Reduction on Model Operations Complexity of Simplified Volterra in Digital Predistortion for Wireless Communications

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***Abstract*—The evolution of Digital Predistortion (DPD) for Power Amplifier (PA) linearization in wireless communications has seen a natural progression towards increasingly complex DPD models. This complexity directly correlates with improved accuracy in modeling PAs, thereby enhancing the linearization performance of DPD systems. As the demand for bandwidth and PA operation frequencies continues to rise, there is a growing trend towards achieving higher accuracy at the expense of increased complexity. This trend poses challenges for DPD design, necessitating the introduction of more complex algorithms to ensure effective PA linearization. This phenomenon is especially apparent in Volterra Series DPD algorithms, which are widely used for modelling nonlinear systems like PAs. This paper showcases the steps of reducing the modelling operations complexity of the Simplified Volterra (SV) method. The treated SV method presented in this paper is compared against the original SV method through simulation. The DPD model operations complexity is evaluated using the number of required multiplication operations. The DPD PA linearization performance is evaluated with respective model's Normalized Mean Square Error (NMSE). A 57% reduction on the number of multiplication operations is observed in the treated SV against the original SV. Measured NMSE is similar for both models, indicating the treated SV model achieves this reduction in complexity without sacrificing linearization performance.**

***Index Terms*—digital predistortion, memory polynomial, PA linearization, Volterra Series, wireless communications**

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I. INTRODUCTION

THE rise of Digital Predistortion (DPD) as one of the most reliable Power Amplifier (PA) Linearization method is not surprising when it is compared against the other PA linearization methods. DPD offers distinct advantages, such as straightforward implementation, power efficiency, and favourable trade-offs that lead to improved linearization outcomes [1], [2], [3], [4], [5].

The DPD system aims to mitigate the non-linearity effects exhibited by the PA, encompassing amplitude distortions, phase distortions, adjacent channel interference and the notorious Memory Effects [6]. The non-linearity of the PA becomes evident when it operates beyond its saturation point, causing the output to plateau despite increasing input signal power. Today's communication systems, with their escalating bandwidth demands, induce fluctuations in the PA's input signal power, leading to undesirable spurious peaks and a high Peak to Average Power Ratio (PAPR) [7], [8]. Variations in electrical thermals introduce Memory Effects, where pass and present input signal values influence the current output signal values at the PA. These effects underscore the significance of DPD as an effective PA linearization tool.

Fig. 1 shows a basic DPD block diagram redrawing from [9]. In the DPD system, the PA input signal is fed into the DPD function that is inversely non-linear to PA's function. Subsequently, the output signal from this DPD block feeds into the PA. This sequential processing yields a linearized PA output, mitigating distortions in both amplitude and phase. For optimized linearizing performance, the DPD function must closely mirror the inversed non-linearity of the PA, further justifies the need for precise PA modelling.

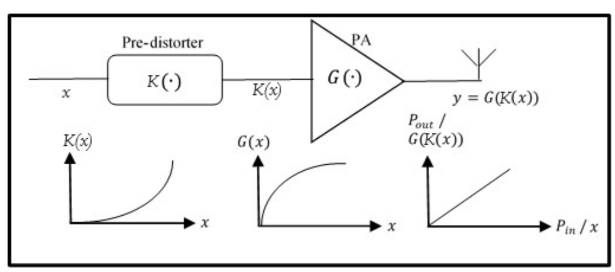

Fig. 1. Diagram of a basic DPD block linearizing the PA. The PA input is pre-distorted with a function that is inversely non-linear with respect to the PA's function (Redrawing based on [9]).

The modeling of PA nonlinearity exemplifies an engineering paradox: as modeling accuracy improves, complexity escalates [10]. This trade-off is evident in widely-used methods like the Volterra Series [11] frequently employed for modeling nonlinear systems such as PAs. Enhanced accuracy necessitates a substantial increase in the number of model coefficients, leading to an exponential rise in model operation complexity [3].

In [12], the diagonal kernels of the Volterra Series are extracted and leads to a more efficient algorithm, known as the Memory Polynomial (MP). Further modelling accuracy enhancing efforts is observed in [13], where a treatment is introduced that considers both leading and lagging elements of the memorial point. This treatment results in Generalized MP (GMP), a modelling algorithm structure with a greater number of total summation branches compared to MP.

GMP introduced in [13] is subsequently expanded into Simplified Volterra (SV) in [14], which serves as the focal point of this paper. The objective is to refine the SV modeling algorithm while preserving its linearization efficacy, with a specific focus on reducing model operations complexity.

Section II of the paper describes the various DPD algorithms, starting with Volterra Series, Memory Polynomial (MP), Generalized MP (GMP), and finally to Simplified Volterra (SV). The treatment of SV is shown with the optimization steps. Section III describes the performance metrics used to measure the linearization performance of the treated SV algorithm against the original SV algorithm. Ensuring equitable comparison involves matching the number of model coefficients between the original and enhanced models, indicating identical model sizes. The widely used Normalized Mean Square Error (NMSE) calculation formula is presented to assess linearization performance. To evaluate model operations complexity, the multiplication operations required for SV and treated SV is each calculated and presented. Section IV presents the results harvested together with analysis. Section V concludes the paper and potential future research directions.

II. Model Description

A. Volterra Series

The notable Volterra Series is shown below:

$$y(t) = \sum_k \int \cdots \int h_{2k+1}(\tau_{2k+1}) \prod_{i=1}^{k+1} z(t-\tau_i) \prod_{i=k+2}^{2k+1} z^*(t-\tau_i) d\tau_{2k+1} \quad (1)$$

where

$$h_{2k+1}(\tau_{2k+1}) = \frac{1}{2^{2k}} \binom{2k+1}{k} \tilde{h}_{2k+1}(\tau_{2k+1}) e^{-j2\pi\left(\sum_{i=1}^{k+1}\tau_i - \sum_{i=k+2}^{2k+1}\tau_i\right)}$$

In discrete-time domain, (1) becomes the following:

$$y(n) = \sum_k \sum_{l_1} \cdots \sum_{l_{2k+1}} h_{2k+1}(l_1, l_2, \cdots, l_{2k+1}) \prod_{i=1}^{k+1} z(n-l_i) \prod_{i=k+2}^{2k+1} z^*(n-l_i) \quad (2)$$

The number of model coefficients in Volterra Series increases exponentially when the model dimensions increase. This introduces exponential increment in model complexity when the algorithm is configured to model PAs operating in high demand environments with high Memory Effects. This encourages researchers to find solutions to the complexity challenge presented.

