Pricing Geometric Asian Power Option under the Mixed Weighted Fractional Brownian Motion Environment

Xianghong Meng, Mingxuan Shen

Abstract—**This paper investigates the pricing formula for geometric Asian option where the underlying asset is driven by the mixed weighted fractional Brownian motion(MWFBM) with jumps. We obtain a closed form expression for the price of a geometric Asian option by quasi-conditional expectation pricing method under the risk-neutral measure. Moreover, we also consider Asian power options when the payoff function is a power function. Numerical experiments are performed to analyze the influence of various factors such as the strike price, jump intensity, power index on the valuation of Asian power options.**

Index Terms—**Asian option, mixed weighted fractional Brownian motion, jump diffusion, numerical analysis.**

I. Introduction

BLACK and Scholes [1] introduced the renowned Black-Scholes model, which assumes that the risk asset price follows a stochastic process driven by standard geometric Brownian motion. This model has laid the groundwork for a plethora of option pricing research. However, these models rely on the assumption of standard geometric Brownian motion, which has significant limitations when applied to real financial markets. Specifically, they fail to capture phenomena such as scale effects, seasonal patterns, heavy tails, and long-term correlations in asset price fluctuations. To address these limitations, fractional Brownian motion (FBM) was introduced. Kolmogorov [2] was the first to propose the fractional Brownian motion model, which offers a more nuanced representation of financial market behaviors by capturing long-term correlation and self-similarity. Regarding the application of FBM in option pricing, Necula [3] employed the Fourier transform method and Girsanov transformation under the risk-neutral measure to study European option pricing. More recently, Kalantari et al. [4] utilized the finite difference method to explore the pricing model of American put options within the fractional Brownian motion framework. The finite difference method is a numerical technique that discretizes partial differential equations to solve option pricing problems iteratively. This approach offers a more accurate and flexible way to model the complex dynamics of financial markets, thereby improving the precision of option pricing models.

Bojedcki [5] introduced the weighted fractional Brownian motion (WFBM) as an extension of the FBM. This novel model exhibits unique properties that set it apart from both Markov processes and semimartingales. These properties include self-similarity and orbital continuity, which enable a more nuanced representation of complex dynamics. Notably, the WFBM is distinguished from the FBM by its non-stationary increments, which provide a more flexible dependency structure. This flexibility allows the WFBM to better capture the intricate behaviors observed in financial markets compared to the traditional FBM. Sun [6] employed probability and actuarial methods to derive the pricing formula for European options under the weighted fractional Brownian motion framework, elucidating the rationale behind employing this model for option pricing purposes.

Cheridito [7] addressed the inherent arbitrage issue in the FBM-based market model by introducing the mixed fractional Brownian motion (MFBM). This approach involves constructing a linear combination of Brownian motion and fractional Brownian motion, and it has been proven that the MFBM is equivalent to the standard Brownian motion for certain parameter values, thereby effectively eliminating the arbitrage possibility in the financial market [7], [8], [9]. Building upon this foundation, Sun et al. investigated the pricing of financial derivatives with credit risk and the valuation of European currency options, developing a pricing model based on the mixed fractional Brownian motion and deriving the corresponding fractional-order partial differential equation [10], [11]. Xiao et al. [12] tackled the pricing problem of warrants in a mixed-score Brownian environment, employing numerical methods to arrive at a solution. Furthermore, Khalaf et al [13] constucted a linear combination of standard Brownian motion and weighted fractional Brownian motion, which defined as a mixed weighted fractional Brownian motion for the first time.

The introduction of jump-diffusion processes into the fractional Brownian motion model is a crucial step in capturing the intermittent jumps in stock prices caused by unexpected events, such as financial crises or natural disasters. Kim et al. [14] previously explored the analytical pricing formulas for European currency options and exchange options under the generalized mixed fractional Brownian motion, highlighting the potential of this approach in option pricing. Xu et al. [15], [16] further advanced this research by developing a European option pricing model driven by fuzzy mixed weighted fractional Brownian motion with jumps, successfully deriving explicit solutions for European call and put options by

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transforming the partial differential equation into a Cauchy problem.

