum waiting tolerance time constraint is set so that the passenger flow will not be lost within the waiting tolerance time interval, as in formula (16). To attract more passengers, the passenger flow loss is controlled within the acceptable range of the enterprise, as in formula (17).

$$R_{\min} \le b_l^m - g_u - e_{ul} \le R_{\max} \quad (16)$$

$$\sum_{u\in U}\sum_{l\in L}\sum_{m\in M} r_u^{l,m} a_u^{l,m} \phi \le Q_l \quad (17)$$

Passenger demand constraint

To determine the amount of passenger loss generated, it is necessary to determine whether the waiting time of feeder bus route exceeds the maximum waiting tolerance time, as in formula (18) and (19). All passengers have two possibilities of transfer, and the total passenger flow is balanced, as in equation (20). Passengers arriving at the starting point of feeder bus must wait for passengers who arrived earlier to board the bus before transferring, and there is no queue-jumping problem, as in formula (21), where M is a sufficiently large positive number. In order to ensure that a higher number of passengers transfer to the feeder bus, passengers who arrived at the first bus stop will not transfer to other modes until the maximum waiting tolerance time is exceeded, as in formula (22). Formula (23) is the decision variable relationship constraints, and equation (24) represents the number of passengers who continue to wait for the next bus after the last bus departs.

$$a_u^{l,m} \in \{0,1\}, \forall u \in U, l \in L, m \in \mathrm{M} \quad (18)$$

$$M\left(a_u^{l,m} - 1\right) \le b_l^m - g_u - e_{ul} - R \le M a_u^{l,m}$$
$$\forall u \in U, l \in L, m \in \mathrm{M} \quad (19)$$

$$\sum_{m\in M} d_u^{l,m} + d_u^l = D_u^{l,m} \quad (20)$$

$$d_u^{l,i} \le M\left(1 - \delta_v^{l,j}\right)$$
$$\forall i, j \in \mathrm{M}, u, v \in U, g_u < g_v, b_l^i > b_l^j, l \in L \quad (21)$$

$$d_u^l \le M\left(1 - \delta_v^{l,j}\right), \forall u, v \in U, g_u < g_v, l \in L \quad (22)$$

$$\delta_u^{l,i} \le d_u^{l,i}, \forall u \in U, l \in L, i \in \mathrm{M} \quad (23)$$

$$r_u^{l,m} = \begin{cases} D_u^{l,\mathrm{m}} & u = 1,2,3,\ldots,U; m = 1 \\ D_u^{l,m} - \sum_{k=1}^{m-1}\left(d_u^{l,k} + w_u^{l,k}\right) & u = 1,2,3,\ldots,U; m = 2,3,.,\mathrm{M} \end{cases} \quad (24)$$

The passenger exits the station and chooses a particular line to transfer. The passenger chooses the line only when the feeder bus line l passes through the station z, as in formula (25). Formula (26) is a 0-1 variable decision constraint.

$$\xi_{u,m}^{l,z} \le H_{l,z} \quad (25)$$

$$x_{ij}, \delta_u^{l,m}, \xi_{u,m}^{l,z}, H_{l,z} \in \{0,1\}$$
$$\forall m, i, j \in \mathrm{M}, u \in U, l \in L \quad (26)$$

IV. Model Solution

In this paper, the optimization model proposed is multi-objective, which is solved by heuristic algorithms in most of researches. Some scholars have proved that the bus schedule optimization is an NP-hard problem, which was solved using genetic algorithm (GA)[24]. Due to the GA algorithm used the weighting method of normalization to solve multi-objective problems, the coefficients are difficult to determine. In essence, it is still a single-objective problem solving method, which can not truly reflect the advantages of multi-objective optimization. There is a certain conflict between the optimization objectives, and it is difficult to find a scheme in which multiple objectives are all optimal. Therefore, the GA algorithm is not suitable for solving the model, and the multi-objective optimization algorithm can be used to generate multiple non-dominated Pareto solutions for makers to choose. Based on the GA, the Non-dominated Sorting Genetic Algorithm II that proposed by Kalyanmoy Deb[25] can obtain uniformly distributed Pareto solution set.

A. Algorithm design

Crowding distance calculation

To obtain an estimate of the density around a specific solution, this paper calculates the average distance between the two points on either side of the solution corresponding to the objective. As can be seen from Fig.4, the two objective functions are Z_1 and Z_2 . The crowding distance for the ith solution on its front is the average edge length of the cuboid, and the crowding distance of i is the perimeter of the cuboid formed by the nearest neighbors as vertices.

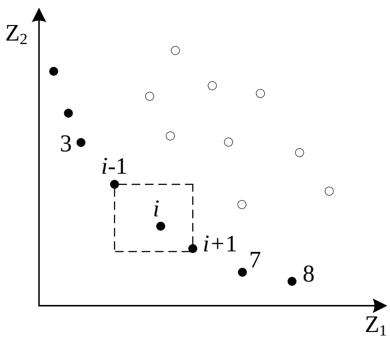


Fig. 4. Individual crowding distance calculation

Taking the allocation of crowding distance based on the non-dominated solution set P as an example. As shown in Table II, the chart illustrates the process of calculating crowding distance. $P[i]_m$ denotes the mth objective function value of the ith individual, while z_m^{max} and z_m^{min} represent the maximum and minimum values of the mth objective function.

