

Severity Assessment of Nonrectangular Wave Voltage Sags from Load Side and Power Side Based on Curve Fitting

Li Ma, Jing-Hui Lou, Yu Li, Ying-Yu Chen, Xin Liu, and Yan-Hu Zhang

Abstract—Aiming at the problem of irregular waveform of nonrectangular wave voltage sags, the least square method is proposed to fit the nonrectangular wave voltage sags. Based on the fitting curve, the severity of nonrectangular wave voltage sag is assessed from load side and power grid side respectively. The proposed method improves the accuracy of nonrectangular voltage sag severity assessment. Firstly, the least square fitting method is used to fit the nonrectangular wave voltage sag. Secondly, the nonrectangular wave's amplitude and duration are used to generate a rectangular wave. The weight function method is used to assess the severity of the rectangular wave voltage sags. According to the area proportion of nonrectangular wave in rectangular wave, the severity of nonrectangular wave voltage sags from the equipment side is assessed. Concurrently, combined with the fitted nonrectangular wave mathematical model, the energy index function is used to assess the severity of nonrectangular wave voltage sag from the power side. A MATLAB model of the IEEE33 node system is built, and the severity of nonrectangular wave voltage sags obtained by simulation is assessed. The results indicate that the voltage sag severity assessment method is simple and efficient in calculation. And the assessment method is easy to operate and implement in engineering, and has high engineering application value.

Index Terms—nonrectangular wave voltage sags; severity assessment; curve fitting; weight function method; energy index

I. INTRODUCTION

Voltage sags have become one of the most serious power quality problems in power supply systems. [1]-[2]. According to the authoritative statistics of the American Electric Power Research Institute (EPRI), voltage sags account for up to 92 % of the total power quality events, causing huge economic losses. The waveform of multiple

voltage sag events at different voltage levels is analyzed statistically. The results show that nonrectangular waves account for 32.08% of all sag events. And the lower the voltage level, the greater the nonrectangular wave occupation ratio, accounting for almost 50% at a voltage level of 35kV [3]. Therefore, it is very important to accurately assess the severity of nonrectangular wave voltage sag. It is of great significance in reducing the economic losses caused by voltage sag and improving power supply reliability

In recent years, the research results of voltage sag assessment at home and abroad are quite abundant [4]-[7]. At the same time, IEC, IEEE and so on also recommend many indicators for voltage sags. Based on the existing assessment indicators, the voltage sags assessment system is divided into two categories. One is to assess the severity of the voltage sags event from load side considering the tolerance characteristics of the equipment [8]. And the other is to directly assess the severity of the voltage sags event only based on the characteristics of the voltage sags event from power side [9]. Based on the tolerance curve of sensitive equipment, experts and scholars have proposed typical indicators for the assessment of voltage sags severity from load side, including voltage sags severity index Se , voltage sags comprehensive index MDSI, and voltage sags impact index D [10]. While the Se index does not satisfy monotonicity [11], the MDSI index does not satisfy continuity [12], and the calculation process of index D is more complicated. Reference [13] used the multi-amplitude duration function to quantify the nonrectangular wave voltage sags, but did not consider the tolerance of sensitive equipment. In [14], a multi-threshold description and time series method is proposed to describe voltage sags, which avoids overestimation of nonrectangular wave voltage sags. However, the severity of a certain interval is regarded as the severity of the whole event, which will cause underestimation of nonrectangular wave voltage sags. In Reference [15], the weight function method is proposed to assess the severity of voltage sags. The weight function parameters are determined according to the tolerance characteristics of the equipment. At the same time, the severity of voltage sags duration and amplitude are considered, and the influence degree of each voltage sag characteristic quantity is truly described. However, the nonrectangular wave voltage sags are not considered. Combining the weight function method to assess the severity of nonrectangular wave voltage sags can make the assessment result of nonrectangular wave voltage sags more reasonable. The assessment indicators of power grid side

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include voltage sags incidence index SI, missing voltage-time area index MVTA, energy index E_{vs} , the SARFI index, and the voltage sags table IEC61000-2-8 for nodes and systems. The energy index takes into account both the duration and amplitude of the sag, and considers the waveform characteristics of the sag, which is reasonable [16]-[17]. When using the energy index to assess the nonrectangular wave voltage sags, if the mathematical expression of the voltage changing with time is used, the assessment result will be more accurate.

Considering the insufficient and inaccurate assessment of the severity of nonrectangular wave voltage sags mentioned above. This paper takes the nonrectangular wave voltage sag caused by induction motor starting as an example. The least square method is proposed to fit the nonrectangular wave voltage sag curve. Based on the fitting function curve, the severity of nonrectangular wave voltage sag is assessed from the load side and the power grid side respectively. The assessment results are of great significance for formulating power supply schemes, rationally planning and transforming power grids, and improving power supply quality.

This paper includes the following contents. Section 2 analyzes the waveform characteristics of nonrectangular waves. In section 3, the process of fitting nonrectangular wave voltage sags based on the least square method is introduced. In section 4, combined with the weight function method and area ratio, a method is proposed to evaluate the severity of nonrectangular wave voltage sag from load side. Section 5, comprehensively utilize two indicators: fitting function and energy index. The severity of nonrectangular wave voltage sag on the system side can be evaluated. In section 6, the IEEE33 node system is used for simulation, and the nonrectangular wave voltage sag severity assessment is obtained to verify the proposed method. Finally, Section 7 gives the conclusion.

