Best-Other Algorithm: A Metaheuristic Combining Best Member with Other Entities

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***Abstract*— This article introduces a novel stochastic optimization method termed the Best-Other Algorithm (BOA). The nomenclature reflects its reliance on the best member, which is amalgamated with other entities. BOA, a metaphor-free swarm-based metaheuristic, comprises three directed searches. The first involves subtracting the best member from a randomly selected member. The second entails determining the midpoint between the best member, and another randomly chosen member. The third centers around the midpoint between the best member and a random solution along the space. The efficacy of BOA is evaluated by challenging it to solve a collection of 23 functions. In this evaluation, BOA is pitted against five other metaheuristics: Northern Goshawk Optimization (NGO), Zebra Optimization Algorithm (ZOA), Coati Optimization Algorithm (COA), Migration Algorithm (MA), and Osprey Optimization Algorithm (OOA). The findings indicate the superiority of BOA over its counterparts. BOA outperforms NGO, ZOA, COA, MA, and OOA in 21, 15, 16, 15, and 17 functions, respectively. These results underscore the pivotal role of the best member as a reference and the comparatively lesser significance of the neighborhood search as the search space diminishes during the iteration.**

***Index Terms*—computational intelligence, swarm intelligence, optimization, metaheuristic, stochastic.**

I. Introduction

METAHEURISTIC represents a widely employed search method, particularly in the realm of optimization, notably within engineering applications. He et al. applied multi-particle swarm optimization extensively to enhance online train trajectory planning, with a specific focus on minimizing energy consumption [1]. Adeola et al. integrated the genetic algorithm (GA) and long short-term memory (LSTM) to forecast bankruptcy [2]. Wang et al. merged the non-dominated sorting genetic algorithm (NSGA II) with quantum coding, leading to the development of the quantum genetic algorithm. This innovative approach was employed to address route optimization challenges for travelers requiring intermodal transportation [3]. The amalgamation of Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) has been employed to optimize the allocation of storage locations within an e-commerce warehouse. The primary objectives of this optimization endeavor are the minimization of both the average and maximum response time, as well as the daily walking distance covered by pickers [4]. Wan et al., exploited the classic variable neighborhood search (VNS) to optimize the batching assignment in e-commerce warehouse to minimize the response time, picking time, and walking distance [5].

In recent years, swarm-based metaheuristics have gained popularity, with some drawing inspiration from animal behaviors. Examples include the Elephant Clan Optimization (ECO) [6], African Vultures Optimization Algorithm (AVOA) [7], Northern Goshawk Optimization (NGO) [8], Marine Predator Algorithm (MPA) [9], Modified Honey Badger Algorithm (MHBA) [10], Pelican Optimization Algorithm (POA) [11], Osprey Optimization Algorithm (OOA) [12], and Coati Optimization Algorithm (COA) [13], Zebra Optimization Algorithm (ZOA) [14], Golden Jackal Optimization (GJO) [15], Fennec Fox Optimization (FFO) [16], Walrus Optimization Algorithm (WaOA) [17], Whale Optimization Algorithm (WOA) [18], Siberian Tiger Optimization (STO) [19], Clouded Leopard Optimization (CLO) [20], Green Anaconda Optimization (GAO) [21], White Shark Optimization (WSO) [22], Snake Optimization (SO) [23], Komodo Mlipir Algorithm (KMA) [24], Cheetah Optimization (CO) [25], and so on. Some of them exploited the social behavior of human, such as Migration Algorithm (MA) [26], Mother Optimization Algorithm (MOA) [27], Modified Social Forces Algorithm (MSFA) [28], Driving Training-Based Optimization (DTBO) [29], Chef-Based Optimization Algorithm (CBOA) [30], Election-Based Optimization Algorithm (EBOA) [31], and so on. Certain metaheuristics incorporate their references in the directed search process, as seen in examples like Three Influential Members-Based Optimization (TIMBO) [32], Mixed Leader-Based Optimization (MLBO) [33], Hybrid Leader-Based Optimization (HLBO) [34], and others. On the contrary, some metaheuristics abstain from employing metaphors altogether, as evident in approaches like Total Interaction Algorithm (TIA) [35], Attack Leave Optimization (ALO) [36], Adaptive Balance Optimization (ABO) [37], ASBO [38], and similar methods.

