Exploring Cotangent Similarity Measures for Enhanced Fault Diagnosis in Steam Turbines

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Abstract—**In the context of multi-criteria decision-making problems within single-valued neutrosophic set environments, this study introduces a simplified version of two similarity measures and develops two aggregation operators to synthesize results based on the proposed measure. Using a turbine generator fault diagnosis problem as a case study, we demonstrate that our proposed aggregation operators effectively generate diagnoses that align with checking reports. Additionally, we conduct a comprehensive examination of two cotangent similarity measures, investigating their properties and uncovering their inherent complexity as transformations of our proposed similarity measures. As a result, we advise researchers to avoid employing these more intricate measures. Our findings provide valuable guidance for practitioners dealing with multi-criteria decision-making problems in single-valued neutrosophic set environments.**

Index Terms—**Steam turbine, Fault diagnosis, Cotangent similarity measure, Single valued neutrosophic set**

I. Introduction

Since Zadeh [1] developed the original concept of fuzzy sets more than sixty thousand articles have been written using FSs to make sense of unclear, unresolved, deficient, and divergent information that was dealt in real environments. There are many important generalizations of FSs that have found been theoretical developed and found practical applications in quality control, economic evaluation and advanced information technology. One such extension is the intuitionistic fuzzy sets proposed by Atanassov [2] to generalize from one membership function to two functions: membership function and non-membership function. Smarandache [3] defined neutrosophic sets further extending the application to three membership functions: truth, indeterminacy, and falsity. Later on, Wang et al. [4] constructed single-valued neutrosophic sets to simplify the application for Neutrosophic sets. Tian [5] constructed a cotangent similarity measure for intuitionistic fuzzy sets. Motivated by Tian [5], Ye [6] developed two cotangent similarity measures for single-valued neutrosophic sets.

In this paper, we provide two new similarity measures to simplify the computation procedure of the cotangent similarity measures. Using the same steam turbine fault diagnosis, we demonstrate that our proposed two similarity measures can derive the ranking that is supported by the actual checking.

The rest of this paper is organized as follows. Section 2 introduced some previous results related to this paper. Section 3 presents our proposed two similarity measures. Our theoretical development for two similarity measures is based on maximum norm and arithmetic mean (average) and two aggregation operators are presented. In Section 4, we use steam turbine fault diagnosis of Ye [7] and Ye [6] to illustrate our two proposed aggregation operators to show that both derive the same diagnosis results as Ye [6]. In Section 5, we provide a detailed analysis for the cotangent similarity measures to show that they are transformations of our proposed similarity measures. We conclude our discussion in Section 6.

II. Review of Related Results for Previous Results

Atanassov [2] developed the intuitionistic fuzzy sets as

$$A = \left\{\left(x, \mu_A(x), v_A(x)\right) : x \in X\right\}, \quad (2.1)$$

with $v_A(x): X \to [0,1]$ and $\mu_A(x): X \to [0,1]$ are the non-membership and membership functions that satisfies $\mu_A(x) + v_A(x) \le 1$, for $x \in X$, where X is the universe of discourse,.

For two Intuitionistic fuzzy sets, $A = \left\{\left(x, \mu_A(x), v_A(x)\right) : x \in X\right\}$ and $B = \left\{\left(x, \mu_B(x), v_B(x)\right) : x \in X\right\}$, Tian [5] defined a cotangent similarity measure for A and B as follows:

$$CS(A,B) = \frac{1}{n}\sum_{k=1}^{n} \cot\frac{\pi}{4}(\alpha_k + 1), \quad (2.2)$$

where α is an abbreviation proposed by us to simplify the expressions, to denote as

$$\alpha_k = \max\left\{|v_A(x_k) - v_B(x_k)|, \left|\mu_A(x_k) - \mu_B(x_k)\right|\right\}, \quad (2.3)$$

and $X = \{x_1, x_2, \ldots, x_n\}$ is the universe of discourse.

Wang et al. [4] define a single-valued neutrosophic sets as follows

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}, \quad (2.4)$$

where X is a universe of discourse with a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ that satisfies $T_A(x)$, $I_A(x)$, and $F_A(x)$ are functions from X to [0,1].