TABLE II
MODEL PARAMETER VALUES

The process of calculating crowding distance	
$l = \|P\|$	The quantity of solutions within set P
for each i , set P[i]distance=0	Initialize distance
for each objective m	
$P = sort(P, m)$	Sort each target value
$P[1]_{distance} = P[l]_{distance} = \infty$	Boundary points are always selected
for i = 2 to (l-1)	For all other points
$P[i]_{distance} = P[i]_{distance} + (P[i+1].m - P[i-1].m) / (Z_m^{max} - Z_m^{min})$	

Chromosome coding

This paper employs real number encoding method, the chromosome is composed of the departure time of feeder bus routes. Each gene value represents the start time of the first station for a bus, and forms a chromosome with x_{1n} elements. The departure time of the first station for each bus can constitute a feasible solution m. Fig.5 illustrates an example of encoding, the first gene value is 5, indicating that the feeder bus starts its service at the 5th minute of the study period, and the second gene value is 15, indicating that the feeder bus starts its service at the 15th minute of the study period, and so on.

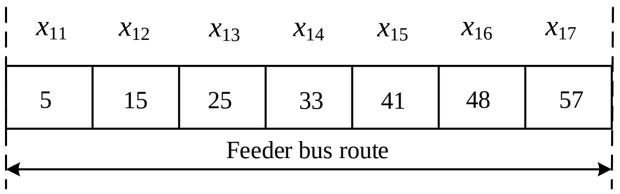


Fig. 5. Chromosome coding diagram

Crossover and mutation

In the feeder bus scheduling problem, the gene segments represent the departure times of specific bus. The crossover operation must take into account the feasibility of bus departure times and the constraints. Taking the crossover process of individuals P_1 and P_2 as an example, feasible bus departure times are generated through the crossover, with the chromosomal restructuring process explicitly illustrated in Fig.6.

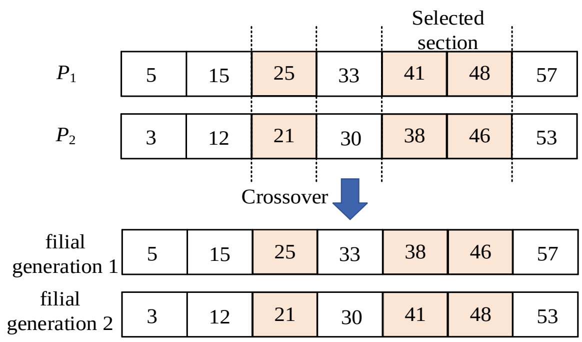


Fig. 6. Schematic diagram of Crossover operation

In the context of feeder bus scheduling optimization, the implementation of mutation operations is crucial, directly affecting the search capability of the algorithm and the quality of the solutions. Mutation can introduce diversity, creating characteristics that are distinctly different from the parent population. For instance, when mutating the departure time of a specific bus trip in individual P_1, it is first necessary to determine whether to mutate a particular gene of the individual based on a preset mutation probability. Once it is decided to mutate, a new departure time value is generated randomly, replacing the original gene value in accordance with the problem's constraints and optimization objectives. This process must ensure that the mutated departure times meet specific constraints, they satisfy Equation (27). Through Equation (27), the randomly generated feeder bus departure will replace the originally selected genes.

$$x_l^m \in [0, h_l] \tag{27}$$

As illustrated in Fig.7, the mutation process of individual P is depicted, taking the mutation operation on the 2nd, 3rd, and 5th genes as an example. Initially, a set of mutation masks is randomly generated based on the number of feeder bus departure times from the first station. According to the mutation mask, it is determined whether the genes on the chromosome undergo mutation.

As shown in Fig.7, if a gene does not mutate, the mutation mask corresponding to the gene value is 0, meaning that the departure time of the corresponding feeder bus remains unchanged after mutation. If a gene mutates, the mutation mask corresponding to the gene value is 1, indicating that the departure time of the corresponding feeder bus changes after mutation.

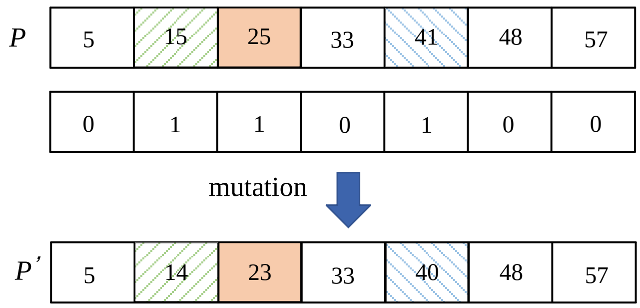


Fig. 7. Schematic diagram of mutation operation

B. NSGA-II algorithm solution process

The NSGA-II algorithm employs a dual-mechanism framework comprising fast non-dominated sorting to determine Pareto dominance hierarchies among solutions, and crowding distance operators that preserve population diversity through density estimation. An elite preservation strategy systematically retains high-quality solutions across generations while enhancing solution distribution uniformity via objective-space coordination mechanisms. As a solution algorithm, this approach demonstrates particular efficacy in resolving multi-dimensional trade-offs inherent in feeder bus scheduling problems, effectively balancing operational constraints and passenger service requirements.

The flowchart of the NSGA-II algorithm is shown in Fig.8.

