Extended Belief Rule Base Inference Model Based on KD Tree

Dan Qu, Huafei Chen, Hongyi Li, Hualin Xiao

Abstract—**The extended belief rule base (EBRB) system has been widely used in decision-making problems for its accuracy and efficiency. However, EBRB system needs to traverse all the rules in the rule base and has the problems of inefficiency and inconsistency. In view of this, an extended belief rule base system inference method based on a k-dimensional (KD) tree is implemented in this paper. First, the KD tree is introduced in the construction of rule base. Then, the K-Nearest Neighbor (KNN) query optimization algorithm, based on the space indexing technique of the KD tree, is used to search for key rules. Next, the obtained key rules are activated to participate in the inference process. In addition, several experiments are conducted on function fitting, oil pipeline leakage simulation and classification datasets from UCI to verify the inference performance of the proposed method. The experimental results illustrate that the extended belief rule base system based on the KD tree can effectively improve the accuracy and stability of EBRB reasoning.**

Index Terms—**Extended belief rule base, Evidential reasoning approach, KD tree, K-Nearest Neighbor algorithm**

I. INTRODUCTION

EXPERT systems are one of the most active application fields in artificial intelligence [1]. In 2006, Prof. Yang *et al.* proposed a belief rule-based inference methodology using the evidence reasoning approach(RIMER), which is based on D-S evidence theory [2], [3], decision theory [4], [5], fuzzy theory [6] and the traditional if-then rule base. It employs a belief rule base (BRB) to represent knowledge and utilizes evidence reasoning (ER) for knowledge inference. The RIMER method has a clearer inference mechanism compared with neural network algorithms [7] and support vector machines [8].

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Dan Qu is a teaching assistant at the College of Mathematics Education, China West Normal University, Nanchong Sichuan 637009, P. R. China and with the College of Mathematics and Statistics, Sichuan University of Science & Engineering, Zigong Sichuan 643002, P. R. China (e-mail: 13980748735@163.com).

Huafei Chen is a teaching assistant at the College of Mathematics and Statistics, Sichuan University of Science & Engineering, Zigong Sichuan 643002, China(e-mail: huafeichen0502@163.com).

Hongyi Li is a teaching assistant at the College of Mathematics and Statistics, Sichuan University of Science & Engineering, Zigong Sichuan 643002, China(e-mail: 1261234217 @qq.com).

Hualin Xiao is a teaching assistant at the College of Mathematics Education, China West Normal University, Nanchong Sichuan 637009, P. R. China(corresponding author, e-mail: hualin_xiao688@163.com).

Belief rule base is an essential carrier of the RIMER method, and its reasoning ability depends on the system's parameters and structure. Among them, the BRB system's parameters are usually determined by experts' knowledge or experience in the field. However, actual parameter values cannot be obtained in complex BRB systems. Based on this gap, Yang [9] presented a single-parameter learning model and optimized the parameters utilizing the Fmincon function, but the learning efficiency was unsatisfactory. Since then, Chang *et al.* [10] proposed to use gradient descent method for parameter learning. However, this method is only suitable for some BRB systems due to its complex derivation process. To overcome these shortcomings, some scholars introduced parameter learning methods based on the swarm intelligence algorithm [11], [12]. Nevertheless, these parameter optimization methods require iterative iterations and cannot simultaneously consider both inference accuracy and efficiency of the rule base. Therefore, some scholars used k-means clustering algorithm, principal component analysis, and reduction algorithms for rough sets and other methods [13]-[15] to optimize the structure of BRB, which solves the combination explosion problem to some extent but the BRB system's reasoning ability cannot be guaranteed. To address this limitation, Liu *et al.* [16] integrated distributed belief representation into the antecedent term to enhance the performance of the BRB system. The improved rule base is referred to as the extended belief rule base (EBRB) system. The rule generation mechanism of the system is a data-driven approach. When combining rules, the entire rule base needs to be traversed to calculate their activation weights, which leads to the low reasoning efficiency of the EBRB system. To address this issue, Su *et al.* [17] proposed to build a BK tree index to search the rule base of the EBRB system, which can improve the inference performance of the EBRB system to some extent. Lin *et al.* [18] applied the VP index structure in the EBRB system and employed a clustering algorithm [19] to achieve the automatic selection of indexing parameters. Furthermore, Liu *et al.* [20] introduced a locality sensitive hashing algorithm based on normal distribution to find indexing hash buckets and select neighbor rules for calculating activation weights, which improved the inference efficiency of the algorithm. Chen *et al.* [21], [22] proposed a K-means clustering tree optimization method to solve the problems of inconsistent rules and inefficient search. In summary, the above studies have contributed to improving the performance of EBRB systems to some extent. Furthermore, although the accuracy of the EBRB system is high, it is still a significant potential to upgrade the EBRB. In this paper, a structure optimization framework based on the KD tree is proposed. It proposed assigning an index for each rule using the KD tree to improve retrieval efficiency and find critical rules involved in reasoning.

The structure of the paper is as follows. Section II reviews the inference methods of the extended belief rule base system and the extended belief rule base. Section III details the structural optimization of the proposed EBRB. Section IV illustrates three case studies to demonstrate excellent performance of the proposed approach, and Section V concludes the paper.

II. EBRB EXPERT SYSTEM

A. Representation of EBRB

In this section, some necessary definitions related to EBRB system are provided to facilitate a better understanding of the paper.

