Fuzzy Portfolio with Multi-objective Approach Using the Treynor Ratio

Padrul Jana, Dedi Rosadi*, and Epha Diana Supandi

Abstract—**Optimization theory in finance is developing rapidly, especially in forming portfolio optimization by including fuzzy elements. However, some fuzzy methods encountered problems in their application when obtaining optimal portfolio weights. Hence, a multi-objective approach was required to solve the fuzzy portfolio. This research focused on compiling a fuzzy portfolio using large-scale data without considering short-selling. Then, the data consisting of opening, closing, highest, and lowest prices were modeled into non-linear adaptive fuzzy numbers. The incoming fuzzy number was a trapezoidal fuzzy number. The data was obtained from 491 trading days of the ten most active stocks in LQ45, respectively. This research also employed the Treynor ratio (TR) to optimize the process. As a control for the Treynor ratio, the Sharpe ratio (SR), which had been employed, was also first introduced. Based on empirical data, there were differences in the composition of the weights resulting from the formed fuzzy portfolio. The research results significantly indicated that TR had a more even diversification level across all stocks when constructing a fuzzy portfolio with a multi-objective approach. It indeed reduced the systematic risk in the portfolio formed.**

Index Terms—**Expected Return, Fuzzy Portfolio, Multi-objective, Risk.**

I. Introduction

In the portfolio risk management process, the more stocks or other assets included in a portfolio are directly proportional to reduced risk [1]–[3]. A portfolio with minimum risk is efficient [1], [4]. Portfolio diversification (spread of assets in some types of investment) by an investor provides minimal risk because it can remove unsystematic risks; thus, systematic risks are only left, which are challenging to eliminate. In fact, eliminating unsystematic risks is difficult because more stocks are combined to obtain a minimum risk. Hence, a statistical method is required to calculate the minimum risk in stock diversification to obtain an efficient and effective combination. The ultimate goal of investors is to get a return according to investor expectations.

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The portfolio's risk measurement has been introduced by [5] and has been extensively studied in recent years. The methods used to calculate the Value at Risk (VaR), which are widely employed, consist of semi-absolute deviation [6], [7] covariance variance method [8], [9], mean-variance [10], mean semi-variance, and mean-variance skewness [11]–[13]. Some research revealed that VaR measurement was considered a random variable based on stochastic analysis of historical data and employed several methods in their estimation [8].

In fact, besides the expected data not always being available, several other factors, such as company stability, market demand and supply, geopolitical conditions, natural disasters, and disease outbreaks that impact the investment world, affect it. For instance, at the end of 2019, the world was hit by the COVID-19 pandemic in various countries. Indeed, this situation significantly affects the investment world. Research [14] stated that foreign direct investment (FDI) was affected by pandemic control in a country. In addition, the pandemic has significantly affected FDI in the service sector compared to FDI in other sectors. Moreover, COVID-19 also had an impact on company performance. The research results [15] indicated that COVID-19 negatively impacted company performance. This negative impact was more perceived when the company's investment scale or sales revenue was smaller. Therefore, due to various factors and aspects, multi-objective portfolios and fuzzy theories' constructions in the stochastic case were needed to determine VaR and future returns [16].

The multi-objective portfolio construction is an optimization method considering more than one objective function [17]. The large number of objective functions implies that no one solution can dominate. Thus, multi-objective optimization is the best solution offered. The objective function is the objective to be optimized by an investor. The portfolio's correlation is undoubtedly the problem of asset portfolio optimization with the mean-variance, which can be considered as a multi-objective optimization problem [18], [19]. The multi-objective optimization process is solved using scalarization to find optimal points for each vector optimization problem [20], [21].

Furthermore, the fuzzy portfolio selection model was previously developed by [22]–[25] in the fuzzy theory development. Risk measurement is generally classified into unsystematic and systematic. However, these two approaches cannot immediately eliminate VaR because, in principle, VaR can be minimized if investors direct them to portfolios with the lowest VaR. Therefore, a solution is offered by applying multiple risk measurements that simultaneously evaluate the portfolio, i.e., employing multi-objective fuzzy on selected portfolios [26].

Fuzzy methods were still complex to be applied in some cases because an efficient and robust approach was needed to obtain optimal weights from fuzzy models for accurate data [27], [28]. Findings of [10] [29] reported that pattern search (PS) and particle swarm optimization (PSO) algorithms outperformed other algorithms in terms of robustness and sparsity to build a portfolio with optimal minimum fuzzy variance (tangency). Indeed, it became a trigger for combining the preparation of a multi-objective portfolio with a fuzzy approach that maintained robustness. The robustness model could produce a portfolio that maintained minimum variance and achieved the lowest portfolio variance.