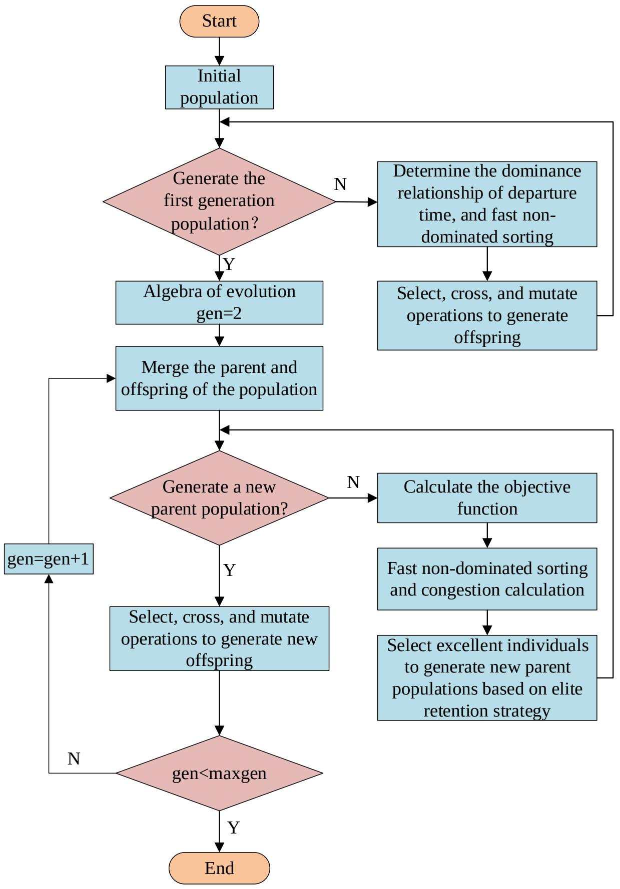


Fig. 8. NSGA-II algorithm flowchart

The solution steps for the multi-objective optimization algorithm based on NSGA-II are as follows:

Step 1. initialization. Parameters such as population size N, crossover probability p_c, mutation probability p_m, Iteration times gen=1 and maximal evolutionary algebra $maxgen$ were randomly selected to generate the initial population P_n.

Step 2. Determine whether to generate the first generation of the population. If it is generated, then gen=2. Otherwise, apply nondominated ordering to all individuals in P_n and obtain nondominated ordering values for all individuals. Simultaneously perform selection, crossover and mutation to generate the offspring population Q_n with a population size of N, and make the evolutionary generations gen=2.

Step 3. Parent population P_n and offspring population Q_n were merged to create a new population $R_n = P_n \cup Q_n$ with a population size of $2N$.

Step 4. Determine whether a new parent population has been generated. If not, then calculate the value of each objective function in turn, perform a fast non-dominated sorting of R_n based on the function values and calculate the crowding degree.

Step 5. Through the elite retention strategy, the best individuals in R_n are selected to join the new parent population P_{n+1}, which undergoes selection, crossover and mutation operations to calculate the value of the objective function produced by the offspring population Q_{n+1}.

Step 6. If the maximum number of iterations $maxgen$ is reached, the Pareto solution set is output and the algorithm ends. Otherwise, $gen = gen + 1$ and go back to Step3.

V. Case Study

A. Example Description

This paper investigates the optimization of feeder bus scheduling based on coordinated transportation capacity, using Line 1 of urban rail transit and two feeder bus routes as a case study to verify the effectiveness of the model and solution algorithm. The study period is from 11:00 to 13:00, which urban rail transit arrives at the station 35 times with a train departure interval of 7 minutes and a station stop time of 2 minutes. The passenger walking time for transfer is 4 minutes. The starting station of feeder bus is located near Station A, and the buses are parked in the same yard. The operating time of feeder bus routes 1 and 2 is 64 minutes and 70 minutes respectively. According to surveys, there are 1007 passengers transferring during the study period. This paper analyzes the passenger flow of urban rail transit transferring to feeder bus route 1 and 2 in 14 minute intervals, and the passenger flow distribution is shown in Fig.9.

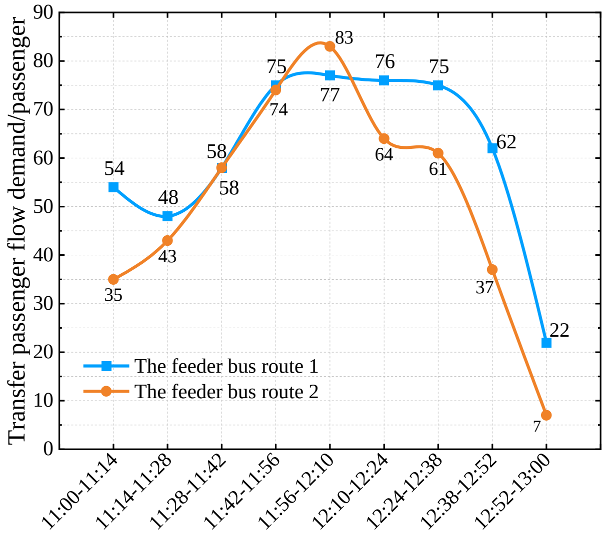


Fig. 9. Distribution of transfer passenger flow demand

TABLE III
MODEL PARAMETER VALUES

Parametric	Value	Parametric	Value
h_{min}	5min	TS_{min}	11:00
h_{max}	14min	Q_l	200
R_{min}	4min	ϕ	66%
R_{max}	15min	e_{ul}	4min
p_{max}	60%	N_{max}	24 vehicles

In the study time period, the feeder bus routes adopt a fixed-time, equal headway scheduling approach, with an average bus speed of 30 km/h and an average headway of 10 minutes. The parameters for the feeder bus in the model are referenced from existing literature [13][17], as shown in Table III. This article refers to relevant standards, the rated passenger capacity for a standard public transit vehicle is 40 passengers. During peak hours, the average load factors should be less than 85%, and during off-peak hours, it should be below 60%. Based on actual data statistics, the arrival time of the train and the distribution of transfer passenger flow during the study period are shown in Table IV.

B. Example Result

Analysis of optimization results

The article uses the NSGA-II algorithm proposed in Section 4 to solve the model, which can obtain the departure times to different numbers of vehicles. The NSGA-II is mainly implemented using Python programming language. The parameter settings are as follows: N =100, $maxgen$ = 500, p_c= 0.8, and p_m= 0.2. When the number of iterations reaches $maxgen$, the algorithm stops and the output result is the Pareto optimal solution. The Pareto approximate optimal solution set obtained by multiple independent operations is shown in Fig.10.