Despite the extensive development of stochastic optimization or in more general terms metaheuristics, the no-free-lunch (NFL) theory still poses the opportunity of potential for further development as there is not any solution that will become general solution for all kinds of problems [20]. There is always a weakness where a metaheuristic

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performs mediocre or poorly despite its superiority in many problems [13].

The existence of the best member for becoming the reference in the directed search in many swarm-based metaheuristics gives guidance for further improvement by exploiting it with many other entities. The list consisting of some recent swarm-based metaheuristics that exploit the existence of the best member is presented in Table 1. On the other hand, the benefaction of the local or neighborhood search enriched in many recent swarm-based metaheuristics can be questioned.

As presented in Table 1, there are a lot of variations in utilizing the best member as a reference or one of the references during the directed search. Some of them utilize the best member in a dedicated manner while some others mix the best member with other entities or use the best member in a stochastic manner.

Considering the identified problem and circumstances, this study aims to introduce a novel metaphor-free swarm-based metaheuristic named Best-Other Algorithm (BOA). The conceptualization of BOA is driven by the intention to harness the presence of the best member and amalgamate it with other entities to create fresh references for the directed search conducted by members in swarm-based metaheuristics. BOA incorporates three distinct references. The first reference is defined as the gap between the best member and a randomly selected member. The second reference is established as the midpoint between the best member, and another randomly chosen member. The third reference is identified as the midpoint between the best member and a randomized solution within the space. It is noteworthy that BOA diverges from the trend observed in many contemporary swarm-based metaheuristics by excluding the implementation of local search with a reduced search space during iteration. Concurrently, BOA adheres to a stringent replacement rule, allowing a new solution candidate to replace the existing one only if it enhances the quality of the solution.

The scientific contributions of this study are briefed as follows.

- A new swarm-based metaheuristic which is free from metaphor called best-other algorithm (BOA) which is constructed by the three directed searches and utilizes the mixture between the best member and other entities is introduced.
- A detailed description encompassing all considerations in the algorithm including the fundamental concept, pseudocode, and mathematical formulation is presented.
- The effectiveness of BOA is evaluated through its application to an optimization problem, specifically addressing a set of 23 functions chosen as the use case.
- Additionally, the performance of BOA is subjected to comparison with five recently developed swarm-based metaheuristics. This comparative analysis aims to discern and highlight the potential contributions of BOA within the broader trajectory of metaheuristic development over the long term.

TABLE I
SOME RECENT SWARM BASED METAHEURISTICS THAT UTILIZE THE BEST MEMBER

No	Metaheuristic	How the Best member is Used	Existence of Local Search with Reduced Local Space
1	GAO [21]	Each member moves toward one of the female anacondas where the female anacondas are all members whose quality is better than the corresponding member. It means that the best member is one of the female anacondas.	yes
2	ASBO [38]	The algorithm involves three distinct searches. In the initial search, the member adjusts its position relative to the gap between the best member and the worst member. In the second search, the member shifts in relation to the midpoint between the best member and the worst member. In the third search, the member deliberately avoids proximity to the best member.	no
3	ZOA [14]	Each member moves toward the best member in the foraging behavior (first phase).	yes
4	COA [13]	Half of the swarm moves toward the best member (iguana) in the hunting-attacking strategy (first phase).	yes
5	MOA [27]	Each member moves toward the best member (mother) in the education stage (first phase).	yes
6	MA [26]	Each member moves toward a randomly picked better member in the choosing and moving to the migration destination (first phase). The best member is one of the better members.	yes
7	WaOA [17]	Each member moves toward the best member (the strongest walrus) during the feeding strategy (first phase).	yes
8	KMA [24]	The members with moderate performance (females) perform crossover with the best swam member. The poor-performance members move toward the resultant of good-performance members (the best member is one in the group of good-performance members). The high-performing members move toward the result of better high-performing members.	no
9	GJO [15]	The best member and the second-best member get closer to or evade the member.	no
10	HLBO [34]	Each member moves relative to a hybrid leader consisting of the mixture of the member itself, the best member, and a randomly picked member.	yes
11	MLBO [33]	In the first half of the iteration, each member moves relative to the mixture between the best swam member and a randomized solution within space. In the second half of the iteration, each member moves relative to the best swam member.	no
12	TIMBO [32]	Each member moves toward the best member in the first search.	no
13	ALO [36]	During the initial search, the member either advances toward the best member, or conversely, the best member retreats from the member. In the second search, the reference point is established as the midpoint between the best member and a randomly chosen member, or alternatively, it is determined as the midpoint between two randomly selected members.	no
14	TIA [35]	Each member performs a directed search relative to all other members and one of them is the best member except the member is the best member itself.	no
15	ABO [37]	Each member performs the motion toward the best member if the improvement takes place.	yes

- The effectiveness of BOA is also evaluated through convergence assessment so that the speed of the optimization process is known.