The extended belief rule base $R=\{R_1,R_2,...,R_L\}$ comprises a series of belief rules. Generally, an extended belief rule is depicted as follows:

$$R_k : IF\ \{A,\alpha^k\}, THREN \{(D_1,\beta_{1,k}),(D_2,\beta_{2,k}),...,(D_N,\beta_{N,k})\} \quad (1)$$

where (A,α^k) denotes belief distribution of the antecedent term, which is equivalent to $\{\{(A_{i,j},\alpha^k_{i,j}), j=1,2,...,J\} | i=1,2,...,T; k=1,2,...,L\}$. k is the index of rules, and L denotes the number of rules. $A_{i,j}$ denotes the jth referential values of the ith antecedent attribute. In a rule, the belief degree of the ith antecedent attribute assessed to the jth referential value is represented by $\alpha^k_{i,j}$. Here, J_i represents the number of referential values for the ith antecedent attribute and T represents the total number of antecedent attributes in the rule. D_j is jth referential value of decision attribute. $\beta_{j,k}$ $(j=1,2,...,N, k=1,2,...,L)$ is the belief degree of the jth consequent of the kth rule. N is the number of decision attributes.

B. Construction of EBRB

Suppose the input vector is $x_k=(x_{k,1},x_{k,2},...,x_{k,T})$, where $x_{k,i}$ represents ith antecedent attribute of the kth input data. For combination schemes, the corresponding methods such as the rule- or utility-based transformation techniques are employed by the decision-maker or expert to obtain the belief structure. It is considered that the reference value of the antecedent attribute $A_{i,j}$ is equivalent to the quantity value $\gamma_{i,j}$.

$$\gamma_{i,j} \text{ means } A_{i,j}, i=1,2,...,T; j=1,2,...,J_i \quad (2)$$

where $\gamma_{i,1}$, γ_{i,J_i} are the minimum and maximum numerical values corresponding to $A_{i,j}$ respectively. If the preference of experts for the value $\gamma_{i,j+1}$ is higher than the value $\gamma_{i,j}$. Then the input value x_i can be equivalently converted into the expected representation of the belief degree distribution form as follows:

$$E(x_i)=\{(A_{i,j},\alpha_{i,j}), i=1,2,...,T_k, j=1,2,...,J_i\} \quad (3)$$

where $\alpha_{i,j}$ is obtained by equations (4-6):

$$\alpha_{i,j}=\frac{\gamma_{i,j+1}-x_i}{\gamma_{i,j+1}-\gamma_{i,j}}, \gamma_{i,j}\le x_i\le\gamma_{i,j+1}, j=1,2,...,J_{i-1} \quad (4)$$

$$\alpha_{i,j+1}=1-\alpha_{i,j}, \gamma_{i,j}\le x_i\le\gamma_{i,j+1}, j=1,2,...,J_{i-1} \quad (5)$$

$$\alpha_{i,s}=0, s\ne j,j+1, s=1,2,...,J_i \quad (6)$$

The distribution of the antecedent attributes of the extended belief rules can be generated by equations (3-6). Similarly, the belief degree distribution form of the consequent attribute takes the following form:

$$E(y)=\{(D_j,\beta_{j,k}), j=1,2,...,N\} \quad (7)$$

C. Reasoning of EBRB

The EBRB system performs combination reasoning on the generated rules through the ER method. In the data-driven scheme, S^k_i is an individual matching degree of the input for the ith antecedent attribute of the kth rule, which can be calculated by the following formula:

$$d^k_i=\sqrt{\frac{1}{2}*\sum_{j=1}^{J_i}(\alpha_{i,j}-\alpha^k_{i,j})^2} \quad (8)$$

$$S^k_i=1-d^k_i \quad (9)$$

Based on the above equations, the individual matching degree S^k_i is calculated based on the Euclidean distance of two belief distributions. Therefore, the activation weight for the kth rule can be calculated as follows:

$$\omega_k=\frac{\theta_k\sum_{i=1}^{T_k}(S^k_i)^{\bar{\delta}_i}}{\sum_{l=1}^{L}\left[\theta_l\prod_{i=1}^{T_k}(S^l_i)^{\bar{\delta}_i}\right]} \quad (10)$$
$$\bar{\delta}_i=\frac{\delta_i}{\max_{i=1,..,T_k}\{\delta_i\}}$$

where $0\le\omega_k\le1$, $k=1,2,...,L$, $\sum_{i=1}^{L}\omega_i=1$. $\omega_k=0$ indicates that the kth rule is not activated. The belief degree is converted to a basic belief value by means of equations (11-13):

$$m_{j,k}=\omega_k\beta_{j,k} \quad (11)$$

$$\tilde{m}_{H,k}=\omega_k(1-\sum_{j=1}^{N}\beta_{j,k}) \quad (12)$$

$$\bar{m}_{H,k}=1-\omega_k \quad (13)$$

The ER approach [23] is performed to calculate the basic credible value of the evaluation result D_j after the combination of activation rules, and the results can be obtained as follows:

$$C_j=K\prod_{k=1}^{L}(m_{j,k}+\bar{m}_{H,k}+\tilde{m}_{H,k})-K\prod_{k=1}^{L}(\bar{m}_{H,k}+\tilde{m}_{H,k}) \quad (14)$$

$$\tilde{C}_H=K\left[\prod_{k=1}^{L}(\bar{m}_{H,k}+\tilde{m}_{H,k})-\prod_{k=1}^{L}\bar{m}_{H,k}\right] \quad (15)$$

