Interval Valued Secondary k-Range Symmetric Fuzzy Matrices with Generalized Inverses

H. Prathab, N. Ramalingam, E. Janaki, A. Bobin, V. Kamalakannan and M. Anandhkumar

***Abstract*—This research examines an interval-valued secondary k-Range symmetric fuzzy matrix. It discusses the relationships between different types of matrices, specifically interval-valued s-k Range symmetric, interval-valued k-Range symmetric, and interval-valued Range symmetric matrices. The study establishes the necessary and sufficient criteria for an interval-valued s-k Range symmetric fuzzy matrix. It is demonstrated that s-symmetry implies s-Range symmetric and the reverse is necessarily true. Also, we illustrate a graphical representation of Kernel symmetric, Column symmetric, and Range symmetric adjacency and incidence fuzzy matrices.**

Every adjacency fuzzy matrix is symmetric, Range symmetric, Column symmetric, and Kernel symmetric, but the incidence matrix satisfies only the KS conditions. Similarly, every Range symmetric adjacency fuzzy matrix is a Kernel symmetric adjacency fuzzy matrix, but a Kernel symmetric adjacency fuzzy matrix need not be a Range symmetric fuzzy matrix. Also, in every isomorphic graph, its adjacency fuzzy matrix is a Kernel symmetric, Range symmetric, and Column symmetric adjacency fuzzy matrix, but the converse need not be true.

Additionally, equivalent criteria for various g-inverses of an interval-valued s-k Range symmetric fuzzy matrix being interval-valued s-k Range symmetric matrices are also established. The generalized inverses of an interval-valued s-k Range symmetric matrix corresponding to the sets A{1,2}, A{1, 2, 3} and A{1, 2, 4} are characterized.In this paper, we present an application of soft graphs in decision-making through the use of the adjacency matrix of a soft graph. We have developed an algorithm for this purpose and provide an example to demonstrate its application.

***Index Terms*—Interval-valued fuzzy matrix, Range symmetric Interval-valued fuzzy matrix, s-k-Range symmetric Interval-valued fuzzy matrices.**

Abbreviations

IV = Interval-valued
RS = Range symmetric
CS = Range symmetric
KS = Kernel symmetric
IFM = Intuitionistic fuzzy matrix
NFM = Neutrosophic fuzzy matrix

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I. Introduction

ALl matrices used in this study are of an IV fuzzy matrix type. Jaya Shree [**1**] has studied secondary k-KS fuzzy matrices. Anandhkumar et.al [**2**] have studied Secondary K-RS NFM. An IV fuzzy matrix is defined as A = (a_{ij})= $[a_{ijL}, a_{ijU}]$, where each a_{ij} is a subinterval of the $[0, 1]$ interval. If F_{nn}, where x + y = max{x,y} and x.y = min{x,y} is the collection of each and every nxn fuzzy matrix supported by 0 and 1 under the procedure. A1 denotes a regular fuzzy matrix A's set of all g-inverses. If A^{+} exists for a fuzzy matrix, then it coincides with A^{+}. A fuzzy matrix A is RS and KS is denoted by $R(A^T) = R(A)$ and $N(A^T) = N(A)$. It is commonly known that for complex matrices, the concepts of range and KS are equivalent. Additionally, this fails for IV fuzzy matrices.

Meenakshi [**3**] has studied fuzzy matrix theory and applications. Matrices having symmetric entries relative to the secondary diagonal, referred to as secondary symmetric matrices, were first studied by Ann Lee [**4**]. Antoni, Cantoni, and Butler Paul [**5**] conducted an examination of the relevance of per-symmetric matrices, which are matrices symmetric with respect to both diagonals, in the field of communication theory. Studies on generalized fuzzy matrices have been conducted by Kim and Roush [**6**].

Water and Hill [**7**] have developed a theory pertaining to s-real and s-Hermitian matrices as a generalization of k-real and k-Hermitian matrices. Meenakshi and Krishnamoorthy [**8**] have studied secondary k-Hermitian matrices. Meenakshi, Krishnamoorthy, and G. Ramesh [**9**] have discussed s-k-EP matrices. Meenakshi and Jaya Shree [**10**] have studied k-KS matrices. Jaya Shree [**11**] has characterized K-RS matrices. Anandhkumar et al. [**12**] have studied various inverses of neutrosophic fuzzy matrices. Shyamal and Pal [**13**] have studied IV fuzzy matrices. Meenakshi and Kalliraja [**14**] have focused on regular IV fuzzy matrices.

Anandhkumar et al. [**15**] have discussed generalized symmetric neutrosophic fuzzy matrices. Xiaomin Gong et al. [**16**] have studied a BWM-TODIM-based integrated decision framework for financial technology selection with interval type-2 fuzzy sets. Qianhong Zhang and Bairong Pan [**17**] have focused on qualitative analysis of k-order rational fuzzy difference equations. Wenjun Sun et al. [**18**] have studied prospect selection decisions for emergency logistics paths in fuzzy environments. As a specific illustration, building upon and expanding the findings related to complex matrices, we introduced and extended the notion of IV s-k Hermitian and IV RS matrices in the context of fuzzy matrices. We also expanded many basic conclusions on these two types of matrices.

Jaya Shree [19] has studied secondary k-RS fuzzy matrices. Kaliraja and Bhavani [20] have focused on interval-valued secondary k-RS fuzzy matrices. Anandhkumar et al. [21] have discussed pseudo similarity of neutrosophic fuzzy matrices. Anandhkumar et al. [22] have discussed reverse sharp and left-T, right-T partial ordering on neutrosophic fuzzy matrices. Anandhkumar et al. [23] have focused on reverse tilde (T) and minus partial ordering on intuitionistic fuzzy matrices. Anandhkumar et al. [24] have highlighted results on partial orderings, characterizations, and generalization of k-idempotent neutrosophic fuzzy matrices. Anandhkumar et al. [25, 26] have discussed secondary k-CS neutrosophic fuzzy matrices and interval-valued secondary k-RS neutrosophic fuzzy matrices. Punithavalli, Anandhkumar [27] have discussed Kernel And K-Kernel Symmetric Intuitionistic Fuzzy Matrices. Anandhkumar et.al [28] have studied Generalized Symmetric Fermatean Neutrosophic Fuzzy Matrices. Punithavalli1 and Anandhkumar [29] have discussed Reverse Sharp and Left-T Right-T Partial Ordering On Intuitionistic Fuzzy Matrices.

The structure of the article is as follows.

In section I, we present intraduction,
In section II, we discuss Research gap.
In section III, we introduce Notation.
In section IV, we present some elementary definitions.
In section V, we provide graphical representations of RS, CS, and KS adjacency matrices.
In section VI, we introduce interval-valued secondary k-RS fuzzy matrices.
In section VII, we discuss interval-valued s-k RS regular fuzzy matrices.
In section VIII, we provide results on interval-valued s-k RS intuitionistic fuzzy matrices.
In section IX, we discuss KS neutrosophic fuzzy matrices.
In section X, we highlight s-k-KS regular neutrosophic fuzzy matrices.
In section XI, we provide results on IV KS, k-KS, RS Neutrosophic fuzzy matrices.
In section XII, we discuss Application of adjacency Neutrosophic fuzzy matrix of a graph in decision making.

II. Research gap

In the preceding introduction section, Meenakshi and Jaya Shree introduced the concept of s-RS matrices, while Meenakshi and Jayashri further developed the notion of RS fuzzy matrices. In this study, we extend these ideas to interval-valued secondary RS fuzzy matrices with generalized inverses. This framework plays a crucial role in the hybrid real matrix structure, and we apply it to fuzzy matrices, examining specific results in depth.

Initially, we present alternative characterizations of interval-valued secondary RS fuzzy matrices. Subsequently, we provide an example of a secondary s-RS fuzzy matrix. We also explore various g-inverses associated with regular matrices and establish a characterization of the set of all inverses using secondary s-RS fuzzy matrices.

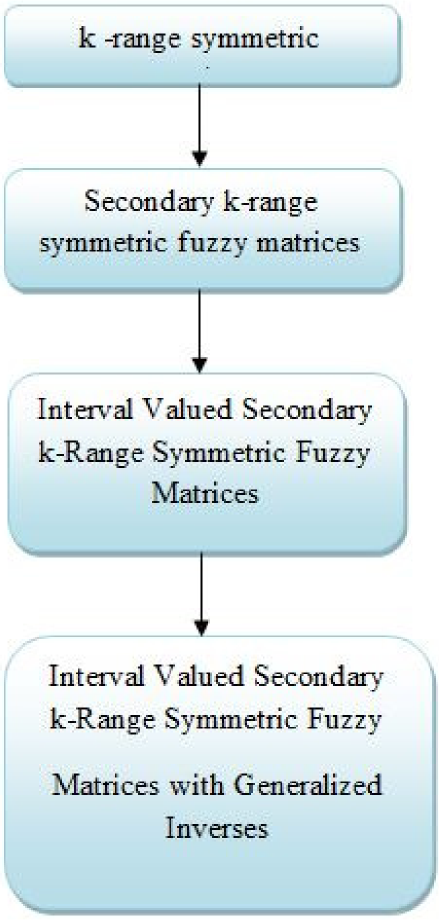

Figure 1:Interrelation of k-RS

III. Notations

A^+ = Moore-penrose inverse of fuzzy matrix A
A^T = Transpose of the fuzzy matrix A
R(A) = Row space of fuzzy matrix A
C (A) = Column space of fuzzy matrix A
N(A) = Null space of fuzzy matrix A
F_{mn} = IV fuzzy matrix

IV. Preliminaries

Definition IV.1. *The ij^{th} entry of the matrix A is an interval reflecting the membership values of an IV fuzzy matrix of order mn.*
Each interval in IVFM is an element and a subinterval of the interval [0,1]. E and F can be used to represent any two IVFMs. In the case of any two elements, $e \in E$ and $f \in F$, where e= $[e_L, e_U]$ and f= $[f_L, f_U]$ are intervals between 0 and 1,
so, $e_L < e_U$, $f_L < f_U$.
(i) $e + f = [max\{e_L, f_L\}, max\{e_U, f_U\}]$

(ii) $e + f$ = $[min\{e_L, f_L\}, min\{e_U, f_U\}]$
For X =(x_{ij}) = $[x_{ijL}, x_{ijU}]$, *Z* =(z_{ij}) = $[z_{ijL}, z_{ijU}]$ *of order mn with their sum denoted as*
X+Z =$(x_{ij} + z_{ij})$ = $[x_{ijL} + z_{ijL}, x_{ijU} + z_{ijU}]$
For X =$(x_{ij})_{mn}$ *and Z* =$(z_{jj})_{np}$ *with their product denoted by*
XZ =$(y_{ij})_{mp}$=$\sum_{k=1}^{10} a_{ik}b_{kj}$, *i=1,2,...m*

XZ =*[*$(y_{ij})_{mp}$=$\sum_{k=1}^{10} a_{ikL}b_{kjL}\sum_{k=1}^{10} a_{ikU}b_{kjU}$*]*,

X≤Z iff x_{ijL}≤z_{ijL} *and* x_{ijU}≤z_{ijU}
x_{ijL}=z_{ijL} *and* x_{ijU}=z_{ijU} *maximum and minimum fuzzy matrix composition.*

Definition IV.2. If $k(y)$=$(y_{k[1]}, y_{k[2]}, y_{k[3]} \cdots, y_{k[n]})$∈$F_{n1}$, where K is involuntary, the corresponding permutation matrix is satisfied using the following.

$(P_{2.1})KK^T$ = K^TK = I_n, K = K^T, K^2=I
By definition of V, and R(y) = Ky

$(P_{2.2})$ V =V^T, VV^T =V^TV = I_n and V^2 = I

$(P_{2.3})$ N(A) = N (AV), N (A) = N (AK)

$(P_{2.4})$ $(AV)^T$ = VA^T, $(VA)^T$ =A^TV

$(P_{2.5})$ If A^+ exists, then $(AV)^T$ = VA^T, $(VA)^T$ =A^TV.

Definition IV.3. For IV fuzzy matrix A is KS fuzzy matrix iff N (A) = $N(A^T)$.

Definition IV.4. Let A be a fuzzy matrix, if R[A]= $R[A^T]$, then A is called as RS.

Example IV.1. Let us consider fuzzy matrix,

$$A = \begin{bmatrix} 0.2 & 0 & 0.7 \\ 0 & 0 & 0 \\ 0.7 & 0 & 0.3 \end{bmatrix}$$

The following matrices does not satisfies the RS condition

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(1 1 0)∈ R(A), (1 1 0)∈ $R(A^T)$
(0 1 1)∈ R(A), (0 1 1)∈ $R(A^T)$
(0 0 1)∈ R(A), (0 0 1)∉ $R(A^T)$
R(A)∉ $R(A^T)$

Definition IV.5. Let A be a fuzzy matrix, if C[A]= $C[A^T]$, then A is called as CS.

Definition IV.6. An adjacency matrix is a square matrix that serves as a representation for a finite graph. The matrix's elements convey information regarding whether pairs of vertices's within the graph are connected or not.

In the specific scenario of a finite simple graph, the adjacency matrix can be described as a binary matrix, often denoted as a (0,1)-matrix, where the diagonal elements are uniformly set to zero.

Let $G(V, E)$ be a simple graph with n vertices's. Then the adjacency matrix $A = [a_{ij}]$ is a symmetric matrix defined by

$$A = [a_{ij}] = \begin{cases} 1, & \text{if } v_i \text{ is adjacent to } v_j \\ 0, & \text{otherwise.} \end{cases}$$

It is denoted by $A(G)$ or A_G.

