The Best-Worst Method Based on Single-valued Neutrosophic Numbers and Its Application

Dongsheng Xu, Xue Kang, Liang Zhou, Xinghai Zhang

***Abstract*—In multi-criteria decision-making, the Best-worst method (BWM) enables decision-makers to determine the best and worst criteria, compare the relationship between the best criterion with other criteria and the worst criterion with other criteria, and then obtain the corresponding weight of each criterion based on the two sets of evaluation results obtained in the previous step by constructing a model. However, due to the diversity of decision-making criteria, it is difficult for decision-makers to assign a specific weight to multiple criteria, and the BWM is commonly employed to weight multi-criteria decision-making problems. In previous research, the BWM was extended to a fuzzy environment. In this study, the BWM is extended to neutrosophic sets. By describing language phrases and converting them into intermediate wisdom, decision-makers can determine the reference comparison of the best criterion to other criteria and the worst criterion to other criteria. Next, all the intelligent numbers are converted into real numbers using the calculation formula of the scoring function. Subsequently, based on the BWM, a nonlinear constraint optimization problem is formulated to determine the criteria weights and alternatives relative to different criteria. In this approach, different criterion weights can be obtained directly from the proposed BWM without further transformation. Meanwhile, the agreement ratio of the BWM is proposed to verify the reliability of the preference comparison. Finally, a case study is conducted to fully demonstrate the effectiveness and feasibility of the proposed neutrosophic BWM. The results indicate that the proposed neutrosophic BWM can obtain a reasonable preference ranking of alternatives and has a higher comparative consistency than the BWM.**

***Index Terms*—BWM, single-valued Neutrosophic sets, consistency ratio, multicriteria decision making.**

I. Introduction

THE multi-criteria decision-making problem [1], [2] involves a large number of interrelated and mutually influential criteria. To obtain optimal decision-making, the weights and priorities of multiple criteria need to be considered comprehensively. Such problems are widely present in real life, such as in enterprise management, investment decision-making, and risk management. Meanwhile, with increasing complexity and uncertainty, it is necessary to find a more scientific and reasonable method the multi-criteria decision-making problems. This method needs to consider the multiple criteria in the problem in all aspects, determine the weight of each criterion, provide a valuable reference for decision-makers, and enhance the efficiency and accuracy of decision-making. For a long time, the multi-criteria decision-making problem has become a research hotspot due to its difficulty. Existing studies have made much progress, and many methods have been proposed for solving multi-criteria decision-making problems, such as the analytic hierarchy process(AHP) [3], grey correlation analysis [4], principal component analysis [5], and fuzzy comprehensive evaluation methods [6]. These methods have achieved good results in solving multi-criteria decision-making problems in various fields.

The Best-Worst method(BWM) [7] is crucial in solving multi-criteria decision-making problems. It was proposed by Bryant, an American scholar, in 1952 and has been widely used in various fields [8], [9], [10], [11], such as economic management, engineering design, and health care. In economic management, the BWM is utilized to evaluate the risks and benefits of investment projects and formulate strategic plans. In engineering design, the BWM is employed to select the best design scheme and evaluate the performance and quality of the project. In health care, the BWM is adopted to evaluate the effects and risks of different treatment options and to develop patient treatment options.

In existing research, the BWM is used to solve multi-criteria decision-making problems alone and combined with other methods. For instance, the BWM is combined with the fuzzy comprehensive evaluation method [12] to expand its application scope, combined with the grey correlation analysis method [13] to comprehensively consider the correlation degree of each alternative in each criterion, and combined with the AHP [14] to determine the relative importance of each alternative in each criterion. Additionally, given the uncertainty and ambiguity of practical problems, the BWM is gradually applied to the fuzzy environment [15], [16], [17], [18], [19], [20] and is used to solve multi-objective decision-making problems in fuzzy environments to help decision-makers comprehensively consider the trade-off between different objectives, so that they can choose a relatively optimal solution. In the fuzzy environment, considering the complexity of the problem, this study chooses the single-valued neutrosophic number to extend the BWM.

The rest of this paper is organized as follows. Section 2 reviews single-valued neutrosophic numbers and their scoring functions. Section 3 provides brief reviews and the specific steps of the classical BWM. In Section 4, the BWM based on single-valued neutrosophic numbers is proposed, and it is applied to practical examples and compared with previous methods. Finally, Section 5 concludes this paper.