$$\bar{C}_H=K\prod_{k=1}^{L}\bar{m}_{H,k} \quad (16)$$

$$K^{-1}=\sum_{j=1}^{N}\prod_{k=1}^{L}(m_{j,k}+\bar{m}_{H,k}+\tilde{m}_{H,k}) -(N-1)\prod_{k=1}^{L}(\bar{m}_{H,k}+\tilde{m}_{H,k}) \quad (17)$$

$$\beta_j=\frac{c_j}{1-\bar{C}_H}, j=1,2,...,N \quad (18)$$

$$\beta_H=\frac{\bar{C}_H}{1-\bar{C}_H} \quad (19)$$

According to the above synthesized formulas, the following output of inference can be obtained in the form of belief distribution as follows:

$$f(x)=\{(D_j,\beta_j),j=1,2,...,N\} \quad (20)$$

In the regression problems, the results can be transformed to calculate the expected utility value of the output, and the final output can be expressed as:

$$\tilde{y}=\sum_{j=1}^{N}\mu(D_j)\beta_j \quad (21)$$

On the basis of the above ER algorithm, Wang *et al.* [24] obtained a more intuitive calculation formula which is given by equations (22) and (23).

III. Structural optimization of EBRB

All the rules in the EBRB are stored randomly, which leads to an increase in complexity and a decrease in efficiency during the retrieval of relevant rules in the reasoning process. On this basis, this paper proposes an EBRB system based on KD tree structure optimization method. Firstly, the disordered rules are collected and used to construct a tree structure index. Moreover, the key rules are searched according to the K-Nearest Neighbor (KNN) query optimization algorithm based on the KD tree space indexing technique. The activation weights of the key rules are calculated by equation (10) in section II. Ultimately, the ER algorithm aggregates rules with activation weight greater than 0 to obtain the belief distributions of reasoning. The details are illustrated in the following section.

A. KD Tree Index

KD tree, known as the k-dimensional tree, is a typical partition tree widely used for searching points in high-dimensional space. The process of constructing a KD tree is equivalent to partitioning the k-dimensional space with a hyperplane perpendicular to the coordinate axis to form several k-dimensional hyper-rectangle regions. Each node in the KD tree corresponds to one of these hyper-rectangular regions. The KD tree is recursively searched from the top-down and terminates at a leaf node. Utilizing the KD tree can avoid searching for most data points, thus improving the efficiency of rule retrieval.

Assume that all leaf nodes contain at most n_0 points. There are *n* data points in set *S*, and the depth of the tree is at most about $\log(n/n_0)$. Generally, if the dimension of the point is *D* and the number of points in the set *S* is *N*, the complexity of the KD tree algorithm is $O(D*\log(N))$. The procedure of constructing KD tree is illustrated as follows:

Algorithm 1: The construction of KD tree algorithm

Input: Spatial point objects set *S*
Output: KD tree.
Function MakeTree (*S*)
  If $|S|<n_0$: return (Leaf)
  Rule = ChooseRule (*S*)
  LeftTree = MakeTree $(\{x\in S: Rule(x)=ture\})$
  RightTree = MakeTree $(\{x\in S: Rule(x)=flase\})$
  Return (Rule, LeftTree, RightTree)
Function ChooseRule (*S*)
  Choose a coordinate direction *i*
  Rule $(x)=(x_i\le median(\{z_i : z\in S\}))$
  Return (Rule)

B. Design of KNN Based on KD Tree Index

The K-Nearest Neighbor (KNN) query is one of the most important operation in a spatial database. The purpose of a KNN query is to search for *k* nearest neighbors of the query object. The criterion of nearest neighbor is generally measured by Euclidean distance. To improve the algorithm's efficiency and reduce computational overhead, an indexing technique, i.e., KD tree, is introduced. The algorithm is designed as follows:

Algorithm 2: The construction of KNN algorithm based on KD tree index

Input: an index-unit of the KD tree *node* , the target spatial point *x*, the value of KNN query *k* .
Output: the set *L* denotes the result of KNN query.
Function KNNKD $(node,x,k)$
If $|L|<k$ then
    $L=L\cup\{p_{node.c}\}$
Else if $\max_{p_i\in L}(d(p_i,x))>d(p_{node.c},x)$ then
    $L=(L-p_i)\cup\{p_{node.c}\}$
If $x_{node.r}\le node.v$ then
    KNNKD $(node.left,x,\max_{p_i\in L}(d(p_i,x)))$
    If $|x_{node.r}-node.v|\le\max_{p_i\in L}(d(p_i,x))$ then
    KNNKD $(node.right,x,\max_{p_i\in L}(d(p_i,x)))$
Else if $x_{node.r}>node.v$ then
    KNNKD $(node.right,x,\max_{p_i\in L}(d(p_i,x)))$
    If $|x_{node.r}-node.v|\le\max_{p_i\in L}(d(p_i,x))$ then
    KNNKD $(node.left,x,\max_{p_i\in L}(d(p_i,x)))$

Notation	
node	index-unit in the KD tree
node.r	splitting axis represented as an integer
node.left	index-unit in the left sub-tree of the KD tree
node.right	index-unit in the right sub-tree of the KD tree
$p_{node.c}$	the coordinate of the current node
$x_{node.r}$	The r^{th} value of vector *x* in the current node
node.v	splitting value of the current split node