For two single-valued neutrosophic sets, A and B, with

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}, \quad (2.5)$$

and

$$B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle : x \in X\}, \quad (2.6)$$

with the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, Ye [6] developed two weighted cotangent similarity measures as follows:

$$WCot_1(A,B) = \sum_{k=1}^{n} w_k \cot\left(\frac{\pi}{4}\beta_k + \frac{\pi}{4}\right), \quad (2.7)$$

where β_k is an abbreviation proposed by us to simplify the expressions. β_k is denoted as follows,

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$$\beta_k = \max\{|F_A(x_k) - F_B(x_k)|,$$
$$|I_A(x_k) - I_B(x_k)|, |TF_A(x_k) - T_B(x_k)|\}. \quad (2.8)$$

and

$$WCot_2(A,B) = \sum_{k=1}^{n} w_k \cot\left(\frac{\pi}{4}\gamma_k + \frac{\pi}{4}\right), \quad (2.9)$$

where γ_k is an abbreviation developed by us to simplify the expressions. γ_k is defined as follows,

$$\gamma_k = \frac{1}{3}\{|F_A(x_k) - F_B(x_k)| +$$
$$|I_A(x_k) - I_B(x_k)| + |TF_A(x_k) - T_B(x_k)|\}, \quad (2.10)$$

under the restriction, $\sum_{k=1}^{n} w_k = 1$, and $w_k \geq 0$, for $k = 1,2,\ldots,n$.

For a steam turbine fault diagnosis problem, Ye [6] used two cotangent measures to discover the most probable fault. To compare a testing sample with a set of possible faults, Ye [6] determines the ranking for possible faults and then checks the steam turbine to determine the actual fault.

III. Our Proposed Similarity Measures

For $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle : x \in X\}$, two single-valued neutrosophic sets with the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, we assume that

$$SIM_{max}(A,B) = \sum_{k=1}^{n} w_k(1 - \beta_k), \quad (3.1)$$

where β_k is defined in Equation (2.6).

Our second similarity is assumed as follows,

$$SIM_{ave}(A,B) = \sum_{k=1}^{n} w_k(1 - \gamma_k), \quad (3.2)$$

where γ_k is defined in Equation (2.8).

Hence, $SIM_{max}(A,B)$ is based on the maximum norm and $SIM_{ave}(A,B)$ is defined as the arithmetic mean of (i) a truth-membership function $T_A(x)$, (ii) an indeterminacy-membership function $I_A(x)$, and (iii) a falsity-membership function $F_A(x)$.

It is obvious that our two proposed similarity measures satisfy Tian [5] proposed four axioms for a well defined similarity measure.

IV. Numerical Example

We recall the numerical example of Ye [6]. Ye [7] proposed under intuitionistic fuzzy sets and subsequently Ye [6] converted the data to single-valued neutrosophic sets. There is a fault diagnosis problem of the steam turbine, with a set of ten fault patterns: A_1: Unbalance, A_2: Pneumatic force couple, A_3: Offset center, A_4: Oil-membrane oscillation, A_5 : Radial impact friction of rotor, A_6: Symbiosis looseness, A_7: Damage of anti-thrust bearing, A_8: Surge, A_9: Looseness of bearing block, and A_{10}: Non-uniform bearing stiffness, as the fault knowledge and a set of nine frequency ranges for different frequency spectrum: $C_1: 0.01 - 0.39f$, $C_2: 0.4 - 0.49f$, $C_3: 0.5f$, $C_4: 0.51 - 0.99f$, $C_5: 1f$, $C_6: 2f$, $C_7: 3 - 5f$, C_8: Odd times of f, and C_9: High frequency $> 5f$, under operating frequency f as a characteristic set (criteria set). Then, the information of the fault knowledge A_k , $k = 1,2,\ldots,10$ with respect to the frequency range (criterion) C_i, $i = 1,2,\ldots,9$ can be introduced from Ye [7] in intuitionistic fuzzy sets expression as $[\mu_A, 1 - v_A]$.

Consequently, Ye [6] tried to convert them to single-valued neutrosophic sets with $T_A = \mu_A$, $I_A = 1 - \mu_A - v_A$ and $F_A = v_A$ which is shown in Table 2 of Ye [6].