The subsequent sections of this paper are organized as follows. Section two provides a comprehensive exposition of the proposed Best-Other Algorithm (BOA), encompassing a comprehensive elucidation of its fundamental concept, pseudocode, and mathematical formulation. The assessment scenario and the result are presented in section three. The discussion of the comprehensive analysis including the result, searching capability, limitations, and computational complexity of BOA is presented in section four. The conclusion and tracks for further or future studies are presented in section five.

II. Proposed Model

BOA is developed on the foundational principle of leveraging the presence of the best member as a reference. However, a key aspect of BOA's construction involves amalgamating this best member with other entities within the swarm. This integration is essential to enhance BOA's exploration capability. Relying solely on the best member has the potential to confine members within local optimal solutions. It is noteworthy that the superiority of the current best member is not perpetual, emphasizing the necessity of diversifying references to foster effective exploration in BOA.

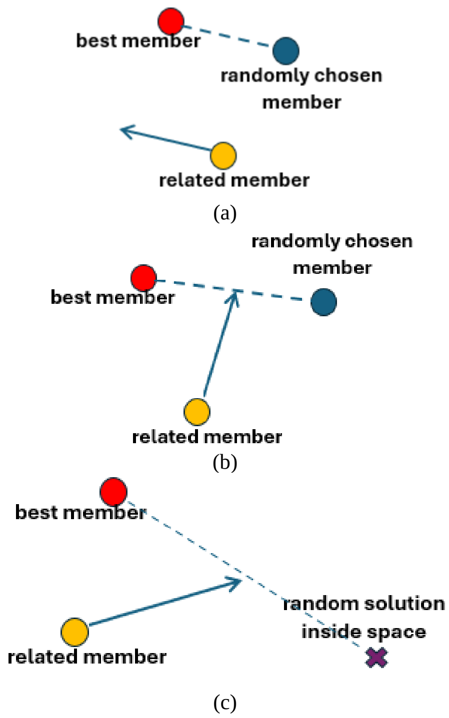

Fig. 1. Illustration of the three motions: (a) first motion, (b) second motion, and (c) third motion.

Based on this reason, BOA incorporates three references to diversify its exploration strategy. The first reference calculates the gap between the best member and a randomly chosen member. The second reference identifies the midpoint between the best member and another randomly selected member. The third reference determines the midpoint between the best member and a randomly generated solution within the space. These references play a pivotal role in three consecutive searches executed by all members in each iteration. During these searches, the direction of movement for each member is determined by comparing its quality with the reference. If the reference surpasses the member in quality, the member adjusts its position towards the reference. Conversely, if the reference is inferior, the member repositions itself to avoid the reference. This dynamic interplay ensures that the members navigate the search space effectively based on the evolving comparison with diverse references. These three motions are depicted using Fig. 1.

In each search, a solution candidate is generated, and this candidate is subsequently compared with the current solution of the member. If the candidate proves to be superior to the member's current solution, an update occurs, replacing the member's current solution with the newly generated candidate. Conversely, if the candidate does not outperform the current solution, the member retains its existing solution. This updating process adheres to a strict replacement rule. The formalization of this concept is outlined in pseudocode shown in Algorithm 1. The accompanying mathematical formulation is then provided in (1) to (10). Prior to delving into these details, the notations employed in this paper are introduced as follows.

d	dimension
f	objective function
i	member's index
j	dimension's index
s	member
S	swarm or set of members
s_b	the best member
s_r	a randomly picked member within the swarm
s_u	a uniformly generated member within the space
s_{ref}	reference
s_{lb}	space lower bound
s_{ub}	space upper bound
s_{ca}	solution candidate
U	uniform random
r_1	floating point random between 0 and 1 (0,1)
r_2	integer random between 1 and 2 [1,2]
t	iteration
t_m	maximum iteration

The initialization stage is presented from line 2 to line 5 in algorithm 1. It consists of two processes. The first process is generating the initial solution for member using (1). Then, an updating process of the best member is performed using (2).