B. Memory Polynomial (MP)

In [3], the diagonal kernels of Volterra Series are extracted, resulted in the Memory Polynomial (MP) with reduced complexity but with matching linearization performance.

$$z(n) = \sum_{\substack{k=1 \\ k\ odd}}^{K} \sum_{q=0}^{Q} a_{kq} x(n-q)|x(n-q)|^{k-1} \quad (3)$$

Where K is the non-linearity order, Q is the memory depth, $x(n)$ is PA input signal, and a_{kq} are the model coefficients

First, the input signal $x(n)$ is replaced with the output signal $y(n)$, then the Least Square Method together with the Moore-Penrose Inverse [15] is used to calculate the model coefficients:

$$z(n) = \sum_{\substack{k=1 \\ k\ odd}}^{K} \sum_{q=0}^{Q} a_{kq} y(n-q)|y(n-q)|^{k-1} \quad (4)$$

(4) in matrix form:

$$z = Y \cdot a \quad (5)$$

Where

$$z = [z(0), z(1), \dots, z(N-1)]^T \quad (6)$$

$$Y = \left[y_{10}, \dots, y_{K0}, \dots, y_{1Q}, \dots y_{KQ}\right] \quad (7)$$

$$y_{KQ} = \left[y_{KQ}(0), y_{KQ}(1), \dots y_{KQ}(N-1)\right]^T \quad (8)$$

$$a = \left[a_{10}, \dots, a_{K0}, \dots, a_{1Q}, \dots a_{KQ}\right]^T \quad (9)$$

To obtain the model coefficients, (5) could be rewritten as:

$$\boldsymbol{a} = (\boldsymbol{Y^{conj}} \cdot \boldsymbol{Y})^{-1} \boldsymbol{Y^{conj}} \boldsymbol{z} \quad (10)$$

C. Generalized MP (GMP)

In [13], MP is further expanded to include the lagging and leading components of the respective input signal, with regards to the memory depth configured in the model. This yields the Generalized MP, shown in the equation below:

$$y_{GMP}(n) = \sum_{k=0}^{K_a-1} \sum_{q=0}^{Q_a-1} a_{kq} x(n-q)|x(n-q)|^k + \sum_{k=1}^{K_b} \sum_{q=0}^{Q_b-1} \sum_{l=1}^{L_b} b_{kqm} x(n-q)|x(n-q-l)|^k + \sum_{k=1}^{K_c} \sum_{q=0}^{Q_c-1} \sum_{l=1}^{L_c} c_{kqm} x(n-q)|x(n-q+l)|^k \quad (11)$$

Where K_a , K_b , K_c, Q_a , Q_b , Q_c are the non-linearity order and memory depth dimensions respectively, and L_b is the dimension for lagging component, while L_c is the leading component model dimension. Respectively, a_{kq}, b_{kqm} and c_{kqm} are the model coefficients.

With a clear increase in model dimensions configurations and summation branches, the GMP increases in complexity compared to MP, but comes with improved linearization performance, strengthening the notion where model complexity is increased conveniently to improve PA modelling accuracies.

D. Simplified Volterra (SV)

GMP from [13] is then further expanded into Simplified Volterra in [14], for improvements in linearization performance, and is coined as Simplified Volterra (SV), as shown below:

$$y_{SV}(n) = \sum_{k=1}^{K_a}\sum_{q=0}^{Q_a} a_{kq}x(n-q)|x(n-q)|^{k-1} + \sum_{k=1}^{K_b}\sum_{q=0}^{Q_b}\sum_{l=1}^{L_b} b_{kql}x(n-q)|x(n-q-l)|^{k} + \sum_{k=1}^{K_c}\sum_{q=0}^{Q_c}\sum_{l=1}^{L_c} c_{kql}x(n-q)|x(n-q+l)|^{k} + \sum_{k=1}^{K_d}\sum_{q=0}^{Q_d}\sum_{l=1}^{L_d} d_{kql}x^*(n-q)x^2(n-q-l)|x(n-q-l)|^{k} + \sum_{k=1}^{K_e}\sum_{q=0}^{Q_e}\sum_{l=1}^{L_e} e_{kql}x^*(n-q)x^2(n-q+l)|x(n-q+l)|^{k} \tag{12}$$

Where K_a , K_b , K_c, K_d and K_e are the non-linearity order dimensions. Q_a , Q_b , Q_c , Q_d and Q_e are the memory depths respectively. L_b and L_d are the model dimension for lagging component, while L_c and L_e are the leading component model dimensions. Similarly, the model coefficients are a_{kq}, b_{kql}, c_{kql}, d_{kql} and e_{kql}.

E. Simplified Volterra with Binomial Reduction (SV-BR)

The optimization of SV through Binomial Reduction is inspired by the work in [16], [17] on MP, and then in [18] on GMP, where evidently the Binomial Reduction optimization method could be applied in MP based DPD algorithms.

SV from (12) is rephrased into below, to have both k and q to start from 0:

$$y_{SV}(n) = \sum_{k=0}^{K_a-1}\sum_{q=0}^{Q_a} a_{kq}x(n-q)|x(n-q)|^{k} + \sum_{k=0}^{K_b-1}\sum_{q=0}^{Q_b}\sum_{l=1}^{L_b} b_{kql}x(n-q)|x(n-q-l)|^{k+1} + \sum_{k=0}^{K_c-1}\sum_{q=0}^{Q_c}\sum_{l=1}^{L_c} c_{kql}x(n-q)|x(n-q+l)|^{k+1} + \sum_{k=0}^{K_d-1}\sum_{q=0}^{Q_d}\sum_{l=1}^{L_d} d_{kql}x^*(n-q)x^2(n-q-l)|x(n-q-l)|^{k+1} + \sum_{k=0}^{K_e-1}\sum_{q=0}^{Q_e}\sum_{l=1}^{L_e} e_{kql}x^*(n-q)x^2(n-q+l)|x(n-q+l)|^{k+1} \tag{13}$$

The absolutes in the basis functions are expanded:

$$y_{SV}(n) = \sum_{k=0}^{K_a-1}\sum_{q=0}^{Q_a} a_{kq}x(n-q)\sqrt{x(n-q)_{real}{}^2 + x(n-q)_{imag}{}^2}^{\,k} + \sum_{k=0}^{K_b-1}\sum_{q=0}^{Q_b}\sum_{l=1}^{L_b} b_{kql}x(n-q)\sqrt{x(n-q-l)_{real}{}^2 + x(n-q-l)_{imag}{}^2}^{\,k+1} + \sum_{k=0}^{K_c-1}\sum_{q=0}^{Q_c}\sum_{l=1}^{L_c} c_{kql}x(n-q)\sqrt{x(n-q+l)_{real}{}^2 + x(n-q+l)_{imag}{}^2}^{\,k+1} + \sum_{k=0}^{K_d-1}\sum_{q=0}^{Q_d}\sum_{l=1}^{L_d} d_{kql}x^*(n-q)x^2(n-q-l)\sqrt{x(n-q-l)_{real}{}^2 + x(n-q-l)_{imag}{}^2}^{\,k+1} + \sum_{k=0}^{K_e-1}\sum_{q=0}^{Q_e}\sum_{l=1}^{L_e} e_{kql}x^*(n-q)x^2(n-q+l)\sqrt{x(n-q+l)_{real}{}^2 + x(n-q+l)_{imag}{}^2}^{\,k+1} \tag{14}$$