The multi-objective portfolio selection based on fuzzy VaR using the Sharpe Ratio has been developed by [24]. The Sharpe Ratio selection has a disadvantage, i.e., it is only appropriate to employ if an investor intends to place all (or almost all) of his or her wealth in one security [30]. Conversely, if an investor considers adding investments to a well-diversified portfolio, the Treynor ratio is considered more appropriate because it is only based on systematic risk. Implementing the asset return model as a non-linear adaptive fuzzy number includes trapezoidal fuzzy numbers [27]. Meanwhile, selecting a multi-objective fuzzy portfolio that maximizes portfolio returns and minimizes portfolio risk employs an interactive fuzzy approach [31]. However, this model has not been able to eliminate and remove systematic risks.

This research filled a gap in the literature by examining the development of the optimization model construction for selected portfolios, especially in a fuzzy approach. A complete description of the fuzzy methods development employed for data analysis and simulation was also provided. It helped determine themes and research direction topics that could be further developed. Assumedly by the author, Treynor's ratio studies to measure and improve the performance of multi-objective fuzzy portfolios in the Indonesian stock market had not been studied. The main contributions of this research consisted of (1) developing and applying the effective Treynor ratio index in the fuzzy portfolio construction; and (2) analyzing the characteristics of using the Treynor ratio index with the Sharpe ratio. The effective index of the critical Treynor ratio was employed to construct portfolio modeling that facilitated the division of assets into several securities. In addition, the Treynor ratio could solve the problem of systematic risk in the assets owned.

II. Fuzzy Portfolio

R is the set of real numbers. Non-linear adaptive fuzzy number $A(x), x \in R$ has a membership function [27], [32]–[35]:

$$A\begin{cases} f(x)\ , if\ \ x \in [p,q] \\ 1\ , if\ x \in [q,r] \\ g(x)\ , if\ \ x \in [r,s] \\ 0\ , if\ \ x \text{ other} \end{cases} \quad (1)$$

The characteristic of $f(x)$ has a real value function, increases, and continues to the right. Meanwhile, the $g(x)$ function has real value, decreases, and continues to the left. The p, q, r, s values are real numbers with the $p < q < r < s$ characteristics. The fuzzy number $A(x)$ which contains the $f(x)$ and $g(x)$ functions, is defined as:

$$f(x) = \left(\frac{x-p}{q-p}\right)^n \quad (2)$$

$$g(x) = \left(\frac{s-x}{s-r}\right)^n \quad (3)$$

In which $n > 0$, then denoted by $A = (p, q, r, s)_n$ as a class of adaptive non-linear fuzzy number. If $n = 1$, the special form is the generalized trapezoidal fuzzy number with $A = (p, q, r, s)$ as illustrated in Fig. .

α-cut operation on A given by:

$$[A]_\alpha = [A_1(\alpha), A_2(\alpha)] = \left[p + \alpha^{\frac{1}{n}}(q-p), s - \alpha^{\frac{1}{n}}(q-r)\right]$$

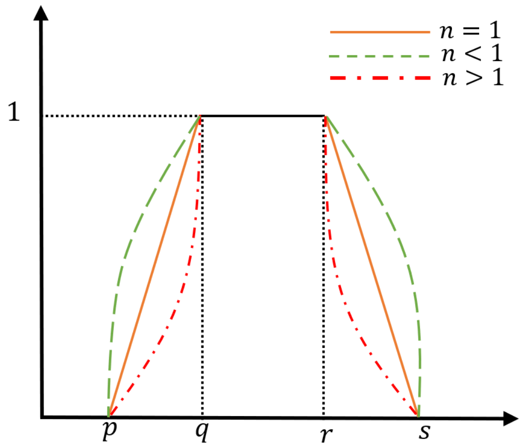

Fig. 1. Membership Functions of Adaptive Fuzzy Numbers

Notably, $n = 1$, n case, the expected value and variance of the trapezoidal fuzzy number $A = (p, q, r, s)$ are as follow:

$$E(A) = \frac{p + 2q + 2r + s}{6}, \quad (4)$$

and

$$Var(A) = \frac{(p + 2q - 2r - s)^2}{36} + \frac{(p - q + r - s)}{72} \quad (5)$$

The evidence can be seen in [27].

The arithmetic operation of trapezoidal fuzzy numbers is based on the interval of its α-cut, which is described as follows [27]:

Addition operation:

$$\begin{aligned} A_1 + A_2 &= (p_1, q_1, r_1, s_1) + (p_2, q_2, r_2, s_2) \\ &= (p_1 + p_2, q_1 + q_2, r_1 + r_2, s_1 + s_2). \end{aligned} \quad (6)$$

Subtraction operation:

$$\begin{aligned} A_1 - A_2 &= (p_1, q_1, r_1, s_1) - (p_2, q_2, r_2, s_2) \\ &= (p_1 - s_2, q_1 - r_2, r_1 - q_2, s_1 - p_2). \end{aligned} \quad (7)$$

Multiplication operation with a k-constant:

$$k \cdot A_1 = k \cdot (p, q, r, s) = (kp, kq, kr, ks). \quad (8)$$

Multiplication operation:

$$\begin{aligned} A_1 \cdot A_2 &= (p_1, q_1, r_1, s_1) \cdot (p_2, q_2, r_2, s_2) \\ &= (p_3, q_3, r_3, s_3). \end{aligned} \quad (9)$$

Where:

- $p_3 = \min(p_1p_2, p_1s_2, s_1p_2, s_1s_2)$,

- $q_3 = \min(q_1q_2, q_1r_2, r_1q_2, r_1r_2)$,
- $r_3 = \max(q_1q_2, q_1r_2, r_1q_2, r_1r_2)$,
- $s_3 = \max(p_1p_2, p_1s_2, s_1p_2, s_1s_2)$,

The arithmetic operation of the trapezoidal fuzzy numbers still maintains the non-linear adaptive fuzzy number class.