Two testing samples were present by Ye [7] in intuitionistic fuzzy sets expression, and then Ye [6] converted them in single-valued neutrosophic sets as follows:

$$B_1 = \{\langle 0,0,1\rangle, \langle 0,0,1\rangle, \langle 0.1,0,0.9\rangle,$$
$$\langle 0.9,0,0.1\rangle, \langle 0,0,1\rangle, \langle 0,0,1\rangle,$$
$$\langle 0,0,1\rangle, \langle 0,0,1\rangle, \langle 0,0,1\rangle\}, \quad (4.1)$$

and

$$B_2 = \{\langle 0.39,0,0.61\rangle, \langle 0.07,0,0.93\rangle, \langle 0,0,1\rangle,$$
$$\langle 0.06,0,0.94\rangle, \langle 0,0,1\rangle, \langle 0.13,0,0.87\rangle,$$
$$\langle 0,0,1\rangle, \langle 0,0,1\rangle, \langle 0.35,0,0.65\rangle\}, \quad (4.2)$$

We follow Ye [6] to assume that the weight of each characteristic C_i is $w_i = 1/9$ for i=1,2,...,9.

We list computation results for the testing sample B_1 of Equation (4.1) with A_k for $k = 1,2,\ldots,10$, in the second column of Table 2, by our proposed measure, SIM_{max}, of Equation (4.1). Moreover, we list computation results for B_1 of Equation (4.1) with A_k for $k = 1,2,\ldots,10$, in the third column of Table 2, by our proposed measure, SIM_{ave}, of Equation (3.2).

For the second testing example, B_2, we list computation results for B_2 of Equation (4.2) with A_k for $k = 1,2,\ldots,10$, in the fourth column of Table 2, by our proposed measure, SIM_{max}, of Equation (4.1). At last, we list computation results for testing sample B_2 of Equation (4.2) with A_k for $k = 1,2,\ldots,10$, in the fifth column of the Table 2, by our proposed measure, SIM_{ave}, of Equation (3.2).

From Table 1, based on our proposed two similarity measures, we derive that the most possible fault for the testing sample B_1 is A_7 and for the testing sample B_2 is A_9; these findings are consistent with Ye [6].

Based on Ye [7] and Ye [6], by actual checking for the testing sample B_1, it was found that one of antithrust bearings was damaged such that the diagnosis result of A_7 (Damage of antithrust bearing) is supported by the real examination.

For the testing sample B_2, Ye [7] and Ye [6] both mentioned that through actual inspection, they discovered the friction between the rotor and cylinder body in the turbine, and then the vibration values of four ground bolts of the bearing between the turbine and the gearbox were very different. Moreover, they also discovered that the gap between the nuts and the bearing block was outside of tolerance. Thus, the diagnosis result of A_9 (Looseness of the bearing block) is consistent to the actual examination.

V. Further Discussion for Cotangent Similarity Measures

Based on our abbreviations of β_k of Equation (2.6) and γ_k of Equation (2.8), we can rewrite $WCot_1(A,B)$ of Equation (2.5) and $WCot_2(A,B)$ of Equation (2.7) as follows,

$$WCot_1(A,B) = \sum_{k=1}^{n} w_k \cot\frac{\pi}{4}(\beta_k + 1), \quad (5.1)$$

and

$$WCot_2(A,B) = \sum_{k=1}^{n} w_k \cot\frac{\pi}{4}(\gamma_k + 1). \quad (5.2)$$

Table 1. (Reproduction of Table 2 of Ye [6]) Fault knowledge with single-valued neutrosophic values.

A_k	Frequency range (*f*: operating frequency)				
	C_1	C_2	C_3	C_4	C_5
A_1	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>	<0.85,0.15,0>
A_2	<0,0,1>	<0.28,0.03,0.69	<0.09,0.03,0.88>	<0.55,0.15,0.3>	<0,0,1>
A_3	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>	<0.30,0.28,0.42>
A_4	<0.09,0.02,0.89>	<0.78,0.04,0.18>	<0,0,1>	<0.08,0.03,0.89>	<0,0,1>
A_5	<0.09,0.03,0.88>	<0.09,0.02,0.89>	<0.08,0.04,0.88>	<0.09,0.03,0.88>	<0.18,0.03,0.79>
A_6	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>	<0.18,0.04,0.78>
A_7	<0,0,1>	<0,0,1>	<0.08,0.04,0.88>	<0.86,0.07,0.07>	<0,0,1>
A_8	<0,0,1>	<0.27,0.05,0.68>	<0.08,0.04,0.88>	<0.54,0.08,0.38>	<0,0,1>
A_9	<0.85,0.08,0.07>	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>
A_{10}	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>
	C_6	C_7	C_8	C_9	
A_1	<0.04,0.02,0.94>	<0.04,0.03,0.93>	<0,0,1>	<0,0,1>	
A_2	<0,0,1>	<0,0,1>	<0,0,1>	<0.08,0.05,0.87>	
A_3	<0.40,0.22,0.38>	<0.08,0.05,0.87>	<0,0,1>	<0,0,1>	
A_4	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>	
A_5	<0.08,0.05,0.87>	<0.08,0.05,0.87>	<0.08,0.04,0.88>	<0.08,0.04,0.88>	
A_6	<0.12,0.05,0.83>	<0.37,0.08,0.55>	<0,0,1>	<0.22,0.06,0.72>	
A_7	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>	
A_8	<0,0,1>	<0,0,1>	<0,0,1>	<0,0,1>	
A_9	<0,0,1>	<0,0,1>	<0.08,0.04,0.88>	<0,0,1>	
A_{10}	<0.77,0.06,0.17>	<0.19,0.04,0.77>	<0,0,1>	<0,0,1>	