Based on the results of the algorithm's execution, the bus departure times are determined, and the Pareto solution set is obtained. The passenger flow loss, the number of feeder buses, and the passenger transfer times are shown in Table V. It can be seen that the minimum passenger flow loss among all solutions is 105 passengers, accounting for 10.4% of the total passenger transfer demand, and the average waiting time for transfer is within the tolerance interval.

It can be seen from Table V that the NSGA-II algorithm can be used to solve the model, which can obtain multiple optimal solutions in the Pareto solution set that are in a nondominated position with each other. The objective values corresponding to different solutions are different, but all of them can make the overall efficiency of multiple objectives reach the optimal, and the Pareto solutions with different objective preferences can provide the decision makers with different solutions. When the decision maker focuses on operating cost, the fifth solution can be chosen to develop a feeder bus scheduling. When the decision maker favors the cost of passenger waiting time and attracting interchange traffic, the first solution can be chosen to develop a feeder bus scheduling.

TABLE IV
ARRIVAL TIME AND TRANSFER DEMAND OF RAIL TRANSIT TRAINS

Direction of travel	Train service u	Arrival time	Time g_u/s	Transfer demand $d_u^{l,m}$ / passenger		Train service u	Arrival time	Time g_u /s	Transfer demand $d_u^{l,m}$ / passenger	
				The feeder route 1	The feeder route 2				The feeder route 1	The feeder route 2
The upside	U1	11:00	0	12	6	U10	12:03	3780	22	21
	U2	11:07	420	14	8	U11	12:10	4200	17	23
	U3	11:14	840	10	7	U12	12:17	4620	20	18
	U4	11:21	1260	16	10	U13	12:24	5040	21	13
	U5	11:28	1680	9	14	U14	12:31	5460	20	15
	U6	11:35	2100	13	15	U15	12:38	5880	16	11
	U7	11:42	2520	21	17	U16	12:45	6300	17	9
	U8	11:49	2940	26	20	U17	12:52	6720	20	9
	U9	11:56	3360	20	19	U18	12:59	7140	13	4
The downside	V1	11:02	120	10	10	V10	12:05	3900	20	21
	V2	11:09	540	8	4	V11	12:12	4320	19	17
	V3	11:16	960	12	11	V12	12:19	4740	16	16
	V4	11:23	1380	11	8	V13	12:26	5160	23	18
	V5	11:30	1800	13	12	V14	12:33	5580	16	17
	V6	11:37	2220	11	14	V15	12:40	6000	14	9
	V7	11:44	2640	9	13	V16	12:47	6420	11	10
	V8	11:51	3060	20	22	V17	12:54	6840	9	3
	V9	11:58	3480	16	18	—	—	—	—	—

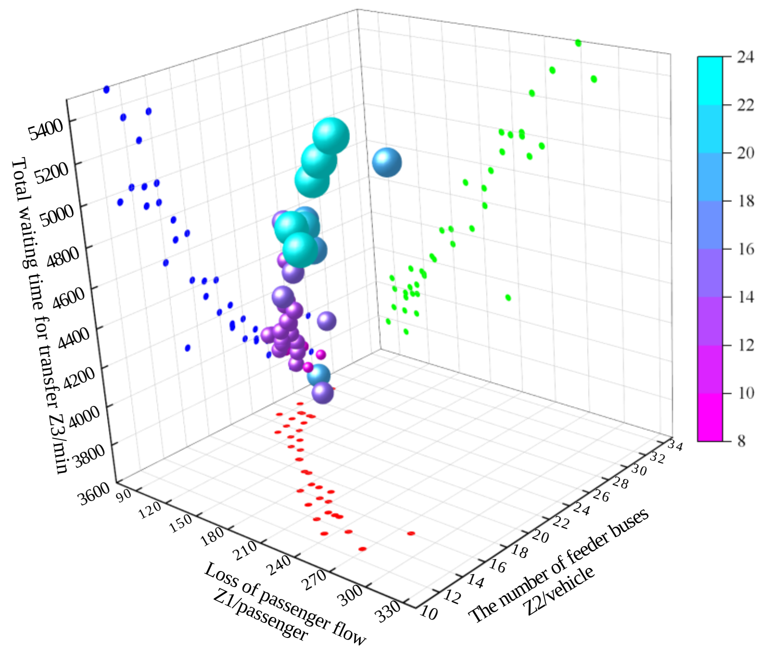


Fig. 10. Schematic diagram of the obtained Pareto solution set

TABLE V
TARGET VALUE OF PARETO SOLUTION SET

No.	Loss of passenger flow Z_1/ passenger	Number of feeder buses Z_2/vehicle	Total waiting time Z_3/min	Average waiting time /min
1	105	22	4020	4.0
2	131	20	4218	4.2
3	150	19	4329	4.3
4	169	18	4440	4.4
5	200	16	4884	4.9

If the decision maker weighs multiple objectives together, the results of the second to fourth solutions can be selected to prepare a schedule that can be adjusted in real time according to the situation. As an example of the second solution, the feeder bus travel schedule scheme is given as shown in Table VI. The turnaround time of public transport vehicles meets the departure demand, and there will be no vehicle dispatching in the depot.

As shown in Table VI, when the number of feeder buses is 20, the total transfer waiting time for passengers to transfer is 4218 minutes, and the average waiting time is 4.2 minutes. During the study period, both feeder bus route 1 and route 2 dispatch 15 buses, and all passengers successfully transferred without any passenger were stranded at the transfer station.