$$s_{i,j} = s_{lb,j} + r_1\left(s_{ub,j} - r_2 s_{lb,j}\right) \quad (1)$$

$$s_b' = \begin{cases} s_i, f(s_i) < f(s_b) \\ s_b, else \end{cases} \quad (2)$$

The iteration stage is presented from line 6 to line 15 in algorithm 1. In the iteration, the outer loop is the loop from the first iteration until the maximum iteration. Then, the loop for whole members is performed to conduct the three sequential searches. Each search consists of two processes. The first process is generating a solution candidate and the replacement of the current solution. The second process is updating the best member.

algorithm 1: best-other algorithm

begin
  for i=1 **to** $n(S)$
    initialize is using (1)
    update s_b using (2)
  end for
  for t=1 **to** t_m
    for i=1 **to** $n(S)$
      perform the first search using (3) to (6)
      update s_b using (2)
      perform a second search using (3), (7) to (9)
      update s_b using (2)
      perform a third search using (10) to (13)
      update s_b using (2)
    end for
  end for
  return s_b
end

The first search is the motion relative to the gap between the best member and a randomly picked member. Equation (3) is used to determine the randomly picked member within the swarm. Equation (4) is used to determine the gap between the best member and the randomly picked member. Equation (5) is employed to compute the first solution candidate, while (6) is utilized to update the member's current solution based on the first solution candidate. These equations play a crucial role in the iterative process, where the generation and evaluation of solution candidates contribute to the refinement of the member's solution.

$$s_r = U(S) \tag{3}$$

$$s_{re1,i,j} = s_{b,j} - s_{r,j} \tag{4}$$

$$s_{ca1,i,j} = \begin{cases} s_{i,j} + r_1\left(s_{re1,i,j} - r_2 s_{i,j}\right), f\left(s_{re1,i}\right) < f(s_i) \\ s_{i,j} + r_1\left(s_{i,j} - r_2 s_{re1,i,j}\right), else \end{cases} \tag{5}$$

$$s_i' = \begin{cases} s_{ca1,i}, f\left(s_{ca1,i}\right) < f(s_i) \\ s_i \end{cases} \tag{6}$$

In the second search, the movement is determined relative to the midpoint between the best member and a randomly chosen member. Equation (7) formally defines the second reference as the middle point between the best member and a randomly selected member from within the swarm. The calculation of the second solution candidate is described by Equation (8), specifying how the member's position evolves relative to the second reference. Subsequently, Equation (9) is applied to facilitate the updating process of the member, contingent on the quality comparison between the second solution candidate and the member's current solution. These equations collectively govern the dynamics of the second search, influencing the iterative optimization process.

$$s_{re2,i,j} = \frac{s_{b,j} + s_{r,j}}{2} \tag{7}$$

$$s_{ca2,i,j} = \begin{cases} s_{i,j} + r_1\left(s_{re2,i,j} - r_2 s_{i,j}\right), f\left(s_{re2,i}\right) < f(s_i) \\ s_{i,j} + r_1\left(s_{i,j} - r_2 s_{re2,i,j}\right), else \end{cases} \tag{8}$$

$$s_i' = \begin{cases} s_{ca2,i}, f\left(s_{ca2,i}\right) < f(s_i) \\ s_i \end{cases} \tag{9}$$

The third search is the motion relative to the middle between the best member and a randomly generated solution within the space. Equation (10) is used to determine the randomly generated member within space. Equation (11) is used to determine the middle between the best member and the randomly generated solution. Equation (12) is used to determine the third solution candidate. Equation (13) is used to update the member based on the third solution candidate.

$$s_u = s_{lb,j} + r_1\left(s_{ub,j} - r_2 s_{lb,j}\right) \tag{10}$$

$$s_{re3,i,j} = \frac{s_{b,j} + s_{u,j}}{2} \tag{11}$$

$$s_{ca3,i,j} = \begin{cases} s_{i,j} + r_1\left(s_{re3,i,j} - r_2 s_{i,j}\right), f\left(s_{re3,i}\right) < f(s_i) \\ s_{i,j} + r_1\left(s_{i,j} - r_2 s_{re3,i,j}\right), else \end{cases} \tag{12}$$

$$s_i' = \begin{cases} s_{ca3,i}, f\left(s_{ca3,i}\right) < f(s_i) \\ s_i \end{cases} \tag{13}$$