Now we compare Equation (3.1) with Equation (5.1) to imply a general expression,

$$\mathrm{SIM}_1(A,B)=\sum_{k=1}^{n} w_k f(\beta_k), \quad (5.3)$$

as in Equation (3.1) for our proposed $\mathrm{SIM}_{\max}$ aggregation operator,

$$f(x)=1-x, \quad (5.4)$$

and in Equation (5.1), for WCot_1,

$$f(x)=\cot((1+x)\pi/4). \quad (5.5)$$

By the same argument, we compare Equation (3.2) with Equation (5.2) to obtain a general expression,

$$\mathrm{SIM}_2(A,B)=\sum_{k=1}^{n} w_k g(\gamma_k), \quad (5.6)$$

as in Equation (3.2) for our proposed aggregation operator, $\mathrm{SIM}_{\mathrm{ave}}$, with

$$g(x)=1-x, \quad (5.7)$$

and in Equation (5.2), for WCot_2, with

$$g(x)=\cot((1+x)\pi/4). \quad (5.8).$$

Hence, we compare Equations (3.1) and (5.1) and find that they are related to the maximum norm, β_k of Equation (2.6). By the same observation, we find that Equations (3.2) and (5.2) are related to the arithmetic mean, γ_k of Equation (2.8).

For the fundamental property of a similarity measure, say SIM, that is, SIM(0)=1, and SIM(1)=0 and SIM is a decreasing function. Hence, in Equations (3.1) and (5.1), we apply the most natural approach to select a strictly decreasing function, $f(x)=1-x$, of Equation (5.4), and $g(x)=1-x$ of Equation (5.7).

In Ye [6], he adopted $f(x)=\cot((1+x)\pi/4)$ of Equation (5.5) which is a decreasing function satisfying $f(0)=1$ and $f(1)=0$.

Moreover, there are infinite decreasing functions that satisfy $f(0)=1$ and $f(1)=0$. Tian [5] and Ye [6] did not provide any explanation for why they selected $f(x)=\cot((1+x)\pi/4)$ to develop their similarity measures.

Since our proposed approach uses the simplest form $f(x)=1-x$ that can derive the desired diagnosis results, we can advice researchers to not apply the more complicated method proposed by Tian [5] and Ye [6] with $f(x)=\cot((1+x)\pi/4)$ to simplify the computation process.

The application of our proposed approach can be applied to another paper of Ye and Fu [8]. In Ye and Fu [8], they used a tangent function with a shift. Following our approach, we can simplify their tangent function by our proposed maximum norm of Equation (3.1) and mean of Equation (3.2).

VI. Our Proposed Geometric Procedures

Recently, there are several papers, for example, Lin [9], Wang and Chen [10], Yang and Chen [11], Yen [12], and Yen [13], discussed to apply geometric methods to solve inventory models. We follow this research trend to present our version by a geometric approach.

Table 2. Computation results for testing samples B_1 and B_2 by our methods.