Sensitivity analysis of maximum bus number

For models that all optimization objectives are minimized, if two of these objectives are positively correlated, there is no conflict between the two optimization objectives, meaning that only one optimization objective is effective. In the multi-objective optimization model constructed in this paper, the number of feeder buses directly affects the passenger flow loss and the total transfer waiting time. The number of buses was adjusted between 16 and 24, the changes in passenger flow loss and the total transfer waiting time are shown in Fig.11. It can be seen from Fig.11, in the Pareto optimal solutions obtained that the more buses there are, the shorter the transfer waiting time for passengers. It suggests that the number of buses and the transfer waiting time are conflicting objectives. In the same way, the number of buses and the passenger flow loss are contradictory objectives, and the optimization objective selected in this paper is reasonable.

As shown in Fig.11, when the number of buses is 16, the passenger flow loss attains the upper limit. If the number of buses is less than 16, the passenger flow loss will exceed the maximum allowable limit. As the number of buses increases, the passenger flow loss and the total waiting time for transfer passengers gradually decrease. When the number of buses reaches 22, the variations in passenger flow loss and total waiting time for transfer passengers tend to level off. If the number of buses continues to increase, the operational cost for the bus company will rise, but the reduction in passenger flow loss and total waiting time for transfer passengers will be minimal. At this point, the number of schedulable buses can be determined by balancing the operational cost of the company and the time cost for passengers.

TABLE VI
OPTIMIZATION PLAN FOR BUS TRANSPORTATION TIMETABLE (THE SECOND PARETO OPTIMAL SOLUTION)

The feeder bus route 1				The feeder bus route 2			
Vehicle No.	Departure time	Arrival Time	Departure interval /s	Vehicle No.	Departure time	Arrival Time	Departure interval /s
A1	11:04	11:36	—	B1	11:06	11:41	—
A2	11:14	11:46	602	B2	11:16	11:51	602
A3	11:23	11:55	542	B3	11:27	12:02	662
A4	11:32	12:04	540	B4	11:34	12:09	422
A5	11:40	12:12	480	B5	11:43	12:18	542
A6	11:48	12:20	490	B6	11:51	12:26	482
A7	11:56	12:28	485	B7	11:59	12:34	482
A8	12:03	12:35	422	B8	12:05	12:40	360
A9	12:10	12:42	437	B9	12:12	12:47	362
A10	12:16	12:48	362	B10	12:18	12:53	362
A11	12:22	12:54	388	B11	12:25	13:00	422
A12	12:30	13:02	482	B12	12:33	13:08	482
A13	12:38	13:10	482	B13	12:43	13:18	602
A14	12:48	13:20	610	B14	12:53	13:28	602
A15	12:58	13:30	600	B15	13:00	13:35	420

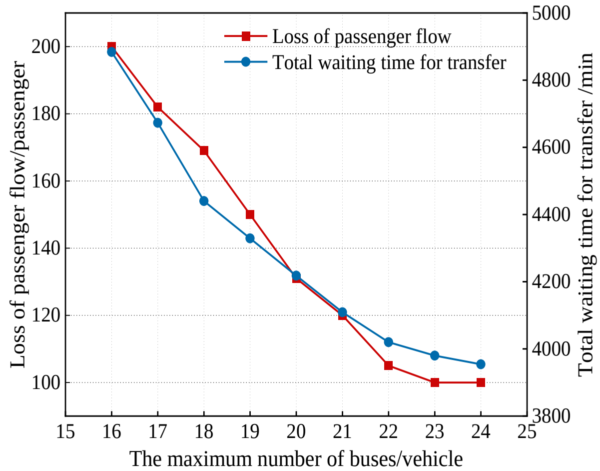


Fig. 11. Relationship diagram of three optimization objectives

C. Comparative analysis with uniform departure interval

Analysis of uniform departure scheme

In actual operations, public transit companies often adopt uniform headway schedules. To verify the effectiveness of the models and methods presented in this paper, this section calculates the headway based on passenger demand and the rated passenger capacity of buses to determine a uniform headway schedule. In this section, equation (6) to (8) are utilized as the objective functions, while formula (10) to (26) are employed as the constraints to formulate a model for uniform headway scheduling, as illustrated in equation (28).

$$\begin{cases} \min Z_1 = \min \sum_{u\in U}\sum_{l\in L}\sum_{m\in M}\left(D_u^{l,m} - d_u^{l,m}\right) \\ \min Z_2 = \min \sum_{l\in L}\sum_{t} q_l^t \\ \min Z_3 = \min \sum_{l\in L}\sum_{u\in U}\sum_{m\in M}\left(b_l^m - g_u - e_{ul}\right) D_u^l \delta_u^{l,m} \\ s.t \qquad formula(10)\sim formula(16) \end{cases} \tag{28}$$

In this section, the headway interval is set between 5 and 15 minutes, and the maximum number of buses is determined based on the interval, with a range of 9 to 24 buses. For comparative analysis, the schemes with the maximum number of buses at 16, 18, 19, 20, and 22 are analyzed, with the results presented in Table VII. Based on the results in Table VII, if a uniform headway schedule is adopted, the maximum passenger loss is 200 passengers, and the minimum number of buses required for scheduling is 16. The bus departure frequency is correlated with passenger loss and the number of buses, and negatively correlated with the headway interval and the total waiting time for transfers. In this section, the passenger flow loss and waiting time are analyzed based on Table VII.

The frequency of bus departures is at an extreme level, which can lead to excessive passenger flow or waiting time for passengers. To balance the relationship between passenger flow loss and waiting time, and to facilitate comparison between plans, among the five schemes in Table VII, the fourth scheme is selected as the optimized plan. It can be seen from Table VIII that the feeder bus schedule is designed and the times of the two feeder bus routes are 15.