In this paper, the present study seeks to extend these findings by applying the mixed weighted fractional Brownian motion with jumps to the pricing of geometric Asian power option. By incorporating both fractional Brownian motion and jump-diffusion processes, the proposed model can more accurately reflect the complex dynamics of stock prices in the real financial market. This paper aims to derive pricing formulas for geometric Asian power options under this enhanced model, providing valuable insights for practitioners and researchers in the field of option pricing.

II. Asian options and mixed weighted fractional Brownian motion

Asian option, also known as average-price option, is among the most actively traded options in the financial derivatives market. Compared with the standard option, the main difference between them is the yield of maturity, which is based on the average of the price of the underlying asset during a certain period of the option contract. It is precisely because of this feature that the risk of artificial manipulation that may exist in the market when Asian options are settled is reduced. According to whether the strike price is fixed or not, Asian option can be divided into fixed strike price and floating strike price. According to the average method of the underlying asset price, Asian options can be divided into arithmetic average and geometric average. This paper mainly considers the geometric Asian option pricing under the fixed strike price. The payoff of a fixed strike price Asian option is$(G(T)-K)^+$and$(K-G(T))^+$for a call and put option, where $G(T)=\exp[\frac{1}{T}\int_0^T \ln S_t dt]$.

Let(Ω, F_t, P)be a complete space with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions, where P represents physical probability measure.

Definition 1. The mixed weighted-fBm $\zeta_t^{a,b} = \{\zeta_t(\delta_1,\delta_2)\}_{t\geq 0}$ is a linear combination of the Brownian motion and the weighted fractional Brownian motion $\{B_t\}_{t\geq 0}$, which can be expressed as [16]

$$\zeta_t(\delta_1,\delta_2)=\delta_1 B_t+\delta_2 B_t^{a,b}, \forall t\geq 0, \tag{1}$$

where a,b are the index and satisfy the condition $a>-1, |b|<1$ and $|b|<a+1$. δ_1,δ_2 are positive constants, $\{B_t\}_{t\geq 0}$ and $\{B_t^{a,b}\}_{t\geq 0}$ are independent of each other.

For $a=0, |b|<1$, the mixed weighted-fBm corresponds to the celebrated fractional Brownian motion with Hurst index $\frac{b+1}{2}$, as well as to the well-known Brownian motion when $a=0, b=0$.

The mixed weighted-fBm $\zeta_t^{a,b}=\{\zeta_t(\delta_1,\delta_2)\}_{t\geq 0}$ has the following properties:
1) $\{\zeta_t(\delta_1,\delta_2)\}_{t\geq 0}$. is a central Gaussian process.
2) When $t=0, \zeta_0(\delta_1,\delta_2)=\delta_1 B_0+\delta_2 B_0^{a,b}=0$.
3) $\forall t,s\geq 0$, the covariance of $\zeta_t(\delta_1,\delta_2)$ and $\zeta_s(\delta_1,\delta_2)$is

$$\begin{aligned}&\operatorname{cov}(\zeta_t(\delta_1,\delta_2),\zeta_s(\delta_1,\delta_2))\\&=\delta_1^2(t\wedge s)+\delta_2^2\int_0^{t\wedge s}u^a[(t-u)^b+(s-u)^b]du,\end{aligned} \tag{2}$$

where $t\wedge s=\frac{1}{2}(t+s-\mid t-s\mid)$.

4)$\forall t\geq 0, E[(\zeta_t(\delta_1,\delta_2))^2]=\delta_1^2 t+2\delta_2^2\int_0^t u^a(t-u)^b du.$ (3)

Lemma 1. The price at every $t\in[0,T]$ of a bounded F_t^H measureable claim $U\in L^2$ is given by $U(t)=e^{-r(T-t)}\tilde{E}_t[U]$, where $\tilde{E}[\cdot]$ denotes the quasi-conditional expectation with respect to the risk-neutral measure [9].