II. ANALYSIS OF THE CAUSES AND WAVEFORM CHARACTERISTICS OF NONRECTANGULAR WAVE VOLTAGE SAGS

The voltage sags sources are mainly divided into three categories: short-circuit fault, transformer switching, and large inductive motor starting. The voltage sag waveform caused by the short-circuit fault is a rectangular wave, and the amplitude of the sag remains unchanged during the sag, and the duration is generally short, as shown in Fig. 1 (a). The voltage sags waveforms caused by the induction motor starting and the transformer switching are nonrectangular waves, the amplitude changes with time during the sag, and the duration is generally longer. Since the induction motor is a symmetrical load, a three-phase symmetrical voltage sag waveform could be caused by the induction motor starting, as shown in Fig. 1 (b). Due to the three-phase core of the transformer has varying saturation degrees, a three-phase asymmetric voltage sag waveform could be caused by transformer switching, as shown in Fig. 1 (c).

The amplitude of voltage sag caused by short circuit fault is basically unchanged in the duration, which can be regarded as rectangular wave. Therefore, most of the current traditional voltage sag severity assessment studies are only suitable for rectangular wave voltage sags.

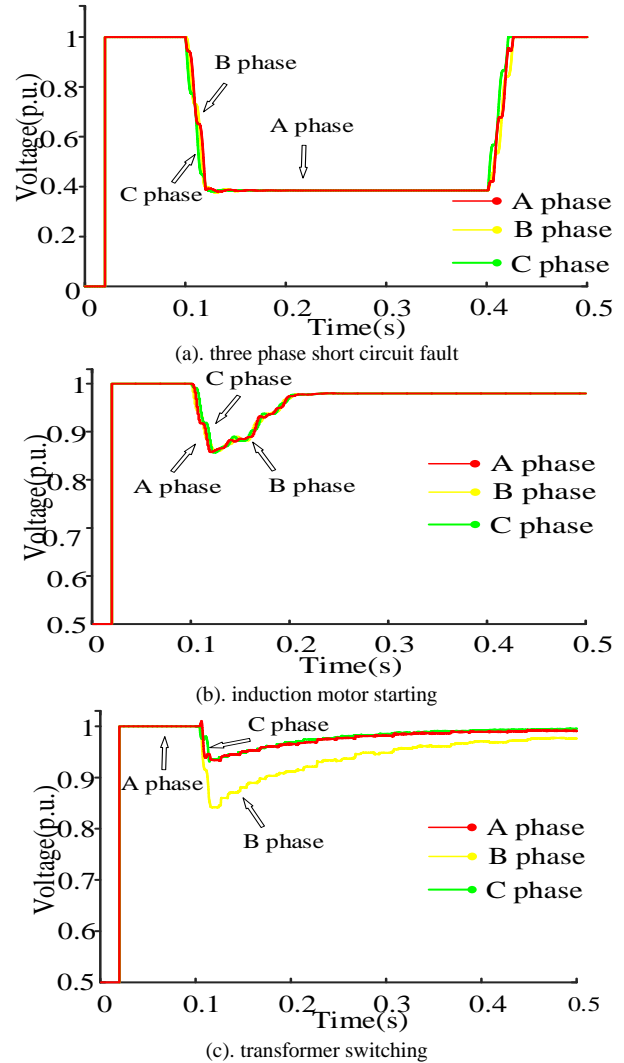


Fig. 1. Voltage sags waveforms caused by three different voltage sag sources

When assessing the severity of nonrectangular wave voltage sags, if the nonrectangular wave is only considered as a rectangular wave, the severity of nonrectangular wave voltage sags will be overestimated, because the voltage change during the sag duration is ignored. In this paper, based on the least squares fitting method, the nonrectangular wave voltage sags waveform is fitted, and the nonrectangular wave voltage sag is described more accurately, so that the assessment of the severity of nonrectangular wave voltage sag is more accurate and reasonable.

III. FITTING NONRECTANGULAR VOLTAGE SAGS WAVEFORM BASED ON THE LEAST SQUARE METHOD

Starting a large induction motor can cause voltage sag, which is the main cause of nonrectangular wave voltage sag in the power system. In this paper, taking the voltage sag caused by the induction motor starting as an example, the method of assessing the severity of nonrectangular wave voltage sag based on least square fitting is introduced. The simulation model of the induction motor starting is shown in Fig. 2. When the motor starts, the current value is 5-6 times the current value during normal operation. Before the motor reaches the rated speed, the motor current will be at a higher value. Assuming that the source impedance and the

line impedance constitute the system impedance, the system impedance voltage increases. At this time, the PCC bus will experience a voltage sag.

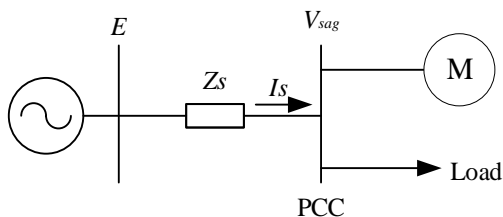


Fig. 2. Simulation model of induction motor starting

Where I_s is the starting current of the motor, and Z_s is the system impedance.