Based on the data from the feeder bus schedules in Table VI and Table VIII, it is possible to create bus operation diagrams for both uneven and uniform headway schedules, as shown in Fig.12 and Fig.13.

Comparative analysis of departure interval

In actual operations, departure interval is an important indicator for evaluating the service level of feeder bus, and departure interval is an important factor affecting passenger time and enterprise operation cost. The service level classification of feeder bus corresponding to departure interval is shown in Table IX [14]. To analyze the impact of different departure interval schemes on the service level of feeder buses, this paper calculates the changes in departure interval for different schemes based on the results in Table VI and Table VIII, as depicted in Fig.14.

By analyzing and comparing the departure intervals of feeder bus routes 1 and 2 with the arrival time of urban rail transit and passenger flow demand, it can be seen from Fig.14 that compared with the uniform departure interval. The optimized uneven departure interval is adjusted according to the arrival time of urban rail transit and the change of actual passenger flow, which can better match the arrival time of trains and flow demand. According to the analysis results, the service level of the two feeder bus routes is class A.

TABLE VII
OPTIMIZATION RESULTS OF UNIFORM DEPARTURE INTERVAL SCHEME

No.	Departure frequency/time	Departure interval/min	Loss of passenger flow Z_1/ passenger	Number of feeder buses Z_2/vehicle	Total waiting time Z_3 /min
1	18	13	234	16	5120
2	20	12	193	18	4824
3	24	10	165	19	4530
4	30	8	118	20	4326
5	34	7	89	22	4110

TABLE VIII
OPTIMIZATION PLAN FOR BUS TRANSPORTATION TIMETABLE (THE UNIFORM DEPARTURE)

The feeder bus route 1				The feeder bus route 2			
Vehicle No.	Departure time	Arrival Time	Departure interval /s	Vehicle No.	Departure time	Arrival Time	Departure interval /s
A1	11:04	11:36	—	B1	11:06	11:41	—
A2	11:12	11:44	480	B2	11:14	11:49	480
A3	11:20	11:52	480	B3	11:22	11:57	480
A4	11:28	12:00	480	B4	11:30	12:05	480
A5	11:36	12:08	480	B5	11:38	12:13	480
A6	11:44	12:16	480	B6	11:46	12:21	480
A7	11:52	12:24	480	B7	11:54	12:29	480
A8	12:00	12:32	480	B8	12:02	12:37	480
A9	12:08	12:40	480	B9	12:10	12:45	480
A10	12:16	12:48	480	B10	12:18	12:53	480
A11	12:24	12:56	480	B11	12:26	13:01	480
A12	12:32	13:04	480	B12	12:34	13:09	480
A13	12:40	13:12	480	B13	12:42	13:17	480
A14	12:48	13:20	480	B14	12:50	13:25	480
A15	12:56	13:28	480	B15	12:58	13:33	480

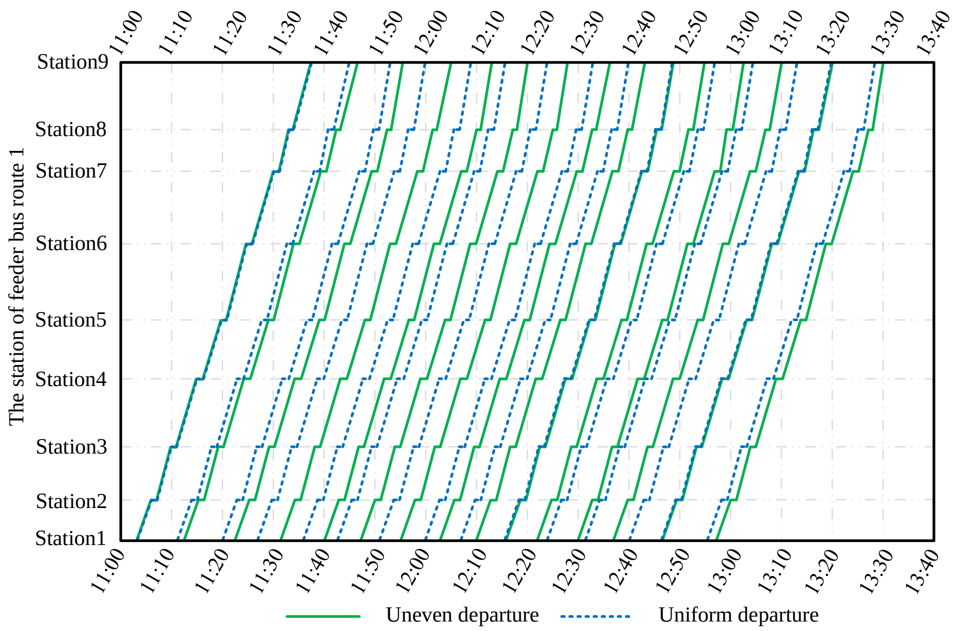


Fig. 12. Bus operation diagram of different departure modes (The feeder bus route 1)