III. Financial market modeling

It is supposed that the following assumptions are hold:
1) There are no transaction costs or taxes in buying or selling the stocks or options.
2) The transaction time and amount of assets are continuous.
3) The interest rate of deposits is same as that for loans.
4) The option can be exercised only at the maturity time.
5) The return of risk-free assets in time period is

$$\mathrm{d}M_t=rM_t dt, M_0=1, 0\leq t\leq T, \tag{4}$$

where constant r is the risk-free interest rate.
6) The stock price S_t is driven by the mixed weighted-FBM which satisfies the following equation:

$$\begin{aligned}dS_t=&(r-\lambda k)S_t dt+S_t d\zeta_t(\delta_1,\delta_2)+(e^{J_t}-1)S_t dN_t\\=&(r-\lambda k)S_t dt+\delta_1 S_t dB_t+\delta_2 S_t dB_t^{a,b}\\&+(e^{J_t}-1)S_t dN_t,\end{aligned} \tag{5}$$

where δ_1 and δ_2 represent the volatility of stock price. N_t is a passion process with rate λ, k represents the expected value of its change rate when the stock price jumps, and $k=E_Q(e^{J_i}-1)$, J_t is the jump size percent at time which is a sequence of independent identically distributed random variables, $J_i\sim N(\mu_J,\delta_J^2)$. $\{B_t\}_{t\geq 0}$, $\{B_t^{a,b}\}_{t\geq 0}$ and $\{J_t\}_{t\geq 0}$ are independent of each other.

By the Itô formula, we have

$$\begin{aligned}S_t=&S_0\exp\{(r-\frac{1}{2}\delta_1^2-\lambda k)t-\delta_2^2\int_0^t u^a(t-u)^b du\\&+\delta_1 B_t+\delta_2 B_t^{a,b}+\sum_{i=1}^{N_t}J_i\},\end{aligned} \tag{6}$$

while $N_t=n$, we have $\sum_{i=1}^{N_i}J_i\sim N(n\mu_J, n\delta_J^2)$, the above formula can be rewrite as follows:

$$\begin{aligned}S_t=&S_0\exp\{(r-\frac{1}{2}\delta_1^2-\lambda k)t-\delta_2^2\int_0^t u^a(t-u)^b du\\&+\delta_1 B_t+\delta_2 B_t^{a,b}+n\mu_J+\sqrt{n}\delta_J Z\},\end{aligned} \tag{7}$$

where $Z\sim N(0,1)$.

IV. Pricing Formula for geometric Asian option

We now obtain a closed form for the price of the geometric Asian call option with fixed strike price K and maturity time T.

Theorem 1. Suppose the stock price S_t follows the model given by (7) under the risk-neutral probability measure and the payoff function at the time of maturity is $(G(T)-K)^+$. Then the price of a geometric Asian call option $C(S_0,T)$ is given by

$$\begin{aligned}C(S_0,T)=&e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}e^{\hat{\mu}_n+\frac{\hat{\delta}_n^2}{2}}N(d_1)\\&-Ke^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}N(d_2),\end{aligned} \tag{8}$$

where $d_1 = d_2 + \sqrt{\frac{1}{3}\delta_1^2 T + \frac{\delta_2^2 \tilde{T}}{T^2} + n\delta_J^2}$,

$$\begin{aligned} d_2 =& \frac{\ln \frac{S_0}{K} + \frac{1}{2}(r - \lambda k - \frac{1}{2}\delta_1^2)T - \delta_T + n\mu_J}{\sqrt{\frac{1}{3}\delta_1^2 T + \frac{\delta_2^2 \tilde{T}}{T^2} + n\delta_J^2}}, \\ \delta_T =& \frac{\delta_2^2}{T}\int_0^T \int_0^t u^a (t-u)^b du dt, \\ \tilde{T} =& \int_0^T \int_0^t \int_0^s u^a[(t-u)^b + (s-u)^b] du ds dt \\ &+ \int_0^T \int_t^T \int_0^t u^a[(t-u)^b + (s-u)^b] du ds dt. \end{aligned}$$

