Machine Learning-Based Adaptive Moment Gradient for Electrical Impedance Tomography

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Abstract—**Electrical Impedance Tomography (EIT) reconstructs the electrical properties of cellular tissues by taking measurements at their boundaries. Its non-invasive nature, safety, and potential for an extensive variety of therapeutic uses have sparked significant interest. The generation of EIT images involves solving an inverse problem through iterative optimization techniques. As a result, the effectiveness of various optimization strategies for EIT may vary. This study evaluates the efficacy of the Adam optimizer in addressing the EIT inverse problem. Our simulations reveal that Adam demonstrates superior convergence rates, faster reconstruction times, and higher-quality images compared to traditional gradient descent-based optimizers. Specifically, the Adam optimizer produces images of superior quality with improved anomaly localization, all while achieving faster reconstruction speeds and higher convergence rates. Additionally, we provide insights into the optimal parameter configurations for Adam in the context of EIT, offering valuable guidance for future research endeavors in this domain.All things considered, our findings unequivocally demonstrate that the Adam optimizer is a valuable tactic for resolving the EIT inverse problem.**

Index Terms—**Electrical impedance tomography, inverse problem, adam optimizer, image reconstruction.**

I. Introduction

DIFFERENT methods for diagnosing diseases have been developed as a result of recent advances in medical technology. Among these, tomography and medical imaging have gained popularity. Tomography involves creating images by examining the internal structures of living organisms. Different tomography methods, such as CT scans and MRI, are available, but they have drawbacks like high cost and potential radiation exposure. To address these limitations, there's a growing demand for non-invasive, painless, quick, and cost-effective imaging techniques [18]. Electrical Impedance Tomography (EIT) is one such method that has attracted attention. It's a non-invasive imaging approach that introduces small electrical currents into biological tissue and measures resulting voltage potentials on the tissue surface. EIT has potential applications in medical imaging, including brain activity evaluation, breast tumor identification, and lung function monitoring [2], [3], [4], [19]. An inverse problem is resolved with Electrical Impedance Tomography (EIT) by analyzing observed voltage data with mathematical approaches to recreate the distribution of electrical conductivity, or impedance, within tissue. However, the inverse problem in EIT is complicated, being both ill-posed and nonlinear, posing considerable hurdles to achieving accurate and reliable reconstructions. To solve this, gradient-based techniques are often used, and several optimization strategies have been proposed to increase convergence and efficacy. [5]. In Electrical Impedance Tomography (EIT), commonly employed methods consist of the Levenberg-Marquardt algorithm, gradient descent with regularization, and the Gauss-Newton method. Recently, adaptive optimization algorithms have gained traction to tackle convergence challenges linked with conventional optimization techniques. These adaptive approaches, exemplified by the Adam method, dynamically adapt the learning rate throughout the optimization process[25].

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The Adam optimizer is an adaptive moment optimizer that integrates the AdaGrad and RMSprop algorithms. It was first presented by Kingma and Ba [21]. Adam keeps a moving average of gradients, only uses first-order gradients to add momentum, and modifies learning rates according to square gradients. Because of this feature, Adam is a good fit for real-time image capture parameter modifications that aim to lower phase estimation mistakes. Better spatial resolution and sharper, more accurate images are the end result. Adam is well renowned for its memory efficiency, great computational capability, and ease of use. Electrical Impedance Tomography (EIT) is one application for it.Please see the following sources [7], [8], [9] for additional details on the specifics of using the Adam optimizer. Implementation of Adam's optimizer requires the use of a suitable loss function that evaluates the disparity between measured and observed potential values. The aim of this optimizer is to accelerate convergence of the loss function to the lowest value by combining gradient descent with adaptive momentum and learning rates. The learning rate depends on the moments of the first and second gradients. It is particularly effective for solving inverse optimization problems.[10], [11].This work aims to highlight the Adam technique's potential to enhance the efficiency and precision of Electrical Impedance Tomography (EIT) by conducting a comprehensive investigation into its application [24]. The contents of this document are as follows: In Section 2, the mathematical concepts on which TIS is based are outlined. The inverse problem is examined in detail in Section 3, where the approach used to reconstruct the conductivity distribution is explained. Our numerical simulations are presented in Section 4, while an in-depth analysis and explanation of the results is given in Section 5.