TABLE II
FUNCTIONS

No	Function	Dim	Space	Target
1	Sphere	70	[-100, 100]	0
2	Schwefel 2.22	70	[-100, 100]	0
3	Schwefel 1.2	70	[-100, 100]	0
4	Schwefel 2.21	70	[-100, 100]	0
5	Rosenbrock	70	[-30, 30]	0
6	Step	70	[-100, 100]	0
7	Quartic	70	[-1.28, 1.28]	0
8	Schwefel	70	[-500, 500]	-418.9 x *dim*
9	Ratsrigin	70	[-5.12, 5.12]	0
10	Ackley	70	[-32, 32]	0
11	Griewank	70	[-600, 600]	0
12	Penalized	70	[-50, 50]	0
13	Penalized 2	70	[-50, 50]	0
14	Shekel Foxholes	2	[-65, 65]	1
15	Kowalik	4	[-5, 5]	0.0003
16	Six Hump Camel	2	[-5, 5]	-1.0316
17	Branin	2	[-5, 5]	0.398
18	Goldstein-Price	2	[-2, 2]	3
19	Hartman 3	3	[1, 3]	-3.86
20	Hartman 6	6	[0, 1]	-3.32
21	Shekel 5	4	[0, 10]	-10.1532
22	Shekel 7	4	[0, 10]	-10.4028
23	Shekel 10	4	[0, 10]	-10.5363

III. SIMULATION AND RESULT

The evaluation of BOA is performed in two ways. The first assessment is the comparative assessment where BOA is confronted with several existing metaheuristics. Its objective is to assess the improvement of the proposed BOA in the development of metaheuristics. The second assessment is the convergence assessment. This assessment is conducted to investigate the speed of BOA to achieve the convergence state.

The first evaluation of BOA's performance involves pitting it against five contemporary metaheuristics: NGO, ZOA, COA, MA, and OOA. NGO debuted in 2021 [8], followed by the introduction of ZOA in 2022 [14]. Subsequently, COA [13], MA [26], and OOA [12] were launched in 2023. Like other metaheuristics, BOA undergoes a challenge to tackle a collection of functions representative of the optimization problem.

For this study, the selected set comprises 23 functions, encompassing seven high-dimensional unimodal functions, six high-dimensional multimodal functions, and ten fixed-dimensional multimodal functions. Refer to Table 2 for a description of these functions. The assessment is carried out with a fixed swarm size and maximum iteration set to 10.

The first assessment results are detailed in Tables 3 to 6, each focusing on distinct aspects of the optimization problem. Tables 3 to 5 specifically present outcomes related to high-dimensional unimodal functions, high-dimensional multimodal functions, and fixed-dimensional multimodal functions, respectively. In contrast, Table 6 offers a comprehensive summary of BOA's superiority compared to its contenders within each function cluster.

Tables 3 to 5 furnish three key parameters for each function: the average fitness score, standard deviation, and mean rank. Conversely, the summary in Table 6 is based on the mean rank. Explaining Table 3, it is evident that BOA excels in addressing high-dimensional unimodal functions. BOA secures the top rank in solving five functions (f_1, f_2, f_4, f_5, and f_6). However, in solving f_7, where ZOA claims the first rank, BOA is positioned second. Unfortunately, in tackling f_3, BOA holds the fifth rank, surpassing only NGO. Notably, all metaheuristics achieve identical results in solving f_2.

Continuing the analysis, Table 4 underscores BOA's proficiency in addressing high-dimensional multimodal functions. BOA secures the top rank in solving four functions (f_{10}, f_{11}, f_{12}, and f_{13}). Meanwhile, BOA is in the second rank in solving two functions (f_8 and f_9). COA is on the first rank in solving f_8 while ZOA is on the first rank in solving f_9.

Table 5 indicates the suitable performance of BOA on the one hand and the tough competition among metaheuristics on the other hand in solving the fixed dimension multimodal functions. In this cluster, BOA is on the first rank in solving six functions (f_{14}, f_{15}, f_{16}, f_{17}, f_{18}, and f_{19}). But, as all metaheuristics achieve the same result in f_{19}, it means that BOA is on the distinct first rank in four functions. Simultaneously, in this category, BOA is positioned at the third rank for one function (f_{20}), the fourth rank for one function (f_{22}), and the fifth rank for two functions as well (f_{21} and f_{23}).