Proof. Let $A(T) = \frac{1}{T}\int_0^T \ln S_t dt$, $G(T) = \exp[A(T)]$. Since the stock price S_t is log-normally distributed, the random variable $A(T)$ has Gaussian distribution under the risk-neutral probability measure. We calculate its mean and variance at first. Let $\hat{\mu}_n$ and $\hat{\delta}_n^2$ denote the mean and the variance of the random variable $A(T)$ under the risk-neutral probability measure. We notice that

$$\begin{aligned} \hat{\mu}_n =& \tilde{E}[A(T)] = \tilde{E}[\frac{1}{T}\int_0^T \ln S_t dt] = \frac{1}{T}\int_0^T \tilde{E}[\ln S_t] dt \\ =& \frac{1}{T}\int_0^T [\ln S_0 + rt - \lambda k t - \frac{1}{2}\delta_1^2 t \\ &- \delta_2^2 \int_0^t u^a (t-u)^b du + n\mu_J] dt \\ =& \ln S_0 + \frac{1}{2} rT - \frac{1}{2}\lambda k T - \frac{1}{4}\delta_1^2 T - \delta_T + n\mu_J, \end{aligned}$$

and

$$\begin{aligned} \hat{\delta}_n^2 =& \mathrm{var}[A(T)] = \tilde{E}[A(T) - \hat{\mu}_n]^2 \\ =& \tilde{E}[\frac{1}{T}\int_0^T (\delta_1 B_t + \delta_2 B_t^{a,b} + \sqrt{n}\delta_J Z) dt]^2 \\ =& \frac{\delta_1^2}{T^2}\int_0^T \int_0^T \tilde{E}[B_t B_s] ds dt \\ &+ \frac{\delta_2^2}{T^2}\int_0^T \int_0^T \tilde{E}[B_t^{a,b} B_s^{a,b}] ds dt + \frac{n\delta_J^2}{T^2}\tilde{E}[\int_0^T Z dt]^2 \\ =& \frac{\delta_1^2}{T^2}\int_0^T \int_0^T \min(t,s) ds dt + n\delta_J^2 \\ &+ \frac{\delta_2^2}{T^2}\int_0^T \int_0^T \int_0^{t\wedge s} u^a[(t-u)^b + (s-u)^b] du ds dt \\ =& \frac{1}{3}\delta_1^2 T + n\delta_J^2 \\ &+ \frac{\delta_2^2}{T^2}\Big\{ \int_0^T \int_0^t \int_0^s u^a[(t-u)^b + (s-u)^b] du ds dt \\ &+ \int_0^T \int_t^T \int_0^t u^a[(t-u)^b + (s-u)^b] du ds dt \Big\} \\ =& \frac{1}{3}\delta_1^2 T + \frac{\delta_2^2 \tilde{T}}{T^2} + n\delta_J^2. \end{aligned}$$

Hence the random variable $A(T)$ is log-normally distributed and the random variable $\log A(T)$ has the Gaussian distribution with the mean $\hat{\mu}_n$ and the variance $\hat{\delta}_n^2$ as obtained above. For the geometric Asian option, the price of a call option is

$$\begin{aligned} &C(S_0,T) = e^{-rT}\tilde{E}[(G(T)-K)^+] \\ =& e^{-rT}\sum_{n=0}^{\infty}\tilde{E}[(G(T)-K)^+ | N(T)=n] P[N(T)=n] \\ =& e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}\int_{D_1}(e^x - K)\frac{1}{\sqrt{2\pi}\hat{\delta}_n} e^{-\frac{(x-\hat{\mu}_n)^2}{2\hat{\delta}_n^2}} dx, \end{aligned}$$