Rearranging (14):

$$y_{SV}(n) = \sum_{k=0}^{K_a-1}\sum_{q=0}^{Q_a} a_{kq}x(n-q)\left[x(n-q)_{real}{}^2 + x(n-q)_{imag}{}^2\right]^{\frac{k}{2}} +$$

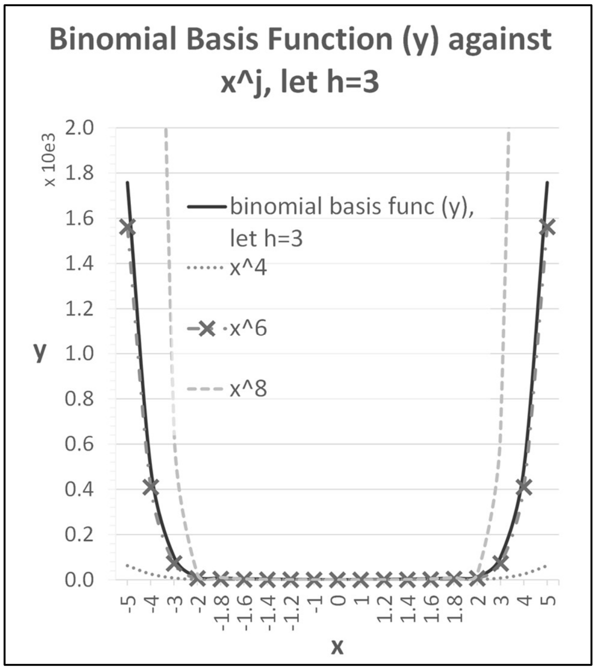

Fig. 2. Plot of Binomial Basis Function (y) in (22) against x^j, when h is 3.

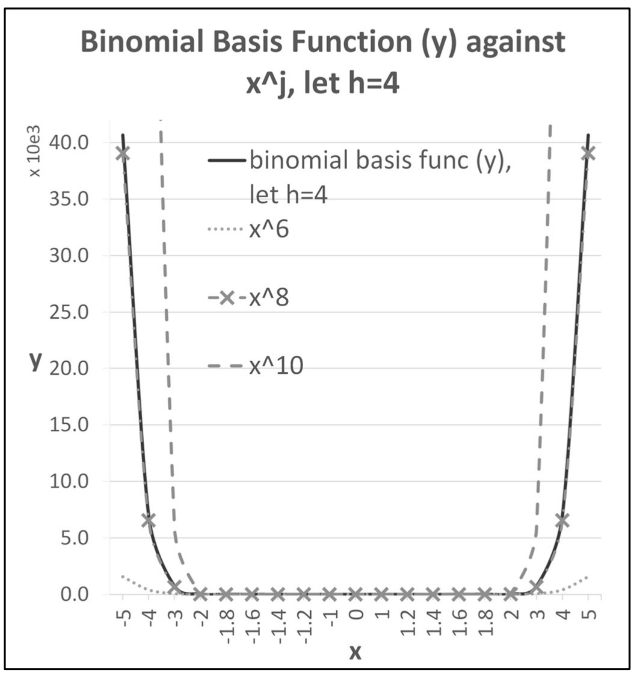

Fig. 3. Plot of Binomial Basis Function (y) in (22) against x^j, when h is 4.

$$\sum_{k=0}^{K_b-1}\sum_{q=0}^{Q_b}\sum_{l=1}^{L_b} b_{kql}x(n-q)\left[x(n-q-l)_{real}{}^2 + x(n-q-l)_{imag}{}^2\right]^{\frac{1}{2}}\left[x(n-q-l)_{real}{}^2 + x(n-q-l)_{imag}{}^2\right]^{\frac{k}{2}} +$$

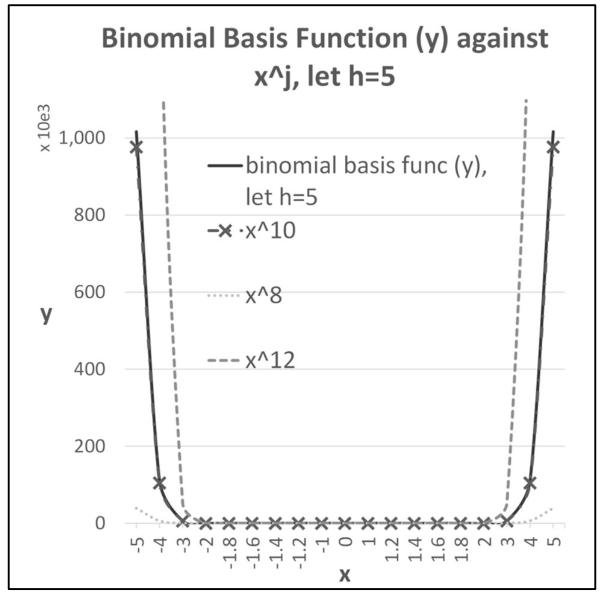

Fig. 4. Plot of Binomial Basis Function (y) in (22) against x^j, when h is 5.

$$\sum_{k=0}^{K_c-1}\sum_{q=0}^{Q_c}\sum_{l=1}^{L_c} c_{kql}x(n-q)\left[x(n-q-l)_{real}{}^2 + x(n-q-l)_{imag}{}^2\right]^{\frac{1}{2}}\left[x(n-q+l)_{real}{}^2 + x(n-q-l)_{imag}{}^2\right]^{\frac{k}{2}} +$$

$$\sum_{k=0}^{K_d-1}\sum_{q=0}^{Q_d}\sum_{l=1}^{L_d} d_{kql}x^*(n-q)x^2(n-q-l)\left[x(n-q-l)_{real}{}^2 + x(n-q-l)_{imag}{}^2\right]^{\frac{1}{2}}\left[x(n-q-l)_{real}{}^2 + x(n-q-l)_{imag}{}^2\right]^{\frac{k}{2}} +$$

$$\sum_{k=0}^{K_e-1}\sum_{q=0}^{Q_e}\sum_{l=1}^{L_e} e_{kql}x^*(n-q)x^2(n-q+l)\left[x(n-q+l)_{real}{}^2 + x(n-q+l)_{imag}{}^2\right]^{\frac{1}{2}}\left[x(n-q+l)_{real}{}^2 + x(n-q+l)_{imag}{}^2\right]^{\frac{k}{2}} \quad (15)$$

Using the binomial theorem:

$$Since\ (a+b)^h = \sum\nolimits_{i=0}^{h}\binom{h}{i}a^i b^{h-i} = \sum\nolimits_{i=0}^{h}\binom{h}{i}b^i a^{h-i} \quad (16)$$