A. Fuzzy Numbers Construction on Stocks

Fuzzy numbers are compiled and constructed using the opening, closing, highest, and lowest prices on the assets traded daily. The formations are consisted of:
Opening Price $\{P_{it}^{open}, t = 1,2,3,\dots,T, i = 1,2,\dots,n.\}$,
Closing Price $\{P_{it}^{close}, t = 1,2,3,\dots,T, i = 1,2,\dots,n.\}$,
Highest Price $\{P_{it}^{high}, t = 1,2,3,\dots,T, i = 1,2,\dots,n.\}$,
Lowest Price $\{P_{it}^{low}, t = 1,2,3,\dots,T, i = 1,2,\dots,n.\}$.

The daily return of the $i-th$ asset and $t-th$ time is formed using P_{it}^{open}, P_{it}^{close}, P_{it}^{high}, P_{it}^{low}. Each asset $i, i = 1,\dots,n$ and time $t, t = 1,2,3,\dots,T$ maximum return, minimum return, and the average return on assets calculated by the formula:

$$r_{it}^{max} = \frac{P_{it}^{high} - P_{it}^{low}}{P_{it}^{low}}, \quad (10)$$

$$r_{it}^{min} = \frac{P_{it}^{low} - P_{it}^{high}}{P_{it}^{high}}, \quad (11)$$

$$r_{it}^{av1} = \frac{P_{it}^{low} - P_{it}^{open}}{P_{it}^{open}}, \quad (12)$$

$$r_{it}^{av2} = \frac{P_{it}^{close} - P_{it}^{low}}{P_{it}^{low}}, \quad (13)$$

The daily return of the $i-th$ asset and $t-th$ time is expressed as a trapezoidal fuzzy number as follows:

$$r_{it} = \left(r_{it}^{min}, r_{it}^{av1}, r_{it}^{av2}, r_{it}^{max}\right), \quad (14)$$

The suitable α-cut r_{it} can be described as follows:

$$[r_{it}]_\alpha = \left[r_{it}^{min} + \alpha\left(r_{it}^{av1} - r_{it}^{min}\right), r_{it}^{max} - \alpha\left(r_{it}^{max} - r_{it}^{av2}\right)\right] \quad (15)$$

The daily return dimension corresponds to the number of stocks observed and the observation time. The average fuzzy return from asset i is calculated as a trapezoidal fuzzy number as follows:

$$r_i = \frac{1}{T}\sum_{t=i}^{T} r_{it}. \quad (16)$$

Portfolio weight is expressed by the set $w_1 = (w_1, \dots, w_n)$, and portfolio return is presented with trapezoidal fuzzy numbers as follows:

$$r_P(w) = \sum_{t=i}^{n} w_i r_i \quad (17)$$

The fuzzy covariance of asset i and asset j is calculated as a trapezoidal fuzzy number as follows:

$$s_{ij} = \frac{1}{T}\sum_{t=i}^{T}(r_{it} - r_i)(r_{jt} - r_i), \quad (18)$$

and portfolio variance is expressed by trapezoidal fuzzy numbers as follows:

$$r_P(w) = \sum_{t=i}^{n} w_i r_i \quad (19)$$

The arithmetic operation of trapezoidal fuzzy numbers calculates $r_i, r_P(w), s_{ij}$ and $s_P(w)$ in equations 17 - 19. The expected portfolio return and portfolio variance are defined as mean $r_P(w)$ and mean $s_P(w)$, respectively, and are presented as follows:

$$\mu_P(w) = E[r_P(w)], \sigma_P^2(w) = E[s_P(w)]. \quad (20)$$

Portfolio construction with this minimum variance and tangency portfolio is without any short sales. Data simulation on a set of stocks is carried out without considering short sales. Short-selling often requires a substantial credit qualification. The minimum fuzzy variance portfolio is defined as follows:

$$\min\{\sigma_P^2(w) | l'w = 1, w \geq 0\}, \quad (21)$$

A fuzzy tangency portfolio with a Sharpe ratio is defined by:

maximizing $\{\frac{\mu_P(w)}{\sqrt{\sigma_P^2(w)}}\}$, namely,
with the constraint function:
$l'w = 1,$
$w \geq 0.$

A fuzzy tangency portfolio with a Treynor ratio is defined by:

maximizing $\{\frac{\mu_P(w) - R_f}{\beta_P}\}$, namely,
with the constraint function:
$l'w = 1,$
$w \geq 0.$