The above formula is converted by the total probability formula. The following computations give the explicit formula for the function $C(S_0,T)$. Observe that

$$\begin{aligned} &C(S_0,T) \\ =& e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}\int_{D_1}(e^{\hat{\mu}_n + \hat{\delta}_n y} - K)\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \\ =& e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!} e^{\hat{\mu}_n + \frac{\hat{\delta}_n^2}{2}}\int_{-d_2}^{\infty}\frac{1}{\sqrt{2\pi}} e^{-\frac{(y-\hat{\delta}_n)^2}{2}} dy \\ &- Ke^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}\int_{-d_2}^{\infty}\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \\ =& e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!} e^{\hat{\mu}_n + \frac{\hat{\delta}_n^2}{2}}\int_{-d_2-\hat{\delta}_n}^{\infty}\frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \\ &- K\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!} e^{-rT} N(d_2) \\ =& e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!} e^{\hat{\mu}_n + \frac{\hat{\delta}_n^2}{2}} N(d_1) \\ &- Ke^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!} N(d_2), \end{aligned}$$

where

$$\begin{aligned} D_1 &= \{x : G(T) \geq K\} = \left\{x : e^{A(T)} \geq K\right\} \\ &= \{x : e^x \geq K\} = \{x : x \geq \ln K\} \\ &= \left\{y : \hat{\mu}_n + \hat{\delta}_n y \geq \ln K\right\} = \left\{y : y \geq \frac{\ln K - \hat{\mu}_n}{\hat{\delta}_n}\right\} \\ &= \{y : y \geq -d_2\}. \end{aligned}$$

V. Pricing Formula for geometric Asian power option

We will now consider computation of the price of Asian Power call option under mwfBm environment where the payoff for a call option with strike price K and maturity time T is $(G^\alpha(T) - K)^+$ for some fixed integer $n \geq 1$.

Theorem 2. Suppose the stock price S_t follows the model given by (7) under the risk-neutral probability measure and the payoff function at the time of maturity is $(G^\alpha(T) - K)^+$. Then the price of a geometric Asian call option $C(S_0,T)$ is given by

$$\begin{aligned} C(S_0,T) =& e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!} e^{\alpha\hat{\mu}_n + \frac{1}{2}\alpha^2\hat{\delta}_n^2} N(d_3) \\ &- Ke^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!} N(d_4), \end{aligned} \tag{9}$$

where

$$d_3 = d_4 + \alpha\sqrt{\frac{1}{3}\delta_1^2 T + \frac{\delta_2^2 \tilde{T}}{T^2} + n\delta_J^2},$$

$$d_4 = \frac{\ln \frac{S_0}{\sqrt[\alpha]{K}} + \frac{1}{2}(r - \lambda k - \frac{1}{2}\delta_1^2)T - \delta_T + n\mu_J}{\sqrt{\frac{1}{3}\delta_1^2 T + \frac{\delta_2^2 \tilde{T}}{T^2} + n\delta_J^2}}.$$

Proof. For the geometric Asian power option, the payoff function is $(G^\alpha(T) - K)^+ = (\exp(\alpha A(T)) - K)^+$. Following the arguments given in Theorem 6, it follows that

$$\begin{aligned} C(S_0, T) &= e^{-rT}\tilde{E}[(G^\alpha(T) - K)^+] \\ &= e^{-rT}\sum_{n=0}^{\infty}\tilde{E}[(G^\alpha(T) - K)^+ | N(T) = n]P[N(T) = n] \\ &= e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!} \\ &\quad \times \int_{D_2}(e^{\alpha x} - K)\frac{1}{\sqrt{2\pi}\hat{\delta}_n}e^{-\frac{(x-\hat{\mu}_n)^2}{2\hat{\delta}_n^2}}dx. \end{aligned} \tag{10}$$