Example IV.2. Let us consider adjacency matrix and corresponding graph

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

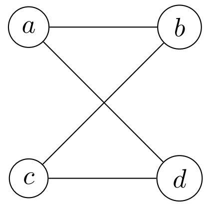

Figure 2: Adjacency Graph

Definition IV.7. *Let $G(V, E)$ be a simple graph with n vertices. Let $V = \{V_1, V_2, \cdots, V_n\}$ and $E = \{e_1, e_2, \cdots, e_m\}$. Then, the incidence matrix $I = [m_{ij}]$ is a matrix defined by*

$$A = [m_{ij}] = \begin{cases} 1, & \textit{if } v_i \textit{ is incident to } e_j \\ 0, & \textit{otherwise.} \end{cases}$$

It is denoted by A(G) or A_G.

Example IV.3. *Let us consider incidence matrix and corresponding graph. The incident matrix is*

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

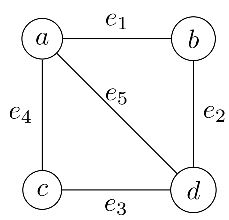

Figure 3: Incidence Graph

Definition IV.8. ***Isomorphism Graph***
Two graphs with the same number of vertices, the same number of edges, and the same degree sequence, and whose fuzzy matrices are equal, are isomorphic.

V. Graphical representation of RS, CS and KS adjacency matrices

Graph A

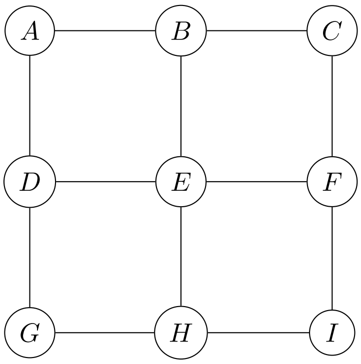

Figure 4: Adjacency Graph

Adjacency Matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

The given graph is RS fuzzy matrix $R(A) = R(A^T)$.

Graph B

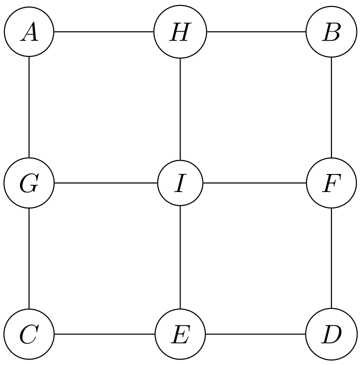

Figure 5: Adjacency Graph

Adjacency Matrix

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

The given graph is CS fuzzy matrix $C(A) = C(A^T)$

Graph C

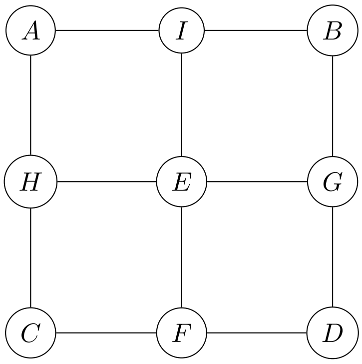

Figure 6: Adjacency Graph

Adjacency Matrix

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The given graph is KS fuzzy matrix $N(A) = N(A^T)$.

Note 1 Every adjacency matrix is symmetric, RS, CS, and KS, but the incidence matrix satisfies only the KS condition.

Note 2 Every fuzzy matrix that exhibits range symmetry is also a fuzzy matrix with kernel symmetry, but it's important to note that not every fuzzy matrix with kernel symmetry necessarily exhibits range symmetry.

Isomorphism Graph The two given graphs have the same number of vertices, the same number of edges, the same degree sequence, and identical adjacency fuzzy matrices. Therefore, the given graphs are isomorphic and their adjacency fuzzy matrices are KS, CS, and RS.

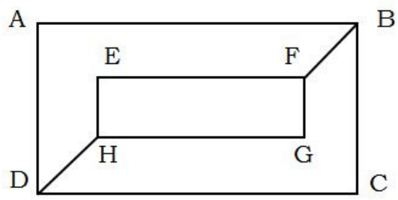

Figure 7: Non isomorphism Graph

$$A_U = \begin{bmatrix} & O & R & P & Q & S & V & T & U \\ O & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ R & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ P & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ Q & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ S & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ V & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ T & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ U & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

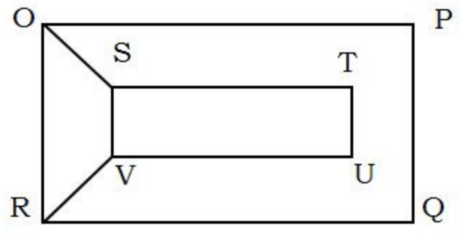

Figure 8: Non isomorphism Graph

$$A_V = \begin{bmatrix} & A & B & D & C & E & F & H & G \\ A & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ B & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ D & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ C & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ E & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ F & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ H & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ G & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

The two given graphs (Figures 7,8) have the same number of vertices, the same number of edges, and the same degree sequence, but the adjacency fuzzy matrices are not equal. Therefore, the graphs in Figure 7,8 are not isomorphic.

Note 3 Every isomorphic graph has KS, RS, and CS adjacency fuzzy matrices, but the converse need not be true.

VI. IV SECONDARY K-RS FUZZY MATRICES

Definition VI.1. For a fuzzy matrix A=$[A_L, A_U]\in IVFM_{nn}$ is an IV s - symmetric fuzzy matrices iff A_L=VA_L^TV and A_U=VA_U^TV.

Definition VI.2. For a fuzzy matrix A=$[A_L, A_U]\in IVFM_{nn}$ is an IV s- RS fuzzy matrix iff $R(A_L) = R(VA_L^TV)$,$R(A_U) = R(VA_U^TV)$.

Definition VI.3. For a fuzzy matrix A=$[A_L, A_U]\in IVFM_{nn}$ is an IV s-k- RS fuzzy matrix iff $R(A_L) = R(KVA_L^TVK)$, $R(A_U) = R(KVA_U^TVK)$.

Lemma VI.1. For a fuzzy matrix A=$[A_L, A_U]\in IVFM_{nn}$ is an IV s-RS fuzzy matrix iff VA=$[VA_L, VA_U]$ IV RS fuzzy matrix iff AV =$[A_LV, A_UV]$ is an IV RS fuzzy matrix.

Proof: A fuzzy matrix A=$[A_L, A_U]\in IVFM_{nn}$ is s- RS fuzzy matrix.

$\iff R(A_L) = R(VA_L^TV)$ [Definition 6.2]
$\iff R(A_LV) = R(A_LV)^T$ [By $P_{2.2}$]
$\iff A_LV$ is RS.
$\iff R(VA_LVV^T) = R(VVA_L^TV)$
$\iff R(VA_L) = R(VA_L)^T$
$\iff VA_L$ is RS.
Similaly
$\iff R(A_U) = R(VA_U^TV)$ [Definition 6.2]
$\iff R(A_UV) = R(A_UV)^T$ [By $P_{2.2}$]
$\iff A_UV$ is RS.
$\iff R(VA_UVV^T) = R(VVA_U^TV)$
$\iff R(VA_U) = R(VA_U)^T$
$\iff VA_U$ is RS.
Therefore, VA=$[VA_L, VA_U]$ IV RS fuzzy matrix
Similarly, AV =$[A_LV, A_UV]$ is an IV RS fuzzy matrix.

Remark VI.1. To be more precise, Definition (6.3) reduces to $R(A_L)$ =$R(VA_L^TV)$,$R(A_U) = R(VA_U^TV)$, meaning that the appropriate fuzzy permutation matrix K is an IV s-RS matrix, where k(i) = i for i = 1, 2, . . . , n.

Remark VI.2. For k(i) = n-i+1, the analogous permutation fuzzy matrix K can be reduced to V. $R(A_L) = R(VA_L^TV)$,$R(A_U) = R(VA_U^TV)$ means that is an IV RS from Definition 6.3.

Remark VI.3. If A is IV s-k-symmetric, then $(A_L) = (KVA_L^TVK)$, $(A_U) = (KVA_U^TVK)$, indicating that it is an IV s-k-RS fuzzy matrix, then $R(A_L) = R(KVA_L^TVK)$, $R(A_U) = R(KVA_U^TVK)$. We note that s-k-symmetric fuzzy matrix is s-k-RS fuzzy matrix. The opposite isn't always true. The example that follows illustrates the same.

Example VI.1. Let us consider fuzzy matrices

$$K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, V = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\text{A}=[A_L, A_U]=\begin{bmatrix} [0.2, 0.2] & [0.6, 0.4] \\ [0.6, 0.4] & [0.2, 0.2] \end{bmatrix}$$

is an IV symmetric, IV s-k symmetric and hence therefore IV s-k KS. Hence

$$A_L=\begin{bmatrix} 0.2 & 0.6 \\ 0.6 & 0.2 \end{bmatrix},$$
$$A_U=\begin{bmatrix} 0.2 & 0.4 \\ 0.4 & 0.2 \end{bmatrix}$$

$$KVA_L^TVK=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0.2 & 0.6 \\ 0.6 & 0.2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0.2 & 0.6 \\ 0.6 & 0.2 \end{bmatrix}$$

$KVA_L^TVK=A_L$.

Similarly, we get $KVA_U^TVK=A_U$
$R(KVA_L^TV)K=R(A_L)$
A=$[A_L, A_U]$ is an IV s-k RS.

Example VI.2. Consider IV fuzzy matrix

$$K = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, V = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A = [A_L, A_L] = \begin{bmatrix} [0, 0.1] & [0.1, 0.1] & [0.2, 0.2] \\ [0.3, 0.3] & [0.3, 0.4] & [0.2, 0.3] \\ [0.0.1] & [0.1, 0.1] & [0, 0.1] \end{bmatrix}$$

is not IV s-k range symmetric and not an IV s-k symmetric.

$$A_L = \begin{bmatrix} 0 & 0.1 & 0.2 \\ 0.2 & 0.3 & 0.2 \\ 0.3 & 0.0 & 0 \end{bmatrix}, A_U = \begin{bmatrix} 0.1 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.1 & 0.1 \end{bmatrix}$$

KVA_L^TVK= $\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&0.1&0.2\\0.2&0.3&0.2\\0.3&0.0&0\end{bmatrix}$
$\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}=\begin{bmatrix}0.3&0.1&0.1\\0.2&0&0.2\\0.2&0.3&0\end{bmatrix}$
KVA_L^TVK is not equal to A_L^T
Hence A is not s- k-symmetric and not s- k- RS.

Theorem VI.1. The subsequent conditions are equivalent for A$\in F_{mn}$.
(i)A =$[A_L, A_U]$ is an IV s-k RS
(ii) KVA =$[KVA_L, KVA_U]$ is an IV RS
(iii) AKV =$[A_LKV, A_UKV]$ is an IV RS
(iv) VA =$[VA_L, VA_U]$ is an IV k-RS
(v) AK =$[A_LK, A_UK]$ is an IV s-RS
(vi) A^T is an IV s-k RS
(vii) $R(A_L)$=$R(A_L^TVK)$,$R(A_U)$=$R(A_U^TVK)$
(viii) $R(A_L^T)$=$R(A_LVK)$,$R(A_U^T)$=$R(A_UVK)$
(ix) $R(KVA_L)$=$R(KVA)^T$,$R(KVA_U)$=$R(KVA)^T$
(x) AVK =$[A_LVK, A_UVK]$ is an IV RS
(xi) AV =$[A_LV, A_UV]$ is an IV RS
(xii) VKA =$[VKA_L, VKA_U]$ is an IV RS
(xii) KA =$[KA_L, KA_U]$ is an IV s-RS