In this case, the problem-solving employs multi-objectives; *first*, solving Sharpe and Treynor ratios; *second*, determining the established portfolio characteristics from these two ratios. This fuzzy portfolio is compiled to minimize $\sigma_P^2(w)$ and maximize $\frac{\mu_P(w)}{\sqrt{\sigma_P^2(w)}}$ and $\frac{\mu_P(w) - R_f}{\beta_P}$. It is equivalent to minimizing the Fuzzy portfolio tangency of both portfolio ratios and portfolio risk $\sigma_P^2(w)$, namely:

$$\min\left(-\frac{\mu_P(w)}{\sqrt{\sigma_P^2(w)}}, \sigma_P^2(w)\right) \quad (22)$$

and

$$\min\left(-\frac{\mu_P(w) - R_f}{\beta_P}, \sigma_P^2(w)\right) \quad (23)$$

with the constraint function $l'w = 1$ and $w \geq 0$. Then, the optimum $w_1, w_2, w_3, \dots, w_n$ values can be obtained by employing the Lingo 20 software.

B. Algorithm

In this section, an algorithm for solving optimization problems in fuzzy portfolios is presented.

Algorithm Fuzzy Portfolio Optimization

Require: The data daily return of the $i-th$ asset and $t-th$ of n stocks: $r_{it} = (r_{it}^{min}, r_{it}^{av1}, r_{it}^{av2}, r_{it}^{max}), i = 1, \dots, n$ and $t = 1, 2, 3, \dots, T$.

1. Calculate fuzzy portfolio returns $r_P(w)$ with formula 17,
2. Calculate the fuzzy covariance of asset s_{ij} based on equation 18,
3. Calculate $s_P(w), \mu_P(w), \sigma_P^2(w)$ based on formula 19, 20.
4. **for** $j \leftarrow 1, \dots, n$ **do**
5. Determine the initial set of weights $w = (w_1, w_2, \dots, w_n)$ with $\sum_{i=i}^{n} w_1 = 1$
6. Determine the objective function of the fuzzy minimum variance portfolio and the fuzzy tangency portfolio referring to 21, 22, and 23.
7. Run "solve" based on the constraint function used to get the "solutionreport" to find the best weights.
8. end for
9. return Get the optimal weight $w = (w_1, w_2, \dots, w_n)$ of each portfolio.

III. Results

A. Data

The research data consisted of the opening (P^{open}), closing (P^{close}), highest (P^{high}), and lowest (P^{low}) stock prices. The data was taken from the ten most active stocks on IDX LQ45 in 461 trading days. LQ45 represented the stock prices of 45 issuers on the Indonesia Stock Exchange (IDX), which were selected based on the highest liquidity and largest market capitalization considerations with other predetermined criteria. Stock selection was also based on the diversity of types of companies, including Construction Services, Banking, Telecommunications, Health, Tobacco Manufacturers, and Food Production. The data was downloaded via Yahoo.com, then employed to calculate fuzzy numbers. The data used was $T = 461$ trading days and $n = 10$ stocks.

Based on the data of the opening price (P^{open}), closing price (P^{close}), highest price (P^{high}), and lowest price (P^{low}) for each observed stock, an adaptive non-linear fuzzy class was formed $r_{it} = (r_{it}^{min}, r_{it}^{av1}, r_{it}^{av2}, r_{it}^{max})$. Each $i-th$ stock asset and $t-th$ time with $i = 1, \dots, n, t = 1, \dots, T$ the value of the maximum return, minimum return, and average return on assets calculated using formula 10-13.

B. Fuzzy Average Return of the Asset

The r_{it}^{max} value is defined as the difference between the highest and the lowest prices with the lowest price on the $i-th$ stock. Conversely, r_{it}^{min} is interpreted as a comparison between the lowest and the highest prices with the highest price on the $i-th$ stock. Meanwhile, r_{it}^{av1} compares the difference between the lowest price and the price-to-book with the price-to-book of the $i-th$ stock. Then, r_{it}^{av2} is formulated as a comparison between the closing and the lowest prices with the lowest price on the $i-th$ stock. The value components from the observed stocks are presented in Fig. 2.