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II. PRELIMINARY

definition 1: [17] In the universe Ω, single-valued neutrosophic sets(SVNS) A is expressed as

$$A = \{(x, t_A(x), i_A(x), f_A(x)) : x \in U\} \tag{1}$$

and, there is

$$0 \le t_A(x) + i_A(x) + f_A(x) \le 3, t_A(x), i_A(x), f_A(x) \in [0, 1]$$

In definition 1, $t_A(x)$ represents the degree of truth or membership; $i_A(x)$ denotes indeterminacy or neutrality; $f_A(x)$ denotes the degree of falsity or non-membership. It is called the neutrosophic component of x and is usually abbreviated as $A < t_A, i_A, f_A >$.

definition 2: [17] If $A =< t_A(x), i_A(x), f_A(x) >$ is a single-valued neutrosophic number, then its score function $s(B)$ can be expressed as follows:

$$s(A) = \frac{t_A(x) + 2 - i_A(x) - f_A(x)}{3}, s(A) \in [0, 1] \tag{2}$$

definition 3: [17] If A and B are two single-valued neutrosophic numbers, then the comparison rules for them are as follows:

(1) If the score function of A is greater than the score function of B, i.e., $s(A) > s(B)$, then $A > B$.

(2) If the score function of A is equal to the score function of B, i.e., s(A) = s(B), then $A = B$.

(3) If the score function of A is less than the score function of B, i.e., $s(A) < s(B)$, then $A < B$.

definition 4: [18] If i is the best element and j is the worst element, then a pairwise comparison $\tilde{a}_{ij}$ is a reference comparison.

definition 5: [18] To obtain a sufficiently consistent pairwise comparison, the equation $\tilde{a}_{Bj} \times \tilde{a}_{jW} = \tilde{a}_{BW}$ needs to hold, where $\tilde{a}_{BW}$ represents the preference of the best criterion relative to the worst criterion, $\tilde{a}_{Bj}$ represents the preference of the best criterion relative to other criteria, and $\tilde{a}_{jW}$ represents the preference of other criteria relative to the worst criterion.

III. A BRIEF REVIEW OF THE BWM

A. The specific steps of the BWM

The BWM was recently proposed in 2015, and it is commonly used in multi-criteria decision problems and provides an effective method for weight calculation. Here, the weighting steps of the standard BWM are briefly reviewed.

The specific steps [7] are as follows:

Step 1: the decision-maker first determines a decision criterion and its collective as $C = \{c_1, c_2, \cdots, c_n\}$.

Step 2: the decision-maker determines the best criterion C_B and the worst criterion C_W.

Step 3: the decision-maker uses the numbers 1-9 to denote the preference of the best criterion relative to other criteria to obtain the best other vectors $A_B = [a_{B1}, a_{B2}, \cdots, a_{Bn}]$. It is noteworthy that $a_{Bj}(j = 1, 2, \cdots, n)$ represents the preference of the best criteria a_B relative to other criterion $a_j(j = 1, 2, \cdots, n)$, and $a_{BB} = 1$.

Step 4: as in the third step, the decision-maker needs to use 1-9 to denote the preference of other criteria relative to the worst criterion, to obtain the other-worst vectors $A_W = [a_{1W}, a_{2W}, \cdots, a_{nW}]$, where $a_{jW}(j = 1, 2, \cdots, n)$ represents the preference of the other vectors $a_j(j = 1, 2, \cdots, n)$ relative to the worst vector a_W, and $a_{WW} = 1$.

Step 5: the optimal weight is obtained as $w^* = [w_1^*, w_2^*, \cdots, w_n^*]$, where w_j^* represents the optimal weight of the criterion c_j, and $j = 1$, so the following model can be established:

$$\begin{gathered} \min\max_j\{\left|\frac{w_B}{w_j} - a_{Bj}\right|, \left|\frac{w_j}{w_w} - a_{jw}\right|\} \\ s.t. \begin{cases} \sum\limits_{j=1}^{n} w_j = 1 \\ w_j \ge 0 (j = 1, 2, \cdots n) \end{cases} \end{gathered} \tag{3}$$

By transforming the mathematical model established above, the following new model is obtained:

$$\begin{gathered} \min \xi \\ s.t. \begin{cases} \left|\frac{w_B}{w_j} - a_{Bj}\right| \le \xi (j = 1, 2, \cdots n) \\ \left|\frac{w_j}{w_w} - a_{jw}\right| \le \xi (j = 1, 2, \cdots n) \\ \sum\limits_{j=1}^{n} w_j = 1, w_j \ge 0 (j = 1, 2, \cdots n) \end{cases} \end{gathered} \tag{4}$$

By using the lingo software to solve the above model, i.e., Eq.(4), the optimal weights $w_1^*, w_2^*, \cdots, w_n^*$ of criteria $c_1, c_2, \cdots, c_n$ can be obtained respectively. Meanwhile, the optimal values ξ can be obtained.