Proof: (i) iff (ii) iff (iv)
Let A =$[A_L, A_U]$ is an IV s-k RS
Let A_L is an s-k RS
$\iff R(A_L)$ =$R(KVA_L^TVK)$,
$R(A_U)$ =$R(KVA_U^TVK)$
(By Definition 3.3)
$\iff R(KVA_L)$ =$R(KVA_L)$,
$R(A_U)$ =$R(KVA_U)$
(By $P_{2.2}$)
KVA =$[KVA_L, KVA_U]$ is an IV RS
VA =$[VA_L, VA_U]$ is an IV K-RS
As a conclusion (i) iff (ii) iff (iv) is true
(i) iff (iii) iff (v)
Let A =$[A_L, A_U]$is an IV s-k RS
$\iff R(A_L)$ =$R(KVA_LVK)$,
$R(A_U)$ =$R(KVA_UVK)$
(By Definition 3.3)
$\iff R(KVA_L)$ =$R(KVA_L)^T$,
$R(A_U)$ =$R(KVA_U)^T$
$\iff R(VK(KVA_L))$ =$R((VK)A_L^T(VK)^T)$,
$R(VK(KVA_U))$ =$R((VK)A_U^T(VK)^T)$
$\iff R(A_LKV)$ =$R(A_LVK)^T$,
$R(A_UKV)$ =$R(A_UVK)^T$
AKV =$[A_LKV, A_UKV]$ is an IV RS
AK =$[A_LK, A_UK]$ is an IV s-RS
As a conclusion (i) iff (iii) iff (v) is true
(ii) iff (ix)
KVA =$[KVA_L, KVA_U]$ is an IV RS
$\iff R(KVA_L)$ =$R(KVA_L)^T$,
$R(KVA_U)$ =$R(KVA_U)^T$
(ii) iff (ix) is true.
(ii) iff (xii)
KVA =$[KVA_L, KVA_U]$ is an IV RS
$\iff R(KVA_L)$ =$R(KVA_L)^T$,
$R(KVA_U)$ =$R(KVA_U)^T$
$\iff R(A_L)$ =$R(KVA_L)^T$,
$R(A_U)$ =$R(KVA_U)^T$
$\iff R(A_L)$ =$R(A_L)$,
$R(A_U)$ =$R(KVA_U)$
$\iff R(A_L)$ =$R(A_L^TVK)$,
$R(A_U)$ =$R(A_U^TVK)$
As a conclusion (ii) iff (xii) is true.
(iii) iff (xiii)
AKV =$[A_LKV, A_UKV]$ is an IV RS
$\iff R(A_LVK)$ =$R(A_LVK)^T$,
$R(A_UVK)$ =$R(A_UVK)^T$
$(By\ P_{2.2})$
$\iff R(A_LVK)$ =$R(A_L)^T$,
$R(A_UVK)$ =$R(A_U)^T$
As a conclusion (iii) iff (viii) is true
(i) iff (vi)
A =$[A_L, A_U]$ is an IV s-k RS
$\iff R(A_L)$ =$R(KVA_L^TVK)$,
$R(A_U)$ =$R(KVA_U^TVK)$
$\iff (KVA)^T$ =$(KVA_L, KVA_U)^T$ is an IV RS
$\iff (AVK)^T$ =(A_LVK, A_UVK) is an IV RS
$\iff (A)^T$ =(A_L^T, A_U^T) is an IV RS
As a conclusion (i) iff (vi) is true
(i) iff (X) iff (xi) A =$[A_L, A_U]$is an IV s-k RS
Consider A_L is a s-k RS
$\iff R(A_L)$ =$R(KVA_L^TVK)$,
$R(A_U)$ =$R(KVA_U^TVK)$
(By Definition 3.3)
$\iff R(A_LVK)$ =$R(A_LVK)^T$,
$R(A_UVK)$ =$R(A_UVK)^T$
$\iff$ AVK =$[A_LVK, A_UVK]$is an IV RS
$\iff$ AV =$[A_LV, A_UV]$is an IV RS
As a conclusion (i) iff (x) iff (xi) is true
(i) iff (xii) iff (xiii)
A =$[A_L, A_U]$is an IV s-k RS
$\iff R(A_L)$ =$R(KVA_L^TVK)$,
$R(A_U)$ =$R(KVA_U^TVK)$
(By Definition 3.3)
$\iff R(A_LVK)$ =$R(A_LVK)^T$,
$R(A_UVK)$ =$R(A_UVK)^T$
$\iff R(VK(KVA_L))$ =$R((VK)A_L^T(VK)^T)$,
$R(VK(KVA_U))$ =$R((VK)A_U^T(VK)^T)$
$\iff$ VKA =$[VKA_L, VKA_U]$is an IV RS
$\iff$ KA =$[KA_L, KA_U]$ is an IV RS
As a conclusion (i) iff (xii) iff (xiii) is true
The above statement can be reduced to the equivalent requirement that a matrix be an IV s- RS for K = I in particular.

Corollary VI.1. The subsequent conditions are equivalent for A$\in F_{mn}$.
(i) A =$[A_L, A_U]$ is an IV s- RS
(ii) VA =$[VA_L, VA_U]$ is an IV RS
(iii) AV =$[A_LV, A_UV]$ is an IV RS
(iv) A^T is an IV RS
(v) $R(A_L)$=$R(A_L^TV)$,$R(A_U)$=$R(A_U^TV)$
(vi) $R(A_L^T)$=$R(A_LV)$,$R(A_U^T)$=$R(A_UV)$
(vii) KVA =$[(VA_L)^T, (VA_U)^T]$is an IV RS

Theorem VI.2. The subsequent conditions are equivalent for A$\in F_{mn}$.
(i)A =$[A_L, A_U]$ is an IV k- RS
(ii)VA =$[A_L, A_U]$ is an IV s-k RS
(iii)$R(A_L^T)$=$R(VKA_LV)^T$, $R(A_U^T)$=$R(VKA_U)^T$

Proof: (i) and (ii) implies (iii)

A =$[A_L, A_U$]is an IV k- RS
$\Rightarrow R(A_L)$=$R(A_L^T V)$, $R(A_U)$=$R(A_U^T V)$
$\Rightarrow$ $R(A_L)^T$=$R(VKA_L^T)$, $R(A_U)^T$=$R(VKA_U^T)$
(i) and (ii) implies (iii) is true
(i) and (iii) implies (ii)
A =$[A_L, A_U]$ is an IV k- RS
$\Rightarrow R(A_L)$=$R(KA_L^T K)$, $R(A_U)$=$R(KA_U^T K)$
Therefore (ii) and (iii)
$\Rightarrow R(KA_L K)$=$R(VA_L K)^T$, $R(KA_U K)$=$R(VA_U K)^T$
$\Rightarrow R(A_L)$=$R(VA_L^T VK)$, $R(A_U)$=$R(VA_U^T VK)$
$\Rightarrow R(A_L)$=$R(KVA_L)^T$, $R(A_U)$=$R(KVA_U^T)^T$
A =$[A_L, A_U]$ is an IV s-k RS (By Theorem 3.1)
(ii) is true
(ii) and (iii) implies (i)
A =$[A_L, A_U]$is an IV s-k RS
$\Rightarrow R(A_L)$=$R(VA_L^T VK)$, $R(A_U)$=$R(VA_U^T VK)$
$\Rightarrow R(KA_L K)$=$R(KA_K^T)$, $R(KA_U K)$=$R(KA_U^T K)$
Therefore,(ii) and (iii)
$\Rightarrow R(KA_L K)$=$R(A_L^T)$,$R(KA_U K)$=$R(A_U^T)$
$\Rightarrow R(A_L)$=$R(KA_L^T K)$, $R(A_U)$=$N(KA_U^T K)$
A =$[A_L, A_U]$ is an IV k- RS
Therefore, (i) is true
Hence the theorem.

VII. Interval valued s-k RS regular fuzzy matrices

In this section, we discuss on various generalized inverses of matrices in IVFM. The comparable standards for various g-inverses of an IV s-k RS fuzzy matrix to be an IV s-k RS are also established. The generalized inverses of an IV s-K RS corresponding to the sets A{1,2}, A{1, 2, 3}, and A{1, 2, 4} are characterized.

Theorem VII.1. Let A$\in F_{mn}$ X $\in A\{1,2\}$ and XA, AX, are an IV s-K-RS. Then A is an IV s- k- RS iff X= $[X_L, X_U]$ is an IV s- k-RS.
Proof: Let $R(KVA_L)$=$R(KVA_L XA_L)$$\subseteq$$R(XA_L)$
(Since (A_L)=(A_L)X(A_L))
=$R(XVVVA_L)$=$R(XVKKVVA_L)$$\subseteq$$R(KVA_L)$
Hence, $R(KVA_L)$=$R(XA_L)$
=$R(KV(XA_L)^T VK)$ [XA is s-k-RS]
=$R(A_L^T X_L^T VK)$
=$R(X_L^T VK)$
=$R(KVX_L^T)$
$R(KVA_L)^T$=$R(A_L^T VK)$
=$R(X_L^T A_L VK)$
=$R(KVA_L X_L^T)^T$
=$R(KVA_L X_L^T)$
=$R(KVX_L^T)$ [VA is s-k-IV RS]
Similarly,
Hence, $R(KVX_U)$=$R(KVA_U)^T$ [KVX is an IV RS]
$\Longleftrightarrow$ $R(KVA_L)$=$R(KVA_L)^T$,
$R(KVA_U)$=$R(KVA_U)^T$
$\Longleftrightarrow$ $R(KVX_L)$=$R(KVX_L)^T$,
$R(KVX_U)$=$R(KVX_U)^T$
$\Longleftrightarrow$ $NKVX$=$N[KVX_L, KVX_U]$ is an IV RS
$\Longleftrightarrow$ NX=$N[X_L, X_U]$ is an IV RS.

Theorem VII.2. Let A$\in F_{mn}$ X $\in A\{1,2,3\}$, $R(KVA_L)$ = $R((KVX_L)^T)$, $R(KVA_U)$ = $R((KVX_U)^T)$. Then A= $[A_L, A_U]$ is s-k-RS iff X= $[X_L, X_U]$ is IV s-k-RS.
Proof: Given $A\{1,2,3\}$, Hence $A_L X_L A_L = A_L$, $X_L A_L X_L = X_L$,$(A_L X_L)^T$=$A_L X_L$
Consider $R(KVA_L)^T$ = $R(X_L^T A_L^T VK)$
[By using $AXA = A$]
=$R(KV(A_L)^T)$
=$R((A_L X_L)^T)$ $[By P_2.2]$
=$R(A_L X_L)$ $[(A_L X_L)^T$=$A_L X_L]$
=$R(X_L)$ [By using $X_L A_L X_L = X_L$]
=$R(KVX_L)$
Similarly,
=$R(KVA_U)^T$ = $R(X_U^T A_U^T VK)$
=$R(KV(A_U)^T)$
=$R((A_U X_U)^T)$ $[By P_2.2]$
=$R(A_U X_U)$ [$(A_U X_U)^T$=$A_U X_U$]
=$R(X_U)$ [By using $X_U A_U X_U = X_U$]
=$R(KVX_U)$
If KVA is an IV RS
$\Longleftrightarrow$ $R(KVA_L)$=$R(KVA_L)^T$,
$R(KVA_U)$=$R(KVA_U)^T$
$\Longleftrightarrow$ $R(KVX_L)$=$R(KVX_L)^T$,
$R(KVX_U)$=$R(KVX_U)^T$
$\Longleftrightarrow$ KVX=$[KVX_L, KVX_U]$ is an IV RS.
$\Longleftrightarrow$ X=$[X_L, X_U]$ is an IV s-k RS.

Theorem VII.3. Let A$\in F_{mn}$ X belongs to $A\{1,2,4\}$, $R(KVA_L)^T$ = $R(KVX_L)$, $R(KVA_U)^T$ = $R(KVX_U)$. Then KVA is an IV s-K-KS iff X= $[X_L, X_U]$ is IV s-k-RS.
Proof: Given $A\{1,2,4\}$,
Hence $A_L X_L A_L = A_L$,
$X_L A_L X_L = X_L$, $(X_L A_L)^T$=$X_L A_L$
Consider $R(KVA_L)$ = $R(A_L)$
$[By P_{2.2}]$
=$R(X_L A_L)$ $(X_L A_L)^T$=$X_L A_L$
=$R((A_L^T X_L^T))$
=$N(X_L)^T$ [$(A_L X_L)^T$=$A_L X_L$]
=$R(KVX_L)^T$ $[By P_{2.2}]$
Similarly,
$R(KVA_U)$ = $R(A_U)$ $[By P_{2.2}]$
=$R(X_U A_U)$ $(X_U A_U)^T$=$X_U A_U$
=$R((A_U^T X_U^T))$
=$R(X_U)^T$ [$(A_U X_U)^T$=$A_U X_U$]
=$R(KVX_U)^T$ $[By P_{2.2}]$
If KVA is an IV RS
$\Longleftrightarrow$ $R(KVX_L)$=$R(KVA_L)^T$,
$R(KVX_U)$=$R(KVA_U)^T$
$\Longleftrightarrow$ $R(KVA_L)$=$R(KVA_L)^T$,
$R(KVA_U)$=$R(KVA_U)^T$
$\Longleftrightarrow$ KVX=$[KVX_L, KVX_U]$ is an IV RS.
$\Longleftrightarrow$ X=$[X_L, X_U]$ is an IV s-k RS.

The aforementioned Theorems reduce to comparable criteria, in particular for K = I, for different g-inverses of IV s-RS to be IV secondary RS.

Corollary VII.1. For A$\in F_{mn}$ X $\in A\{1,2\}$, and $R(AX)$ =$R[A_L X_L, A_U X_U]$, $R(XA)$ =$[X_L A_L, X_U A_U]$ are an IV s-RS. Then A is an IV s- KS iff X=$[X_L, X_U]$ is an IV s-KS.

Corollary VII.2. For A$\in F_{mn}$ X $\in$ $A\{1,2,3\}$, and $R(KVA_L) = R(VX_L)^T$, $R(KVA_U) = R(VX_U)^T$. Then A is an IV s-RS iff X=$[X_L, X_U]$ is an IV s- RS.

Corollary VII.3. For A$\in F_{mn}$ X $\in$ $A\{1,2,4\}$, and

$R(VA_L)^T = R(VX_L)$, $R(VA_U)^T = R(VX_U)$. Then A is an IV s-RS iff X=$[X_L, X_U]$ is an IV s- RS.