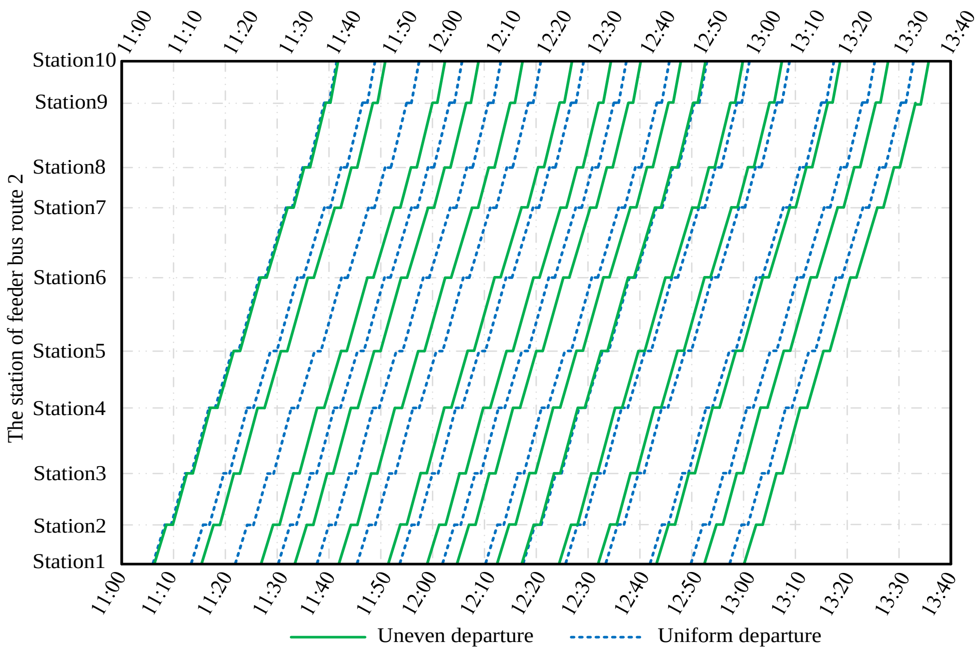


Fig. 13. Bus operation diagram of different departure modes (The feeder bus route 2)

TABLE IX
DEPARTURE INTERVAL SERVICE LEVEL LEVEL

Service level	A	B	C	D	E	F
departure interval/min	0~10	10.1~14.0	14.1~20.0	20.1~30.0	30.1~60.0	>60.0

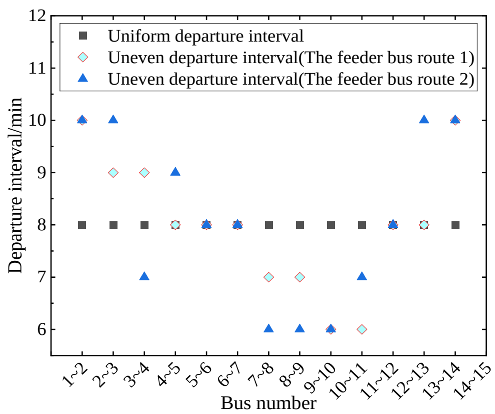


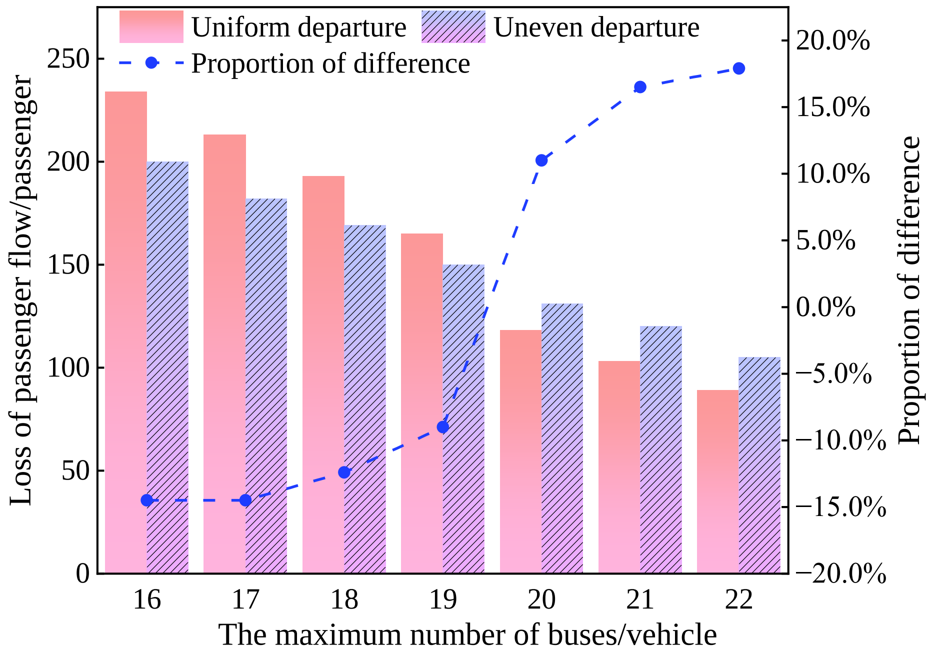
Fig. 14. Departure interval comparison diagram