The voltage sag caused by the induction motor is three-phase symmetrical, and Figure 3 is the voltage sag of A phase waveform of the induction motor starting power. The voltage sag waveform caused by the starting of the induction motor is a nonrectangular wave, and the voltage sag amplitude is small and the duration is long. Where t_0 , t_1 and t_2 are the starting time of voltage sag, the time corresponding to the lowest point of voltage sag amplitude, the time when voltage amplitude returns to steady state, and the U_1 is the voltage sag amplitude.

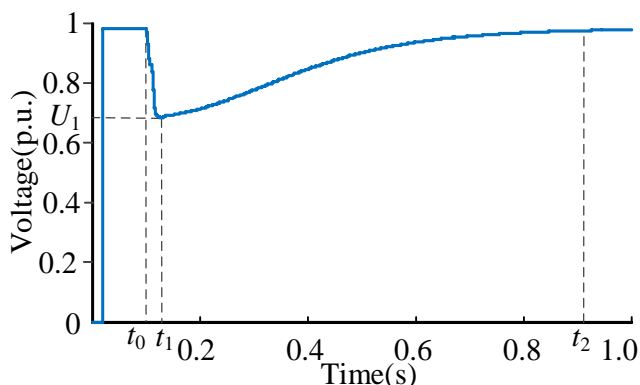


Fig. 3. Voltage sags waveform caused by induction motor starting

Taking the sag amplitude lowest point as the segmentation point, the sag waveform is fitted into a segmentation function. Observing the waveform of Fig. 3, it can be seen that the left section of the curve is approximately linear. Therefore, the left section of the curve is fitted with a linear function, and the general formula of the fitting is shown in (1). Since the right half of the curve is irregular, by comparing the fitting effects of several function curves, the curve fitted by the least squares polynomial is closest to the original curve. When the fitting is a quartic polynomial, the computational complexity is low and the fitting effect is good. The general formula of the fitting is shown in (2).

$$U_{1(t)} = kt + b \quad (1)$$

$$U_{2(t)} = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 \quad (2)$$

Where $U_{1(t)}$ is the left segment fitting function of the voltage sag waveform, and $U_{2(t)}$ is the right segment fitting function of the voltage sag waveform.

To solve the k and b parameters of formula (1), two data

points in the waveform need to be read. To solve the a_0, a_1, a_2, a_3, a_4 in formula (2), five data points of the right waveform need to be read. The data points of the left waveform are the sags starting point and the sag amplitude lowest point, respectively. The data points of the right curve take the sag amplitude lowest point and the voltage steady state point. The other three data points are evenly distributed between the sag amplitude lowest point and the voltage steady state point. As shown in Fig.4

The steps of least squares polynomial fitting are as follows:

Given the data points $(x_i, y_i), i = 1, 2, \dots, N$, find an n -degree polynomial (3) to make the formula (4) take the minimum value, and call the $p_n(x)$ that satisfies the above conditions as the least squares fitting polynomial.

$$p_n(x) = a_0 + a_1x + \dots + a_nx^n, n < N \quad (3)$$

$$S(a_0, a_1, \dots, a_n) = \sum_{i=1}^N (p_n(x_i) - y_i)^2 = \sum_{i=1}^N (\sum_{j=0}^n a_j x_i^j - y_i)^2 \quad (4)$$

S is nonnegative and is a quadratic polynomial of a_0, a_1, \dots, a_n , so it must have a minimum value. According to the necessary condition for obtaining the extreme value of a multivariate function, formula (5) can be obtained., and thus equations (6) can be obtained.

$$\frac{\partial S}{\partial a_k} = 2 \sum_{i=1}^N (\sum_{j=0}^n a_j x_i^{k+j} - y_i) x_i^k \quad (5)$$

$$= 2 (\sum_{i=1}^N \sum_{j=0}^n a_j x_i^{k+j} - \sum_{i=1}^N y_i x_i^k) = 0, k = 0, 1, 2, \dots, n$$

$$\sum_{j=0}^n (\sum_{i=1}^N x_i^{k+j}) a_j = \sum_{i=1}^N y_i x_i^k, k = 1, 2, \dots, n \quad (6)$$

By introducing formula (7), equation system (6) can be written as equation system (8).

$$s_k = \sum_{i=1}^N x_i^k, u_k = \sum_{i=1}^N y_i x_i^k \quad (7)$$

$$\sum_{j=0}^n s_{k+j} a_j = u_k, k = 1, 2, \dots, n \quad (8)$$

The system of equations (8) is a normal equation, which is expressed in matrix form as (9).

$$\begin{pmatrix} s_0 & s_1 & s_2 & \dots & s_n \\ s_1 & s_2 & s_3 & \dots & s_{n+1} \\ s_2 & s_3 & s_4 & \dots & s_{n+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_n & s_{n+1} & s_{n+2} & \dots & s_{2n} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{pmatrix} \quad (9)$$

It can be proved that when x_1, x_2, \dots, x_N are different from each other, the coefficient matrix of equation (9) is nonsingular, so equation (9) has a unique solution. According to equation (9), a_0, a_1, \dots, a_n is solved and the polynomial (10) is obtained.

$$p_n(x) = \sum_{j=0}^n a_j x^j \quad (10)$$

The waveform based on the least squares fitting is shown in Fig. 4.