	$SIM_{max}(A_k, B_1)$	$SIM_{ave}(A_k, B_1)$	$SIM_{max}(A_k, B_2)$	$SIM_{ave}(A_k, B_2)$
k = 1	0.7633	0.8422	0.7744	0.8496
k = 2	0.9089	0.9393	0.8011	0.8674
k = 3	0.7411	0.8274	0.7700	0.8467
k = 4	0.7944	0.8630	0.8244	0.8830
k = 5	0.8011	0.8674	0.8556	0.9037
k = 6	0.7644	0.8430	0.8478	0.8985
k = 7	**0.9878***	**0.9919**	0.7856	0.8570
k = 8	0.9200	0.9467	0.8000	0.8667
k = 9	0.7722	0.8481	**0.8589**	**0.9059**
k = 10	0.7711	0.8474	0.8000	0.8667

* The boldface indicates the maximum value.

VII. OUR PROPOSED GEOMETRIC PROCEDURES

Recently, there are several papers, for example, Lin [9], Wang and Chen [10], Yang and Chen [11], Yen [12], and Yen [13], discussed to apply geometric methods to solve inventory models. We follow this research trend to present our version by a geometric approach.

Our proposed problem is to minimize

$$f(r) = r^2(v+h) - 2vr + v, \qquad (7.1)$$

for $0 \le r \le 1$.

We assume a new parameter, say

$$s = 1 - r, \qquad (7.2)$$

with $0 \le s \le 1$, and then we convert $f(r)$ to $F(r,s)$ with

$$F(r,s) = hr^2 + vs^2, \qquad (7.3)$$

under the restrictions $0 \le r, s \le 1$ and $r + s = 1$.

From geometric point, the minimum will happen at the parabola,

$$hr^2 + vs^2 = c, \qquad (7.4)$$

where c is the constant to specific the desire parabola, touch the straight line,

$$r + s = 1, \qquad (7.5)$$

at one point. It follows that the determinant of intersection $hr^2 + v(1-r)^2$ is zero, that is,

$$(-2v)^2 - 4(h+v)(v-c) = 0. \qquad (7.6)$$

We obtain that

$$c = hv/(h+v). \qquad (7.7)$$

After we find the minimum value, we try to locate the minimum point to solve

$$f(r) = hv/(h+v), \qquad (7.8)$$

to derive that

$$r^* = v/(h+v). \qquad (7.9)$$

From Equation (7.1), we know that

$$f(r) = r^2(r+h) - 2vr + v, \qquad (7.10)$$

and then we use the new parameter $s = 1 - r$ then

$$f(s) = s^2(r+h) - 2hs + h. \qquad (7.11)$$

Now, we compare Equations (7.1) and (7.11) to find that if we interchange h with v and replace r by s then Equation (7.1) is converted to Equation (7.11) to reveal the minimum solution of Equation (7.1) is symmetric with respect to h, and v. Our observation is supported by the findings of Equation (7.9).

VIII. DIRECTIONS FOR FUTURE RESEARCH

Several recently published papers are very important that can help practitioners to know directions for the hot spots in the future research such that we cite them in the following. Chen et al. [14] enhanced transformer models by time sequence estimation through efficiency and accuracy. By deep and shallow data, under attention fusion and dynamic convolution, Hu et al. [15] examined Siamese network tracker. Referring to a few information collection, Berot et al. [16] examined least square time market network for the parameters selection. Using K-mean nearest neighbor classifier and discrete grasshopper optimization algorithm, Qi et al. [17] developed optimal feature selection. Using strip surface defects, Hou et al. [18] developed biological geography optimized procedure for multilayer perception identification. Kakarlapudi et al. [19] examined optimal iterative schemes to deal with nonlinear equations and then to apply to the real case appliance under attraction of valleys and basins. With combination of multi-level concentration, Zhang and Guo [20] considered Cold Rolled Steel to calculate mechanical assets. Rakhmawati et al. [21] constructed multiple graphs with respect to intuitionistic fuzzy number for interval value under optimal path. For deprived person, Chinnadurai et al. [22] constructed a mathematical system with respect to the management of wine and drug abuse. With holiday disruption and two-stage trip, Tian et al. [23] studied stability investigation with the M/M/1 unpredictable waiting in line. Under the restriction of three-dimension space, Zheng, and Chen [24] considered Riesz-type space and Caputo-type time dispersed array transmission formula to find analytic solutions Referring to bounded disturbances and directed topology, Zhou et al. [25] examined universal nonlinear multi-purpose models to derive consensus without leader under fully scattered system. Based on our above discussion, practitioners will locate some hot spots for their future research topics.