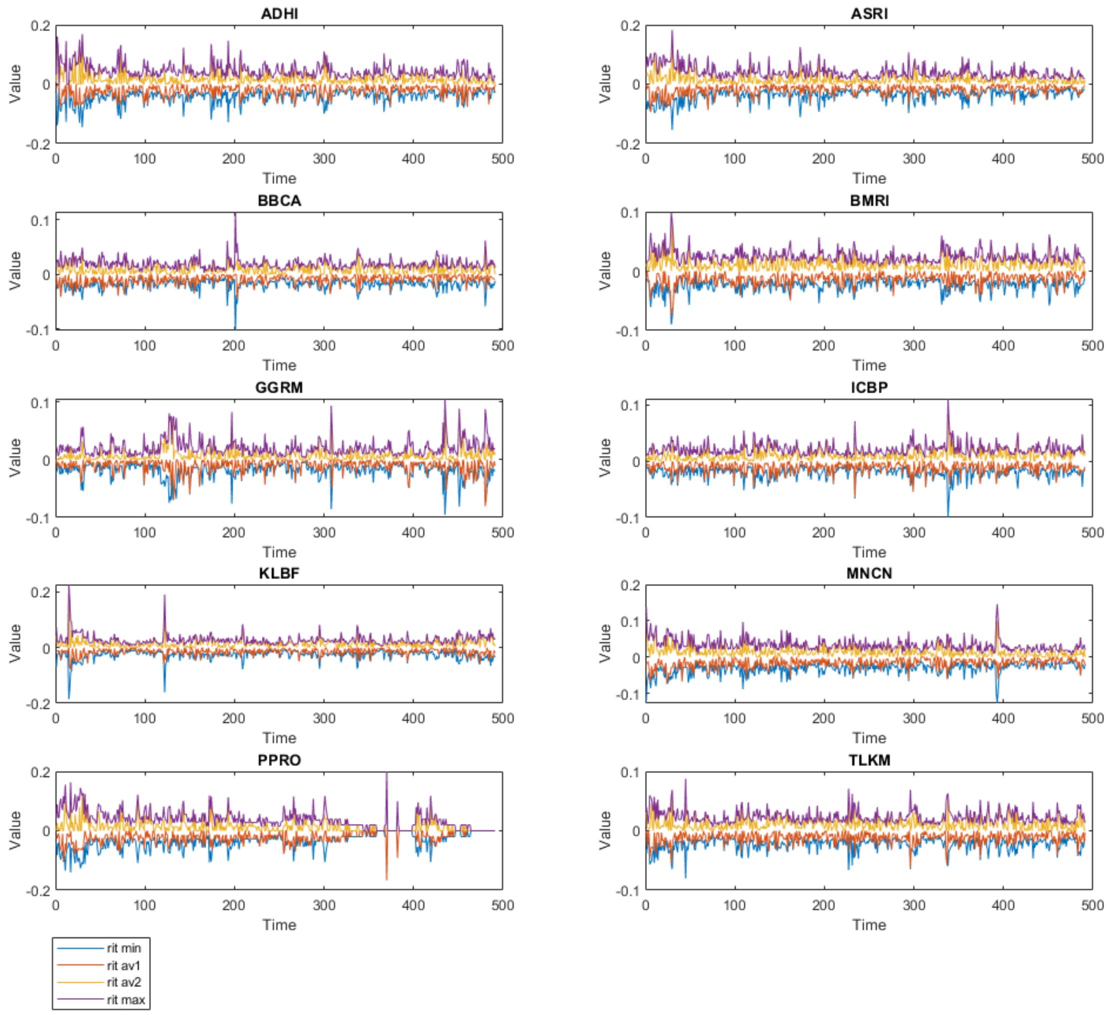

Fig. 2. The r_{it}^{max}, r_{it}^{min}, r_{it}^{av1}, r_{it}^{av2} values of the observed stock samples

Fig. 2 illustrates the r_{it} value display of stocks representing the construction services sector (ADHI.JK), property (ASRI.JK and PPRO.JK), banking (BBCA.JK and BMRI.JK), food and tobacco manufacture (GGRM.JK and ICBP.JK), health (KLBF.JK), and telecommunications (MNCN.JK and TLKM.JK). It indicates that stocks in the construction, property, and telecommunications services sectors relatively tend to have the same r_{it} distribution. Meanwhile, the banking and health sectors tend to have a narrower data of r_{it} on the distribution of fuzzy numbers. This condition is interpreted as the range of fuzzy number data between r_{it}^{min} to r_{it}^{max} has a competitive distance.

The average fuzzy return of the $i-th$ asset (ri) is calculated as a trapezoidal fuzzy number. The formula for calculating the average fuzzy return from the $i-th$ asset employs the formula 16. The calculation process refers to trapezoidal fuzzy numbers 6 and 8 arithmetic operations. The r_i value for each observed stock is in the form of an adaptive non-linear fuzzy number class written in Table I.

TABLE I
CLASS COMPOSITION OF ADAPTIVE NON-LINEAR FUZZY NUMBERS IN THE OBSERVED STOCKS

	Composition r_i			
Stocks	p	q	r	s
ADHI.JK	-0.0424	-0.0223	0.0196	0.0449
ASRI.JK	-0.0382	-0.0192	0.0165	0.0402
BBCA.JK	-0.0188	-0.0096	0.0095	0.0192
BMRI.JK	-0.0242	-0.0134	0.0122	0.0249
GGRM.JK	-0.0203	-0.0106	0.0084	0.0209
ICBP.JK	-0.0207	-0.0114	0.0112	0.0213
KLBF.JK	-0.0272	-0.0148	0.0146	0.0283
MNCM.JK	-0.0320	-0.0172	0.0145	0.0333
PPRO.JK	-0.0362	-0.0207	0.0136	0.0384
TLKM.JK	-0.0229	-0.0118	0.0118	0.0235

Each stock's r_i value of p, q, r, s illustrates $p < q < r < s$. It fulfills the characteristic of adaptive non-linear fuzzy numbers. The r_i value is the average fuzzy return from asset i. A clear illustration indicating r_i as a trapezoidal fuzzy number of the observed stocks is presented in Fig. 3.