Let $h = \frac{k}{2}$, $a = x(n-q)_{real}{}^2$ and $b = x(n-q)_{imag}{}^2$, Basis function of SV is restructured as:

$$\left[x(n-q)_{real}{}^2 + x(n-q)_{imag}{}^2\right]^h = \sum\nolimits_{i=0}^{h} \binom{h}{i} \left[x(n-q)_{real}{}^2\right]^i \left[x(n-q)_{imag}{}^2\right]^{h-i} \quad (17)$$

$$\left[x(n-q)_{real}{}^2 + x(n-q)_{imag}{}^2\right]^h = \sum\nolimits_{i=0}^{h} \binom{h}{i} \left[x(n-q)_{imag}{}^2\right]^i \left[x(n-q)_{real}{}^2\right]^{h-i} \quad (18)$$

Where

$$\binom{h}{i} = \frac{h!}{i!\,(h-i)!}$$

Restructuring (17) into the following:

$$\sum\nolimits_{i=0}^{h} \binom{h}{i} \left[x(n-q)_{real}{}^2\right]^i \left[x(n-q)_{imag}{}^2\right]^{h-i} = \sum\nolimits_{i=0}^{h} \binom{h}{i} \left[x(n-q)_{real}{}^{2i}\right] \left[\frac{x(n-q)_{imag}{}^{2h}}{x(n-q)_{imag}{}^{2i}}\right] = x(n-q)_{imag}{}^{2h} \sum\nolimits_{i=0}^{h} \binom{h}{i} \left[\frac{x(n-q)_{real}}{x(n-q)_{imag}}\right]^{2i} \quad (19)$$

Restructuring (18) into the following:

$$\sum\nolimits_{i=0}^{h} \binom{h}{i} \left[x(n-q)_{imag}{}^2\right]^i \left[x(n-q)_{real}{}^2\right]^{h-i} = \sum\nolimits_{I=0}^{h} \binom{h}{i} \left[x(n-q)_{imag}{}^{2i}\right] \left[\frac{x(n-q)_{real}{}^{2h}}{x(n-q)_{real}{}^{2i}}\right] = x(n-q)_{real}{}^{2h} \sum\nolimits_{i=0}^{h} \binom{h}{i} \left[\frac{x(n-q)_{imag}}{x(n-q)_{real}}\right]^{2i} \quad (20)$$

The binomial basis function of (19) is extracted and let

$$\sum\nolimits_{i=0}^{h} \binom{h}{i} \left[\frac{x(n-q)_{real}}{x(n-q)_{imag}}\right]^{2i} = \sum\nolimits_{i=0}^{h} \binom{h}{i} x^{2i} \quad (21)$$

The binomial basis function of (20) is extracted and let

$$\sum\nolimits_{i=0}^{h} \binom{h}{i} \left[\frac{x(n-q)_{imag}}{x(n-q)_{real}}\right]^{2i} = \sum\nolimits_{i=0}^{h} \binom{h}{i} x^{2i} \quad (22)$$

Let

$$y = \sum\nolimits_{i=0}^{h} \binom{h}{i} x^{2i} \approx x^{j} \quad (23)$$

Equation (23) is then expanded and explored in Figs. 2 to 4 and summarized in Table I.

Let

$$y = \sum\nolimits_{i=0}^{h} \binom{h}{i} x^{2i} \approx x^{2h} \quad (24)$$

Substituting (24) into (21) and (22), then to (19) and (20), and finally in (15), results in SV with Binomial Reduction below

$$\begin{aligned} y_{SV-BR}(n) = &\sum_{k=1}^{K_a}\sum_{q=0}^{Q_a} a_{kq} x(n-\mathrm{q}) x(n-q)_{real}{}^{k-1} + \\ &\sum_{k=1}^{K_b}\sum_{q=0}^{Q_b}\sum_{l=1}^{L_b} b_{kql} x(n-q) x(n-q-l)_{real}{}^{k} + \\ &\sum_{k=1}^{K_c}\sum_{q=0}^{Q_c}\sum_{l=1}^{L_c} c_{kql} x(n-q) x(n-q+l)_{real}{}^{k} + \\ &\sum_{k=1}^{K_d}\sum_{q=0}^{Q_d}\sum_{l=1}^{L_d} d_{kql} x^{*}(n-q) x^{2}(n-q-l) x(n-q-l)_{real}{}^{k} \\ &+ \\ &\sum_{k=1}^{K_e}\sum_{q=0}^{Q_e}\sum_{l=1}^{L_e} e_{kql} x^{*}(n-q) x^{2}(n-q+l) x(n-q+l)_{real}{}^{k} \end{aligned} \quad (25)$$

$$\begin{aligned} y_{SV-BR}(n) = &\sum_{k=1}^{K_a}\sum_{q=0}^{Q_a} a_{kq} x(n-\mathrm{q}) x(n-q)_{imag}{}^{k-1} + \\ &\sum_{k=1}^{K_b}\sum_{q=0}^{Q_b}\sum_{l=1}^{L_b} b_{kql} x(n-q) x(n-q-l)_{imag}{}^{k} + \\ &\sum_{k=1}^{K_c}\sum_{q=0}^{Q_c}\sum_{l=1}^{L_c} c_{kql} x(n-q) x(n-q+l)_{imag}{}^{k} + \\ &\sum_{k=1}^{K_d}\sum_{q=0}^{Q_d}\sum_{l=1}^{L_d} d_{kql} x^{*}(n-q) x^{2}(n-q-l) x(n-q-l)_{imag}{}^{k} \\ &+ \\ &\sum_{k=1}^{K_e}\sum_{q=0}^{Q_e}\sum_{l=1}^{L_e} e_{kql} x^{*}(n-q) x^{2}(n-q+l) x(n-q+l)_{imag}{}^{k} \end{aligned} \quad (26)$$

TABLE I
SUMMARY OF FIG 2, 3 AND 4

When h=3	When h=4	When h=5
$y = \sum_{i=0}^{3} \binom{3}{i} x^{2i} \approx x^{6}$	$y = \sum_{i=0}^{4} \binom{4}{i} x^{2i} \approx x^{8}$	$y = \sum_{i=0}^{5} \binom{5}{i} x^{2i} \approx x^{10}$

In the following sections, (26) will be referred as the treated SV: SV with Binomial Reduction (SV-BR). Based on the binomial reduction of MP in [19], (26) is predicted to have better linearization performance compared to (25)

III. Performance Metrics

A. Model Coefficients

To achieve meaningful comparison between the treated algorithm against the original algorithm, it is crucial that both algorithms are configured with identical numbers of model coefficients. When the two algorithms possess the same number of model coefficients, their model sizes are identical. This ensures the comparison of model operations complexity remains valid, unaffected by differences in model sizes.

The model coefficients calculation of SV as obtained from [20] is as follows:

$$K_a(M_a+1)+K_b(M_b+1)L_b+K_c(M_c+1)L_c+ K_d(M_d+1)L_d+K_e(M_e+1)L_e \tag{27}$$

Similarly, the model coefficients for the treated SV method, SV with Binomial Reduction has an identical calculation formula as above (24).