As can be seen from Figure 4. The curve obtained based on least squares fitting basically coincides with the original voltage sag curve. Therefore, based on the fitted curve to evaluate the severity of nonrectangular wave voltage sags, this method has high accuracy and rationality.

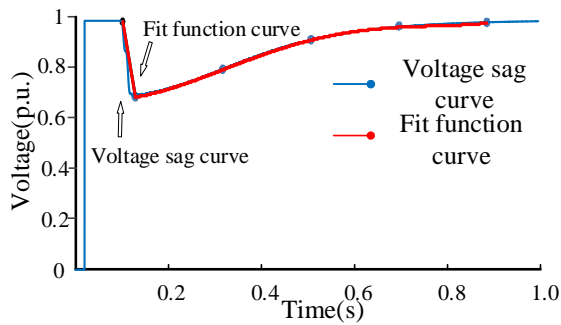


Fig.4. Nonrectangular voltage sags curve fitted by the least square method

IV. SEVERITY ASSESSMENT OF NONRECTANGULAR WAVE VOLTAGE SAGS FROM LOAD SIDE BASED ON WEIGHT FUNCTION METHOD

A. Severity assessment of rectangular wave voltage sags from load side based on weight function method

After fitting the nonrectangular wave voltage sags based on the least square fitting method, the rectangular wave voltage sag is constructed to assess the severity of the nonrectangular wave voltage sags. As shown in Fig. 5, the sag amplitude of the rectangular wave is the lowest point U_1 of the nonrectangular wave sag amplitude, the duration is the time from the nonrectangular wave sag start time t_0 to the voltage recovery steady state time t_2 , and t_1 is the time when the sag amplitude reaches the lowest point. The weight function method is used to assess the rectangular wave voltage sags severity.

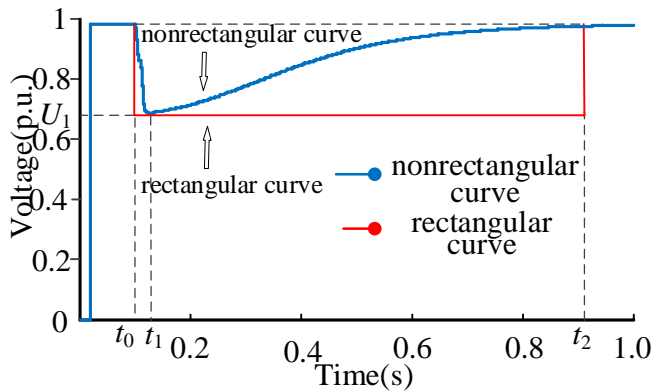


Fig. 5. Nonrectangular wave and its corresponding rectangular wave

According to the analysis of reference [15], it can be concluded that the duration and amplitude of voltage sag in the generalized tolerance curve of sensitive equipment have the following characteristics on the impact of the equipment: the two ends are gentle, and the middle changes quickly. The logistic function is used to construct the voltage sags duration influence weight function D_T , the voltage sags amplitude influence weight function D_M and the voltage sags event influence weight function D . As shown in formula (11-13).

$$D_T = \frac{1}{1 + e^{c_1 - d_1 T}} \quad (11)$$

$$D_M = \frac{1}{1 + e^{c_2 - d_2 M}} \quad (12)$$

$$D = \frac{\sqrt{D_T^2 + D_M^2}}{2} \quad (13)$$

Where T and M are the duration and amplitude of the rectangular wave voltage sags, respectively, and c_1 , c_2 , d_1 , and d_2 are the control parameters of the influence function.

Each duration interval in the IEC61000-2-8 statistical table is in the same status, but the length of the intervals varies greatly. To avoid the calculation error caused by this, the duration is standardized and each duration interval is mapped to the interval $[0,1]$. The standardized mapping interval is used for subsequent calculations. The mapping intervals corresponding to each duration are shown in Table I.

TABLE I
VOLTAGE SAGS DURATION STANDARDIZED MAPPING

Duration(s)	mapping range	Duration(s)	mapping range
(0.02, 0.1]	(0, 0.125]	(1, 3]	(0.5, 0.625]
(0.1, 0.25]	(0.125, 0.25]	(3, 20]	(0.625, 0.75]
(0.25, 0.5]	(0.25, 0.375]	(20, 60]	(0.75, 0.875]
(0.5, 1]	(0.375, 0.5]	(60, 180]	(0.875, 1]

According to the interval statistics of voltage sag amplitude and voltage sag duration, as sag events are mainly distributed within the duration range of (0.02,1) s and amplitude range of (0.7,0.9) p.u.. So it is considered that the influence degree value of voltage sags duration 0.02s is 0.05, and the influence degree value of voltage sags duration 1s is 0.9; the influence degree value of voltage sags amplitude 0.8p.u. is 0.15, and the influence degree value of voltage sags amplitude 0.1p.u. is 0.95. By solving the formula (11-12), the influence function parameters of voltage sag amplitude and voltage sag duration are obtained as follows $c_1=3$, $d_1=10$, $c_2=-4$, and $d_2=-7$.