VIII. IV Secondary k-RS Intuitionistic fuzzy matrices

Definition VIII.1. *An IV IFM (IVIFM) P of order $m \times n$ is defined as $P = [x_{ij}, < p_{iju}, p_{ijv} >_{m\times n}$, where p_{iju} and $p_{ijv} >$ are both subsets of $[0,1]$ which are denoted by $p_{iju} = [p_{ijuL}, p_{ijuU}]$ and $p_{ijv} = [p_{ijvL}, p_{ijvU}]$ with the condition $0 \le p_{ijuU} + p_{ijvU} \le 1$, $0 \le p_{ijuL} + p_{ijvL} \le 1$, $0 \le p_{uL} \le p_{uU} \le 1$, $0 \le p_{vL} \le p_{vU} \le 1$ for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$*

Example VIII.1. Let us consider IV intuitionistic fuzzy matrices

$$P = \begin{bmatrix} <[0.4, 0.4], [0.5, 0.5]> & <[0.4, 0.5], [0.2, 0.4]> \\ <[0.4, 0.5], [0.2, 0.4]> & <[0.4, 0.4], [0.3, 0.3]> \end{bmatrix},$$

Hence, Lower Limit IFM,$P_L = [P_{uL}, P_{vL}]$

$$= \begin{bmatrix} <0.4, 0.4> & <0.4, 0.5> \\ <0.4, 0.5> & <0.4, 0.4> \end{bmatrix},$$

Upper Limit IFM, $P_U = [P_{uU}, P_{vU}]$

$$= \begin{bmatrix} <0.5, 0.5> & <0.2, 0.4> \\ <0.2, 0.4> & <0.3, 0.3> \end{bmatrix}$$

Definition VIII.2. *If $k(X)$=$(X_{k[1]}, X_{k[2]}, X_{k[3]}, ..., X_{k[n]})$ $\in F_{n\times 1}$, where K is involuntary, the corresponding permutation matrix is satisfied using the following.*
$(P._{2.1})KK^T = K^TK = I_n, K = K^T, K^2 = I$ and $R(x) = kx$
By Definition of V ,

$$(P._{2.2})V = V^T, VV^T = V^TV = I_n \text{ and } V^2 = I$$

$(P._{2.3})R([P_{uL,P_{vL}}]) = R([P_{uL,P_{vL}}]V)$,
$R([P_{uL,P_{vL}}]) = R([P_{uL,P_{vL}}]K)$
$R([P_{uU,P_{vU}}]) = R([P_{uU,P_{vU}}]V)$,
$R([P_{uU,P_{vU}}]) = R([P_{uU,P_{vU}}]K)$

$(P._{2.4})R([P_{uL,P_{vL}}]V)^T = R(P[P_{uL,P_{vL}}]^T)$,
$R(V[P_{uL,P_{vL}}]^T) = R([P_{uL,P_{vL}}]^TV)$
$R([P_{uU,P_{vU}}]V)^T = R(V[P_{uU,P_{vU}}]^T)$,
$R(V[P_{uU,P_{vU}}]^T) = R([P_{uU,P_{vU}}]^TV)$

Definition VIII.3. *For an IFM P =< $[P_{uL}, P_{uU}], [P_{vL}, P_{vU}]$ >$\in IVIFM_{nn}$ is an IV s - symmetric fuzzy matrix iff $[P_{uL}, P_{vL}] = V[P_{uL}, P_{vL}]$ $= V[P_{uU}, P_{vU}]^TV$.*

Definition VIII.4. *For an IFM P = < $[P_{uL}, P_{uU}], [P_{vL}, P_{vU}]$ > $\in IVIFM_{nn}$ is an IV s -RS IFM iff $R([P_{uL}, P_{vL}]) = R(V[P_{uL}, P_{vL}]^TV), R([P_{uU}, P_{vU}])$ $= R(V[P_{uU}, P_{vU}]^TV)$.*

Definition VIII.5. *For an IFM P =< $[P_{uL}, P_{uU}], [P_{vL}, P_{vU}]$ >$\in IVIFM_{nn}$ is an IV s - k - RS fuzzy matrix iff $R([P_{uL}, P_{vL}])$ $= R(KV[P_{uL}, P_{vL}]^TVK), R([P_{uU}, P_{vU}])$ $= R(KV[P_{uU}, P_{vU}]^TVK)$.*

Lemma VIII.1. For P =< $[P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ > P^+= < $[P_{uL}, P_{vL}]^+, [P_{uU}, P_{vU}]^+$ > exists iff $(K[P_{uU}, P_{vU}])^+ existsiff (VK[P_{uL}, P_{vL}])^+$, $(VK[P_{uU}, P_{vU}])^+ exists.$

Proof: For P =< $[P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ >,
P^+ =< $[P_{uL}, P_{vL}]^+$ exists
then $[P_{uL}, P_{vL}]^+ = [P_{uL}, P_{vL}]^T$ which $[P_{uL}, P_{vL}]^T$ is a generalized inverse of $[P_{uL}, P_{vL}]$
Consider $[P_{uL}, P_{vL}]^+, [P_{uU}, P_{vU}]^+$ exists
$\iff (K[P_{uL}, P_{vL}])^+, (K[P_{uU}, P_{vU}]^+)$
$\iff (K[P_{uL}, P_{vL}])(K[P_{uL}, P_{vL}])^+(K[P_{uL}, P_{vL}])$, $(K[P_{uU}, P_{vU}])(K[P_{uU}, P_{vU}])^+(K[P_{uU}, P_{vU}])$
$\iff (K[P_{uL}, P_{vL}])^+(K[P_{uU}, P_{vU}])^+$ exists.
$(VK[P_{uL}, P_{vL}])(VK[P_{uL}, P_{vL}])^T(VK[P_{uL}, P_{vL}])$, $(VK[P_{uU}, P_{vU}])([P_{uU}, P_{vU}])^T(VK[P_{uU}, P_{vU}])$
$\iff (VK[P_{uL}, P_{vL}])^T \in (VK[P_{uL}, P_{vL}])\{1\}$, $(K[P_{uU}, P_{vU}])^T \in (VK[P_{uU}, P_{vU}])\{1\}$
$\iff (VK[P_{uL}, P_{vL}])^+(VK[P_{uU}, P_{vU}])^+$ exists.

Remark:8.1 For $P = \langle [P_{uL}, P_{vL}], [P_{uU}, P_{vU}] \rangle$, P^+ =< $[P_{uL}, P_{vL}]^+, [P_{uU}, P_{vU}]^+$ > and $(KV[P_{uL}, P_{vL}])^+, (KV[P_{uU}, P_{vU}])^+$ exists.

Lemma VIII.2. For an IFM P =< $[P_{uL}, P_{uU}], [P_{vL}, P_{vU}]$ >$\in IVIFM_m$ is interval valued s-RS IFM $\iff VP$ =< $V[P_{uL}, P_{vL}], V[P_{uU}, P_{vU}]$ > IV RS intuitionistic fuzzy matrix $\iff PV$ =< $[P_{uL}, P_{vL}]V, [P_{uU}, P_{vU}]V$ > is IV RS IFM.

Proof: An IFM P =< $[P_{uL}, P_{uU}], [P_{vL}, P_{vU}]$ >$\in IVIFM_m$ is s-RS fuzzy matrix
$\iff R([P_{uL}, P_{vL}]) = R(V[P_{uL}, P_{vL}]^TV)$
$\iff R([P_{uL}, P_{vL}]V) = R([P_{uL}, P_{vL}]V)^T$
$\iff [P_{uL}, P_{vL}]V$ is RS [By $P_{2.2}$]
$\iff R(V[P_{uL}, P_{vL}]VV^T) = R(VV[P_{uL}, P_{vL}]^TV)$
$\iff R(V[P_{uL}, P_{vL}]) = R(V[P_{uL}, P_{vL}])^T$
$\iff V[P_{uL}, P_{vL}]$ is RS.
Similarly
$\iff R([P_{uU}, P_{vU}]) = R(V[P_{uU}, P_{vU}]^TV)$
$\iff [P_{uU}, P_{vU}]V$ is RS.
$\iff R(V[P_{uU}, P_{vU}]VV^T) = R(VV[P_{uU}, P_{vU}]^TV)$
$R(V[P_{uU}, P_{vU}]) = R(V[P_{uU}, P_{vU}])^T$ $V[P_{uU}, P_{vU}]$ is RS.
Therefore, VP =< $V[P_{uL}, P_{vL}], V[P_{uU}, P_{vU}]$ > is an IV symmetric.

Remark: If P is IV s-k-symmetric, then $[P_{uL}, P_{vL}] = KV[P_{uL}, P_{vL}]^TVK$, and $P_U = KVA_U^TVK$, indicating that it is IV (IV) s-k-RS IFM, then $R([P_{uL}, P_{vL}]^TVK), R([P_{uU}, P_{vU}]) = R(KV[P_{uU}, P_{vU}]^TVK)$.
We note that s-k-symmetric IFM is s-k-RS IFM.

The converse not always true, though. The example that follows illustrates this V.

Example VIII.2. Let us consider IVIFM

$$K = \begin{bmatrix} <1, 0> & <0, 1> \\ <0, 1> & <1, 0> \end{bmatrix},$$
$$V = \begin{bmatrix} <0, 1> & <1, 0> \\ <1, 0> & <0, 1> \end{bmatrix} \text{ and}$$

$P =< [P_{uL}, P_{uU}], [P_{vL}, P_vU] >\in IVIFM_m$

$$P = \begin{bmatrix} < [0.4, 0.4], [0.5, 0.5] > & < [0.4, 0.5], [0.2, 0.4] > \\ < [0.4, 0.5], [0.2, 0.4] > & < [0.4, 0.4], [0.3, 0.3] > \end{bmatrix},$$

is an IV symmetric, IV $s-k$ symmetric and hence therefore IV $s-k$ RS. Hence,

$$P_L = \begin{bmatrix} < [0.4, 0.4] > & < [0.4, 0.5] > \\ < [0.4, 0.5] > & < [0.4, 0.4] > \end{bmatrix},$$

$$P_U = \begin{bmatrix} < [0.5, 0.5] > & < [0.2, 0.4] > \\ < [0.2, 0.4] > & < [0.3, 0.3] > \end{bmatrix}$$

$$KV = \begin{bmatrix} < 1, 0 > & < 0, 1 > \\ < 0, 1 > & < 1, 0 > \end{bmatrix} \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix}$$

$$= \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix}$$

$$VK = \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix} \begin{bmatrix} < 1, 0 > & < 0, 1 > \\ < 0, 1 > & < 1, 0 > \end{bmatrix}$$

$$= \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix}$$

$$KVP_L^TVK = \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix}$$

$$\begin{bmatrix} < [0.4, 0.4] > & < [0.4, 0.5] > \\ < [0.4, 0.5] > & < [0.4, 0.4] > \end{bmatrix}$$

$$= \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix} = P_L$$

$$KVP_L^TVK = \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix}$$

$$\begin{bmatrix} < [0.5, 0.5] > & < [0.2, 0.4] > \\ < [0.2, 0.4] > & < [0.3, 0.3] > \end{bmatrix}$$

$$= \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix} = P_U$$

$R(P_L) = R(KVP_L^TVK)$

$P = [P_L, P_U]$ is an IV $s-k$ RS.

Example VIII.3. For $k = (1, 2)$,

$$K = \begin{bmatrix} < 1, 0 > & < 0, 1 > \\ < 0, 1 > & < 1, 0 > \end{bmatrix},$$
$$V = \begin{bmatrix} < 0, 1 > & < 1, 0 > \\ < 1, 0 > & < 0, 1 > \end{bmatrix} \text{ and}$$

$P =< [P_{uL}, P_{uU}], [P_{vL}, P_vU] >\in IVIFM_m$

$$P = \begin{bmatrix} < [0, 0.02], [0, 1] > & < [0.2, 0.5], [0.2, 0.4] > \\ < [0.2, 0.5], [0.2, 0.4] > & < [0.2, 0.2], [0.3, 0.4] > \end{bmatrix},$$

$$P_U = \begin{bmatrix} < [0, 1] > & < [0.2, 0.4] > \\ < [0.2, 0.4] > & < [0.3, 0.4] > \end{bmatrix},$$

$KVP_U^TVK \neq P_U$

Here $P = KP_U^TK$

Therefore P is symmetric IFM, k- symmetric IFM, $s-k$-RS IFM but not $s-k$-symmetric IFM.

Theorem VIII.1. The following conditions are equivalent for $P \in IVIFM_n$.

(i) $P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU}] >$ is an IV s-k RS

(ii) $KVP =< KV[P_{uL}, P_{vL}], KV[P_{uU}, P_{vU}] >$is an IV RS

(iii) $PKV =< [P_{uL}, P_{vL}]KV, [P_{uU}, P_{vU}]KV >$ is an IV RS

(iv) $VP =< [P_{uL}, P_{vL}], V[P_{uU}, P_{vU}] >$ is an IV k-RS

(v) $PK =< [P_{uL}, P_{vL}]K, [P_{uU}, P[vU]]K >$ is an IV s-RS

(vi) P^T is an s-k KS

(vii) $R([P_{uL}, P_{vL}]) = (R[P_{uL}, P_{vL}]^TVK)$,
$R([P_{uU}, P_{vU}]) = (R[P_{uU}, P_{vU}]^TVK)$

(viii) $R([P_{uL}, P_{vL}]^T) = (R[P_{uL}, P_{vL}]VK)$,
$R([P_{uU}, P_{vU}]^T) = (R[P_{uU}, P_{vU}]VK)$

(ix) $C(KV[P_{uL}, P_{vL}]) = C(KV[P_{uL}, P_{vL}]^T)^T$,
$C(KV[P_{uU}, P_{vU}]) = C(KV[P_{uU}, P_{vU}]^T)^T$

(x) $[P_{uL}, P_{vL}] = VK[P_{uL}, P_{vL}]^TVKH_1, [P_{uU}, P_{vU}]$
$= VK[P_{uU}, P_{vU}]^TVKH_1 for H_1 \in IVIFM$

(xi) $[P_{uL}, P_{vL}] = H_1VK[P_{uL}, P_{vL}]^TVK, [P_{uU}, P_{vU}]$
$= H_1K[P_{uU}, P_{vU}]^TVK for H_1 \in IVIFM$

(xii) $[P_{uL}, P_{vL}]^T = KV[P_{uL}, P_{vL}]KVH_1, [P_{uU}, P_{vU}]^T$
$= KV[P_{uU}, P_{vU}]VKH_1 for H_1 \in IVIFM$

(xiii) $[P_{uL}, P_{vL}]^T = H_1KV[P_{uL}, P_{vL}]KV, [P_{uU}, P_{vU}]^T$
$= H_1KV[P_{uU}, P_{vU}]VK for H_1 \in IVIFM$

Proof: (i) iff (ii) iff (iv)

Let $P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU} >]$ is an IV s-k RS