IX. A Related Algebraic Problem

In this section, we would like to highlight the work of Luo and Chou [26], who published a paper addressing inventory models using an algebraic approach. While examining their derivation, we have identified a potentially problematic step. They rely on the assumption that the difference of two positive terms will always yield a positive result. However, it should be noted that the difference of two positive terms can sometimes result in a negative number. Therefore, we will propose a brief improvement to address this issue and rectify any potential shortcomings in their methodology.

To begin, let us revisit Equation (14) from the study conducted by Luo and Chou [26],

$$4(a-1)^2\left(x-\frac{2f(x)-b}{2(a-1)}\right)^2+\left(\frac{a-1}{a}\right)(4ac-b^2)$$
$$=4a\left(f(x)-\frac{b}{2a}\right)^2. \quad (9.1)$$

In their publication, Luo and Chou [26] discussed the condition for obtaining a positive minimum value based on Equation (14). They noted that "due to the coefficients of two square terms, namely $4(a-1)^2$ and 4a, being positive, the following condition is established:

$$4ac-b^2>0. \quad (15)"$$

Remark: Equation (14) serves as the reference index in Luo and Chou's study [26]. Additionally, Equation (15) is also cited from their work.

X. Our Improvements

In this revised section, we aim to present an alternative derivation for the condition stated in Equation (15) without relying on the coefficients $4(a-1)^2$ and 4a from Equation (9.1).

Building upon the findings of Luo and Chou [26], they have already derived the conditions for

$$a>1, \quad (10.1)$$

and

$$c>0. \quad (10.2)$$

We endeavor to identify the restrictions for

$$f(x)=\sqrt{ax^2+bx+c}-x. \quad (10.3)$$

In order to ensure the condition $f(x)>0$, for $x>0$ and to find a unique minimum solution of f(x) within the range $0<x<\infty$as an interior minimum point, we examine the function $\sqrt{ax^2+bx+c}$, which involves taking the square root of ax^2+bx+c. For this function to be valid, it is necessary that $ax^2+bx+c\geq 0$, for $x>0$.

We will discuss this further in two cases: (I) when $b^2-4ac\geq 0$, and (II) when $b^2-4ac<0$.

Under case (I), where $b^2-4ac\geq 0$, we solve the equation $ax^2+bx+c=0$, for the entire range of $-\infty<x<\infty$. In this scenario, there exist two real roots, which may be repeated.

$$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a}, \quad (10.4)$$

and

$$x_2=\frac{-b+\sqrt{b^2-4ac}}{2a}. \quad (10.5)$$

We can further subdivide case (I) into two sub-cases: (Ia) when b < 0, and (Ib) when b > 0. This distinction arises due to the conditions a > 1, c > 0, and $0\leq b^2-4ac$, which imply that b cannot be equal to 0.

In the case of (Ia), we observe that $x_2>0$ and $f(x_2)=-x_2<0$ which contradicts our requirement of $f(x)>0$, for $x>0$. Consequently, (Ia) falls outside the scope of our study.

In the case of (Ib), we will now establish the following inequality:

$$ax^2+bx+c\geq\left(\sqrt{a}x+\sqrt{c}\right)^2. \quad (10.6)$$

Let us proceed with the computation to establish the proof.

$$ax^2+bx+c-\left(\sqrt{a}x+\sqrt{c}\right)^2$$
$$=\left(b-2\sqrt{a}\sqrt{c}\right)x\geq 0, \quad (10.7)$$

given that we are considering the case (Ib) with $b>0$, and also the condition (I) $b^2-4ac\geq 0$, we can proceed with the computation.

Hence, we derive that

$$\sqrt{ax^2+bx+c}\geq\sqrt{a}x+\sqrt{c}, \quad (10.8)$$

and then

$$f(x)\geq\left(\sqrt{a}-1\right)x+\sqrt{c}, \quad (10.9)$$

to imply that

$$f(x)>\sqrt{c}. \quad (10.10)$$

Next, we will demonstrate that as x approaches 0, f(x) can get arbitrarily close to $\sqrt{c}$. To illustrate this, let us evaluate f(1/m) as follows.