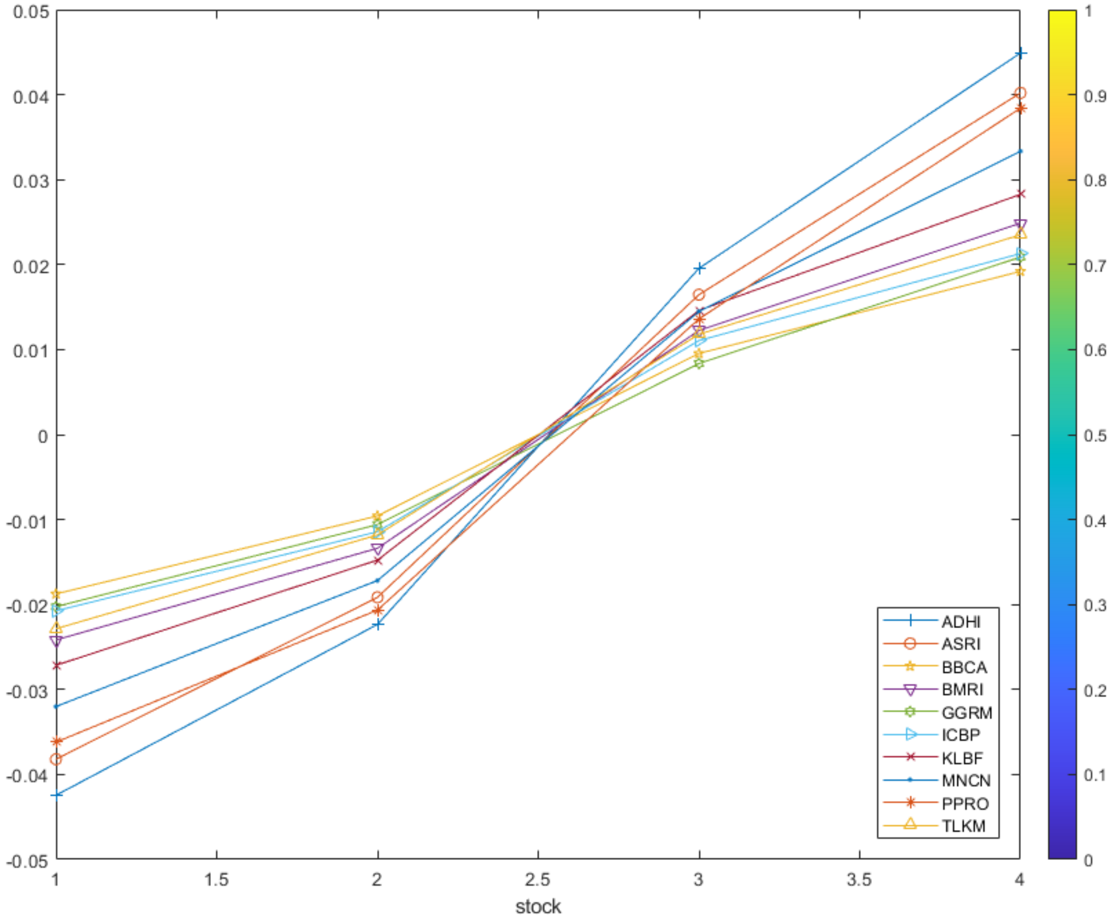

Fig. 3. The average return value of the observed stocks

The r_i value is unique if the lowest p-value results in the highest s-value among other stocks and vice versa. Fig. 3 illustrates each observed stock's p, q, r and s values.

A. Fuzzy Portfolio Return

Portfolios with weights $w = (w_1, \dots, w_n)$, have portfolio returns presented using trapezoidal fuzzy numbers based on formula 17 by considering the nature of the arithmetic operations of trapezoidal fuzzy numbers at 6 and 8.

Based on formula 17, it obtains:

$$\begin{aligned} r_P(w) &= \sum_{t=i}^{n} w_i r_i \\ &= w_1(-0.0424, -0.0223, 0.0196, 0.0449) \\ &+ w_2(-0.0382, -0.0192, 0.0165, 0.0402) \\ &+ w_3(-0.0188, -0.0096, 0.0095, 0.0192) \\ &+ w_4(-0.024, -0.0134, 0.0122, 0.0249) \\ &+ w_5(-0.0203, -0.0106, 0.0084, 0.0209) \\ &+ w_6(-0.0207, -0.0114, 0.0111, 0.0213) \\ &+ w_7(-0.0272, -0.0148, 0.0146, 0.0283) \end{aligned}$$

$$+ \quad w_8(-0.0320, -0.0172, 0.0145, 0.0333)$$
$$+ \quad w_9(-0.0362, -0.0206, 0.0136, 0.0384)$$
$$+ \quad w_{10}(-0.0228, -0.0118, 0.0118, 0.0235).$$

The expected value of $r_P(w)$ is maximized as one of the objective functions in w. The composition of the function $r_P(w)$ is still in the form of a trapezoidal fuzzy number.