Let $[P_{uL}, P_{vL}]$ is an s-k RS

$\iff R([P_{uL}, P_{vL}]) = R(KV[P_{uL}, P_{vL}]^TVK)$,
$R([P_{uU}, P_{vU}]) = R(KV[P_{uU}, P_{vU}]^TVK)$,

(By Definition 8.5)

$\iff R(KV[P_{uL}, P_{vL}]) = R(KV[P_{uL}, P_{vL}])^T$,
$R([P_{uU}, P_{vU}]) = R(KV[P_{uU}, P_{vU}])^T$ By $(P_{2.3})$

$\iff KVP =< KV[P_{uL}, P_{vL}], KV[P_{uU}, P_{vU}] >$ is an IV RS

$\iff VP =< V[P_{uL}, P_{vL}], V[P_{uU}, P_{vU} >]$ is an IV k-RS

As a conclusion (i) iff (ii) iff (iv) is true

(i) iff (iii) iff (v)

Let $P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU} >]$ is an IV s-k RS

$\iff R([P_{uL}, P_{vL}]) = R(KV[P_{uL}, P_{vL}]^TVK)$,
$R([P_{uU}, P_{vU}]) = R(KV[P_{uU}, P_{vU}]^TVK)$,

(By Definition 8.5)

$\iff R(KV[P_{uL}, P_{vL}]) = R(KV[P_{uL}, P_{vL}])^T$,
$R([P_{uU}, P_{vU}]) = R(KV[P_{uU}, P_{vU}])^T$ By $(P_{2.3})$

$R(VK(KV[P_{uL}, P_{vL}]))$
$= R((VK)[P_{uL}, P_{vL}]^TVK(VK)^T)$,
$R(VK(KV[P_{uU}, P_{vU}]))$
$= R((VK)[P_{uU}, P_{vU}]^TVK(VK)^T)$,

$\iff R([P_{uL}, P_{vL}]KV)$
$= R([P_{uL}, P_{vL}]KV)^T, R([P_{uU}, P_{vU}]KV)$
$= R([P_{UU}, P_{vU}]KV)^T$

$\iff PKV = [[P_{uL}, P_{vL}]KV, [P_{uU}, P_{vU}]KV]$ is an IV RS

$\iff PK = [[P_{uL}, P_{vL}]K, [P_{uU}, P_{vU}]K]$ is an IV k-RS

As a conclusion (i) iff (iii) iff (v) is true

(ii) $\iff$ (ix)

$KVA = [KV[P_{uL}, P_{vL}]KV[P_{uU}, vU]]$ is an IV RS

$\iff R(KV[P_{uL}, P_{vL}]) = R((KV[P_{uL}, P_{vL}])^T)$,
$R(KV[P_{uU}, P_{vU}]) = R((KV[P_{uU}, P_{vU}])^T)$

$\iff C(KV[P_{uL}, P_{vL}])^T = R(KV[P_{uL}, P_{vL}])$,
$R(KV[P_{uU}, P_{vU}])^T = R(KV[P_{uU}, P_{vU}])$

$(ii) \iff (ix)$ is true

$(ii) \iff (vii)$

$KVP = [KV[P_{uL}, P_{vL}]KV[P_{uU}, vU]]$ is an IV RS

$\iff R(KV[P_{uL}, P_{vL}]) = R((KV[P_{uL}, P_{vL}])^T)$,
$R(KV[P_{uU}, P_{vU}]) = R((KV[P_{uU}, P_{vU}])^T)$

$\iff R([P_{uL}, P_{vL}]) = R((KV[P_{uL}, P_{vL}])^T)$,
$R([P_{uU}, P_{vU}]) = R((KV[P_{uU}, P_{vU}])^T)$

$\iff R([P_{uL}, P_{vL}]) = R([P_{uL}, P_{vL}]^T)$,
$R([P_{uU}, P_{vU}]) = R([P_{uU}, P_{vU}]^T VK)$
As a conclusion $(ii) \iff (vii)$ is true
$(iii) \iff (viii)$ $PVK = [[P_{uL}, P_{vL}]VK, [P_{uU}, P_{vU}]VK]$
$\iff R([P_{uL}, P_{vL}]VK) = R(([P_{uL}, P_{vL}]VK)^T)$,
$R([P_{uU}, P_{vU}]VK) = R(([P_{uU}, P_{vU}]VK)^T)$
$\iff R([P_{uL}, P_{vL}]VK) = R([P_{uL}, P_{vL}])^T$,
$R([P_{uU}, P_{vU}]VK) = R([P_{uU}, P_{vU}])^T$
As a conclusion $(iii) \iff (viii)$ is true
$(i) \iff (vi)$
Let $P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU} >]$ is an IV s-k RS
$\iff R([P_{uL}, P_{vL}]) = R(KV[P_{uL}, P_{vL}]^T VK)$,
$R([P_{uU}, P_{vU}]) = R(KV[P_{uU}, P_{vU}]^T VK)$,

(By Definition 8.5)
$\iff R(KV[P_{uL}, P_{vL}]) = R((KV[P_{uL}, P_{vL}])^T)$,
$R([P_{uU}, P_{vU}]) = R((KV[P_{uU}, P_{vU}])^T)$
$\iff (KVP)^T = (KV[P_{uL}, P_{vL}]$,
$KV[P_{uU}, P_{vU}])^T$ is an IV RS
$\iff P^T VK = ([P_{uL}, P_{vL}]VK, [P_{uU}, P_{vU}]VK)$ is an IV RS
$\iff P^T = ([P_{uL}, P_{vL}]^T, [P_{uU}, P_{vU}]^T)$ is an IV s-k RS
As a conclusion $(i) \iff (vi)$ is true
$(i) \iff (xii) \iff (11)$
Let $P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU} >]$ is an IV s-k RS
Consider $[P_{uL}, P_{vL}]$ is an s-k RS $\iff R([P_{uL}, P_{vL}])$
$= R(KV[P_{uL}, P_{vL}]^T VK), R([P_{uU}, P_{vU}])$
$= R(KV[P_{uU}, P_{vU}]^T VK)$, (By Definition 8.5)
$\iff R([P_{uL}, P_{vL}]^T)$
$= C(KV[P_{uL}, P_{vL}]VK), R([P_{uU}, P_{vU}])^T$
$= C(KV[P_{uU}, P_{vU}]VK)$
$\iff [P_{uL}, P_{vL}]^T = KV[P_{uL}, P_{vL}]VK, [P_{uU}, P_{vU}]^T$
$= KV[P_{uU}, P_{vU}]VK$
$\iff [P_{uL}, P_{vL}] = H_1 KV[P_{uL}, P_{vL}]^T VK, [P_{uU}, P_{vU}]$
$= KV[P_{uU}, P_{vU}]^T VK for H_1 \in$ IVIFM by $(P_{.2.3})$
As a conslusion $(i) \iff (xii) \iff (xi)$ is true
Therefore, $(i) \iff (x) \iff (11)$ is true
$(ii) \iff (xii) \iff (x)$
$KVP =< KV[P_{uL}, P_{vL}], KV[P_{uU}, vU] >$ is an IV RS
$VP =< V[P_{uL}, P_{vL}], V[P_{uU}, vU] >$ is an IV k-RS
$\iff R(V[P_{uL}, P_{vL}]) = R(K(V[P_{uL}, P_{vL}])^T K)$,
$N[P_{uU}, P_{vU}] = R(V(V[P_{uU}, P_{vU}])^T K)$,
$R([P_{uL}, P_{vL}]) = R([P_{uL}, P_{vL}])^T VK, N[P_{uU}, P_{vU}]$
$= R([P_{uU}, P_{vU}])^T VK$,
$C([P_{uL}, P_{vL}])^T = C(KV[P_{uL}, P_{vL}])^T, C([P_{uU}, P_{vU}])^T$
$= R(KV[P_{uU}, P_{vU}])^T$ [By Definition 8.5]
$([P_{uL}, P_{vL}])^T = HKV[P_{uL}, P_{vL}], ([P_{uU}, P_{vU}])^T$
$= HKV[P_{uU}, P_{vU}] for \in$ IFIFM
$([P_{uL}, P_{vL}])^T = H_1 KV[P_{uL}, P_{vL}]KV, ([P_{uU}, P_{vU}])^T$
$= H_1 KV[P_{uU}, P_{vU}]KV$
$[P_{uL}, P_{vL}] = VK[P_{uL}, P_{vL}]^T VKH_1, [P_{uU}, P_{vU}]$
$= VK[P_{uU}, P_{vU}]^T VKH_1$
As a conclusion $(ii) \iff (xiii) \iff (x)$ are true. As a result, the theorem is valid.

The above statement can be reduced to the equivalent requirement that a matrix be an IV s-RS for $k = I$ in particular.

Theorem VIII.2. For $P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ then any two of the conditions below imply the other

(i) $P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ is an IV $k - RS$
(ii) $P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ is an $s - k - RS$
(iii) $R([P_{uL}, P_{vL}])^T = R(VK[P_{uL}, P_{vL}])^T$,
$R([P_{uU}, P_{vU}])^T = R(VK[P_{uU}, P_{vU}]^T)$
Proof: (i) and (ii) implies (iii)
Let $P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ is an $s - k - RS$
$\Rightarrow R([P_{uL}, P_{vL}]) = R([P_{uL}, P_{vL}]^T VK), R([P_{uU}, P_{vU}]) = R([P_{uU}, P_{vU}]^T VK)$
$\Rightarrow R(K[P_{uL}, P_{vL}]K) = R(K[P_{uL}, P_{vL}]^T K)$,
$R(K[P_{uU}, P_{vU}]K) = R(K[P_{uU}, P_{vU}]^T K)$
$\Rightarrow R([P_{uL}, P_{vL}])^T = R((VK[P_{uL}, P_{vL}])^T)$,
$R([P_{uU}, P_{vU}])^T = R((VK[P_{uU}, P_{vU}])^T)$
(i) & (ii) implies (iii) is true
(i) & (iii) implies (ii)
$P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ is an IV $k - RS$
$\Rightarrow R([P_{uL}, P_{vL}]) = R(K[P_{uL}, P_{vL}]^T K)$,
$R([P_{uU}, P_{vU}]) = R(K[P_{uU}, P_{vU}]^T K)$
$\Rightarrow R(K[P_{uL}, P_{vL}]K) = R(([P_{uL}, P_{vL}])^T)$,
$R(K[P_{uU}, P_{vU}]K) = R(([P_{uU}, P_{vU}])^T)$
Therefore, (i) & (iii)
$\Rightarrow R(K[P_{uL}, P_{vL}]K) = R((V[P_{uL}, P_{vL}]K)^T)$,
$R(K[P_{uU}, P_{vU}]K) = R((V[P_{uU}, P_{vU}]K)^T)$,
$\Rightarrow R([P_{uL}, P_{vL}]) = R([P_{uL}, P_{vL}]K)^T VK$,
$R([P_{uU}, P_{vU}]) = R([P_{uU}, P_{vU}]K)^T VK)$,
$\Rightarrow R([P_{uL}, P_{vL}]) = R(KV[P_{uL}, P_{vL}])^T)$,
$R([P_{uU}, P_{vU}]) = R(KV[P_{uU}, P_{vU}])^T)$,
$P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ is an $s - k - RS$
$\Rightarrow (ii)$ is true
(ii) & (iii) implies (i)
$P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ is an $s - k - RS$
$\Rightarrow R([P_{uL}, P_{vL}]) = R([P_{uL}, P_{vL}]^T VK)$,
$R([P_{uU}, P_{vU}]) = R([P_{uU}, P_{vU}]^T VK)$
$\Rightarrow R(K[P_{uL}, P_{vL}]K) = R(K[P_{uL}, P_{vL}]^T K))$,
$R(K[P_{uU}, P_{vU}]K) = R(K[P_{uU}, P_{vU}]^T K))$
Therefore, (ii) & (iii)
$\Rightarrow R(K[P_{uL}, P_{vL}]K) = R([P_{uL}, P_{vL}]^T)$,
$R(K[P_{uU}, P_{vU}]K) = R([P_{uU}, P_{vU}]^T)$,
$\Rightarrow R([P_{uL}, P_{vL}]) = R(K[P_{uL}, P_{vL}]^T K)$,
$R([P_{uU}, P_{vU}]) = R(K[P_{uU}, P_{vU}]^T K)$,
$P =< [P_{uL}, P_{vL}], [P_{uU}, P_{vU}]$ is an IV $k - RS$
Therefore, (i) is true
Hence the theorem.

IX. KS NEUTROSOPHIC FUZZY MATRICES

Theorem IX.1. The subsequent conditions are equivalent for $\psi \in F_n$

(i) $N(\psi) = N(\psi^T)$.

(ii) $\psi^T = \psi H = K\psi$ for several NFM H, K and $\rho(\psi) = r$.