The weight function method combines the generalized tolerance curve of sensitive equipment, and uses the logistic function as the weight function of duration and amplitude, so that the influence function has both continuity and monotonicity. The weight function method considers the severity of voltage sags duration and amplitude at the same time, and truly describes the influence degree caused by the change of each voltage sag characteristic, which avoids complex operation and makes the evaluation process simpler and more convenient.

B. Severity assessment of nonrectangular wave voltage sags from load side

The weighted function method assesses the severity of rectangular wave voltage sags based on their duration and amplitude characteristics, and the product of duration and sag depth is the area of rectangular wave voltage sags, so there is a correlation between the area and the severity of rectangular wave voltage sags. Similarly, the area and the severity of nonrectangular wave voltage sags are also correlated. Based on the correlation between area and severity, the assessment method for nonrectangular wave voltage sags severity is proposed in this chapter. According to the area ratio of nonrectangular wave in rectangular wave and the severity of voltage sag of rectangular wave, the severity of nonrectangular wave is calculated. The proposed method fully considers the waveform characteristics of

nonrectangular waves to describe voltage sags, and solves the problem of inaccurate assessment of nonrectangular wave voltage sags.

After the nonrectangular wave voltage sags are fitted by the least square method, the area S_{nr} of the nonrectangular wave sags is calculated by integration, as shown in formula (14).

$$S_{nr} = \int_{t_0}^{t_1} (1-U_{1(t)})dt + \int_{t_1}^{t_2} (1-U_{2(t)})dt \quad (14)$$

Taking the voltage sag duration (t_2-t_0) of rectangular wave as long and the voltage sag amplitude $(1-U_1)$ as wide. The calculation formula of rectangular wave area S_r is shown in formula (15).

$$S_r = (t_2 - t_0)(1 - U_1) \quad (15)$$

After the areas of rectangular wave and nonrectangular wave and the severity of rectangular wave voltage sags are calculated, according to the area ratio of nonrectangular wave in rectangular wave, the severity of voltage sag of nonrectangular wave is calculated. The nonrectangular wave voltage sag severity function is established as shown in formula (16).

$$D_{nr} = D(U_1, (t_2 - t_0)) \frac{S_{nr}}{S_r} \quad (16)$$

Where $D(U_1, (t_2 - t_0))$ is the severity of the rectangular wave voltage sag corresponding to the nonrectangular wave.

V. SEVERITY ASSESSMENT OF NONRECTANGULAR WAVE VOLTAGE SAGS FROM GRID SIDE BASED ON ENERGY INDEX

The energy index considers the duration and amplitude of

voltage sags, and uses the integral form to take into account the waveform characteristics of voltage sags, which is reasonable. Based on the nonrectangular wave voltage sag function fitted by the least square method, this chapter uses the energy index to assess the severity of the nonrectangular wave voltage sags from power side.

The energy index E_{VS} of voltage sag events recommended by IEEE in P1564 standard is shown in formula (17).

$$E_{VS} = \int_0^T \left[1 - \left(\frac{U(t)}{U_{nom}} \right)^2 \right] dt \quad (17)$$

Where U_{nom} is the nominal voltage; T is the time of the sag; and $U(t)$ is the RMS of the time-varying voltage during the sagging.

The traditional assessment of nonrectangular wave voltage sag based on energy index usually does not distinguish the waveform of voltage sags, and directly replaces the time-varying voltage with the lowest point of sag amplitude. The method introduces the lowest point of voltage sag amplitude to assess the severity of voltage sags, although it can simplify the calculation, will cause excessive evaluation of nonrectangular wave voltage sag.

In this chapter, the nonrectangular wave voltage sag function obtained by least squares fitting is substituted into the energy index function, as shown in equation (18), which avoids the overestimation of nonrectangular wave voltage sags.

$$E_{VS} = \int_{t_0}^{t_1} \left[1 - \left(\frac{U_{1(t)}}{U_{nom}} \right)^2 \right] dt + \int_{t_1}^{t_2} \left[1 - \left(\frac{U_{2(t)}}{U_{nom}} \right)^2 \right] dt \quad (18)$$

Where $U_{1(t)}$ and $U_{2(t)}$ are the left and right functions of the fitted nonrectangular wave curve, respectively.

The severity assessment flow chart of nonrectangular wave voltage sags from the load side and the power side based on curve fitting, as shown in Fig. 6.