To identify the right model dimensions, the same general sweep method in [14] is applied, where the non-linearity orders are swept from 1 to 8, memory depths are swept from 1 to 5, and the leading/lagging components are swept from 1 to 3.

B. Normalized Mean Square Error (NMSE)

The infamous Normalized Mean Square Error (NMSE) has been used widely to evaluate the linearization performance of DPD models. The predistorted output is compared against the ideal PA output, where a lower number of NMSE (dB) indicates a smaller magnitude of error against the ideal output.

The general NMSE calculation formula for DPD [20] is shown below:

$$NMSE(dB)=10\,log\frac{\sum_{n=1}^{N}|y_{ideal}(n)-y_{pd}(n)|^2}{\sum_{n=1}^{N}|y_{ideal}(n)|^2} \tag{28}$$

The NMSE calculation of SV is shown as:

$$NMSE_{SV}(dB)=10\,log\frac{\sum_{n=1}^{N}|y_{ideal}(n)-y_{pd(SV)}(n)|^2}{\sum_{n=1}^{N}|y_{ideal}(n)|^2} \tag{29}$$

Similarly, the NMSE calculation for the treated SV method: SV with Binomial Reduction (SV-BR), is shown below:

$$NMSE_{SV-B}\ (dB)=10\,log\frac{\sum_{n=1}^{N}|y_{ideal}(n)-y_{pd(SV-BR)}(n)|^2}{\sum_{n=1}^{N}|y_{ideal}(n)|^2} \tag{30}$$

C. Model Operations Complexity (Multiplication Operations Calculation)

To evaluate the model operations complexity of SV, the multiplication operations calculations equation is as follows:

$$\begin{aligned}
&No.of\ Multiplication\ Operations\ in\ SV\\
&=\sum_{k=1}^{K_a}\sum_{q=0}^{Q_a}\left(1+3(k-1)\right)
+\sum_{k=1}^{K_b}\sum_{q=0}^{Q_b}\sum_{l=1}^{L_b}\left(1+3(k)\right)
+\sum_{k=1}^{K_c}\sum_{q=0}^{Q_c}\sum_{l=1}^{L_c}\left(1+3(k)\right)\\
&+\sum_{k=1}^{K_d}\sum_{q=0}^{Q_d}\sum_{l=1}^{L_d}\left(3+3(k)\right)
+\sum_{k=1}^{K_e}\sum_{q=0}^{Q_e}\sum_{l=1}^{L_e}\left(3+3(k)\right)\\
&=\frac{K_a(3K_a-1)(Q_a+1)}{2}
+\frac{K_bL_b(3K_b+5)(Q_b+1)}{2}
+\frac{K_cL_c(3K_c+5)(Q_c+1)}{2}\\
&+\frac{3K_dL_d(K_d+3)(Q_d+1)}{2}
+\frac{3K_eL_e(K_e+3)(Q_e+1)}{2}
\end{aligned} \tag{31}$$

To evaluate the model operations complexity of the treated SV: SV-BR, the multiplication operations calculations equation is shown below:

$$\begin{aligned}
&No.of\ Multiplication\ Operations\ in\ SV-BR\\
&=\sum_{k=1}^{K_a}\sum_{q=0}^{Q_a}\left(1+1(k-1)\right)
+\sum_{k=1}^{K_b}\sum_{q=0}^{Q_b}\sum_{l=1}^{L_b}\left(1+1(k)\right)
+\sum_{k=1}^{K_c}\sum_{q=0}^{Q_c}\sum_{l=1}^{L_c}\left(1+1(k)\right)\\
&+\sum_{k=1}^{K_d}\sum_{q=0}^{Q_d}\sum_{l=1}^{L_d}\left(3+1(k)\right)
+\sum_{k=1}^{K_e}\sum_{q=0}^{Q_e}\sum_{l=1}^{L_e}\left(3+1(k)\right)\\
&=\frac{1}{2}K_a(K_a+1)(Q_a+1)
+\frac{1}{2}K_bL_b(K_b+3)(Q_b+1)
+\frac{1}{2}K_cL_c(K_c+3)(Q_c+1)\\
&+\frac{1}{2}K_dL_d(K_d+7)(Q_d+1)
+\frac{1}{2}K_eL_e(K_e+7)(Q_e+1)
\end{aligned} \tag{32}$$

D. Model Operations Complexity Reduction (Savings in Multiplication Operations)

To evaluate and quantify the complexity reductions in model operations, the following equation is applied:

TABLE II
NMSE AND MULTIPLICATION OPERATIONS OF SIMPLIFIED VOLTERRA (SV) AND SIMPLIFIED VOLTERRA WITH BINOMIAL REDUCTION (SV-BR)

Method	Model Dimensions	No. of Model Coefficients	NMSE (dB)	Multiplication Operations	Model Operations Complexity Reduction
Simplified Volterra (results extracted from [14])	$K_a(9)$ $K_b(3)$ $K_c(3)$ $K_d(1)$ $K_e(1)$ $M_a(3)$ $M_b(0)$ $M_c(0)$ $M_d(1)$ $M_e(1)$ $L_b(1)$ $L_c(1)$ $L_d(1)$ $L_e(1)$	26	-35.38	214	-
Simplified Volterra (simulated using (12))	$K_a(1)$ $K_b(1)$ $K_c(1)$ $K_d(1)$ $K_e(5)$ $M_a(4)$ $M_b(1)$ $M_c(4)$ $M_d(1)$ $M_e(1)$ $L_b(1)$ $L_c(1)$ $L_d(1)$ $L_e(1)$	24	-34.48	195	-
Simplified Volterra with Binomial Reduction (Simulated using (16))	$K_a(1)$ $K_b(1)$ $K_c(1)$ $K_d(1)$ $K_e(4)$ $M_a(4)$ $M_b(1)$ $M_c(2)$ $M_d(2)$ $M_e(1)$ $L_b(1)$ $L_c(1)$ $L_d(2)$ $L_e(1)$	24	-35.12	83	57.43%

$$Model\ Operations\ Complexity\ Reduction\ (\%) = \frac{(\text{No.of Multiplication Operations in SV} - No.of\ Multiplication\ Operations\ in\ SV\text{BR})}{No.of\ Multiplication\ Operations\ in\ SV} x100 \quad (33)$$

IV. RESULTS AND DISCUSSION

Table II presents the linearization performance and model operations complexity reduction results for SV and the treated SV: SV with Binomial Reduction (SV-BR). The results from [14] (where SV is first coined) is presented together as a reference. The respective model dimensions are shown, where the number of model coefficients are calculated using (24) from [20]. The linearization performance is presented using NMSE, while the model operations complexity is shown using multiplication operations calculated using (31). The reference SV from [14] uses 26 model coefficients, yields a reported -35.38dB NMSE, with a complexity cost of 214 multiplication operations.