B. The Fuzzy Covariance of Asset

The fuzzy covariances of assets i and j are trapezoidal fuzzy numbers. The arrangement refers to equation 18 and considers the nature of the arithmetic operations of trapezoidal fuzzy numbers in 6-9. Based on equation 18, it obtains:

$$s_{ij} = \frac{1}{T}\sum_{t=i}^{T}(r_{it} - r_i)(r_{jt} - r_i)$$
$$s_{11} = (-0.0082, -0.0010, 0.0020, 0.0048)$$
$$s_{12} = s_{21} = (-0.0072, -0.0011, 0.0014, 0.0042)$$
$$s_{13} = s_{31} = (-0.0034, -0.0006, 0.0006, 0.0018)$$
$$s_{14} = s_{41} = (-0.0044, -0.0008, 0.0009, 0.0024)$$
$$\vdots$$
$$s_{1010} = (-0.0023, -0.0003, 0.0005, 0.0012).$$

C. Fuzzy Portfolio Variance

Portfolio variances are represented by trapezoidal fuzzy numbers following 19 by considering the nature of the arithmetic operations of trapezoidal fuzzy numbers on 6, 8, and 9. Based on formula 19, the portfolio variance is obtained as follows:

$$S_P(w) = \sum_{t=i}^{n} w_i w_j s_{ij}$$
$$= w_1 w_1 s_{11} + w_1 w_2 s_{12} + w_1 w_3 s_{13} + w_1 w_4 s_{14}$$
$$+ \ldots + w_{10} w_{10} s_{1010}$$
$$= w_1^2(-0.0082, -0.0010, 0.0020, 0.0048)$$
$$+ \ w_1 w_2(-0.0072, -0.0011, 0.0014, 0.0042)$$
$$+ \ w_1 w_3(-0.0034, -0.0006, 0.0006, 0.0018)$$
$$+ \ w_1 w_4(-0.0044, -0.0008, 0.0009, 0.0024)$$
$$\vdots$$
$$+ \ w_{10}^2(-0.0023, -0.0003, 0.0005, 0.0012)$$

Portfolio return expectations and daily portfolio variance are respectively defined according to equation 20, namely:

$$\mu_P(w) = E[r_P(w)]$$
$$= E\left[\sum_{t=i}^{n} w_i r_i\right]$$
$$= E[w_1(-0.0424, -0.0223, 0.0196, 0.0449)$$
$$+ \ w_2(-0.0382, -0.0192, 0.0165, 0.0402)$$
$$+ \ w_3(-0.0188, -0.0096, 0.0095, 0.0192)$$
$$+ \ w_4(-0.0242, -0.0134, 0.0122, 0.0249)$$
$$\vdots$$
$$+ \ w_{10}(-0.0228, -0.0118, 0.0118, 0.0235)].$$

Furthermore, the $\mu_P(w)$ value is solved using formula 4. On the other side, the daily portfolio variance is described on 20 as follows:

$$\sigma_P^2(w) = E[s_P(w)]$$
$$= E\left[\sum_{i,j=i}^{n} w_i w_j s_{ij}\right]$$
$$= E[w_1 w_1 s_{11} + w_1 w_2 s_{12} + w_1 w_3 s_{13}$$
$$+ \ w_1 w_4 s_{14} + \ldots + w_{10} w_{10} s_{1010}]$$
$$= E[w_1^2(-0.0082, -0.0010, 0.0020, 0.0048)$$
$$+ w_1 w_2(-0.0072, -0.0011, 0.0014, 0.0042)$$
$$+ w_1 w_3(-0.0034, -0.0006, 0.0006, 0.0018)$$
$$+ w_1 w_4(-0.0044, -0.0008, 0.0009, 0.0024)$$
$$\vdots$$
$$+ w_{10}^2(-0.0023, -0.0003, 0.0005, 0.0012)].$$

The process of obtaining $\sigma_P^2(w)$ can be solved with equations 4 and 5.

D. Optimization of Fuzzy Portfolio

1) *Beta Portfolio:* In this case, the risk-free interest rate employed is 7.5%. Then, the β-value of each stock is used to determine the Treynor ratio as follows:

TABLE II
BETA VALUE OF STOCK

Stock	Beta
ADHI.JK	1.1125
ASRI.JK	1.0555
BBCA.JK	0.9211
BMRI.JK	1.1753
GGRM.JK	0.3768
ICBP.JK	0.4654
KLBF.JK	0.6258
MNCM.JK	0.7427
PPRO.JK	0.5235
TLKM.JK	0.8895

The calculation outcomes can be seen in Table II. It reveals that the beta value of ADHI.JK, ASRI.JK, BMRI.JK stocks have a beta value of > 1, denoting a volatility price above the Capital Asset Pricing Model (CAPM). Meanwhile, the beta value of other stocks is < 1. It indicates that the change in stock returns is smaller than in the market. Stocks have less volatile returns. Portfolio beta is calculated using β-stocks presented in Table II. The beta portfolio is obtained from the sum of each beta multiplied by the proportion of the share weight.