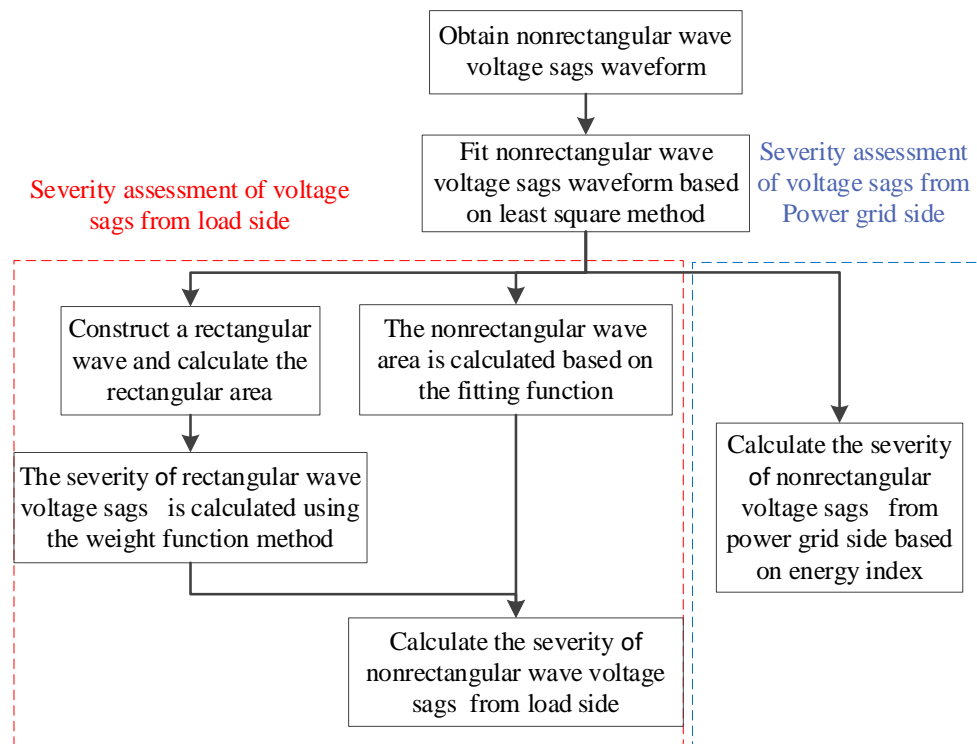


Fig. 6. Flow chart for assessing the severity of nonrectangular wave voltage sags based on curve fitting

VI. CASE ANALYSIS

In order to verify the method proposed, an IEEE33 node system model is built in MATLAB. The IEEE33 node system model is a radial network with 33 bus nodes. The voltage of bus 1 is 12.66 kV, and the transformer capacity is 10 MVA. Transformers and induction motors are connected to bus 7 and bus 29 respectively, as shown in Fig. 7.

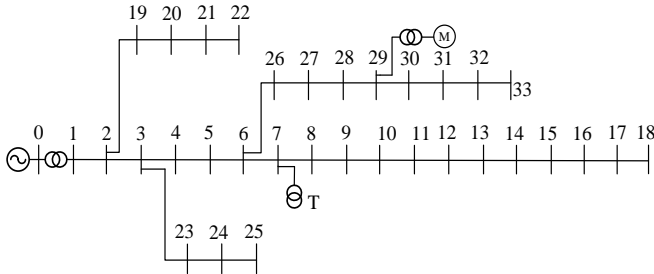


Fig. 7. Schematic diagram of the IEEE 33-bus radial network and the connection points of the transformer and the induction motor

A. Assessment of the severity of voltage sags caused by induction motor starting

The normal operating power of the induction motor is set to 1800 W, the rated voltage is 400 V, the rated frequency is 60 Hz, and the motor starts at 0.1 s.

The voltage sag waveform of bus 29 caused by induction motor starting is shown in Fig. 8. The voltage sags waveform is a nonrectangular wave, the amplitude of the sag is 0.608, and the duration is 1 s. Based on the weight function method, the corresponding rectangular wave severity is calculated as $D(0.608,1)=0.491$. Six points in the curve are read for least squares polynomial fitting of the nonrectangular wave, as shown in Table II.

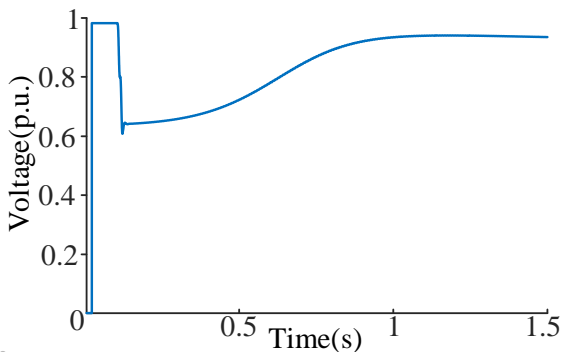


Fig. 8. Voltage sags waveform caused by induction motor starting

TABLE II
LEAST SQUARES POLYNOMIAL FITTING DATA POINTS

Time(s)	Voltage(p.u.)
0.1	0.982
0.116	0.608
0.366	0.673
0.616	0.792
0.866	0.914
1.1	0.941

The nonrectangular wave is segmented and fitted. The left half of the waveform is fitted as a linear function, and the right half is fitted as a quartic function based on the least square method. The fitted function waveform is shown in Fig. 9.