II. The EIT Forward Problem

In Electrical Impedance Tomography (EIT), the forward challenge is to compute voltage measurements along a domain's boundary. This calculation considers the applied currents as well as the distribution of electrical conductivity throughout the domain. In order to do this, the Laplace equation for the domain's electric potential needs to be solved with appropriate boundary conditions that mimic the voltage measurement and current injection carried out by electrodes on the border. The potential distribution that has been determined is then used to use Ohm's law to get the boundary voltages. [17], [20]. The specific reason for this circumstance is crucial for solving the inverse problem in EIT; the forward problem is significant. Reconstructing the conductivity distribution from the measured voltages is the goal of the inverse problem. The forward problem facilitates estimating expected voltage readings given a particular conductivity distribution and applied currents by providing a theoretical framework. Mastery of the forward problem is essential to advance EIT techniques and accurately recreate internal conductivity distributions in various areas, such as industrial processes and medical imaging.Let $\Omega \subseteq \mathbb{R}^n$, n=2,3 be our domain of interest and $\partial\Omega$ the boundary of Ω

$$-\nabla \cdot (\sigma \nabla V) = 0 \quad \text{on } \Omega \tag{1}$$

with the homogeneous Dirichlet and Neumann boundary conditions

$$\sigma \frac{\partial V}{\partial n} = \begin{cases} I_i & \text{at } i\text{th node} \\ 0 & \text{at the others} \end{cases} \tag{2}$$

where V is the electric potential distribution, σ is the conductivity distribution, $\dfrac{\partial}{\partial \eta}$ is the outward normal derivative, I is the injected current through the boundary. The EIT forward model, characterized by the Laplace equation (1) and boundary conditions (2), does not possess a readily available analytical solution, especially for complex geometries. Therefore, a common approach is to tackle the partial differential equation (PDE) numerically. This is typically achieved using the finite element method (FEM), a well-documented technique in the scientific literature.[12], [13], [14].

III. The EIT Inverse Problem

The internal conductivity distribution inside an item is ascertained using surface measurements in the EIT inverse problem. In terms of mathematics, this means resolving the inverse problem related to the elliptic equation (1) with the goal of deriving the internal conductivity distribution in the domain Ω from the measured boundary voltage positions. [15], [25].

$$f(\sigma) = V_s \tag{3}$$

Where V_s measured from the object domain bondaries $\partial\Omega$, $f(\sigma)$ is implicity defined as the forward model based on equation (1)

A. Linearization and Regularization Techniques for Solving EIT Inverse Problem

The EIT inverse problem is challenging due to its ill-posed nature[16], [18], [25], which makes the solution non-unique and sensitive to measurement noise and errors. Linearization and regularization techniques are commonly used to obtain a stable and unique solution.The objective is to minimize the cost function J by determining the optimum approach σ:

$$J(\sigma) = \frac{1}{2} \left\| f(\sigma) - V_s \right\|^2 \tag{4}$$

The EIT problem may be expressed as a problem of optimization:

$$\sigma^* = argmin(\sigma) \tag{5}$$

In order to add a regularization component to the cost function and aid achieve a stable solution, total variation regularization is utilized.

$$J(\sigma) = \frac{1}{2} \left\| f(\sigma) - V_s \right\|^2 + \lambda R(\sigma) \tag{6}$$

Where the term $R(\sigma) = \left\| \sigma - \sigma_0 \right\|^2$of the objective function represents the regularization related to the prior knowledge of σ. Here, λ is the regularization parameter and σ_0 is the initial guess. The forward model$f(\sigma)$ in EIT is approximated using a linearization method, where the linearization is performed around an initial estimate σ_0 by taking the first order of Taylor series expression of the forward problem function[22], [16]. This can be approximed as follows:

$$f(\sigma) = f(\sigma_0) + f^{'}(\sigma_0)(\sigma - \sigma_0) + r \tag{7}$$

$f^{'}$ is the Fréchet derivative of the forward model f and the symbol r is used to represent the residual term obtained from the Taylor series expansion around σ_0.The gradient of objective functional can be written as follows :

$$\nabla J(\sigma) = f(\sigma_0) * f^{'}(\sigma - V_s) + \lambda R^{'}(\sigma) \tag{8}$$

The Fréchet derivative of the regularization operator with respect to the parameter σ is denoted as $R^{'}(\sigma)$.