C. Construction of EBRB based on KD tree

The distance between the two rules is measured by:

$$d(R_p,R_q)=\sqrt{\frac{1}{2T}\sum_{i=1}^{T}\sum_{j=1}^{J_i}\left(\alpha_{i,j}^{p}-\alpha_{i,j}^{q}\right)^2} \quad (24)$$

The KD tree construction algorithm builds a tree-structured index for the unordered storage of extended belief rules. The input of the algorithm is the rule set $R=\{R_1,R_2,...,R_L\}$. Assuming that the target sample is *p*, the algorithm searches for the *k* nearest rules to the target point *p*, which will serve as key rules for inference. It is worth noting that the value of *k* affects the inference result. If *k* is too small, the results will be sensitive to training datasets. If *k* is too large, rules with lower activation weights, which are less relevant to the input data, will be activated, decreasing the efficiency and effectiveness of inference. Therefore, choosing the appropriate *k* value is essential for accurate inference results. Several methods exist to select the k value, such as cross-validation, Bayesian[25] and bootstrapping. In this paper, the ten-fold cross-validation method has been utilized to determine the optimal k value.

Then, the activation weights of the key rules are calculated according to the content of Section II (C), and the ER algorithm is utilized to obtain the belief degree distributions form. In order to distinguish these two types of BRB systems and for the convenience of presentation in this paper, the extended belief rule base system based on the KD tree is referred to as the KD-EBRB system. The flow of the proposed BRB system is shown in Fig. 1. The KD-EBRB system only assigns indexes for the rules, so the rule generation process is the same as the EBRB system.

IV. Case studies

In order to validate model's effectiveness, experiments were conducted on non-linear function fitting, oil pipeline leakage simulation and multi-classified datasets classification. The experimental environment of this paper is 11th generation Intel(R) Core(TM) i5-11320H @ 3.20GHz 3.19 GHz; 16 GB RAM; Windows 11 operating system. The algorithm implementation platform is MATLAB R2015a.

A. *Nonlinear Function*

A nonlinear function is examined, which can be defined as follows:

$$f(x) = x\sin(x^2), 0 \le x \le 3 \quad (25)$$

When constructing a belief rule base, the variable x is regarded as the antecedent attribute, and the corresponding output y is considered as the consequent attribute. The range of values of x is $\{0, 0.5, 1, 1.5, 2, 2.5, 3\}$ and the range of evaluation result y is $\{-2.5, -1, 1, 2, 3\}$. 1500 groups of data are selected evenly from the range of x as a sample data set. In the EBRB system, 500 groups of data are randomly selected from the sample datasets for training, and the remaining 1000 groups of data are used for testing. The mean square error (MSE) is the primary metric to assess performance in the experiment. In order to compare the effectiveness of the KD-EBRB system with other EBRB systems, we conducted tests and compared the results with the conventional Liu-EBRB system and improved EBRB systems with VP method (VP-EBRB) and MVP method (MVP-EBRB) [18]. The fitting results for the KD-EBRB and Liu-EBRB systems are displayed in Fig. 2, while comparative results can be found in Table I.

TABLE I
EXPERIMENT RESULTS OF NONLINERAR FUNCTION PROBLEMS

Method	MSE	Running time/s
Liu-EBRB	0.2976	420.5976
VP_EBRB	0.1446	221.4824
MVP_EBRB	0.1449	225.3062
KD-EBRB	9.8259e-05	109.3354

Fig. 2 shows the comparison results between the estimated values and actual values. Fig. 2 (a) displays the fitting results of the Liu-EBRB system, which shows a notable difference between the output of the system and the actual value. From the fitting curve, it can be seen that the EBRB system does not fit well in this interval. This is attributed to traversing all the belief rules of the Liu-EBRB system, including redundant rules. Fig. 2 (b) shows the fitting results of the KD-EBRB system, which indicates a significant improvement in the fitting effect. Furthermore, the fitting curve is in line with the standard function curve.

As shown in TABLE I, the MSE value of the KD-EBRB system is smaller than that of Liu-EBRB, VP-EBRB, and MVP-EBRB (9.8259e-05 vs 0.2976, 0.1446, and 0.1449, respectively). The running time is also shorter than the Liu-EBRB system. The reason behind this phenomenon is that the KD-EBRB system only visits approximate neighbor rules to participate in the inference process, which undoubtedly improves the efficiency and accuracy of inference. In conclusion, the proposed approach can effectively deal with nonlinear function fitting. Moreover, it has obvious advantages compared with the initial inference model.

B. *Oil pipeline leakage*

This experiment is conducted on a 100-kilometer oil pipeline installed in the United Kingdom. The actual leak datasets of this pipeline are used to illustrate the effectiveness of the proposed KD-EBRB system, and the initial EBRB is used for comparison. Flow difference between input and output (FD), average pressure difference (PD) of oil to pipeline and leak size (LS) are taken as the antecedent attributes. To define FD, eight reference grades are selected, with the corresponding values $\{-10, -5, -3, -1, 0, 1, 2, 3\}$. Similarly, PD and LS are defined by choosing seven and five reference levels, with corresponding values $\{-0.042, -0.025, -0.01, 0, 0.01, 0.025, 0.042\}$ and $\{0, 2, 4, 6, 8\}$. 2008 groups of data are collected as the sample data set. Then, 1500 groups of the sample data are selected as training data, which are randomly selected from three periods according to a certain proportion to generate belief rules. The reference results are evaluated by the mean absolute error (MAE) metric. Fig. 3 and Fig. 4 show the experimental results.