The following computations give an explicit formula the function $C(S_0, T)$. Observe that

$$\begin{aligned} C(S_0, T) &= e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!} \\ &\times \int_{D_2}(e^{\alpha(\hat{\mu}_n + \hat{\delta}_n y)} - K)\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy \\ =& e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}e^{\alpha\hat{\mu}_n + \frac{1}{2}\alpha^2\hat{\delta}_n^2} \\ &\times \int_{-d_4}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{(y-\alpha\hat{\delta}_n)^2}{2}}dy \\ &- Ke^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}\int_{-d_4}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy \\ =& e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}e^{\alpha\hat{\mu}_n + \frac{1}{2}\alpha^2\hat{\delta}_n^2}\int_{-d_4-\alpha\hat{\delta}_n}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz \\ &- Ke^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}N(d_4) \\ =& e^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}e^{\alpha\hat{\mu}_n + \frac{1}{2}\alpha^2\hat{\delta}_n^2}N(d_3) \\ &- Ke^{-rT}\sum_{n=0}^{\infty}\frac{(\lambda T)^n e^{-\lambda T}}{n!}N(d_4), \end{aligned}$$

where

$$\begin{aligned} D_2 &= \{x : G^\alpha(T) \geq K\} = \left\{x : e^{\alpha A(T)} \geq K\right\} \\ &= \{x : e^{\alpha x} \geq K\} = \left\{x : x \geq \frac{1}{\alpha}\ln K\right\} \\ &= \left\{y : \hat{\mu}_n + \hat{\delta}_n y \geq \ln \sqrt[\alpha]{K}\right\} = \left\{y : y \geq \frac{\ln \sqrt[\alpha]{K} - \hat{\mu}_n}{\hat{\delta}_n}\right\} \\ &= \{y : y \geq -d_4\}. \end{aligned}$$

VI. Numerical Experiments

In this section, a series of numerical experiments are performed to investigate the impact of various factors on geometric Asian option. Specifically, the effect of the strike price K, the jump intensity λ, the power exponent α, and the parameters a and b are examined.

Firstly, parameters are assumed as follows: $S_0 = 10, T = 0.5, r = 0.05, \delta_1 = \delta_2 = 0.2, n = 8, \mu_J = 0.03, \delta_J = 0.06$.

Fixed parameters $\alpha = 2, a = b = 0.2$, according to Theorem 2, the value of Asian power option $C(S_0, T)$ under different strike price K and jump intensity λ can be obtained, as shown in Table I.

TABLE I: The value of Asian power option for different strike price and jump intensity

K	C				
	$\lambda_1 = 1$	$\lambda_2 = 3$	$\lambda_3 = 5$	$\lambda_4 = 7$	$\lambda_5 = 9$
70	34.550	38.532	42.538	45.916	47.347
75	30.039	34.064	38.105	41.544	43.130
80	25.768	29.826	33.892	37.379	39.105
85	21.805	25.874	29.940	33.456	35.302
90	18.206	22.248	26.286	29.802	31.743
95	15.008	18.978	22.949	26.436	28.442
100	12.225	16.074	19.939	23.363	25.406
105	9.851	13.529	17.252	20.583	22.633
110	7.861	11.328	14.876	18.085	20.117

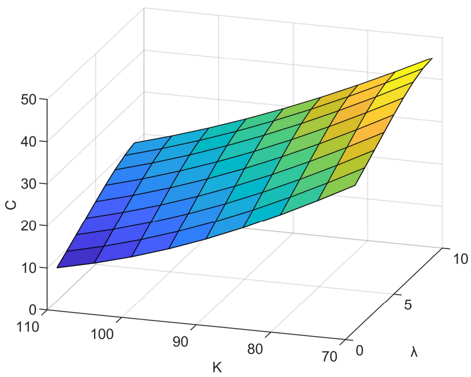

Fig. 1: Asian power option value against different strike price and jump intensity values