2) *Portfolio Weight:* An essential part of the research results was the determination of the weighting of each observed stock. Stock weighting was performed based on two ratios, i.e., the Sharpe and Treynor ratios. The optimum weight with the multi-objective method can be obtained using equations 22 and 23. Table III displays the constructed portfolio's weight, expected return, and risk.

TABLE III
WEIGHT, EXPECTED RETURN, AND RISK PORTFOLIO

Stock	Weight with SR	Weight with TR
ADHI.JK	0.0400	0.1200
ASRI.JKT	0.0000	0.0800
BBCA.JK	0.0400	0.1400
BMRI.JK	0.3200	0.1800
GGRM.JK	0.0200	0.1000
ICBP.JK	0.4200	0.1900

Stock	Weight with SR	Weight with TR
KLBF.JK	0.1600	0.1100
MNCM.JK	0.0000	0.0400
PPRO.JK	0.0000	0.0200
TLKM.JK	0.0000	0.0200
Expected Return	0.0723	0.0681
Risk	0.0234	0.0172

Based on Table III, the fuzzy portfolio construction involved the Treynor ratio, which tended to be more diversified. It was considered from the distribution of weights on each observed stock. In comparison, the fuzzy portfolio employed the Sharpe ratio. Several stocks did not get an allocation in the portfolio construction process. Minimally, four out of ten stocks weighed 0.0000. This condition is interpreted by the absence of investment funds for these shares. The SR tended to allocate investment funds in a cluster of specific stocks. This information is presented graphically in Fig. 4.

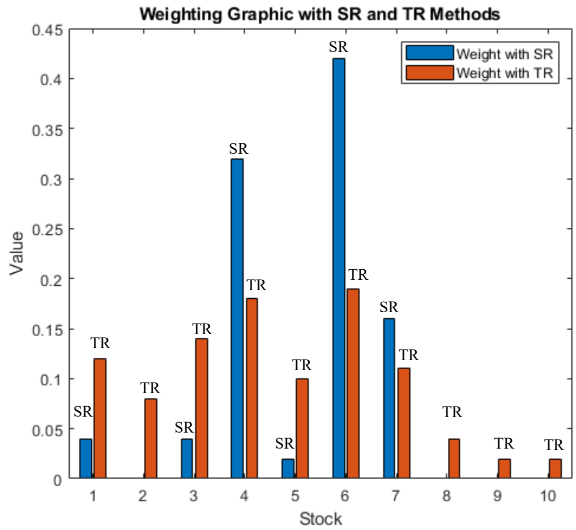

Fig. 4. Weighting Graphic with SR and TR Methods

There are also differences in the involvement of SR and TR in calculating expected return and risk. The application of SR tends to have a higher expected return than TR. Based on the risk outcomes, it indicates that the involvement of SR is higher than TR. This condition is under the basic principle of high risk and high return. Find it more clearly in Fig. 5.

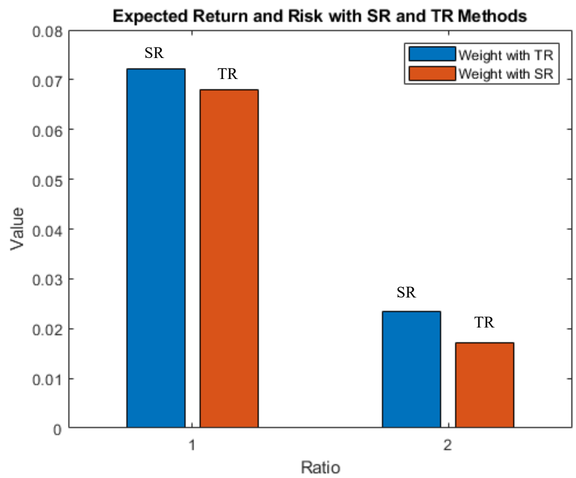

Fig. 5. Comparison of Expected Return and Risk with SR and TR Methods

Furthermore, when examined in more detail, the difference in expected return between SR and TR is at an interval of 0.42%. Meanwhile, the risk difference between SR and TR is at a 0.62% interval. These results expand investors' preferences in spreading investment to several options. In fact, the TR method is more adaptable in the distribution of investment weights.

IV. Conclusion

In short, SR and TR ratios applied in preparing a multi-objective fuzzy portfolio gave the weighted observed stocks color. These results were an extension of the research results of [24], [27]. The SR and TR ratios included in the objective function provided different results in the weight portion. The results indicated that TR provided better diversification. It could be seen from the weights distributed throughout the observed stocks. These results confirmed [36]–[38], but previous research has not included fuzzy concepts. This situation certainly could provide a more flexible choice for an investor in determining investment steps. An open problem that might be worked on as a follow-up to this research was to apply Annealing simulation algorithms and genetic algorithms in the optimization process and expand the proposed method to accommodate short selling.

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