Comparative analysis of optimization target values

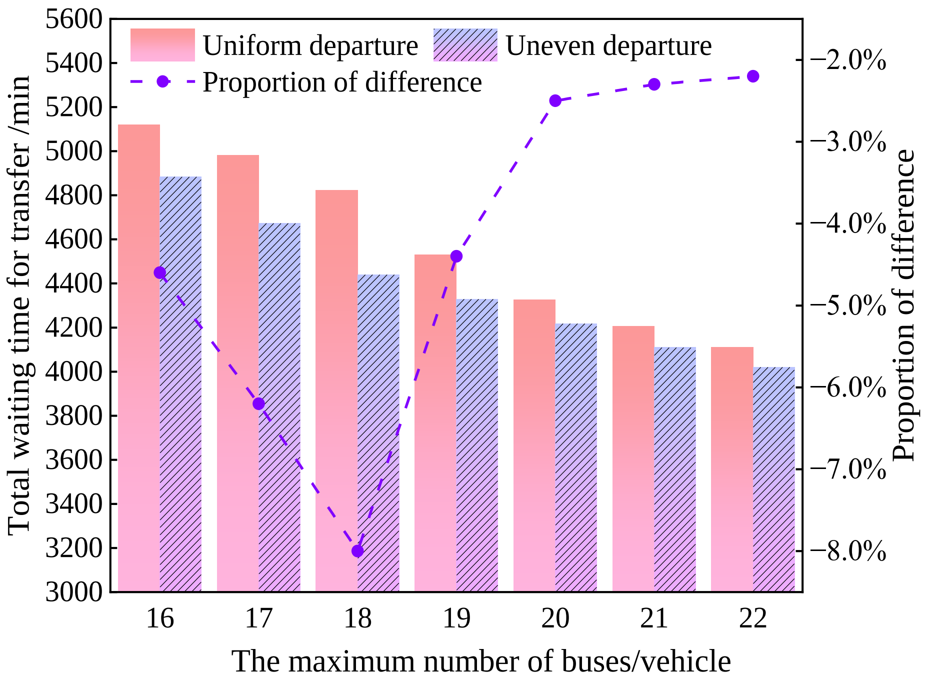
In order to evaluate the effectiveness and applicability of the proposed method, it is compared with the uniform departure scheme. Based on the same number of dispatchable feeder buses, the passenger flow loss and the total transfer waiting time of the two options are compared, and the comparison results are shown in Table X. Among them, a positive difference ratio indicates that the target value of the uneven departure interval exceeds that of the even departure interval, suggesting that the uneven departure is more optimal. Otherwise, a negative difference ratio implies the opposite. The comparative analysis of the uniform departure and uneven departure interval is shown in Fig.15.

TABLE X
COMPARISON OF OPTIMIZATION RESULTS FOR DIFFERENT DEPARTURE PLANS

NO.	The number of buses/vehicle	Loss of passenger flow/passenger			Total waiting time for transfer /min		
		Uniform departure	Uneven departure	Proportion of difference	Uniform departure	Uneven departure	Proportion of difference
1	16	234	200	-14.5%	5120	4884	-4.6%
2	17	213	182	-14.5%	4982	4673	-6.2%
3	18	193	169	-12.4%	4824	4440	-8.0%
4	19	165	150	-9.0%	4530	4329	-4.4%
5	20	118	131	11.0%	4326	4218	-2.5%
6	21	103	120	16.5%	4206	4109	-2.3%
7	22	89	105	17.9%	4110	4020	-2.2%



(a)



(b)

Fig. 15. Comparison diagram of uniform departure interval and uneven departure interval. (a) Comparison of loss of passenger flow; (b) Comparison of total waiting time for transfer

According to Fig.15(a), when $16 \leq N_{\max} < 20$, compared with the uniform departure scheme, the passenger flow loss of the uneven departure scheme is reduced by 14.5%, 12.4% and 9.0% respectively. If the number of buses increases, the bus departure interval becomes smaller, and the probability of passengers who giving up queuing will be reduced. According to equation (3), it will also result in a reduction in passenger flow loss, which is consistent with the changes in passenger flow loss. At the same time, with the increase of feeder bus that can be dispatched, the passenger flow loss of the two departure schemes gradually decreases, indicating that the optimization of departure interval by increasing public transport vehicles is getting smaller and smaller.

Based on the comparison of total transfer waiting time, it can be observed that as the number of feeder buses increases, all optimization rates are less than 0. It indicates that the total transfer waiting time of uneven departure interval is consistently lower than that of uniform departure. If the number of buses is reduced, the total time difference between the departure schemes will gradually increase, which further illustrates the advantages of uneven departure interval.

VI. CONCLUSION

(1) This paper considered factors such as the arrival times of urban rail transit, capacity coordination, and passenger flow loss. The objectives aimed to minimize passenger flow loss, the number of buses, and total transfer waiting time. Constraints including departure time intervals, maximum passenger flow loss, and waiting tolerance time are selected to construct a multi-objective optimization model for the scheduling of feeder bus services.

(2) The Pareto solution set was obtained by solving the model using the NSGA-II algorithm. The calculation results of the example show that the model and NSGA-II could obtain multiple optimal feeder bus schedules and vehicle scheduling schemes within a limited time for decision makers to choose, and verify the effectiveness of the model and algorithm.

(3) From a comprehensive perspective that considers both bus operation costs and passenger transfer waiting time costs, the model constructed in this chapter has certain advantages. Compared with the uniform departure scheme, the bus schedule model constructed in this chapter can take into account the cost of bus operation and passenger transfer time. By adopting uneven departure intervals, this model can synchronize with the arrival times of rail transit. Under the normal operation, the bus schedule can be adjusted in real time according to the passenger flow demand and transport capacity, so as to avoid the increase of enterprise operation cost due to too small departure interval during the operation period, the increase of passenger flow loss due to too large departure interval, and the reduction of enterprise benefit.

(4) If the number of feeder buses is increased, the change rate of passenger flow loss in the two departure schemes gradually decreases and tends to be gentle , indicating that the optimization of departure interval by increasing the number of buses becomes smaller gradually. If the feeder buses are the same, compared with the uniform departure scheme, the optimization model in this paper increases the optimization effect of the total waiting time of passengers.

The model in this paper examined the scheduling of feeder bus services under the coordination of transportation capacity, which all bus vehicles adopted a single model. However, we consider scenarios with high passenger flow such as peak hours and sports events, a single vehicle model can not be able to meet the demand. In the next stage, the research will focus on maximizing the evacuation of passenger flow, and addressing the joint optimization of the feeder bus schedules and multiple vehicle models under capacity constraints.

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Mingming Chen was born in Shanxi, China, in 1982. In June 2016, he obtained a doctoral degree in Transportation Planning and Management from Lanzhou Jiaotong University. Currently, he is a professor of the School of Traffic and Transportation of Lanzhou Jiaotong University. He has rich scientific research achievements, and has published more than a few papers in domestic academic journals and international conferences.