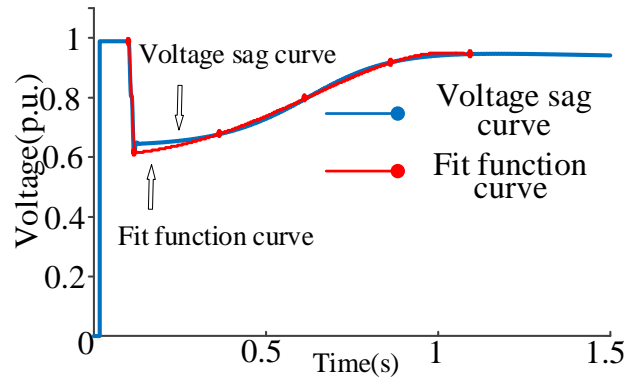


Fig. 9. The original voltage sags waveform and the fitted function waveform

The expressions of the segmentation function fitting are shown in equations (19)-(20).

$$U_{1(t)} = -23.375t + 3.3195 \quad (0.1 \leq t \leq 0.116) \quad (19)$$

$$U_{2(t)} = -0.5463t^4 + 0.5289t^3 + 0.3245t^2 + 0.042t + 0.598 \quad (0.116 \leq t \leq 1.1) \quad (20)$$

In Fig. 9, the fitting curve based on the least squares and the original voltage sags curve are basically coincident, so it is reasonable, accurate and effective to use the fitting curve instead of the nonrectangular wave voltage sags waveform to calculate.

1) Assessment of the severity of voltage sags caused by induction motor starting from load side

The functions $U_{1(t)}$ and $U_{2(t)}$ are obtained by using the formula (19-20), and they are substituted into the formula (14) to calculate the area of nonrectangular waves, and the area of rectangular wave is calculated according to formula (15), as shown in formula (21-22). Since the initial steady state value of the voltage is 0.982, 1 in formula (14) is replaced by 0.982.

$$S_{nr} = \int_{0.1}^{0.116} (0.982 - U_{1(t)}) dt + \int_{0.116}^{1.1} (0.982 - U_{2(t)}) dt \quad (21)$$

$$S_r = (1.1 - 0.1)(0.982 - 0.608) \quad (22)$$

The area of nonrectangular wave is calculated by formula (21) as $S_{nr}=0.206$, and the area of the rectangular wave is calculated by formula (22) as $S_r=0.374$. Furthermore, according to formula (16), the severity of the voltage sag caused by induction motor starting from load side is calculated to be $D_{nr}=0.49*0.206/0.374=0.27$.

2) Assessment of the severity of voltage sags caused by induction motor starting from power grid side

The functions $U_{1(t)}$ and $U_{2(t)}$ are obtained by using the formula (19-20), and they are substituted into the formula (18) to calculate the energy index value for this voltage sag.

$$E_{VS} = \int_{0.1}^{0.116} \left[1 - \left(\frac{U_{1(t)}}{0.982} \right)^2 \right] dt + \int_{0.116}^{1.1} \left[1 - \left(\frac{U_{2(t)}}{0.982} \right)^2 \right] dt \quad (23)$$

The power side energy index value of this voltage sag event is calculated by formula (23) as $E_{VS}=0.342$.

3) Compared with the weight function method and the energy index function assessment results

The assessment results of the assessment method in this paper are compared with the direct assessment results of the traditional use weight function method. The comparison results are shown in Table III.

TABLE III
COMPARISON OF SEVERITY ASSESSMENT RESULTS BASED ON CURVE FITTING AND UNCURVE FITTING

Assessment method	Severity of load Side (Weight Function)	severity of Grid side (energy index)
uncurve fitting	0.49	0.617
curve fitting	0.27	0.342

The traditional load side is based on the weight function method and the grid side is based on the energy index function voltage sag severity assessment method. When assessing the nonrectangular wave voltage sag waveform, the waveform characteristics are not considered, and the voltage sag minimum point is directly used instead of the sag amplitude. Although this assessment method can simplify the assessment steps and reduce the amount of calculation, it can be seen from the comparison with the method in this paper that the direct use of the lowest point of voltage sag as the amplitude of voltage sag will cause the over evaluation of the severity of nonrectangular wave voltage sag. Therefore, the assessment results of the nonrectangular wave voltage sag severity assessment method based on curve fitting are more accurate.

B. Assessment of the severity of voltage sags caused by transformer switching

Since the voltage sag caused by transformer switching is a three-phase asymmetric voltage sag, to avoid underestimation of voltage sag, the most serious phase of voltage sag is used to assess the severity of voltage sag caused by transformer switching. The transformer connected to bus 7 is a three-phase two-winding transformer with a saturated core. The rated capacity of the transformer is 1MVA, the load size is set to 5kW, and the transformer is connected at 0.2s.

The A-phase voltage sag waveform of bus 7 caused by transformer switching is shown in Fig. 10. The voltage sags waveform is a nonrectangular wave, the amplitude of the sag is 0.77, and the duration is 1s. Based on the weight function method, the corresponding rectangular wave severity is calculated as $D(0.77,1)=0.45$. Six points in the curve are read for least squares polynomial fitting of the nonrectangular wave, as shown in Table 3.