In this paper, the SV algorithm is simulated as per (12) gives a yield of -34.48dB NMSE, with 24 model coefficients and a complexity cost of 195 multiplication operations. The simulated SV results are in near proximity of the reference results extracted from [14], with a minor lag in NMSE, justified by the reduced number of model coefficients and hence number of multiplication operations.

The treated SV (SV-BR) is simulated with a set of model dimensions that has an identical number of model coefficients against the original SV algorithm simulated: 24. SV-BR yields a similar linearization performance of -35.12dB in NMSE, with a complexity cost of 83 multiplication operations. Using (33), the SV-BR yields an improvement of 57.43% in model operations complexity reduction. The model operations complexity reduction is achieved in SV-BR without compromising linearization performance in terms of NMSE and matching model coefficients against the original SV algorithm.

The model dimensions in SV and SV-BR were swept across values from 1 to 8 to further stress-test both models. This results in model coefficient ranges of 1 to 200 for both SV and SV-BR models. This ensures a fair comparison of similar model sizes, where both models are compared using matching model coefficient numbers, despite differences in model dimensions. As a result, a total of 231105 SV and 264780 SV-BR simulation samples are harvested. The following figures show various comparisons between SV-BR against SV, on number of multiplication operations, NMSE, improvement percentages and the cumulative average of improvement percentages across the 200 model coefficients.

Fig. 5 shows the number of multiplication operations against number of model coefficients for SV and SV-BR. In Fig. 5, as number of model coefficients increases, SV-BR requires fewer number of multiplication operations compared to SV. This indicates savings in number of multiplication operations, which results in reduction of model operations complexity in SV-BR against SV.

Fig. 6 shows the best NMSE performance for SV and SV-BR with respect to number of model coefficients. The differences in best NMSE performance of the two models are better represented in Fig. 7, where a positive graph bar indicates a better performing NSME of SV-BR compared against SV. A negative graph bar indicates the opposite, which is where SV was performing better than SV-BR for that respective number of model coefficient. The length of the bar indicates the magnitude of NMSE differences in dB. In the middle of Fig. 7, a graph line is drawn to show the cumulative average of NMSE differences as the number of model coefficients increases. The final cumulative average of NMSE difference of SV-BR against SV is at -0.16dB. The small number of NMSE difference indicates that the linearization performance of SV-BR is similar to the original SV algorithm.

Following the best NMSE of SV and SV-BR shown previously in Fig. 6, Fig. 8 shows the respective number of multiplication operations of each simulation sample that produces the best NMSE results and is plotted against number of model coefficients. Aligning with what was shown previously in Fig. 5, SV requires higher number of multiplication operations compared to SV-BR. Fig. 8 also shows that SV-BR requires fewer number of multiplications operations, while capable of achieving similar linearization performance in SV as shown in Fig. 6 and Fig. 7. The crude visual inspection of Fig. 8 shows almost 50% reduction in the required number of multiplication operations in SV-BR compared to SV. This estimation is proven to be true in Fig. 9, where the percentage of model operations complexity reduction is calculated using (33), and is plotted against number of model coefficients. Similar to Fig. 7, a line is again plotted in the middle of Fig. 9, where it shows the cumulative average of model operations complexity reduction percentage, as model coefficients increase. The cumulative average of model operations complexity reduction percentage is 55.38%. This value almost matches with the simulation results presented earlier in Table II, where both SV and SV-

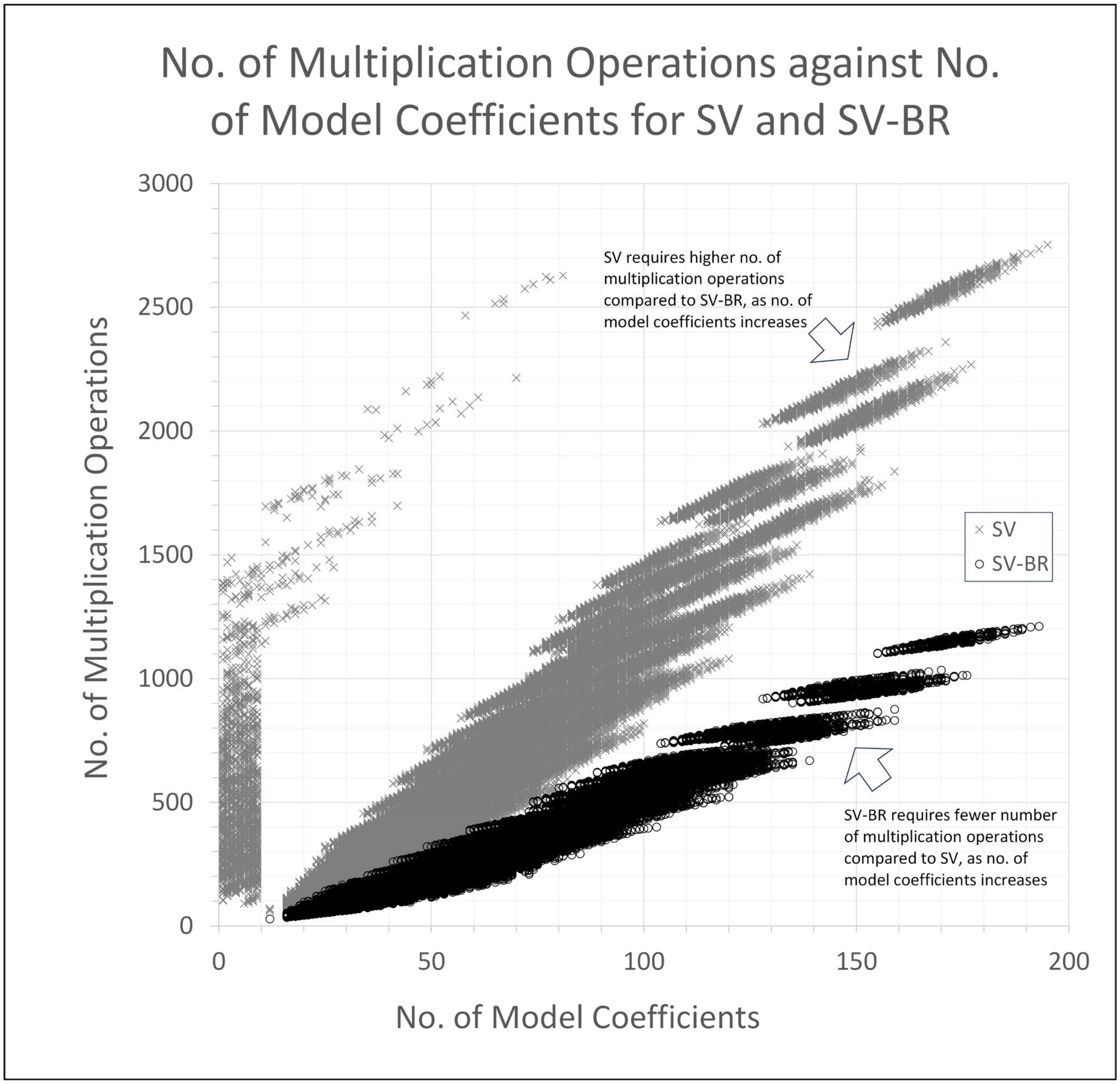

Fig. 5. No. of multiplication operations against no. of model coefficients for SV and SV-BR. SV-BR requires fewer no. of multiplication operations compared to SV, as no. of model coefficients increase

BR models are ran at a similar number model coefficient referred in [14] where SV is first coined.