B. Iterative Inverse Problem Solution

This paragraph discusses the convergence of an iterative optimization algorithm denoted by I for solving the inverse problem in electrical impedance tomography. The algorithm aims to iteratively get closer to the solution until it reaches the desired accuracy.

$$\begin{cases} \sigma = \sigma_0 \\ \sigma_{n+1} = L(\sigma_n; J) \end{cases} \tag{9}$$

Convergence of the algorithm is proven to be achieved when the limit of $L(\sigma; J)$ as n approaches infinity is equal to the true solution σ.

$$lim_{n \to +\infty} L(\sigma_n; J) = \sigma^* \tag{10}$$

Determining the rate of convergence is a major challenge, especially for complex problems such as electrical impedance tomography, which require many multidimensional parameters. Various elements, such as the algorithm selected and the problem configuration, have an influence on the speed of convergence in optimization. The convergence properties of different algorithms have an effect on the convergence rate. In addition, the starting point or initial guess is also a determining factor in convergence speed..

TABLE I: Meaning of Parameters and Hyperparameters

Parameters and hyperparameters	Description
α	Learning rate
σ_0	Initial guess
$\beta 1$, $\beta 2$	Exponential decay rates
ε	Tiny value used in ADAM
λ	Regularization parameter

For example, gradient descent serves as a prevalent optimization algorithm known for its rapid convergence in convex problems with smooth objectives. Conversely, evolutionary algorithms may converge at a slower pace but excel in handling complex and non-convex optimization challenges. In the realm of electrical impedance tomography, gradient descent-based optimization algorithms are commonly employed, with the learning rate hyperparameter significantly influencing convergence speed. Various factors related to the structure of the problem, such as the number of electrodes, water properties, etc., influence the value of $J(\sigma)$, as described above. The value of $J(\sigma)$ also depends on the choice of regularization and the initial guess. Table 1 provides an illustration of the parameters and hyperparameters that can be used to solve the practical optimization problem, including the hyperparameters of the iterative algorithm.

Algorithm 1 ADAM Optimizer

Input: α
Input: $b_1, b_2 \in [0, 1)$
Input: $J(\sigma)$
Input: σ_0
$m_0 \leftarrow 0$
$v_0 \leftarrow 0$
$k \leftarrow 0$
while J not converged **do**
  $k \leftarrow k + 1$
  $g_k \leftarrow \nabla J(\sigma_{k-1})$
  $m_k \leftarrow b_1 \cdot m_{k-1} + (1 - b_1) \cdot g_k$
  $v_k \leftarrow b_2 \cdot v_{k-1} + (1 - b_2) \cdot g_k^2$
  $\hat{m} \leftarrow \frac{m_k}{1-b_1^k}$
  $\hat{v} \leftarrow \frac{v_k}{1-b_2^k}$
  $\sigma_k \leftarrow \sigma_{k-1} - \alpha \cdot \frac{\hat{m}}{\sqrt{\hat{v}}+\epsilon}$
end while
Output: return σ_k

The code above presents Adam's optimization algorithm, common for stochastic gradient descent, in pseudo-code form. The approach uses hyper-parameters such as the learning rate α, the decay rates of the exponential moving averages of the gradients b_1 and b_2,as well as a little constant for numerical stability, ε. The aim of the approach is to update the model parameters σ step by step until the loss function J is convergent.[6]. The method evaluates the gradient of the loss function as a function of the current parameters σ_{k-1} for each iteration. Then, using the decay coefficients b_1 and b_2, we update the exponential moving averages of the gradient m_k and the squared gradient v_k. Bias is also taken into account in the procedure. subsequently, the average movement and the learning rate α are used to update the model parameters σ_k. Additionally, a little constant ε is included to provide numerical stability.Our results show that non-convergent or delayed minimization occurs when the learning rate is selected outside of the interval [0.001, 0.1]. Following the literature [24], we sampled the learning rate for this simulation equally from the range [0.001, 0.1]. We also kept the momentum $\beta_1 = 0.9$ and $\beta_2 = 0.999$ at the recommended hyperparameter values.