Theorem IX.2. The subsequent conditions are equivalent for $\psi \in F_n$

(i) $N(\psi) = N(KV\psi^T VK)$
(ii) $N(KV\psi) = N((KV\psi)^T)$
(iii) $N(\psi KV) = N((\psi KV)T)$
(iv) $N(V\psi) = N(K(V\psi)^T K)$
(v) $N(\psi K) = N(V(\psi K)^T V)$
(vi) $N(\psi^T) = N(KV(\psi)VK)$

(vii) $N(\psi) = N(\psi^T VK)$
(viii) $N(\psi^T) = N(\psi KV)$
(ix) $\psi = VK\psi^T VKH_1 \ \ for \ \ H_1 \in F_n$
(x) $\psi = H_1 KV\psi^T VK \ \ for \ \ H_1 \in F_n$
(xi) $\psi^T = KV\psi VKH \ \ for \ \ H \in F_n$
(xii) $\psi^T = HKV\psi KV \ \ for \ \ H \in F_n$.
Proof:
$(i) \Leftrightarrow (ii) \Leftrightarrow (iv)$
$\psi \ \ is \ \ s-k-KS$
$\Leftrightarrow N(\psi) = N(KV\psi^T VK)$
$\Leftrightarrow N(KV\psi) = N((KV\psi)^T$
$\Leftrightarrow KV \ \ \ \psi \ \ is \ \ KS$
$\Leftrightarrow VP \ \ is \ \ k-KS$
Hence, $(i) \Leftrightarrow (ii) \Leftrightarrow (iv)$ $h$old.
$(i) \Leftrightarrow (iii) \Leftrightarrow (v)$
$\lambda \ \ is \ \ s-k-KS \Leftrightarrow N(\psi) = N(KV\psi^T VK)$
$\Leftrightarrow N(KV\psi) = N((KV\psi)^T)$
$\Leftrightarrow N(VK(KV\psi)(VK)^T = N((VK)\psi^T VK(VK)^T)$
$\Leftrightarrow N(KV) = N((KV)T)$
$\Leftrightarrow \psi KV \ \ is \ \ \ KS$
$\Leftrightarrow \psi K \ \ is \ \ s-KS$
Hence, $(i) \Leftrightarrow (iii) \Leftrightarrow (v)$ hold.
$(ii) \Leftrightarrow (vii)$
$KV\psi \ \ is \ \ KS \Leftrightarrow N(KV\psi) = N((KV\psi)^T)$
$\Leftrightarrow N(\psi) = N((KV\psi)^T)$
$\Leftrightarrow N(\lambda) = N(\lambda^T VK)$
Hence, $(ii) \Leftrightarrow (vii)$ hold.
$(iii) \ \ iff \ \ (viii):$
$\psi VK \ \ is \ \ KS \Leftrightarrow N(\psi VK) = N((\psi VK)^T)$
$\Leftrightarrow \ N(\psi VK) \ = \ N(\psi^T)$
Hence, $(iii) \Leftrightarrow (viii)$ hold.
$(i) \ \ \ \ iff \ \ \ \ (vi)$
$\psi \ \ is \ \ s-k-KS \Leftrightarrow N(\psi) = N(KV\psi^T VK)$
$\Leftrightarrow N(KV\psi) = N((KV\psi)^T)$
$\Leftrightarrow (KV\psi)^T \ \ is \ \ KS$
$\Leftrightarrow \psi^T VK \ \ is \ \ KS$
$\Leftrightarrow \psi^T is \ \ \ s-k-KS$
Hence, $(i) \Leftrightarrow (vi) \ hold.$
$(i) \ \ \ iff \ \ \ (xi) \ \ \ iff \ \ \ (x)$
$\psi \ \ is \ \ s-k-KS \Leftrightarrow N(\lambda) = N(KV\lambda^T VK)$
$\Leftrightarrow N(\psi^T) = N(KVVK)$
$\Leftrightarrow \psi^T = KV\psi VKH$
$\psi = H_1 KV\psi^T VK \ for \ H_1 \in F_n$
Hence,$(i) \Leftrightarrow (xi) \ iff \Leftrightarrow (x)$ hold.
$(ii) \Leftrightarrow (xii) \ iff \Leftrightarrow (ix)$
$KVP \ \ is \ \ RS \Leftrightarrow V\psi \ \ is \ \ k-KS$
$\Leftrightarrow N(V\psi) = N(K(V\psi)^T K)$
$\Leftrightarrow (\psi) = N(\psi^T VK)$
$\Leftrightarrow N(\psi^T) = N(KV\psi)$
$\Leftrightarrow \psi^T = HKV\psi \ \ \ for \ \ \ H \in F_n$
$\Leftrightarrow \psi^T = HKV\psi KV$
$\Leftrightarrow \psi = VK\psi^T VKH_1 \ \ \ for \ \ \ H_1 \in F_n$
Hence, $(ii) \Leftrightarrow (xii) \ iff \Leftrightarrow (ix)$ hold.

Theorem IX.3. For then, any pair of the following statements indicates the other one.
(i) $N(\psi) = N(K\psi^T K)$
(ii) $N(\psi) = N(VK\psi^T \psi KV)$
(iii) $N(\psi^T) = N((VK\psi)^T)$
Proof:
$(i) \ and \ (ii) \ \ iff \ \ \ (iii)$
$\psi \ \ is \ \ s-k–KS$
$\Rightarrow R(\psi) = R(\psi^T VK)$
$\Rightarrow N(K\psi K) = N(K\psi^T K)$
Hence $(i) \ \ and \ \ (ii) \Rightarrow N(\psi^T) = N((V\psi K)^T)$
Hence, (iii) hold.
$(i) \ \ and \ \ (iii) \ \ iff \ \ \ (ii)$
$\psi \ \ is \ \ k-KS \Rightarrow N(\psi) = N(K\psi^T K)$
$\Rightarrow N(K\psi K) = N(\psi^T)$
Hence $(i) \ \ and \ \ (iii) \Rightarrow N(K\psi K) = N((V\psi K)^T)$
$\Rightarrow N(\psi) = N(\psi^T VK)$
$\Rightarrow N(\psi) = N((KV\psi)^T)$
$\Rightarrow \psi \ \ is \ \ s-k-KS$ Therefore,(ii) hold.
$(ii) \ \ and \ \ (iii) \Leftrightarrow (i)$
$\psi \ \ is \ \ s-k–KS \Rightarrow N(\psi) = N(\psi^T VK)$
$\Rightarrow N(K\psi K) = N(K\psi^T V)$
Hence $(ii) \ \ and \ \ (iii) \Rightarrow N(K\psi K) = N(\psi^T)$
$\Rightarrow N(\psi) = N(K\psi^T K)$
$\Rightarrow \psi \ \ is \ \ k \ \ \ -KS$
Therefore, (i) hold. Hence the Theorem.

X. s- k-KS Regular NFM

We show the existence of several generalized inverses of NFM in F_n and determine the conditions for different g-inverses of a s-k-KS NFM to be s-k KS NFM. Generalized inverses belonging to the sets $\psi\{1,2\}, \psi\{1,2,3\}$ and $\psi\{1,2,4\}$ of s-k-KS NFM are characterized.

Theorem X.1. Let $\psi \in F_n, Z \in F_n\{1,2\}$ and $\psi Z, Z\psi$, are s-k-KS NFM. Then ψ is s - k - KS NFM $\Leftrightarrow Z$ is s -k - KS NFM.
Proof:
$N(KV\lambda) = N(KV\psi Z\psi) \subseteq N(Z\psi)[since \ \psi = \psi Z\psi]$
$= N(ZVV\psi) = N(ZVKKV\psi) \subseteq N(KV\psi)$
Hence, $N(KV\psi) = N(Z\psi)$
$= N(KV(Z\psi)^T VK)$
$= N(\psi^T Z^T VK)$
$= N(Z^T VK)$
$= N((KVZ)^T)$
$N((KV\psi)^T) = N(\psi^T VK)$
$= N(Z^T \psi^T VK)$
$= N((KV\psi Z)^T)$
$= N(KV\lambda Z) = N(KVZ)$
$KVZ \ \ is \ \ KS \Leftrightarrow N(KV\psi) = N((KV\psi)^T)$
$\Leftrightarrow N((KVZ)^T) = N(KVZ)$
$\Leftrightarrow KVZ \ \ is \ \ KS$
$\Leftrightarrow Z$ is s- k- KS.

Theorem X.2. Let $Z \in \{1,2,3\}, N(KV) = N((KVZ)^T)$. Then ψ is s-k-KS NFM $\Rightarrow Z$ is $s-k-$ KS NFM.
Proof:
Since $Z \in \psi\{1,2,3\}$,
$Hence$
$\psi Z\psi = \psi, Z$
$\psi Z = Z,$
$(\psi Z)^T = \psi Z$
$N((KV\psi)^T)$
$= N(Z^T \psi^T VK)$
$[By \ using \ \psi Z\psi = \psi]$
$= N(KV(\lambda Z)^T)$
$= N((\lambda Z)^T)$
$= N(Z)$
$[(\psi Z)^T = \psi Z]$

$= N(KVZ)$
$KV\psi \ \ is \ \ KSNFM \Leftrightarrow N(KV\lambda) = N((KV\lambda)^T)$
$\Leftrightarrow N((KVZ)^T) = N(KVZ)$
$\Leftrightarrow KVZ \ \ is \ \ KS$
$\Leftrightarrow Ziss - k - KS.$

Theorem X.3. Let $\psi \in F_n, Z \in \psi\{1,2,4\}$, $N((KVP)^T) = N(KVZ)$. Then Z is s-k-KS NFM $\Leftrightarrow Z$ is s- k- KS NFM.
Proof:
Since $Z \Leftrightarrow \psi\{1,2,4\}$,
we have
$\psi Z\psi = \psi,$
$Z\psi Z = Z,$
$(Z\psi)^T = Z\psi$
$N(KV) = N(\psi)$
$= N(Z\psi)[Z\psi Z = Z, \psi Z\psi = N((Z\psi)^T)[(Z\psi)^T = Z\psi]$
$= N(\psi Z^T)$
$= N(Z^T)$
$= N((KVZ)^T)$
$KV\psi \ \ is \ \ KSNFM \Leftrightarrow N(KV\lambda) = N((KV\lambda)^T$
$\Leftrightarrow N((KVZ)^T) = N(KVZ)$
$\Leftrightarrow KVZ \ \ is \ \ KSNFM$
$\Leftrightarrow Z$ is s- k – KS NFM.

XI. IV KS, k-KS, RS Neutrosophic Fuzzy Matrices

Theorem XI.1. For an IVNFM $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >, Q =< [Q_\mu, Q_\lambda, Q_v]_L, [Q_\mu, Q_\lambda, Q_v]_U >\in IVNFM_m$ and K be a NFPM if $N([P_\mu, P_\lambda, P_v]_L) = N([Q_\mu, Q_\lambda, Q_v]_L) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$.
$N([P_\mu, P_\lambda, P_v]_U) = N([Q_\mu, Q_\lambda, Q_v]_U)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$.
Proof: Let $w \in N(K[P_\mu, P_\lambda, P_v]_L)K^T)$
$\Rightarrow w(K[P_\mu, P_\lambda, P_v]_L)K^T) = (0.0, 0.0, 0.0)$
$\Rightarrow yK^T = (0,0,0)$ where $y = wK([P_\mu, P_\lambda, P_v]_L)$
$\Rightarrow y \in N(K^T)$
$detK = detK^T > (0.0, 0.0, 0.0)$
Therefore, $N(K^T) = (0.0, 0.0, 0.0)$
Hence, $y = (0.0, 0.0, 0.0)$
$\Rightarrow wK([P_\mu, P_\lambda, P_v]_L) = (0.0, 0.0, 0.0)$
$\Rightarrow wK \in N([P_\mu, P_\lambda, P_v]_L) = N([Q_\mu, Q_\lambda, Q_v]_L)$
$\Rightarrow wK([Q_\mu, Q_\lambda, Q_v]_L)K^T = (0.0, 0.0, 0.0)$
$\Rightarrow w \in N(K([Q_\mu, Q_\lambda, Q_v]_L)K^T)$
$N(K[P_\mu, P_\lambda, P_v]_L K^T) \subseteq N(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$
Similarly, $N(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$
$\subseteq N(K[P_\mu, P_\lambda, P_v]_L K^T)$
Therefore,
$N([P_\mu, P_\lambda, P_v]_L) = N([Q_\mu, Q_\lambda, Q_v]_L)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L K^T)$
$= N(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$
Therefore,
$N([P_\mu, P_\lambda, P_v]_U) = N([Q_\mu, Q_\lambda, Q_v]_U)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$
Conversely, if $N(K[P_\mu, P_\lambda, P_v]_L K^T)$
$= N(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$, $N([P_\mu, P_\lambda, P_v]_L)$
$= N(K^T(K([P_\mu, P_\lambda, P_v]_L)K^T)K)$
$= N(K^T(K([Q_\mu, Q_\lambda, Q_v]_L)K^T)K)$
$N([P_\mu, P_\lambda, P_v]_L) = N([Q_\mu, Q_\lambda, Q_v]_L)$
Similarly,
$N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_U) = N([Q_\mu, Q_\lambda, Q_v]_U)$.