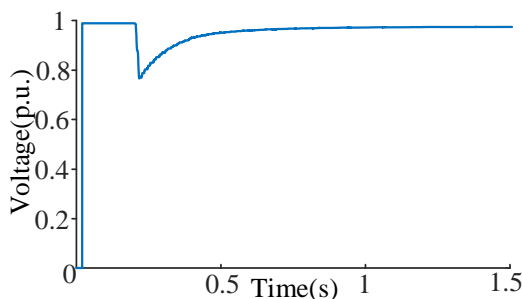


Fig. 10. The voltage sags waveform caused by transformer switching

TABLE III
LEAST SQUARES POLYNOMIAL FITTING DATA POINTS

Time(s)	Voltage(p.u.)
0.2	0.992
0.216	0.77
0.412	0.94
0.608	0.97
0.804	0.978
1.2	0.98

The segmentation function expression obtained by fitting is shown in equations (24)-(25).

$$U_{1(t)} = -13.875t + 3.767 \quad (0.2 \leq t \leq 0.216) \quad (24)$$

$$U_{2(t)} = -2.363t^4 + 7.4325t^3 - 8.5113t^2 + 4.2638t + 0.1764 \quad (0.216 \leq t \leq 1.2) \quad (25)$$

1) Assessment of the severity of voltage sags caused by transformer switching from load side

The functions $U_{1(t)}$ and $U_{2(t)}$ are obtained by using the formula (24-25), and they are substituted into the formula (14) to calculate the area of nonrectangular waves, and the area of rectangular wave is calculated according to formula (15), as shown in formula (26)-(27).

Since the initial steady state value of the voltage is 0.992, 1 in formula (14) is replaced by 0.992.

$$S_{nr} = \int_{0.2}^{0.216} (0.992 - U_{1(t)})dt + \int_{0.216}^{1.2} (0.992 - U_{2(t)})dt \quad (26)$$

$$S_r = (1.2 - 0.2)(0.992 - 0.77) \quad (27)$$

The area of nonrectangular wave is calculated by formula (26) as $S_{nr}=0.0346$, and the area of the rectangular wave is calculated by formula (27) as $S_r=0.222$. Furthermore, according to formula (16), the severity of the voltage sag caused by transformer switching from load side is calculated to be $D_m=0.45*0.0346/0.222=0.07$.

2) Assessment of the severity of voltage sags caused by transformer switching from power grid side

The functions $U_{1(t)}$ and $U_{2(t)}$ are obtained by using the formula (24-25), and they are substituted into the formula (18) to calculate the energy index value for this voltage sag.

$$E_{vs} = \int_{0.2}^{0.216} \left[1 - \left(\frac{U_{1(t)}}{0.992} \right)^2 \right] dt + \int_{0.216}^{1.2} \left[1 - \left(\frac{U_{2(t)}}{0.992} \right)^2 \right] dt \quad (28)$$

The power side energy index value of this voltage sag event is calculated by formula (28) as $E_{vs}=0.07$.

Based on the least squares curve fitting method, this chapter assesses the severity of nonrectangular voltage sags caused by two different voltage sags sources from power grid side and load side. The fitting results show that the least squares method can well fit the nonrectangular wave voltage sags curve and avoid the problem of overestimation and underestimation. The fitting function is used to assess the severity of nonrectangular wave voltage sags, which makes the assessment result more accurate and the assessment process more convenient and faster.

3) Compared with the weight function method and the energy index function assessment results

TABLE IV
COMPARISON OF SEVERITY ASSESSMENT RESULTS BASED ON CURVE FITTING AND UNCURVE FITTING

Assessment method	Severity of load Side (Weight Function)	severity of Grid side (energy index)
uncurve fitting	0.45	0.397
curve fitting	0.007	0.007

It can be seen from Fig.10 that the voltage sag waveform recovery stage caused by transformer switching is convex, that is, the time required for the voltage to return to a higher value is shorter, but the time to return to the steady state value is longer. According to the tolerance characteristics of the sensitive equipment, the higher voltage sag amplitude has little effect on the sensitive equipment. Therefore, the severity of non-rectangular wave

voltage sag is smaller when the voltage amplitude is higher. However, the original evaluation method uses the lowest point of the sag amplitude to replace the time-varying voltage during the sag to evaluate the non-rectangular wave. Through the comparison of Table 5.5, it can be seen that the severity of the voltage sag will be over-evaluated. Therefore, the evaluation method of non-rectangular wave voltage sag severity based on curve fitting in this paper is more accurate.

VII. CONCLUSION

In this paper, a method for assessing the severity of nonrectangular wave voltage sags based on curve fitting is proposed, the severity of nonrectangular wave voltage sag is assessed from load side and power grid side respectively.

1) Aiming at the problem of irregular waveform of nonrectangular wave voltage sags, the least square method is proposed to fit the nonrectangular wave voltage sags, and the severity of nonrectangular wave voltage sags is assessed based on the fitting curve. The waveform characteristics of nonrectangular wave voltage sags are fully considered, and the overestimation and underestimation of the severity of nonrectangular wave voltage sags are avoided.

2) Taking the weight function method as the severity assessment index of the load side, the tolerance characteristics of the sensitive equipment are considered, and the influence degree caused by the change of each feature quantity is truly described. The assessment results provide a reference for the access of user sensitive devices.

3) Taking the energy index as the severity assessment index of the power side, the duration, amplitude and waveform of the voltage sag event are fully considered. The assessment results can reflect the energy loss caused by the voltage sag event and the power supply quality of the grid.

4) The severity of the nonrectangular wave voltage sag obtained by simulation is assessed, it is verified that the assessment process of the proposed method is simple and efficient, the assessment.

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