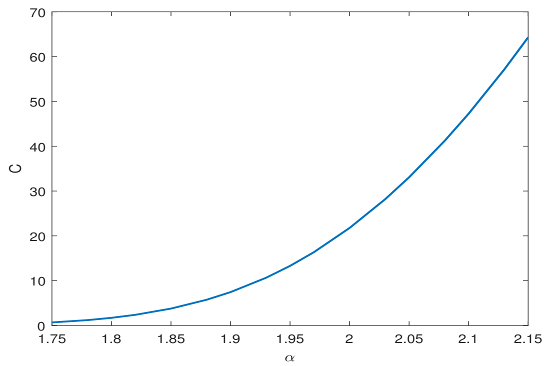

Fig. 2: The impact of power index α on the pricing of Asian power call option

Fig.1 illustrates the variation in the value of Asian power call option as the strike price and jump intensity change. It is evident that there is a negative correlation between the value of the Asian power call option and the strike price. As the strike price increases, the payoff of the option at maturity decreases, which in turn reduces the corresponding option value.

Conversely, the value of the Asian power option moves in the same direction as the jump intensity. The jump intensity reflects the degree of unsystematic risk in the underlying asset. As the jump intensity increases, the underlying asset experiences more pronounced fluctuations, characterized by a higher upper limit while the lower bound remains unchanged. This leads to an increase in the option value.

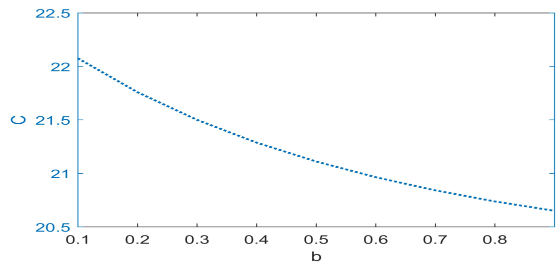

Fig. 3: The impact of parameter b on the pricing of Asian power call option with $a = 0.2, T = 0.5$

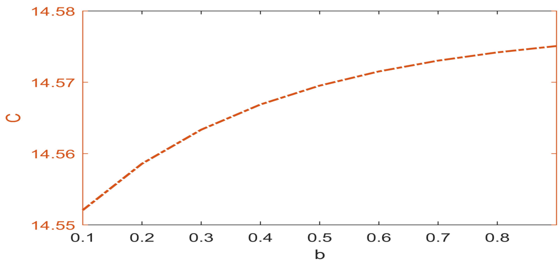

Fig. 4: The impact of parameter b on the pricing of Asian power call option with $a = 0.2, T = 1.5$

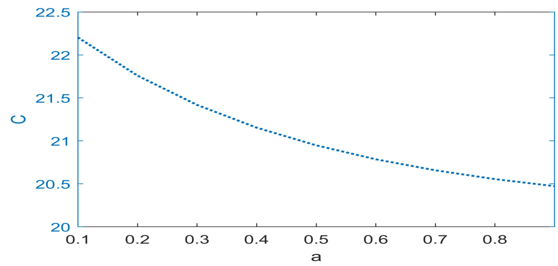

Fig. 5: The impact of parameter a on the pricing of Asian power call option with $b = 0.2, T = 0.5$

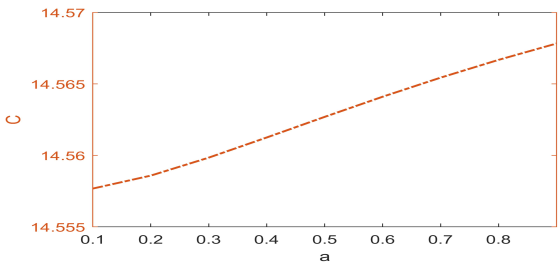

Fig. 6: The impact of parameter a on the pricing of Asian power call option with $b = 0.2, T = 1.5$

Fixed parameters $K = 100, \lambda = 6$ and $a = b = 0.2$, Fig.2 shows the positive correlation between the power index α and the price of Asian power option. It can be seen that the option price also rises with the increase of the α. In particular, when the power index exceeds a certain threshold, the increase of the power index has a more significant impact on the option price, which fully reflects the leverage effect of the power option.