TABLE III
ASSESSMENT RESULT ON HIGH-DIMENSION UNIMODAL FUNCTIONS

F	Parameter	NGO	ZOA	COA	MA	OOA	BOA
1	mean	4.2574×10^3	1.1420×10^1	1.0672×10^3	3.5499×10^2	2.9000×10^2	0.2511
	standard deviation	1.2701×10^3	5.6224	2.2012×10^2	7.9892×10^1	8.8410×10^1	0.1318
	mean rank	6	2	5	4	3	1
2	mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	standard deviation	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	mean rank	1	1	1	1	1	1
3	mean	1.5840×10^5	5.9861×10^3	3.9882×10^4	6.4585×10^4	4.2492×10^4	7.2400×10^4
	standard deviation	8.4210×10^4	3.2299×10^3	1.6462×10^4	3.6705×10^4	2.4638×10^4	4.6731×10^4
	mean rank	6	1	2	4	3	5
4	mean	5.2385×10^1	2.5847	2.2976×10^1	1.1657×10^1	1.1498×10^1	1.0620
	standard deviation	1.0652×10^1	0.7909	5.6536	3.0806	2.9773	0.3598
	mean rank	6	2	5	4	3	1
5	mean	1.4671×10^6	2.4210×10^2	1.4180×10^5	1.2589×10^4	8.0855×10^3	7.3835×10^1
	standard deviation	1.2374×10^6	8.3135×10^1	1.0865×10^5	9.5163×10^3	3.6794×10^3	2.3412
	mean rank	6	2	5	4	3	1
6	mean	4.4579×10^3	2.4210×10^1	1.0603×10^3	3.6995×10^2	2.7214×10^2	1.5624×10^1
	standard deviation	1.8870×10^3	4.6492	4.0950×10^2	1.0024×10^2	7.9106×10^1	0.6286
	mean rank	6	2	5	4	3	1
7	mean	1.5585	0.0341	0.3264	0.1317	0.1167	0.0623
	standard deviation	0.8824	0.0178	0.1401	0.0438	0.0588	0.0248
	mean rank	6	1	5	4	3	2

TABLE IV
ASSESSMENT RESULT ON HIGH-DIMENSION MULTIMODAL FUNCTIONS

F	Parameter	NGO	ZOA	COA	MA	OOA	BOA
8	mean	-3.8129×10^3	-3.5963×10^3	-5.2592×10^3	-4.0822×10^3	-4.3138×10^3	-4.5331×10^3
	standard deviation	8.1613×10^2	6.4787×10^2	7.5276×10^2	4.5054×10^2	5.6243×10^2	3.9293×10^2
	mean rank	5	6	1	4	3	2
9	mean	5.7197×10^2	1.7326×10^1	2.2310×10^2	2.7642×10^2	2.0363×10^2	4.2861×10^1
	standard deviation	4.6779×10^1	1.3658×10^1	5.7295×10^1	1.0646×10^2	6.3769×10^1	1.1817×10^2
	mean rank	6	1	4	5	3	2
10	mean	1.0251×10^1	0.9735	5.8192	4.5814	4.0193	0.0876
	standard deviation	1.1446	0.2042	0.6514	0.5725	0.4234	0.0241
	mean rank	6	2	5	4	3	1
11	mean	4.8355×10^1	0.6619	1.1911×10^1	4.5107	3.3283	0.1994
	standard deviation	2.1352×10^1	0.2853	4.0799	1.0944	1.1577	0.1786
	mean rank	6	2	5	4	3	1
12	mean	9.2162×10^5	1.0501	1.4303×10^1	3.0397	2.8203	0.9649
	standard deviation	4.1480×10^6	0.1043	2.2863×10^1	0.8461	0.5024	0.1099
	mean rank	6	2	5	4	3	1
13	mean	1.0289×10^6	4.1802	7.5608×10^3	1.9008×10^1	1.0402×10^1	3.7247
	standard deviation	1.5262×10^6	0.2914	3.6384×10^4	3.3816×10^1	2.5204	0.1526
	mean rank	6	2	5	4	3	1