V. Conclusion

SV from [14] is optimized and results in the SV with Binomial Reduction algorithm (SV-BR), capable of achieving 57% of reduction in model operations complexity, while retaining most of the linearization performance, evaluated through NMSE. Future SV-based algorithms could be treated using the binomial reduction method, expecting to be able to reduce model operations complexity, while retaining PA linearization performance.

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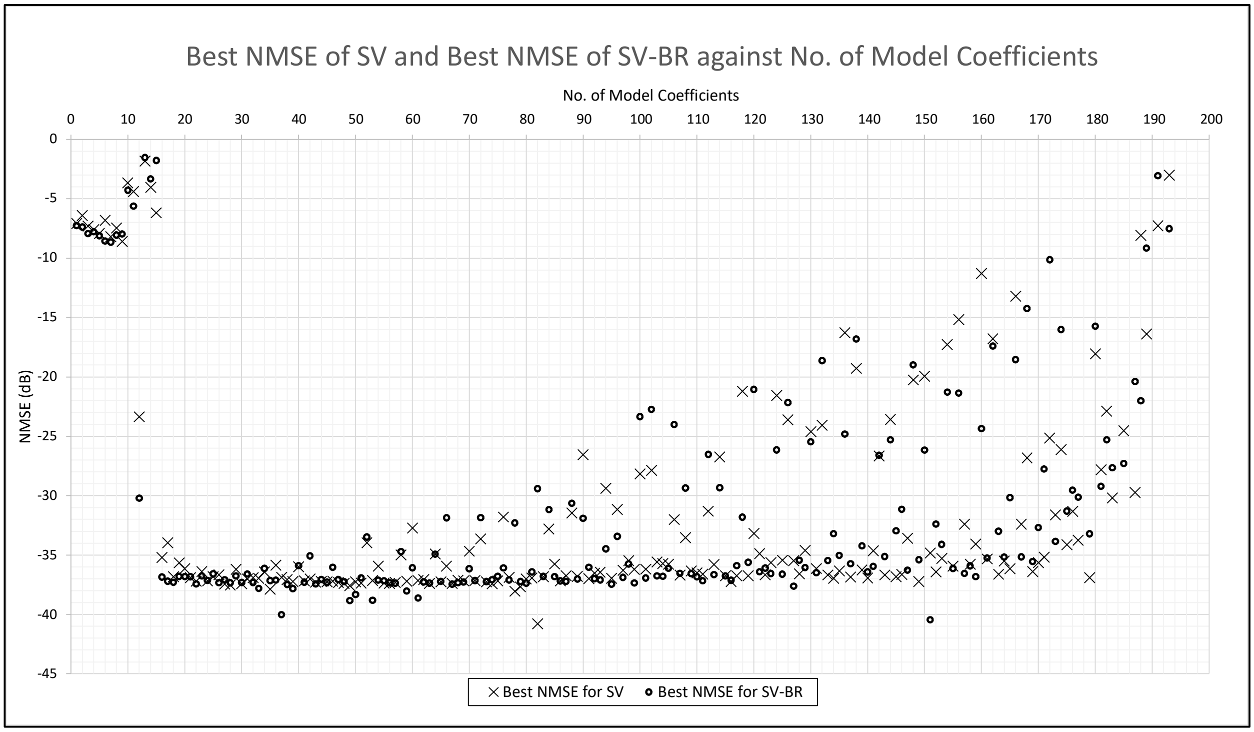

Fig. 6. Best NMSE performance for each SV and SV-BR with respect to the no. of model coefficients.

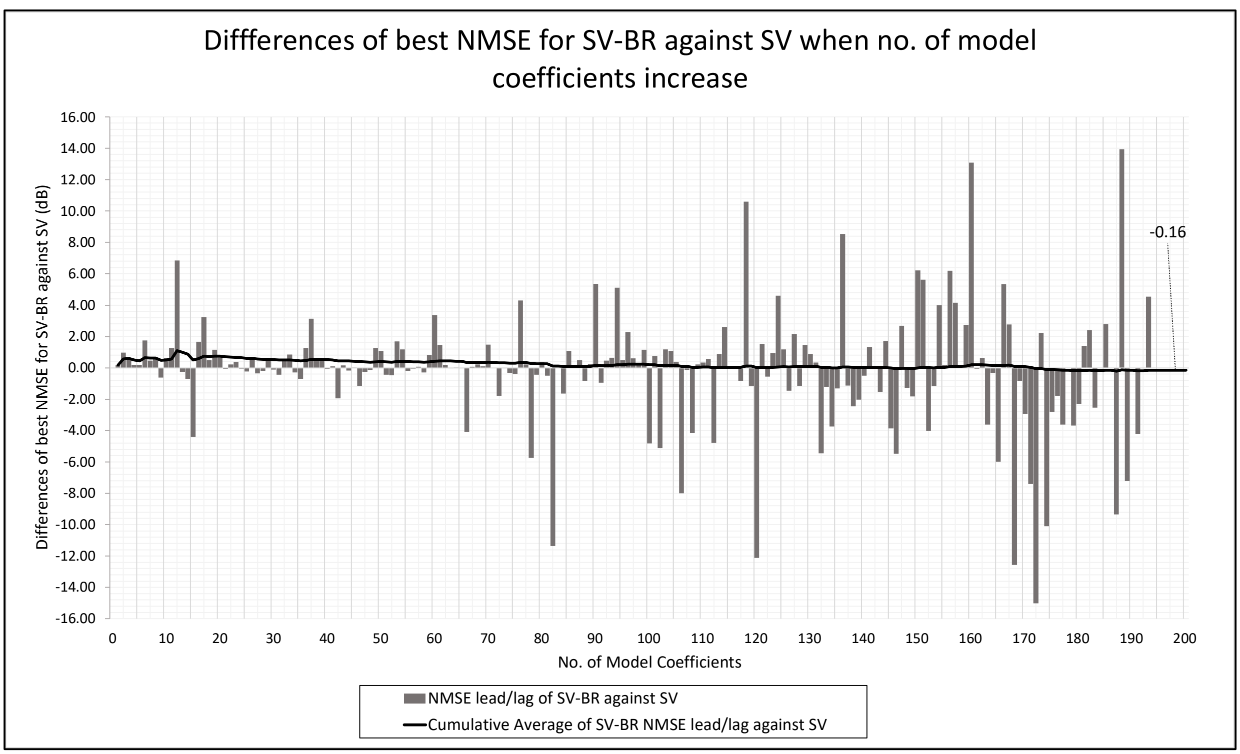

Fig. 7. Differences of best NMSE performance for SV-BR against SV when no. of model coefficients increase