As shown in Fig. 4, the estimated outcomes of the KD-EBRB system match the actual data well. It is worth noting that when $PD \in [-0.02, 0]$ and $FD \in [-10, -5]$, the Liu-EBRB system performs poorly, while the KD-EBRB system fits closely. This is because, during the reduction process, the KD-EBRB system integrates KNN query based on tree index searching of the necessary rules and eliminates redundant rules.

TABLE II
EXPERIMENTAL RESULTS OF OIL PIPELINE LEAKAGE

Method	MAE	Number of search rules
Liu-EBRB	0.9386	2965120
KD-EBRB	**0.1489**	**1527342**

As shown in TABLE II, the MAE of the KD-EBRB system is 0.1489, which indicates that the accuracy of the KD-EBRB system is the best. Considering the number of searching rules, the KD-EBRB method traverses fewer rules in the process due to its KNN query method. In conclusion, the proposed method not only reduces the complexity of the EBRB system but also improves its efficiency in dealing with the pipeline leakage detection problem.

C. Classification problem

Ten practical cases of classification datasets from UCI [26] are selected for testing. The details of these datasets are shown in TABLE III.

TABLE III
DETAILS OF THE CLASSIFICATION BENCHMARKS

Datasets	Number of antecedent attributes	Number of categories	Number of data
Iris	4	3	150
Ecoli	7	2	336
Seeds	7	3	210
Banknote	4	2	1372
Knowledge	5	4	403
Vertebral	6	3	310
Bupa liver	6	2	345
Yeast	8	10	1484
Glass	9	6	214
Diabetes	8	2	768

The experimental inference results are obtained by the average of 10-fold cross-validation. Six reference grades are evenly selected from the range of antecedent attributes. The number of evaluation results is consistent with the number of classifications. The experimental results are shown in TABLE IV and TABLE V.

TABLE IV
CLASSIFICATION RESULTS OF THE PROPOSED METHOD COMPARED WITH LIU-EBRB SYSTEMS

Benchmark	Optimal k	Accuracy of KD-EBRB	Accuracy of Liu-EBRB	Increased
Iris	14	96.67%	94.67%	2%
Ecoli	5	87.17%	79.19%	7.98%
Seeds	23	93.33%	91.43%	1.9%
Banknote	11	99.93%	96.94%	2.99%
Knowledge	7	82.38%	79.70%	2.68%
Vertebral	9	84.52%	74.19%	10.33%
Bupa liver	16	70.68%	66.94%	3.74%
Yeast	7	59.04%	51.68%	7.36%
Glass	1	72.32%	68.66%	3.66%
Diabetes	17	72.15%	70.96%	1.19%

TABLE V
COMPARISON OF THE PROPOSED METHOD AND BK_EBRB SYSTEMS ON UCI CATEGORICAL DATASET

Datasets	Accuracy of KD-EBRB	Accuracy of BK_EBRB		
		(theta=0.8)	(theta=0.6)	(theta=0.4)
Iris	**96.67**(%)	94.67(%)	93.33(%)	**96.67**(%)
Ecoli	**87.17**(%)	78.79(%)	80.61(%)	83.88(%)
Seeds	**93.33**(%)	86.19(%)	90.10(%)	90.95(%)
Banknote	**99.93**(%)	84.67(%)	94.41(%)	97.64(%)
Knowledge	**82.38**(%)	81.63(%)	81.85(%)	82.35(%)
Vertebral	**84.52**(%)	72.56(%)	73.57(%)	73.59(%)
Bupa liver	**70.68**(%)	42.64(%)	42.66(%)	42.53(%)
Yeast	**59.04**(%)	49.49(%)	53.14(%)	55.40(%)
Glass	**72.32**(%)	63.16(%)	63.59(%)	56.62(%)
Diabetes	**72.15**(%)	63.64(%)	65.13(%)	68.26(%)

Based on TABLE IV, it is evident that the KD-EBRB system outperforms the Liu-EBRB system in most classification datasets, with an improvement of over 1% in inference accuracy. Especially for the datasets of Ecoli, Vertebral and Yeast, the accuracy of the proposed method is improved by 7.98%, 10.33% and 7.36%, respectively. Moreover, it is worth noting that the parameter k in the KD-EBRB system has a significant impact on classification benchmark results. In the KD-EBRB system, the parameter k significantly impacts the results for classification benchmarks. Thus, it is necessary to adaptively adjust the value of k in the face of different benchmarks. To better illustrate the impact of k on the inference accuracy of the KD-EBRB system, experiments were conducted on all datasets, and 10-fold cross-validation was used to determine the k value of KNN. In order to ensure neighbor search, the range of k was set as $[1,30]$. The experimental results are shown in TABLE IV. Specifically, we take Iris and Knowledge as examples to plot the relationship between inference accuracy and the value of k. The vertical axis presents the inference accuracy of the KD-EBRB system and the horizontal axis denotes the value of k. It is worth noting that the optimal value of k for KNN is roughly $[5,10]$ in the datasets of Iris and Knowledge. The shape of the curve is roughly an inverted U-shape, as shown in Fig. 5, which indicates that the value of k that is either too small or too large is suitable for the query process.