While parameters $\alpha = 2, K = 100$ and $\lambda = 6$, then we investigate how the parameters a and b affect the pricing model by utilizing the control variable method. Fig.3-Fig.6 are the variation diagrams of the value of Asian power call option with the different index a and b. As depicted in Fig.3-Fig.6, we observe that when the maturity date is set at $T = 0.5$, the price of Asian power call option experiences a decline as parameters a and b increase. However, when the maturity date is set at $T = 1.5$, the option price experiences an increase as parameters a and b increase.

VII. CONCLUSIONS

This paper investigated the geometric Asian option pricing model in the environment of the mixed weighted fractional Brownian motion with jump intensity. We derived explicit solutions for both Asian call option and Asian power option by using the quasi-martingale pricing method. Finally, we simulated the impact of strike price, jump intensity, power index and parameters a, b on the pricing of option.

REFERENCES

[1] F. Black and M. Scholes, "The pricing of options and corporate liabilities," Journal of Political Economy, vol. 81, no. 3, pp. 637–654, 1973.
[2] A. N. Kolmogorov, "Wienersche spiralen und einige andere interessante kurven in hilbertscen raum, cr (doklady)," Academie Sciences URSS (NS), pp. 115–118, 1940.
[3] C. Necula, "Option pricing in a fractional Brownian motion environment," Mathematical Reports, vol. 6, no. 3, pp. 259–273, 2004.
[4] R. Kalantari and S. Shahmorad, "A stable and convergent finite difference method for fractional Black–Scholes model of American put option pricing," Computational Economics, vol. 53, pp. 191–205, 2019.
[5] T. Bojdecki, L. G. Gorostiza, and A. Talarczyk, "Occupation time limits of inhomogeneous poisson systems of independent particles," Stochastic Processes and Their Applications, vol. 118, no. 1, pp. 28–52, 2008.
[6] X. Sun, L. Yan, and H. Jing, "A weighted-fractional model to european option pricing," Theoretical Mathematics & Applications, vol. 2, no. 3, pp. 87–99, 2012.
[7] P. Cheridito, "Regularizing fractional Brownian motion with a view towards stock price modelling," Ph.D. dissertation, ETH Zurich, 2001.
[8] P. Cheridito, "Arbitrage in fractional Brownian motion models," Finance and Stochastics, vol. 7, no. 4, pp. 533–553, 2003.
[9] P. Cheridito, "Mixed fractional Brownian motion," Bernoulli, vol. 7, no. 6, pp. 913–934, 2001.
[10] X. Sun and L. Yan, "Mixed-fractional models to credit risk pricing," Journal of Statistical and Econometric Methods, vol. 1, no. 3, pp. 79–96, 2012.
[11] L. Sun, "Pricing currency options in the mixed fractional Brownian motion," Physica A: Statistical Mechanics and its Applications, vol. 392, no. 16, pp. 3441–3458, 2013.
[12] W. Xiao, W. Zhang, X. Zhang, and X. Zhang, "Pricing model for equity warrants in a mixed fractional Brownian environment and its algorithm," Physica A: Statistical Mechanics and its Applications, vol. 391, no. 24, pp. 6418–6431, 2012.
[13] A.D. Khalaf, A. Zeb, T. Saeed, M. Abouagwa, S. Djilali, and H.M. Alshehri, "A special study of the mixed weighted fractional Brownian motion," Fractal and Fractional, vol. 5, no. 4, pp. 192–192, 2021.
[14] K.H. Kim, S. Yun, N.U. Kim, and J.H. Ri, "Pricing formula for european currency option and exchange option in a generalized jump mixed fractional Brownian motion with time-varying coefficients," Physica A: Statistical Mechanics and its Applications, vol. 522, pp. 215–231, 2019.
[15] F. Xu and X. Yang, "Pricing european options under a fuzzy mixed weighted fractional Brownian motion model with jumps," Fractal and Fractional, vol. 7, no. 12, pp. 859, 2023.
[16] F. Xu and M. Han, "European option pricing in the generalized mixed weighted fractional Brownian motion," Fractals, vol. 32, no. 4, pp. 2440030, 2024.