Theorem XI.2. For an IVNFM $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >\in IVNFM_m$ and K be a NFPM if $N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L) \Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[P_\mu, P_\lambda, P_v]_L K^T)$.
and $N([P_\mu, P_\lambda, P_v]_U) = N([P_\mu, P_\lambda, P_v]_U)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[P_\mu, P_\lambda, P_v]_U K^T)$.
Proof: Let $x \in N(K[P_\mu, P_\lambda, P_v]_L)K^T)$
$\Rightarrow x(K[P_\mu, P_\lambda, P_v]_L)K^T) = (0.0, 0.0, 0.0)$
$\Rightarrow wK^T = (0,0,0)$ where $w = xK([P_\mu, P_\lambda, P_v]_L)$
$\Rightarrow w \in N(K^T)$
$detK = detK^T > (0.0, 0.0, 0.0)$
$N(K^T) = (0.0, 0.0, 0.0)$
Here, $w = (0.0, 0.0, 0.0)$
$\Rightarrow xK([P_\mu, P_\lambda, P_v]_L) = (0.0, 0.0, 0.0)$
$\Rightarrow xK \in N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T)$
$\Rightarrow xK([P_\mu, P_\lambda, P_v]_L^T)K^T = (0.0, 0.0, 0.0)$
$\Rightarrow x \in N(K([P_\mu, P_\lambda, P_v]_L)K^T)$
$N(K[P_\mu, P_\lambda, P_v]_L K^T) \subseteq N(K[P_\mu, P_\lambda, P_v]_L K^T)$
Similarly, $N(K[P_\mu, P_\lambda, P_v]_L K^T)$
$\subseteq N(K[P_\mu, P_\lambda, P_v]_L K^T)$
Therefore,
$N(K[P_\mu, P_\lambda, P_v]_L K^T) = N([P_\mu, P_\lambda, P_v]_L^T K^T)$
Conversely, if $N(K[P_\mu, P_\lambda, P_v]_L K^T)$
$= N(K[P_\mu, P_\lambda, P_v]_L K^T)$,
$N([P_\mu, P_\lambda, P_v]_L) = N(K^T(K([P_\mu, P_\lambda, P_v]_L)K^T)K)$
$= N(K^T(K([P_\mu, P_\lambda, P_v]_L)K^T)K)$
$N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L)$
$N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K^T[P_\mu, P_\lambda, P_v]_L^T K^T)$
Similarly,
$N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[P_\mu, P_\lambda, P_v]_U K^T)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K^T[P_\mu, P_\lambda, P_v]_U^T K^T)$.

Theorem XI.3. For $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >\in IVNFM_m$ is KS IVNFM, then $N([P_\mu, P_\lambda, P_v]_L[P_\mu, P_\lambda, P_v]_L^T)$
$= N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T[P_\mu, P_\lambda, P_v]_L)$
and $N([P_\mu, P_\lambda, P_v]_U[P_\mu, P_\lambda, P_v]_U^T)$
$= N([P_\mu, P_\lambda, P_v]_U) = N([P_\mu, P_\lambda, P_v]_U^T[P_\mu, P_\lambda, P_v]_U)$.
Proof: Let $x \in N([P_\mu, P_\lambda, P_v]_L)$
$\Leftrightarrow x[P_\mu, P_\lambda, P_v]_L = (0.0, 0.0, 0.0)$
$\Leftrightarrow x[P_\mu, P_\lambda, P_v]_L[P_\mu, P_\lambda, P_v]_L^T = (0.0, 0.0, 0.0)$
$\Leftrightarrow X \in N([P_\mu, P_\lambda, P_v]_L[P_\mu, P_\lambda, P_v]_L^T)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L) \subseteq N([P_\mu, P_\lambda, P_v]_L[P_\mu, P_\lambda, P_v]_L^T)$
Similarly, $N([P_\mu, P_\lambda, P_v]_L[P_\mu, P_\lambda, P_v]_L^T)$
$\subseteq N([P_\mu, P_\lambda, P_v]_L)$
Therefore, $N([P_\mu, P_\lambda, P_v]_L)$
$= N([P_\mu, P_\lambda, P_v]_L[P_\mu, P_\lambda, P_v]_L^T)$
Similarly, $N([P_\mu, P_\lambda, P_v]_L)$
$= N([P_\mu, P_\lambda, P_v]_L^T[P_\mu, P_\lambda, P_v]_L)$

Therefore, $N([P_\mu, P_\lambda, P_v]_L[P_\mu, P_\lambda, P_v]_L^T)$
$= N([P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T[P_\mu, P_\lambda, P_v]_L)$

Similarly, $N([P_\mu, P_\lambda, P_v]_U[P_\mu, P_\lambda, P_v]_U^T)$
$= N([P_\mu, P_\lambda, P_v]_U) = N([P_\mu, P_\lambda, P_v]_U^T[P_\mu, P_\lambda, P_v]_U)$.

Theorem XI.4. For $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >, < [Q_\mu, Q_\lambda, Q_v]_L, [Q_\mu, Q_\lambda, Q_v]_L >\in IVNFM_{nm}$ and

KNFM, $R([P_\mu, P_\lambda, P_v]_L) = R([Q_\mu, Q_\lambda, Q_v]_L)$
$\Leftrightarrow R(K[P_\mu, P_\lambda, P_v]_L K^T) = R(K[Q_\mu, Q_\lambda, Q_v]_L K^T)$ and
$R([P_\mu, P_\lambda, P_v]_U) = R([Q_\mu, Q_\lambda, Q_v]_U)$
$\Leftrightarrow R(K[P_\mu, P_\lambda, P_v]_U K^T) = R(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$
Proof: Let $R([P_\mu, P_\lambda, P_v]_L) = R([Q_\mu, Q_\lambda, Q_v]_L)$
Then,$R([P_\mu, P_\lambda, P_v]_L K^T) = R([Q_\mu, Q_\lambda, Q_v]_L K^T)$
$= R([P_\mu, P_\lambda, P_v]_L)K^T$
$= R([P_\mu, P_\lambda, P_v]_L K^T)$
Let $z \in \{R([P_\mu, P_\lambda, P_v]_L K^T)\}$
$z = w(K[P_\mu, P_\lambda, P_v]_L K^T)$ for some $w \in V^n$
$z = r[P_\mu, P_\lambda, P_v]_L K^T, r = wK$
$z \in R(R([P_\mu, P_\lambda, P_v]_L)K^T) = R([Q_\mu, Q_\lambda, Q_v]_L)(K^T)$
$z = u[P_\mu, P_\lambda, P_v]_L K^T$ for some $u \in V^n$
$z = (uK^T)K[Q_\mu, Q_\lambda, Q_v]_L K^T$
$z = vK[Q_\mu, Q_\lambda, Q_v]_L K^T$ for some $v \in V^n$
$z \in R(K([Q_\mu, Q_\lambda, Q_v]_L K^T)$
Therefore, $R(K([P_\mu, P_\lambda, P_v]_L K^T)$
$\subseteq R(K([Q_\mu, Q_\lambda, Q_v]_L K^T)$
Similarly, $R(K([Q_\mu, Q_\lambda, Q_v]_L K^T)$
$\subseteq R(K([P_\mu, P_\lambda, P_v]_L K^T)$
Therefore, $R(K([P_\mu, P_\lambda, P_v]_L K^T)$
$= R(K([Q_\mu, Q_\lambda, Q_v]_L K^T)$
Conversely, Let $R(K([P_\mu, P_\lambda, P_v]_L K^T)$
$\subseteq R(K([Q_\mu, Q_\lambda, Q_v]_L K^T)$
$= R(K^T([Q_\mu, Q_\lambda, Q_v]K^T)K$
$= R([Q_\mu, Q_\lambda, Q_v])$
$R([P_\mu, P_\lambda, P_v]_L) = R([Q_\mu, Q_\lambda, Q_v]_L)$
Similarly,
$R([P_\mu, P_\lambda, P_v]_U) = R([Q_\mu, Q_\lambda, Q_v]_U)$
$\Leftrightarrow R(K[P_\mu, P_\lambda, P_v]_U K^T) = R(K[Q_\mu, Q_\lambda, Q_v]_U K^T)$.

Theorem XI.5. The subsequence conditions are equivalent for $P =< [P_\mu, P_\lambda, P_v]_L, [P_\mu, P_\lambda, P_v]_U >$
$\in IVNFMnn$
$(i)N(K[P_\mu, P_\lambda, P_v]_L K^T) = N(K[P_\mu, P_\lambda, P_v]_L K^T)$,
$N(K[P_\mu, P_\lambda, P_v]_U K^T) = N(K[P_\mu, P_\lambda, P_v]_U K^T)$
$(ii)N(K[P_\mu, P_\lambda, P_v]_L) = N((K[P_\mu, P_\lambda, P_v]_L)^T)$,
$N(K[P_\mu, P_\lambda, P_v]_U) = N((K[P_\mu, P_\lambda, P_v]_L)^T)$
$(iii)N(K[P_\mu, P_\lambda, P_v]_L) = N((K[P_\mu, P_\lambda, P_v]_L)^T)$,
$N([P_\mu, P_\lambda, P_v]_U K) = N(([P_\mu, P_\lambda, P_v]_U K)^T)$
$(iv)N([P_\mu, P_\lambda, P_v]_L^T) = N(K[P_\mu, P_\lambda, P_v]_L)$,
$N[P_\mu, P_\lambda, P_v]_U^T = N(K[P_\mu, P_\lambda, P_v]_U)$
$(v)N([P_\mu, P_\lambda, P_v]_L) = N(([P_\mu, P_\lambda, P_v]_L K)^T)$,
$N([P_\mu, P_\lambda, P_v]_U) = N(([P_\mu, P_\lambda, P_v]_U K)^T)$
$(vi)[P_\mu, P_\lambda, P_v]_L^\dagger isk - KSIVNFM, [P_\mu, P_\lambda, P_v]_U^\dagger \ is \ k - KSIVNFM$
$(vii)N([P_\mu, P_\lambda, P_v]_L) = N[P_\mu, P_\lambda, P_v]_L^\dagger K)$,
$N[P_\mu, P_\lambda, P_v]_U = N[P_\mu, P_\lambda, P_v]_U^\dagger K$
$(viii)K[P_\mu, P_\lambda, P_v]_L^\dagger [P_\mu, P_\lambda, P_v]_L$
$= [P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^\dagger K$,
$K[P_\mu, P_\lambda, P_v]_U^\dagger [P_\mu, P_\lambda, P_v]_U$
$= [P_\mu, P_\lambda, P_v]_U [P_\mu, P_\lambda, P_v]_U^\dagger K$
$(ix)[P_\mu, P_\lambda, P_v]_L^\dagger [P_\mu, P_\lambda, P_v]_L K$
$= K[P_\mu, P_\lambda, P_v]_L [P_\mu, P_\lambda, P_v]_L^\dagger$,
$[P_\mu, P_\lambda, P_v]_U^\dagger [P_\mu, P_\lambda, P_v]_U K$
$= K[P_\mu, P_\lambda, P_v]_U [P_\mu, P_\lambda, P_v]_U^\dagger$.
Proof: $(i) \Leftrightarrow (ii)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L) = N(K[P_\mu, P_\lambda, P_v]_L^T K)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L) = N([P_\mu, P_\lambda, P_v]_L^T K)$
(By P_2)$(K^2 = I)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L) = N((K[P_\mu, P_\lambda, P_v]_L)^T)$
(Because $(KP)^T = P^T(K^T = P^T K)$
$K[P_\mu, P_\lambda, P_v]_L$ is KS,
Similarly, $N([P_\mu, P_\lambda, P_v]_U) = N(K[P_\mu, P_\lambda, P_v]_U^T K)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_U) = N((K[P_\mu, P_\lambda, P_v]_U)^T)$
Condition(ii) is true
Condition $(i) \Leftrightarrow (iii)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L) = N(K[P_\mu, P_\lambda, P_v]_L^T K)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L K) = N(K[P_\mu, P_\lambda, P_v]_L^T)$
(By P_2)$(K^2 = I)$
$\Leftrightarrow \quad N([P_\mu, P_\lambda, P_v]_L K) \quad = \quad N(([P_\mu, P_\lambda, P_v]_L)K)^T)$
(Because $(KP)^T = P^T(K^T = P^T K)$
$K[P_\mu, P_\lambda, P_v]_L K$ is KS,
Similarly, $N([P_\mu, P_\lambda, P_v]_U) = N([P_\mu, P_\lambda, P_v]_U^T K)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_U K) = N(([P_\mu, P_\lambda, P_v]_U K)^T)$
Therefore, (iii) holds
$(ii) \Leftrightarrow (iv)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L) = N(K[P_\mu, P_\lambda, P_v]_U^T)$
$= N(([P_\mu, P_\lambda, P_v]_U)^T K)$
$\Leftrightarrow N(K[P_\mu, P_\lambda, P_v]_L)$
$= N([P_\mu, P_\lambda, P_v]_U^T) \qquad (By\ P_2)$
Similarly,$N(K[P_\mu, P_\lambda, P_v]_U) = N(K[P_\mu, P_\lambda, P_v]_U)^T$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L)^T) = N(K[P_\mu, P_\lambda, P_v]_U)$
Therefore, (iv) holds
$(iii) \Leftrightarrow (iv)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L K) = N(([P_\mu, P_\lambda, P_v]_L K)^T)$
$\Leftrightarrow N([P_\mu, P_\lambda, P_v]_L) = N(([P_\mu, P_\lambda, P_v]_L K)^T)(ByP_2)$
Similarly, $N([P_\mu, P_\lambda, P_v]_U K)$
$= N(K[P_\mu, P_\lambda, P_v]_U)K^T \Leftrightarrow N([P_\mu, P_\lambda, P_v]_L)$
$= N([P_\mu, P_\lambda, P_v]_U K)^T$.
$(ii) \Leftrightarrow (vi)$ holds
Condition $(i) \Leftrightarrow (vii)$ holds.
$(i) \Leftrightarrow (viii)$ holds.
$(viii) \Leftrightarrow (ix)$.
Therefore (ix) are holds.

XII. Application of adjacency Neutrosophic Fuzzy Matrix by using graphs in decision making

In this section, we introduce an algorithm designed to reduce parameters through the use of an adjacency matrix associated with a soft graph. We then apply this algorithm to a decision-making problem.

12.1 Algorithm Consider the product set $\{M_1, M_2, M_3, \ldots..M_n\}$ with parameters $\{P_1, P_2, \ldots, P_k\}$. To select the best product based on these parameters, we propose the following algorithm, which utilizes the adjacency matrix of a soft graph.

Form a bipartite graph G=(V,E) for the given problem.

In this graph: V represents the set of vertices, which includes the products $\{M_1, M_2, M_3, \ldots..M_n\}$ and the parameters $\{P_1, P_2, \ldots, P_k\}$. If E represents the set of edges, which connect each product Mi to the relevant parameters P_i.