$$f\left(\frac{1}{m}\right)=\sqrt{\frac{a}{m^2}+\frac{b}{m}+c}-\frac{1}{m},$$
$$=\frac{\frac{a}{m^2}+\frac{b}{m}+c-\frac{1}{m^2}}{\sqrt{\frac{a}{m^2}+\frac{b}{m}+c}+\frac{1}{m}}, \quad (10.11)$$

such that we consequently obtain that

$$\sqrt{c}<f\left(\frac{1}{m}\right)<\frac{\frac{a-1+b}{m}+c}{\sqrt{c}}$$
$$=\frac{a-1+b}{m\sqrt{c}}+\sqrt{c}. \quad (10.12)$$

By choosing a sufficiently large value for m, we can infer that the value of f(1/m) approaches $\sqrt{c}$. This suggests that f(x) does not possess a minimum solution for $x>0$. The lowest attainable value is $\sqrt{c}$ which is not achieved for any $x>0$.

The original problem addressed by Luo and Chou [26] is to determine the conditions for a, b, and c such that the function $f(x)=\sqrt{ax^2+bx+c}-x$ for x>0, satisfies the requirements $f(x)>0$ and has an interior minimum solution x^* with $0<x^*$. Consequently, we can conclude that case (I) falls outside the scope of our study.

Hence, we propose an enhancement to the work of Luo and Chou [26] specifically addressing case (II) where $b^2-4ac<0$. This improvement aims to advance their subsequent derivations following Equation (14), which is Equation (9.1) of this paper, in Luo and Chou's publication.

Remark: It is widely recognized that the condition for ensuring $ax^2 + bx + c \geq 0$ is $4ac > b^2$ which holds true within the domain of $-\infty < x < \infty$.

However, it should be noted that in the context of Luo and Chou's study [26], the domain considered is specifically $0 < x < \infty$.

Consequently, researchers cannot directly apply the result of $4ac > b^2$ from the domain of $-\infty < x < \infty$ to the domain of $0 < x < \infty$ in Luo and Chou's work.

XI. A Related Fuzzy Problem

The idea of fuzzy sets has been extended to the conception of intuitionistic fuzzy sets. Since it was proposed by Atanassov [2] in 1986, intuitionistic fuzzy sets has been used to deal with the soft or hard decision making problems in the vague and fuzzy environment involving imprecise and unknown information. In the fuzzy and vague environment, the multi-criteria and multi-attribute decision making problems are very difficult to be handled with conventional approach since there exists obscurities in approximating the parameters and recognizing the relationships among criteria and attributes as well as the attributes and criteria with respect to variables. Consequently, to handle the multi-criteria or group decision making problems in the fuzzy and vague environment, Li [27] obtained a linear programming problem to solve the optimal weights among attributes and criteria under the restriction of intuitionistic fuzzy sets environment. Li [27] proposed that the model can be solved by applying the Simplex algorithm. The solution system of Li [27] in the rand new not difficult to use such that more than three hundreds papers had cited Li [27] in their references. Because of the significance of Li [27], a more robust and efficient approach to handle these kind of systems is needed to be developed. Therefore, in this section, we will work on a new approach, which is easier and intuitive then the well-known Simplex method, to handle the solution algorithm mentioned by Li [27].

XII. Applying Fuzzy Weighted Average Algorithm

We claim that from the solution structure of fuzzy weighted average algorithm to develop an example for Li [27] such that we can add fuzzy weighted average algorithm to enhance the solution procedure for Li [27].

We study the following maximum problem,

$$\text{Max } f_U = \frac{w_1 + 6w_2 + 7w_3 + 8w_4}{w_1 + w_2 + w_3 + w_4}, \quad (12.1)$$

under restrictions of

$$2 \leq w_1 \leq 5, \quad (12.2)$$

$$4 \leq w_2 \leq 8, \quad (12.3)$$

$$1 \leq w_3 \leq 4, \quad (12.4)$$

and

$$3 \leq w_4 \leq 6. \quad (12.5)$$

We evaluate the boundary value from the left end, to derive that

$$f_U(2,4,1,3) = \frac{1\cdot 2 + 6\cdot 4 + 7\cdot 1 + 8\cdot 3}{2+4+1+3} = \frac{57}{10} = 5.7. \quad (12.6)$$

We replace the left and of w_4 to its right end, and then we obtain that

$$f_U(2,4,1,6) = \frac{1\cdot 2 + 6\cdot 4 + 7\cdot 1 + 8\cdot 6}{2+4+1+6} = \frac{81}{13} = 6.230. \quad (12.7)$$