IV. Numerical Simulations

We used the Python module Pyeit, an open-source tool for resolving the direct and inverse EIT problems, to create simulated data. We used a unit circular domain for our simulations, as well as separate meshes for the direct and inverse problems. First, we used a circular mesh with 1486 nodes and 2840 elements. Next, we used a redesigned mesh with 4084 nodes and 7890 elements. Equivalent intervals of 32 electrodes were placed on the domain boundary. With a background value of 2, the position, size, and quantity of anomalies in the electrical conductivity σ were selected 10^{-2}, 10^{-4}, 10^{-6} and 10^{-8} regularization parameters λ were used. We employed the pseudocode-explained Algorithm 1 to resolve the minimization issue. For the moment estimates β_1 and β_2, as well as the learning rate α, we employed the exponential decay rates. 10^{-8} was selected as ε, the stabilizing parameter. We used the exponential decay rates for moment estimates β_1 and β_2, and the learning rate α. 10^{-8} was chosen as the stabilizing parameter ε. To ensure accurate parameter control in our simulations, we first focused on minimizing the error of the initial reconstruction guess σ_0. We observed that smaller values of the regularization parameterλ led to better results in our simulations. Specifically, for values of $\lambda = 10^{-6}$ and $\lambda = 10^{-8}$, the reconstructed conductivity images exhibited fewer artifacts and better resolution of the anomalies. These results suggest that a smaller value of the regularization parameter is better suited for reconstructing electrical conductivity anomalies in our setup. Overall, our simulations highlight the importance of carefully selecting the regularization parameter and other parameters in the inverse problem to obtain accurate and reliable results. A correctly selected learning rate α is also crucial for obtaining accurate and reliable results in the minimization problem.

According to our simulations, a learning rate of 0.01 might result in accurate reconstruction of the electrical conductivity distribution and constant convergence. However, oscillations and instability in the reconstructed images were brought on by higher learning rates, such as 0.1. Thus, for reconstruction to be successful, care must be taken in selecting the learning rate and other optimization factors.

Furthermore, the accuracy of the reconstruction is greatly influenced by the choice of mesh, both for the direct and inverse problem. In our experiments, we noticed that the coarser mesh with 1486 nodes and 2840 elements generated less accurate and less detailed images compared to a finer mesh with 4083 nodes and 7954 elements. However, using a finer connection can lead to over-fitting and increased computational expense. So, when selecting the mesh, it's vital to strike a balance between computational efficiency and accuracy. It is possible to use a coarser mesh for larger, simpler anomalies, but in general, it is preferable to use a finer mesh to reconstruct smaller or more complex anomalies.

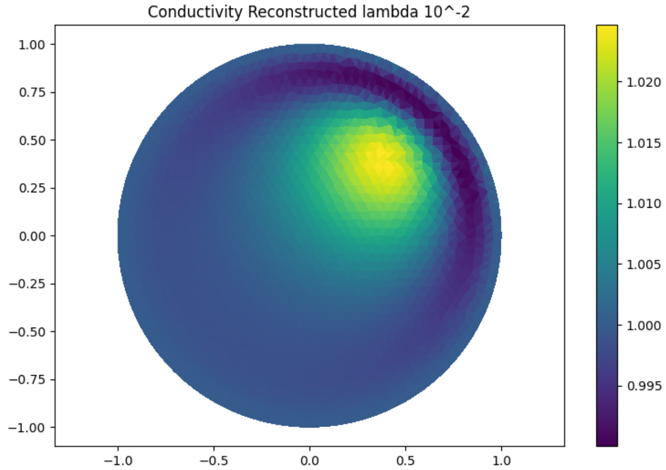

Fig. 1: The image reconstruction of the conductivity distribution was performed using 1486 nodes and 2840 elements, with a step size of 0.001 and regularization parameter set to 10^{-2}.

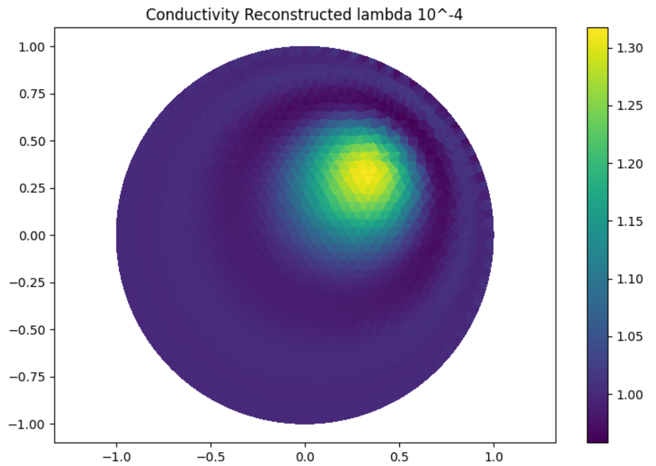

Fig. 2: The image reconstruction of the conductivity distribution was performed using 1486 nodes and 2840 elements, with a step size of 0.001 and regularization parameter set to 10^{-4}.