This bipartite graph effectively illustrates the relationships between the products and their associated parameters.
To construct a soft graph (F,A) with A=$\{M_1, M_2, M_3, \ldots..M_n\}$ follow these steps:

1) Define the Set S(x): For a given vertex x in the graph, define $S(x) = \{z \in V : d(x,z) \leqslant 1\}$, where d(x,z) denotes the distance between vertices x and z.

2) Define the Set T(x): Define $T(x) = \{xu \in E : u \in S(x)\}$, where E represents the set of edges and u is a vertex connected to x.

3) Construct F(x): The soft graph F(x) is then represented as F(x)=(S(x),T(x)), where S(x) is the set of vertices within a distance of (1,1,0) from x and T(x) is the set of edges connecting x to these vertices.

4) • Construct the adjacency matrix of the given soft graph (F,A). In this matrix:

   • The rows correspond to the products M_i.

   • The columns correspond to the parameters P_j.

   Each entry (M_i, P_j) in the matrix represents the relationship or connection between product Mi and parameterP_j. If there is a connection, the entry is (1,1,0); otherwise, it is (0,0,1).

   • If any entry (M_i, P_j) in the adjacency matrix is either (1,1,0) or (0,0,1) for all i=1,2,...,n, then the parameter P_j should be removed. This indicates that the parameter P_j does not contribute to distinguishing between products and can therefore be excluded from consideration.

   • To determine the row totals in the modified adjacency matrix of the soft graph.

   •Modify the Adjacency Matrix: Make any necessary modifications to the original adjacency matrix based on the specific requirements or criteria provided.

   •Calculate Row Totals: For each row in the modified adjacency matrix, sum the entries. This sum represents the total number of connections or relationships for each vertex in the graph.

   •Identify the Last Column: The last column of the matrix should contain these row totals.

   • Determine the product M_i that has the highest row total.

   • The product with the highest row total will be the most favorable option.

12.2 Application

Mr. X is looking to purchase a laptop from the following options:$M_1, M_2, M_3, M_4,$ andM_5, each of which has certain properties $P_1, P_2, P_3,$ and P_4.

• Laptop M_1 features properties P_1 and P_2.

• Laptop M_2 includes properties $P_1, P_2,$ and P_4.

• LaptopM_3 has property P_2.

• Laptop M_4 also includes property P_2.

• Laptop M_5 comes with properties P_2 and P_3.

What would be the best laptop choice for optimal performance?

First, create a graph based on the provided problem as described.

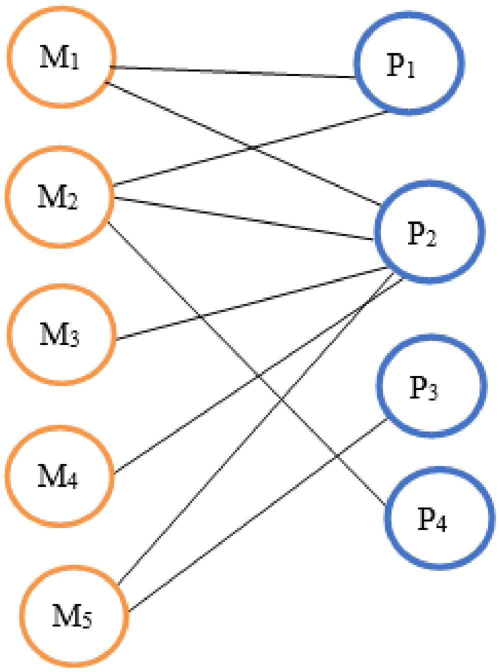

Figure 9:Bipartite Graph

If V =$\{M_1, M_2, M_3, M_4, M_5, P_1, P_2, P_3, P_4\}$,
Select A=$\{M_1, M_2, M_3, M_4, M_5\}$,
Describe $S(x) = \{z \in V : d(x,z) \leqslant 1\}$,
$T(x) = \{xu \in E : u \in S(x)\}$ and
F(x)=(S(x), T(x)),
$F(M_1) = \{M_1, P_1, P_2\}$,
$F(M_2) = \{M_2, P_1, P_2, P_4\}$,
$F(M_3) = \{M_3, P_2\}$,
$F(M_4) = \{M_4, P_2\}$,
$F(M_5) = \{M_5, P_2, P_3\}$.
Thus (F, A) is a soft graph. From graph (Figure 9), the adjaceny matrices are determined as shown in Table 1 and Table 2.

The row total in the final column of the adjacency matrix represents the vertex degree in the soft graph (F,A). Given that the entry (M_i, P_2) is (1,1,0) for i=1,2,3,4,5 we should remove the second column from the matrix.

In the final column of the matrix, examine the row totals for laptops $M_1, M_2, M_3, M_4,$ and M_5. The row total forM_2 is the highest among them. Therefore, M_2 is the optimal choice.

TABLE I
ADJACENCY NEUTROSOPHIC SOFT FUZZY MATRIX

	P_1	P_2	P_3	P_4	M_1	M_2	M_3	M_4	M_5	Row total =Degree of vertex
M_1	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1,\rangle$	$\langle 2\rangle$
M_2	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 3\rangle$
M_3	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1\rangle$
M_4	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1\rangle$
M_5	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 2\rangle$
P_1	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 2\rangle$
P_2	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 5\rangle$
P_3	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 1\rangle$
P_4	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1\rangle$

TABLE II
ADJACENCY NEUTROSOPHIC SOFT FUZZY MATRIX

	P_1	P_3	P_4	M_1	M_2	M_3	M_4	M_5	Row total =Degree of vertex
M_1	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1,\rangle$	$\langle 1\rangle$
M_2	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 2\rangle$
M_3	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0\rangle$
M_4	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0\rangle$
M_5	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1\rangle$
P_1	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 2\rangle$
P_2	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 5\rangle$
P_3	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 1\rangle$
P_4	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1\rangle$

XIII. CONCLUSIONS AND FUTURE WORK

In conclusion, this research has explored the properties and relationships of IV secondary s-RS fuzzy matrices, highlighting on their connections with various other types of IV matrices. We have determined the required and complete criteria for a fuzzy matrix to be classified as IV s-RS. Notably, we've shown that s-symmetry implies s-RS, though the reverse is always true. Furthermore, we've illustrated the equivalent criteria for the g-inverses of IV s-RS fuzzy matrices to retain their IV s-RS. Also, we illustrate a graphical representation of KS, CS and RS adjacency and incidence fuzzy matrices.

Every adjacency NFM is symmetric, RS, CS and KS but incidence matrix satisfies only KS conditions. Similarly, every RS adjacency fuzzy matrices is KS adjacency NFM but KS adjacency fuzzy matrices need not be RS fuzzy matrices. The generalized inverses of an IV s-k RS matrix A corresponding to the sets A$\{1,2\}$, A$\{1, 2, 3\}$ and A$\{1, 2, 4\}$ are characterized. These findings contribute to a deeper understanding of IV matrices and their symmetrical properties within the context of fuzzy matrices.Soft graphs represent a novel area of research in mathematics. In this paper, we explore their application in decision-making by utilizing the adjacency matrix of a soft graph and developing a corresponding algorithm. In future we will work on related properties of IV Secondary k-RS fuzzy matrices with graphical representation.

REFERENCES

[1] D. Jaya shree, Secondary k-KS fuzzy matrices, Intern. J. Fuzzy Mathematical Archive, Vol. 5, no. 2, pp. 89-94, 2014.

[2] M. Anandhkumar, H. Prathab, S. M. Chithra, A. S. Prakaash, A. Bobin, Secondary k-Range Symmetric Neutrosophic Fuzzy Matrices, International Journal of Neutrosophic Science, vol. 23, no. 4, 2024, pp. 23-28.

[3] AR. Meenakshi, fuzzy matrix: Theory and Applications, MJP Publishers, Chennai, 2008.

[4] An Lee, Secondary Symmetric, Secondary Skew Symmetric, Secondary Orthogonal Matrices, Period Math, Hungary, Vol. 7, pp. 63-76, 1976.

[5] C. Antonio and B. Paul, Properties of the eigen vectors of persymmetric matrices with applications to communication theory, IEEE Trans. Comm., Vol. 24, pp. 804-809, 1976.

[6] K. H. Kim and F. W. Roush, Generalized fuzzy matrices, Fuzzy Sets and Systems, Vol. 4, no. 3, pp. 293-315, 1980.

[7] R. D. Hill and S. R. Waters, On k-Real and k-Hermitian matrices, Linear Algebra and its Applications, Vol. 169, pp. 17-29, 1992.

[8] AR. Meenakshi and S. Krishanmoorthy, On Secondary k-Hermitian matrices, Journal of Modern Science, Vol. 1, pp. 70-78, 2009.

[9] AR. Meenakshi, S. Krishnamoorthy and G. Ramesh, On s-k-EP matrices, Journal of Intelligent System Research, Vol. 2, pp. 93-100, 2008.

[10] AR. Meenakshi and D. Jaya Shree, On k-KS matrices, International Journal of Mathematics and Mathematical Sciences, Article ID 926217, 8 Pages, 2009.

[11] AR. Meenakshi and D. Jaya Shree, On k-RS matrices, Proceedings of the National conference on Algebra and Graph Theory, MS University, pp. 58-67, 2009.

[12] M. Anandhkumar, B. Kanimozhi, S. M. Chithra, V. Kamalakannan, Broumi Said, On various Inverse of Neutrosophic fuzzy matrices International Journal of Neutrosophic Science, International Journal of Neutrosophic Science, Vol. 21, no. 2, pp. 20-31, 2023.

[13] A. K. Shyamal and M. Pal, Interval valued fuzzy matrices, Journal of Fuzzy Mathematics, Vol. 14, no.3, pp. 582-592, 2006.

[14] AR. Meenakshi and M. Kalliraja, Regular IV fuzzy matrices, Advance in Fuzzy Mathematics, Vol.5, no. 1, pp.7-15, 2010.

[15] M. Anandhkumar, G. Punithavalli, T. Soupramanien and Said Broumi, Generalized Symmetric Neutrosophic fuzzy matrices, Neutrosophic Sets and Systems, Vol. 57, pp. 114-127, 2023.

[16] Xiaomin Gong, Jingshu Zhang, and Xuemei Zhao, A BWM-TODIM based Integrated Decision Framework for Financial Technology Selection with Interval Type-2 Fuzzy Sets, IAENG International Journal of Applied Mathematics, Vol. 52, no. 4, pp826-837, 2022.

[17] Qianhong ZHANG, and Bairong PAN, Qualitative Analysis of k-order Rational Fuzzy Difference Equation, IAENG International Journal of Applied Mathematics, Vol. 53, no. 3, pp839-845, 2023.

[18] Wenjun Sun, Changfeng Zhu, and Kangru Liu, Prospect Selection Decisions for Emergency Logistics Paths in Fuzzy Environments, IAENG International Journal of Applied Mathematics, Vol. 53, no.1, pp183-193, 2023.

[19] D. Jaya Shree, Secondary k-RS fuzzy matrices, Journal of Discrete Mathematical Sciences and Cryptography 21(1):1-11,2018.

[20] M. Kaliraja and T. Bhavani, IV Secondary k-RS fuzzy matrices, Advances and Applications in Mathematical Sciences Vol. 21, no. 10, August 2022, Pages 5555-5574.

[21] M. Anandhkumar, V.Kamalakannan, S.M.Chitra, and Said Broumi, Pseudo Similarity of Neutrosophic fuzzy matrices, International Journal of Neutrosophic Science, Vol. 20, no. 04, PP. 191-196, 2023.

[22] M. Anandhkumar, T. Harikrishnan, S. M. Chithra, V. Kamalakannan, B. Kanimozhi, Broumi Said, Reverse Sharp and Left-T Right-T Partial Ordering on Neutrosophic fuzzy matrices, International Journal of Neutrosophic Science, Vol. 21, no. 04, PP. 135-145, 2023.

[23] M.Anandhkumar, B.Kanimozhi, S.M. Chithra, V.Kamalakannan, Reverse Tilde (T) and Minus Partial Ordering on Intuitionistic fuzzy matrices, Mathematical Modelling of Engineering Problems, 2023, 10(4), pp. 1427–1432.

[24] M. Anandhkumar, T. Harikrishnan, S. M. Chithra, V. Kamalakannan, B. Kanimozhi, Partial orderings, Characterizations and Generalization of k-idempotent Neutrosophic fuzzy matrices, International Journal of Neutrosophic Science, Vol. 23, no. 2, 2024, pp. 286-295.

[25] M. Anandhkumar, G. Punithavalli, and E.Janaki, secondary k-column neutrosophic fuzzy matrices, Neutrosophic Sets and Systems 64, 2024, pp. 24-37.

[26] M. Anandhkumar, G. Punithavalli, R. Jegan, and S Broumi, Interval valued Secondary k-RS Neutrosophic fuzzy matrices, Neutrosophic Sets and Systems 61, 2024,1.

[27] G. Punithavalli, M. Anandhkumar, Kernel and k-Kernel Symmetric Intuitionistic Fuzzy Matrices,TWMS J. App. and Eng. Math. V.14, N.3, 2024, pp. 1231-1240.

[28] M. Anandhkumar, A. Bobin, S. M. Chithra, and V. Kamalakannan, Generalized Symmetric Fermatean Neutrosophic Fuzzy Matrices. Neutrosophic Sets and Systems 70, 1 (2024).

[29] G. Punithavalli1, M. Anandhkumar, Reverse Sharp and Left-T Right-T Partial Ordering On Intuitionistic Fuzzy Matrices, TWMS J. App. and Eng. Math. V.14, N.4, 2024, pp. 1772-1783.