Following this trend, we replace the left end of w_3 to its right end, and then we find that

$$f_U(2,4,4,6) = \frac{1\cdot 2 + 6\cdot 4 + 7\cdot 4 + 8\cdot 6}{2+4+4+6} = \frac{102}{16} = 6.375. \quad (12.8)$$

We continuously replace the left end of w_2 to its right end, and then we show that

$$f_U(2,8,4,6) = \frac{1\cdot 2 + 6\cdot 8 + 7\cdot 4 + 8\cdot 6}{2+8+4+6}$$

$$f = \frac{126}{20} = 6.3. \quad (12.9)$$

Finally, we replace the left end of w_1 to its right end, and then we get that

$$f_U(5,8,4,6) = \frac{1\cdot 5 + 6\cdot 8 + 7\cdot 4 + 8\cdot 6}{5+8+4+6} = \frac{129}{23} = 5.609. \quad (12.10)$$

Based on our results of Equations (12.6-12.10), we locate the maximum point at $w_1 = 2$, $w_2 = 4$, $w_3 = 4$, and $w_4 = 6$, with the maximum value, $f_U(2,4,4,6) = 6.375$.

Based on our above discussion, we will construct an example for Li [27] as follows.

$$\text{Max } f_U = y_1 + 6y_2 + 7y_3 + 8y_4, \quad (12.11)$$

under the following conditions,

$$\frac{2}{16} \leq y_1 \leq \frac{5}{16}, \quad (12.12)$$

$$\frac{4}{16} \leq y_2 \leq \frac{8}{16}, \quad (12.13)$$

$$\frac{1}{16} \leq y_3 \leq \frac{4}{16}, \quad (12.14)$$

$$\frac{3}{16} \leq y_4 \leq \frac{6}{16}, \quad (12.15)$$

and

$$y_1 + y_2 + y_3 + y_4 = 1. \quad (12.16)$$

We directly check the boundary point of $\frac{2}{16} = y_1$, $\frac{4}{16} = y_2$, $y_3 = \frac{4}{16}$, and $y_4 = \frac{6}{16}$, that satisfies the restriction of Equation (12.16) which is the maximum solution.

Our solution procedure demonstrate that some kind of maximum problem can be solved by the fuzzy weighted average algorithm such that we only need to consider the combination of left and right end point. consequently, we offer a fast solution approach.

XIII. Direction for Future Research

We recall that $\left[\bar{z}_j^l, \bar{z}_j^u\right]$ of Equations (3) and (4) of Li [27] and $\left[z_j^{0l}, z_j^{0u}\right]$ of Equation (15) Li [27], and then based on Theorem 2 of Li [27], we know that

$$\left[\bar{z}_j^l, \bar{z}_j^u\right] \supset \left[z_j^{0l}, z_j^{0u}\right]. \qquad (13.1)$$

If we used fuzzy weighted average algorithm to derive an interval, say $\left[z_j^l, z_j^u\right]$, then we can claim that our findings will have the following property,

$$\left[z_j^l, z_j^u\right] \supset \left[\bar{z}_j^l, \bar{z}_j^u\right] \supset \left[z_j^{0l}, z_j^{0u}\right], \qquad (13.2)$$

which indicates that our derived interval will have the larger interval length. Moreover, we point out that Lin et al. [28] and Hwang and Yoon [29] are valuable for practitioners for their future study.

XIV. Conclusion

Our study introduces two novel similarity measures, derived from the widely used maximum norm and average distances commonly found in various applications. Through a comprehensive analysis, we have demonstrated that our proposed measures offer a simplified and straightforward form compared to the cotangent similarity measures proposed by Ye [6]. By applying our measures to two testing samples, we have successfully obtained fault diagnoses for steam turbines that align with actual inspection results. The effectiveness of our proposed measures highlights their potential as a simplified computation procedure, serving as a valuable resource for researchers in future studies. By leveraging these measures, researchers can streamline their analysis and decision-making processes in the field of fault diagnosis for steam turbines. On the other hand, we raise a new solution approach for a hot research spot that had been considered by Lin [9], Wang and Chen [10], Yang and Chen [11], Yen [12], and Yen [13], with inventory models to derive the optimal solution without using calculus. Our geometric method will encourage researchers to consider alternative solution procedures for their examined problems. At last, we discuss a related fuzzy problem proposed by Li [27]. We offer an alternative solution approach through fuzzy weighted average algorithm.

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