The TABLE V presents the classification accuracy of the KD-EBRB and BK_EBRB system. The accuracy results of the KD-EBRB system correspond to the best accuracy of the method achieved through the optimal k value. The BK_EBRB system is an improvement of the EBRB system based on the BK tree structure proposed by Su *et al.* [17], and θ is the parameter of the BK_EBRB system. Apparently, two versions of the EBRB system have almost the same accuracy on the Iris and Knowledge benchmarks. While on the remaining benchmarks, the accuracy of the KD-EBRB system is obviously higher than the BK_EBRB system.

Based on the comparison results, the proposed KD-EBRB system effectively reduces EBRB size and improves accuracy.

To evaluate the effectiveness of the proposed method in this paper, the study compares the performance of the KD-EBRB system with that of some conventional machine learning techniques [27]. The results are presented in TABLE VI. The results illustrate that the KD-EBRB system reaches generally favorable accuracy on the four datasets according to the average rank. Significantly, the KD-EBRB method achieves an accuracy of 87.17% on the Ecoli benchmark, surpassing any other method. KD-EBRB does not have the best performance on Iris and Seeds, but it also ranks second. The optimization of KD-EBRB effectively improves the competitiveness of the EBRB system compared with traditional machine learning methods. Furthermore, although the proposed method effectively improves the effectiveness of EBRB, it is important to note that no universal method can reach the best accuracy on all benchmarks.

V. Conclusion

In this paper, KD-EBRB system is proposed for optimizing the rule base structure, aiming to solve the problem of unordered rules storage in the rule base. The system implements the KD tree spatial indexing technology with the K-Nearest Neighbor query optimization algorithm to achieve a fast search of desired rules, avoiding time wasted on traversing all belief rules. The proposed KD-EBRB system was tested through case studies and experiments, and the results demonstrate that it is effective and efficient compared with several state-of-the-art methods. In further work, the optimization of parameters in the EBRB system and indexing methods for refinement rules will be investigated.

$$\beta_j = \frac{\mu \prod_{k=1}^{L} (\omega_k \beta_{j,k} + 1 - \omega_k \sum_{i=1}^{N} \beta_{i,k}) - \mu \prod_{k=1}^{L} (1 - \omega_k \sum_{i=1}^{N} \beta_{i,k})}{1 - \mu \prod_{k=1}^{L} (1 - \omega_k)} \quad (22)$$

$$\mu = \left[\sum_{j=1}^{N} \prod_{k=1}^{L} (\omega_k \beta_{j,k} + 1 - \omega_k \sum_{i=1}^{N} \beta_{i,k}) - (N-1) \prod_{k=1}^{L} (1 - \omega_k \sum_{i=1}^{N} \beta_{i,k}) \right]^{-1} \quad (23)$$

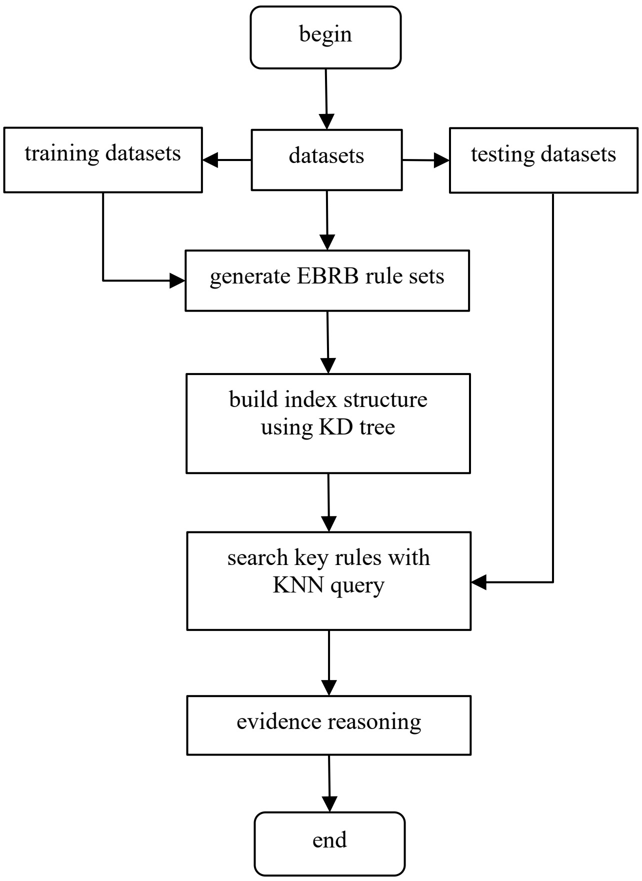
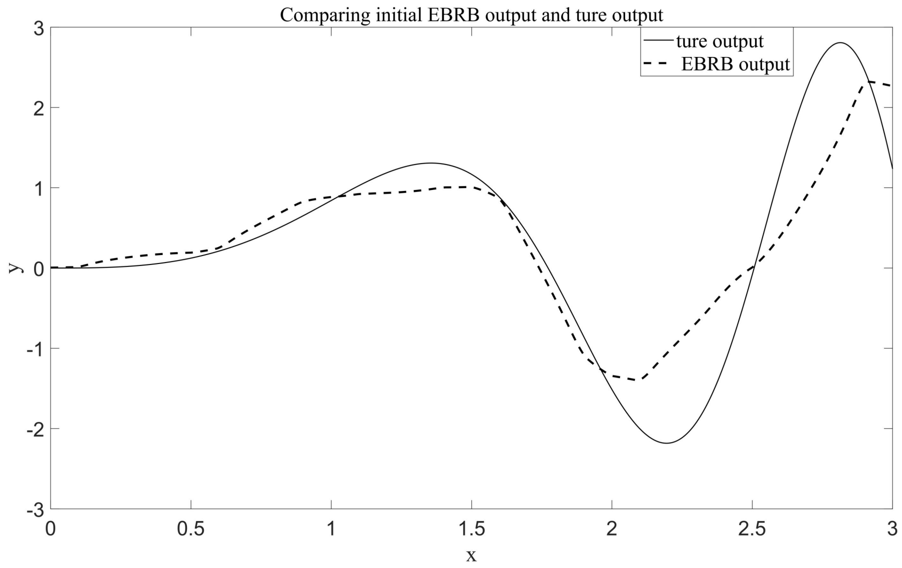
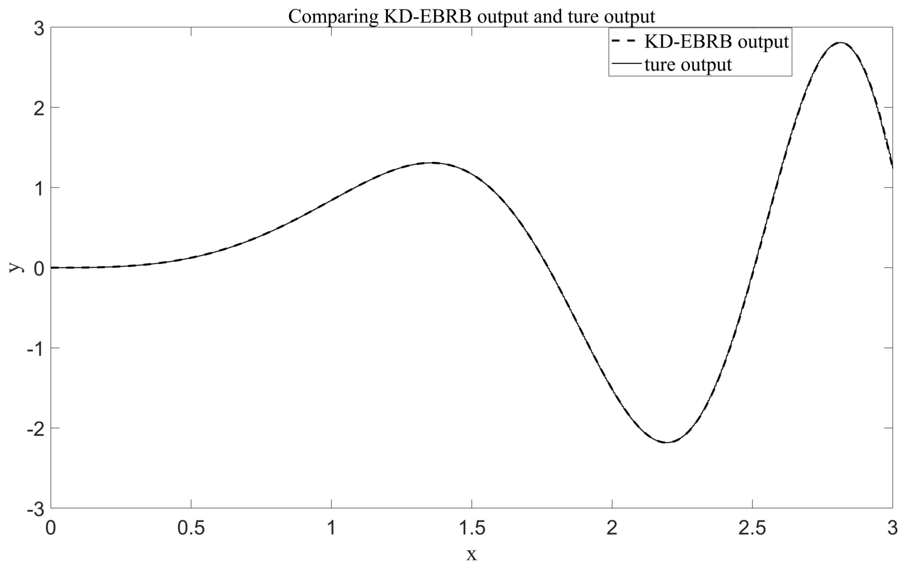