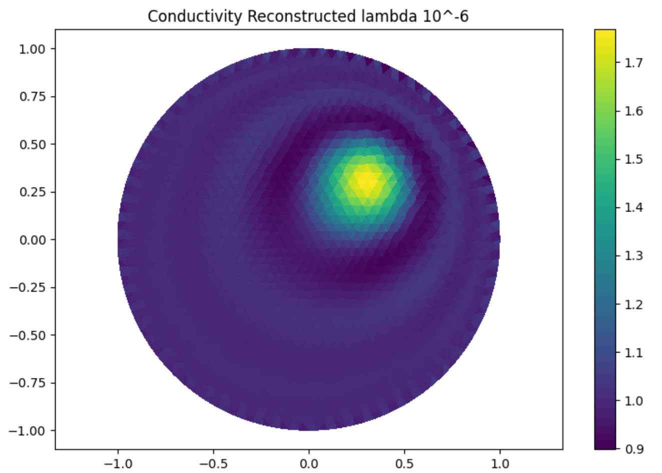

Fig. 3: The image reconstruction of the conductivity distribution was performed using 1486 nodes and 2840 elements, with a step size of 0.001 and regularization parameter set to 10^{-6}.

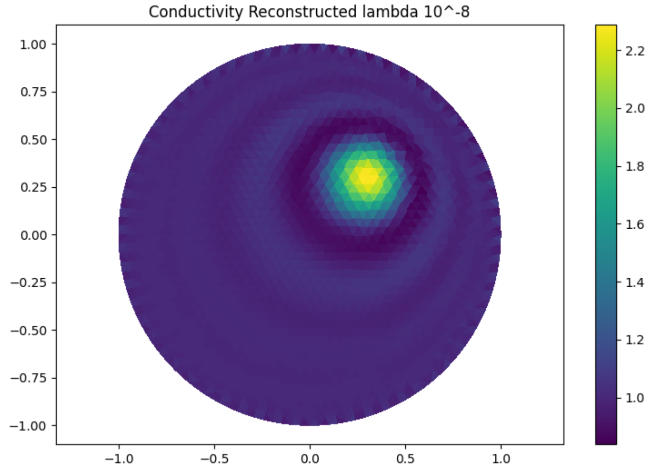

Fig. 4: The image reconstruction of the conductivity distribution was performed using 1486 nodes and 2840 elements, with a step size of 0.001 and regularization parameter set to 10^{-8}.

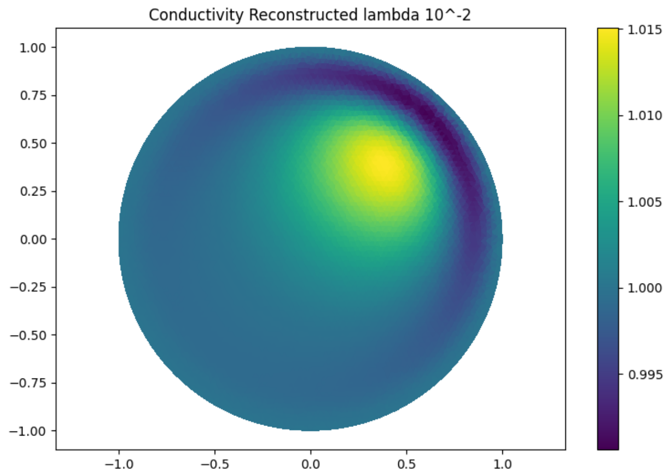

Fig. 5: The image reconstruction of the conductivity distribution was conducted using a denser mesh comprising 4083 nodes and 7954 elements, with a step size of 0.001 and regularization parameter set to 10^{-2}.

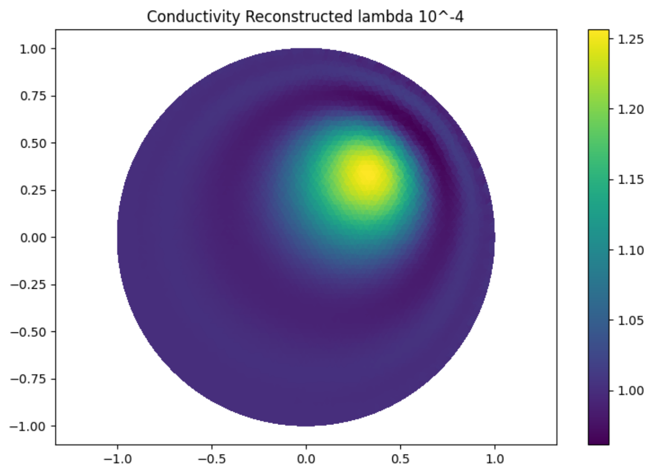

Fig. 6: The image reconstruction of the conductivity distribution was conducted using a denser mesh comprising 4083 nodes and 7954 elements, with a step size of 0.001 and regularization parameter set to 10^{-4}.