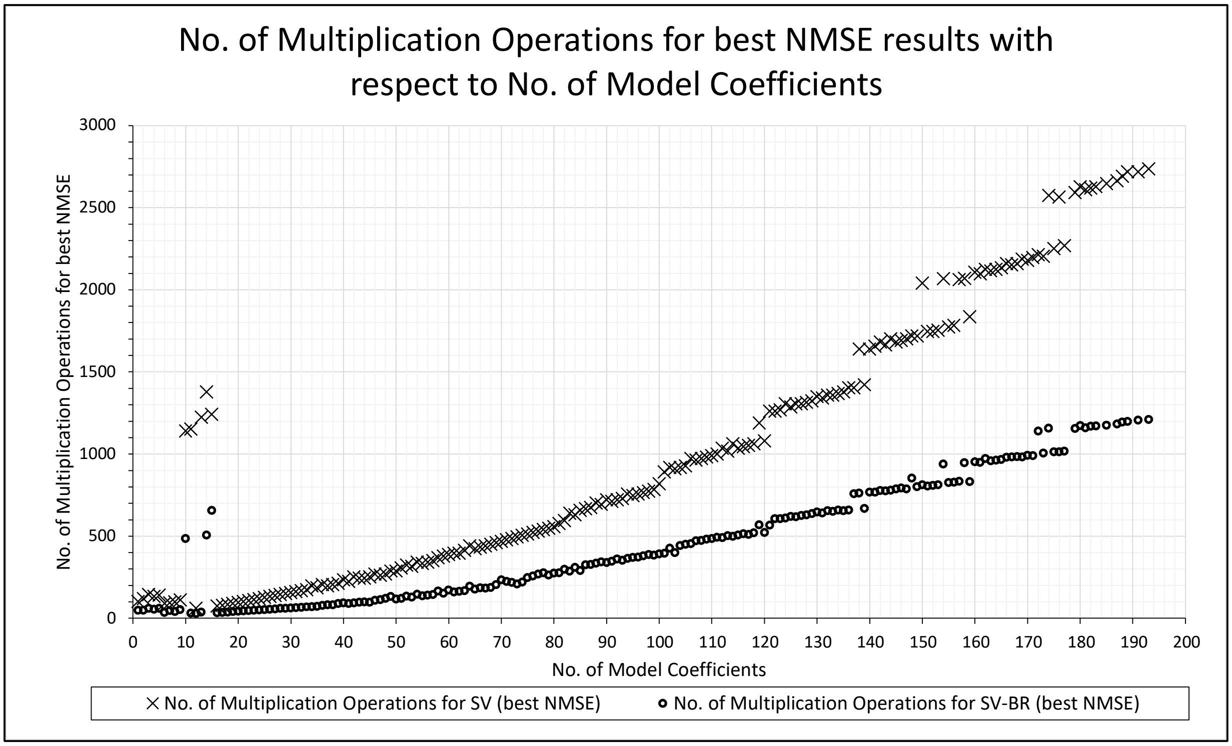

Fig. 8. No. of multiplication operations of SV-BR and SV with best NMSE results when no. of model coefficients increase

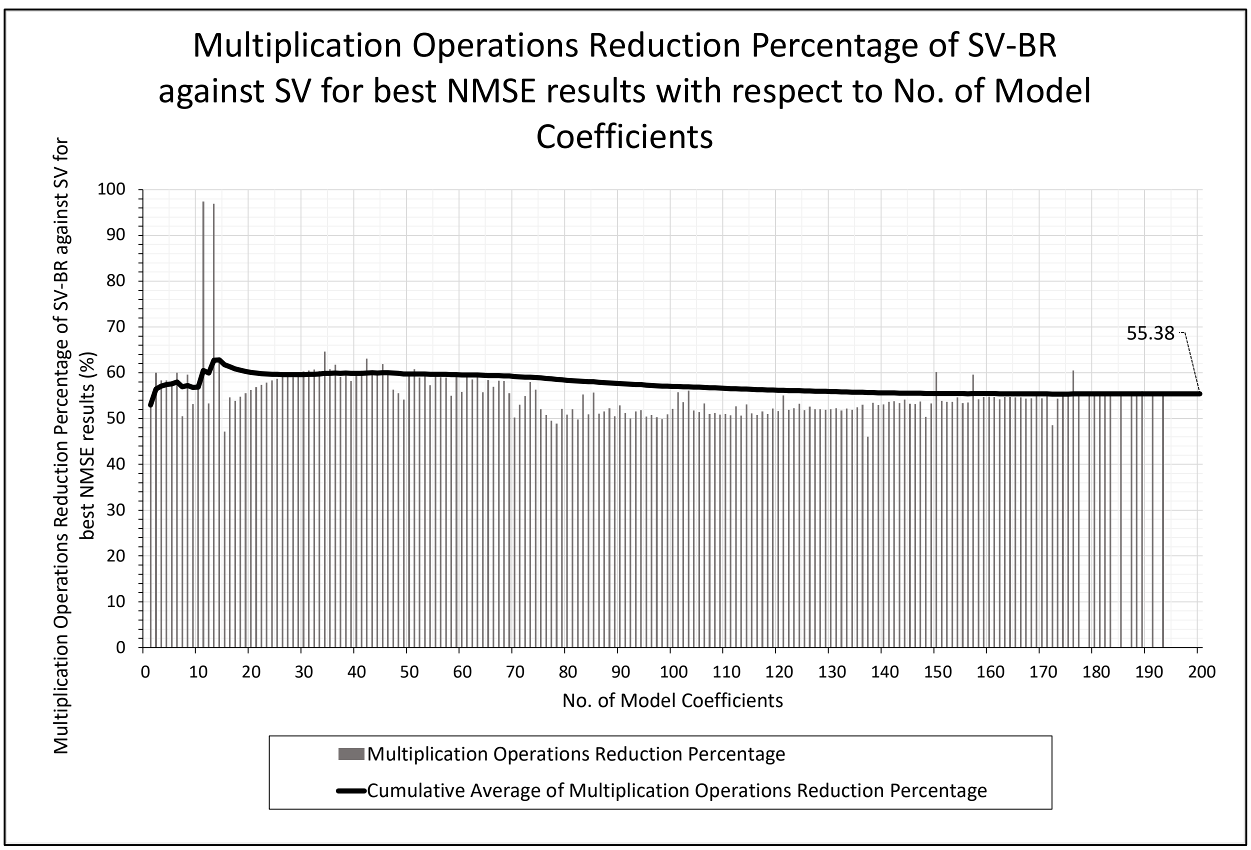

Fig. 9. Multiplication operations reduction percentage of SV-BR against SV for best NMSE results with respect to no. of model coefficients

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