Fig. 1. KD-EBRB system flow chart



(a) Fitting result of Liu-EBRB system



(b) Fitting result of KD-EBRB system

Fig. 2. Comparison between Liu-EBRB (a) and KD-EBRB (b) on function fitting

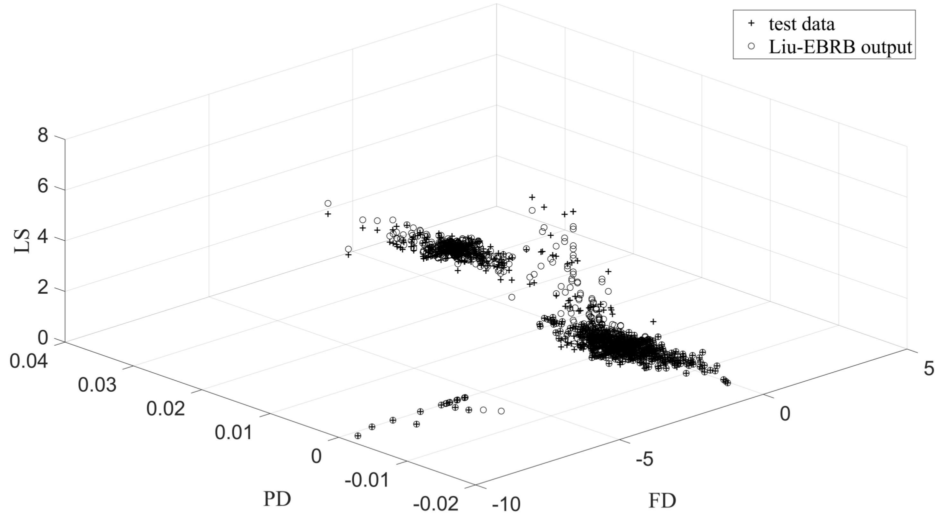


Fig. 3. Output and test data of Liu-EBRB system

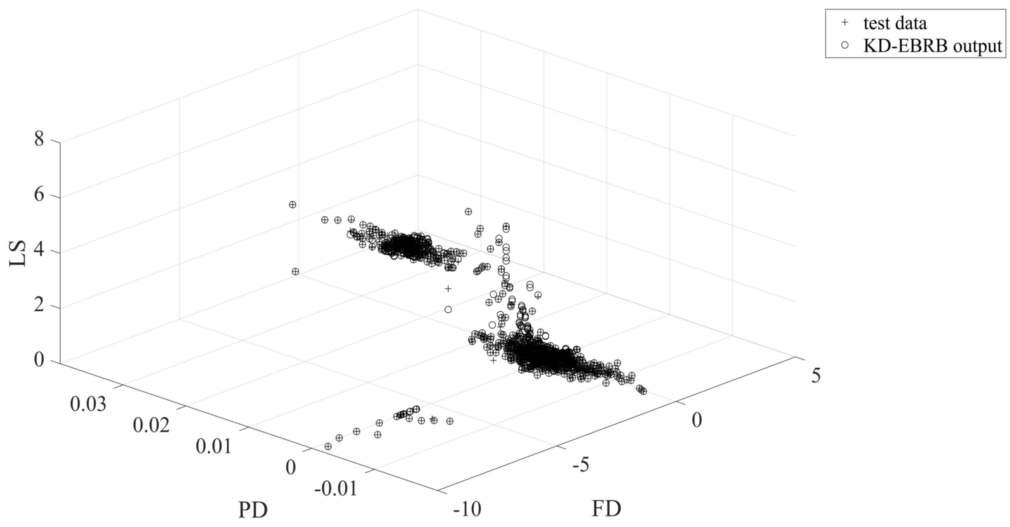
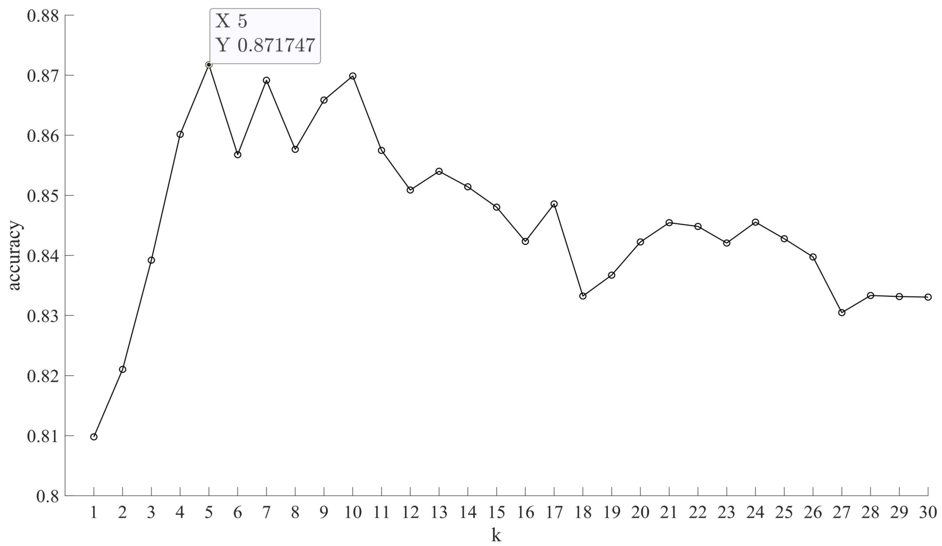
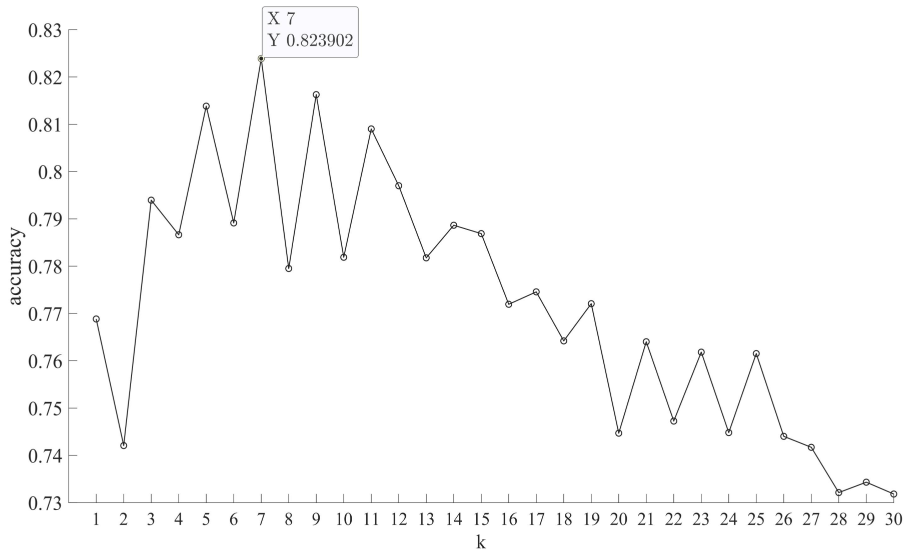


Fig. 4. Output and test data of KD-EBRB system



(a) The optimal k on Iris



(b) The optimal k on Knowledge

Fig. 5. The optimal *k* on Iris (a) and Knowledge (b) benchmark

TABLE VI
COMPARISION OF ACCURACY WITH TRADITIONAL MACHINE LEARNING METHODS

Method	Iris	Ecoli	Seeds	Glass	Average rank
EBRB(%)	94.6(5)	79.19(4)	91.43(4)	63.59(5)	4.5
C4.5(%)[28]	**96.0(3)**	**80.24(3)**	-	**66.82(3)**	3
KNN(%)[29]	85.17(9)	**81.27(2)**	**92.38(3)**	61.21(7)	6.5
BPNN(%)[30]	91.59(8)	75.65(7)	-	64.58(4)	6.33
BRBCS(%)[31]	93.67(6)	78.34(5)	87.00(5)	**69.04(2)**	4.5
AISWNB(%)[32]	94.87(4)	-	-	57.74(8)	6
WLTSVM(%)[33]	**98.00(1)**	-	**96.24(1)**	49.91(9)	3.67
EFRBCS(%)[34]	93(7)	77.79(6)	82.38(6)	61.38(6)	6.25
KD-EBRB(%)	**96.67(2)**	**87.17(1)**	**93.33(2)**	**72.32(1)**	1.5

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