TABLE V
ASSESSMENT RESULT ON FIXED DIMENSION MULTIMODAL FUNCTIONS

F	Parameter	NGO	ZOA	COA	MA	OOA	BOA
14	mean	1.2478×10^1	9.5303	7.5608	8.6018	7.5121	6.0122
	standard deviation	9.6211	4.5508	3.6384	3.9730	4.0032	3.3098
	mean rank	6	5	3	4	2	1
15	mean	0.0177	0.0111	0.0060	0.0119	0.0070	0.0045
	standard deviation	0.0173	0.0230	0.0055	0.0100	0.0052	0.0047
	mean rank	6	4	2	5	3	1
16	mean	-0.9870	-1.0108	-1.0224	-1.0206	-1.0220	-1.0295
	standard deviation	0.0694	0.0612	0.0132	0.0113	0.0165	0.0027
	mean rank	6	5	2	4	3	1
17	mean	0.7238	0.8627	0.4081	0.4248	0.4197	0.4055
	standard deviation	0.4364	0.8065	0.0127	0.0240	0.0223	0.0089
	mean rank	5	6	2	4	3	1
18	mean	8.3575	1.4241×10^1	3.7378	3.8582	3.9785	3.3805
	standard deviation	7.3280	1.9785×10^1	2.6011	1.8860	3.8173	1.3135
	mean rank	5	6	2	3	4	1
19	mean	-0.0495	-0.0495	-0.0495	-0.0495	-0.0495	-0.0495
	standard deviation	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	mean rank	1	1	1	1	1	1
20	mean	-2.5933	-2.5638	-2.8306	-2.9676	-2.9648	-2.8800
	standard deviation	0.2981	0.3460	0.1865	0.1632	0.1452	0.1773
	mean rank	5	6	4	1	2	3
21	mean	-1.1724	-2.7882	-2.4645	-2.3259	-1.7867	-1.7574
	standard deviation	0.5043	1.3170	0.7803	0.8616	0.5186	0.7653
	mean rank	6	1	2	3	4	5
22	mean	-1.6422	-2.7012	-2.6580	-3.1026	-2.2627	-2.4166
	standard deviation	1.6677	1.3422	0.7626	1.3032	1.1237	1.1908
	mean rank	6	3	3	1	5	4
23	mean	-1.5290	-2.4715	-2.6275	-2.8781	-2.7534	-2.2414
	standard deviation	0.4609	0.9699	0.8009	0.8631	0.9470	0.6835
	mean rank	6	4	3	1	2	5

The presentation in Table 6 shows that BOA is superior to the five contenders, especially the NGO. Table 6 shows that BOA is better than NGO, ZOA, COA, MA, and OOA in 21, 15, 14, 15, and 17 functions respectively. It shows that BOA is superior to NGO in almost all clusters. Meanwhile, the superiority of BOA to ZOA, COA, MA, and OOA mostly takes place in the first and second clusters that consist of high dimension functions.

TABLE VI
NUMBER OF FUNCTIONS BEATEN IN EVERY GROUP

Cluster	Contender				
	NGO	ZOA	COA	MA	OOA
1	6	4	5	5	5
2	6	5	5	6	6
3	9	6	6	5	6
Total	21	15	16	15	17

The second assessment is the convergence assessment. In this assessment, the maximum iteration is set to 20. Meanwhile, the data is grabbed four times during iteration, which is the 5^{th}, 10^{th}, 15^{th}, and 20^{th} iteration. In this assessment, BOA is not confronted with other metaheuristics. The result is presented in Table 7.

Table 7 shows that there are thirteen functions where the convergence is achieved when the iteration is under or equal to 20. These functions include three high dimension unimodal functions (f_2, f_5, and f_6), three high dimension multimodal functions (f_8, f_{12}, and f_{13}), and seven fixed dimension multimodal functions (f_{16}, f_{17}, f_{19}, f_{20}, f_{21}, f_{22}, f_{23}). This result shows that BOA the convergence of the optimization occurs in the low iteration in most of functions. Specifically, this circumstance occurs in multimodal functions, especially the fixed dimension ones.

IV. DISCUSSION

The suitable performance of BOA in solving the high dimension unimodal functions shows that BOA has good exploitation capability. The exploitation capability can be defined as the capability to improve by tracing a better solution near the current solution. As each high-dimension unimodal function has only one optimal solution, the main challenge is finding this optimal solution in a fast manner.

Although BOA does not implement the neighborhood search with declining local search space as implemented in its contender, the extensive exploitation of the best member as reference is proven better than this neighborhood search.

The more suitable performance of BOA in solving the high-dimension multimodal functions shows that BOA also has good exploration capability. Table 4 shows that BOA is superior to NGO, MA, and OOA in this cluster. Each function in this second cluster has multiple optimal solutions.

This circumstance may lock the members into the local optimal entrapment. It makes the main challenge in this cluster of functions is avoiding the local optimal entrapment. Although BOA does not implement any random search during the iteration as all its three searches are directed searches, the combination of the best member with other references plays a critical role.