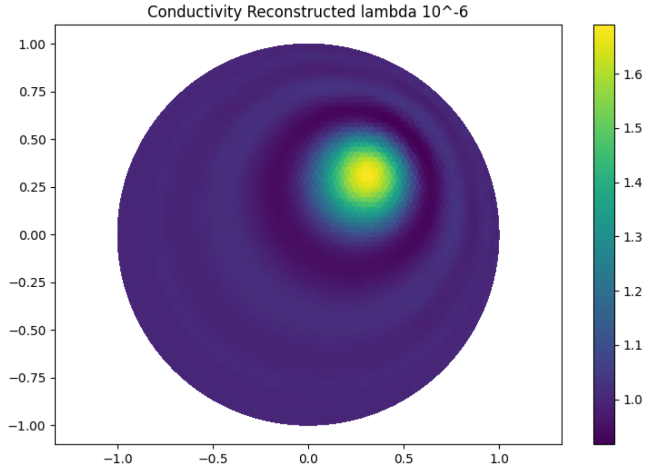

Fig. 7: The image reconstruction of the conductivity distribution was conducted using a denser mesh comprising 4083 nodes and 7954 elements, with a step size of 0.001 and regularization parameter set to 10^{-6}.

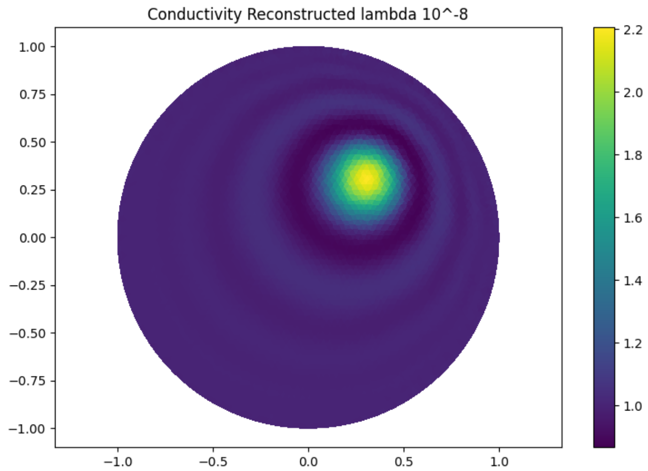

Fig. 8: The image reconstruction of the conductivity distribution was conducted using a denser mesh comprising 4083 nodes and 7954 elements, with a step size of 0.001 and regularization parameter set to 10^{-8}.

V. Conclusion and Future Work

In this study, we propose an innovative method for reconstructing electrical impedance tomography (EIT) images using the adaptive moment estimation (Adam) method. In light of this research, Adam has the potential to improve the efficiency of EIT image reconstruction by highlighting the importance of adjusting hyperparameters, such as learning rate, beta1, beta2 and delta.

Compared to traditional gradient descent algorithms, Adam's approach offers a number of major advantages, including faster convergence and better optimization performance due to his ability to dynamically modify the learning rate based on the gradient. Furthermore, Adam is especially useful for EIT image reconstruction, where the conductivity distribution is frequently unknown and an optimization process must be started with an approximation because to its strong performance and insensitivity to parameter initialization.

The application of Adam holds promise for improving both the accuracy and speed of EIT image reconstruction, with potential clinical implications for disease diagnosis and monitoring. Future research avenues may involve a comparative analysis between Adam and its variant, the Nesterov-accelerated Adaptive Moment Estimation (Nadam) algorithm, focusing on convergence speed, reconstruction accuracy, and sensitivity to different hyperparameters. Furthermore, investigations could explore their performance under diverse EIT imaging scenarios, including variations in electrode configurations, noise levels, and measurement setups.

Further studies may examine how well these algorithms work in more intricate EIT imaging contexts as dynamic imaging, real-time monitoring, or three-dimensional reconstruction. Lastly, evaluating the practical significance of enhanced precision and speed in EIT image reconstruction through Adam and Nadam in diverse medical contexts, like lung function tracking or stroke detection, will offer insightful information about their possible influence in healthcare.

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