TABLE VII
CONVERGENCE ASSESSMENT RESULT

F	Average Fitness Score			
	$t_m = 5$	$t_m = 10$	$t_m = 15$	$t_m = 20$
1	1.5068×10^2	0.2864	0.0004	0.0000
2	0.0000	0.0000	0.0000	0.0000
3	8.2056×10^4	5.0686×10^4	2.8632×10^4	1.9746×10^4
4	1.2076×10^1	1.0185	0.0663	0.0058
5	8.3074×10^3	7.4233×10^1	6.8927×10^1	6.8867×10^1
6	1.6239×10^2	1.5328×10^1	1.3992×10^1	1.3061×10^1
7	0.1437	0.0596	0.0502	0.0423
8	-4.5292×10^3	-4.8877×10^3	-5.1050×10^3	-5.1752×10^3
9	4.2331×10^2	7.6719×10^1	0.4330	0.0011
10	3.5676	0.0895	0.0027	0.0001
11	2.6389	0.1781	0.0282	0.0043
12	2.5903	0.9698	0.8524	0.7549
13	1.1219×10^1	3.7300	3.3233	3.1657
14	9.6172	5.5707	3.4036	2.8400
15	0.0176	0.0111	0.0082	0.0069
16	-1.0163	-1.0273	-1.0308	-1.0312
17	0.4558	0.4213	0.4024	0.4000
18	6.9248	4.0479	3.7430	3.3475
19	-0.0495	-0.0495	-0.0495	-0.0495
20	-2.5759	-2.9179	-2.9820	-3.0560
21	-1.0429	-1.4481	-1.7214	-1.9939
22	-1.5284	-1.9144	-3.2006	-3.3403
23	-2.3495	-2.6900	-2.7795	-3.0972

The still suitable performance of BOA in solving fixed dimension multimodal functions shows that BOA has a good balance between exploitation capability and exploration capability. The fixed-dimension multimodal functions are known for their multiple optimal solutions and ambiguous terrain. In some functions, the terrain of the search space is flat with a narrow slope where the global optimal solution exists. It makes the improvement becomes more difficult.

The almost absolute superiority of BOA to the NGO can be traced back to the existence of the best member as reference. BOA is better than NGO in 21 functions and draw in 2 functions. This result shows that NGO fails to outperform BOA in any functions. Among the contenders, NGO is the only metaheuristic that does not use the best member as reference or one of references [8]. NGO depends on only the randomly picked member as reference in the first search [8].

The assessment result also shows the less significant contribution of the neighborhood search with declining local search space during the iteration. Initialized by MPA [9], this neighborhood search has become very popular in recent metaheuristics, such as ZOA [14], COA [13], NGO [8], OOA [12], and so on. BOA is still superior and competitive even though BOA does not implement this kind of search except the directed search.

There exists a variation in computational complexity between the initialization and iteration stages in BOA. The complexity level during the initialization stage is equivalent to $O(n(S).n(D))$. The rationale behind this stems from the presence of a nested loop in the initialization stage, comprising an outer loop for the entire swarm and an inner loop for all dimensions. Meanwhile, the computational complexity during the iteration stage is equivalent to $O(3t_m.n(S).n(D))$. There are three searches in every iteration. Meanwhile, the iteration runs from the first iteration to the maximum iteration as one of stopping criteria. The nested loop consisting of the loop for whole swarm and loop for whole dimension also occurs in every iteration.

V. CONCLUSION

In this paper, a new swarm-based metaheuristic called Best-Other Algorithm (BOA) has been introduced. A description of BOA including the fundamental concept and formalization through pseudocode and mathematical formulation has been presented. The set consisting of 23 functions has been used as the optimization problem for the performance assessment. The result indicates the suitable performance of BOA in all clusters of these functions. Moreover, BOA outperforms its contenders by better than NGO, ZOA, COA, MA, and OOA in 21, 15, 16, 15, and 17 consecutively. Its superiority comes mostly from the high dimension functions. The assessment result also shows the critical importance of the best swam member as reference in the directed search. The result indicates the less significant contribution of the neighborhood search.

The introduction of BOA opens the possibility of further studies in the metaheuristic development. The first track is the improvement of BOA as there is not any general best metaheuristic due to the NFL theorem. The second track is the utilization of BOA in many practical optimization problems, whether they are the numerical ones or combinatorial ones.

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