B. Consistency ratio of the BWM

When using the BWM to solve the optimal weights, the meaning of vector groups $A_B = [a_{B1}, a_{B2}, \cdots, a_{Bn}]$ and $A_W = [a_{1W}, a_{2W}, \cdots, a_{nW}]$ is already known in the second and third steps of the method. A comparison is entirely consistent when $a_{Bj} \times a_{jW} = a_{BW}(j = 1, 2, \cdots, n)$, where a_{BW} represents the preference of the best criterion over the worst criterion [18]. However, this equation does not hold for certain j, namely

$$a_{Bj} \times a_{jW} \ne a_{BW}(j = 1, 2, \cdots, n) \tag{5}$$

By Eq.(5), it is clear that if a_{Bj} and a_{jw} take the maximum value at the same time, then there will be the highest inequality. Therefore, in this case, a value of ξ is taken. It is subtracted from a_{Bj} and a_{jW} on the left side of the equation and added to a_{BW} on the right side of the equation to achieve the highest inequality, namely:

$$(a_{Bj} - \xi) \times (a_{jW} - \xi) = a_{BW} + \xi \tag{6}$$

As for the minimum consistency $a_{Bj} = a_{jW} = a_{BW}$, the following equation is obtained:

$$(a_{BW} - \xi) \times (a_{BW} - \xi) = (a_{BW} + \xi) \tag{7}$$

Simplifying the above equation yields:

$$\xi^2 - (1 + 2a_{BW})\xi + (a_{BW}^2 - a_{BW}) = 0 \tag{8}$$

By substituting a_{BW} with numbers 1 to 9 into the above equation, the maximum value of ξ can be obtained, which is called the consistency index(CI), and their specific values are listed in Table 1 [7]. The consistency ratio(CR) is used by the obtained consistency index in the following formula:

$$CR = \frac{\xi^*}{\xi} \tag{9}$$

Then, ξ^* in Eq.(5) can be obtained by the formula.

TABLE I
CONSISTENCY INDEX (CI)OF BWM

a_{BW}	1	2	3	4	5	6	7	8	9
CI (max ξ)	3	4.56	6	7.37	8.7	10	11.27	12.53	14.28

TABLE II
THE TRANSFORMATION RULES OF THE LINGUISTIC VARIABLES OF THE DECISION-MAKERS

Linguistic terms	neutrosophic numbers
Equally influential(EI)	(0.5,0.5,0.5)
Sporadic values between EI and SI	(0.4,0.65,0.6)
Slightly influential(SI)	(0.3,0.75,0.7)
Sporadic values between SI and STI	(0.6,0.35,0.4)
Strongly influential(STI)	(0.8,0.15,0.2)
sporadic values between STI and VSI	(0.7,0.25,0.3)
Very strongly influential(VSI)	(0.9,0.1,0.1)
sporadic values between VSI and AI	(0.85,0.1,0.15)
Absolutely influential(AI)	(1,0,0)

IV. BWM BASED ON SINGLE-VALUED NEUTROSOPHIC NUMBERS

A. The specific steps of BWM based on single-valued neutrosophic numbers

This section mainly introduces the proposed BWM based on single-valued neutrosophic numbers. It has the same specific steps as the classical BWM, except that the specific real numbers are replaced with single-valued neutrosophic numbers, and then the mathematical programming model is transformed.

Suppose that a study subject has n criteria, which can be paired and compared according to the decision-maker's linguistic variables(terms), e.g. "Equally important(EI)", "Slightly important(I)", "Strongly(FI)", "Very important(VI)" and "Absolutely important (AI)". Then, the decision-maker's linguistic evaluation needs to be converted into single-valued neutrosophic numbers, as listed in Table 2 [16].

By pairwise comparison, the following nth-order pairwise comparison matrix can be obtained:

$$\tilde{A}=\begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix} \tag{10}$$

The elements represented in the n-order matrix above, such as $\tilde{a}_{12}$, defined in definition 4, which refers to comparing criterion 1 to criterion 2. Also, it should be noted that all elements in the matrix are single-valued neutrosophic numbers. In the later specific steps, this matrix is unnecessary, so a simple understanding can be made. The following presents the specific steps of the proposed BWM based on single-valued neutrosophic numbers.

Step 1: determine the criteria for decision-making.

The decision-maker first determines a decision criterion and its collective as $C=\{c_1,c_2\cdots,c_n\}$. This step is crucial and lays the foundation for the following steps.

Step 2: determine the best and worst decision criteria.

The decision-maker determines the best and worst decision criteria according to the previous series of decision criteria; the best decision criterion is denoted as c_B, and the worst decision criterion is denoted as c_W.

Step 3: dbtained the Best-to-Others vector.

From definition 4, the Best-to-Others vector is obtained by comparing the best criterion with the other criteria, i.e., for a reference comparison a_{ij}, where i represents the best criterion, j refers to the other criteria, and j can be equal to i. Then, the Best-to-Others vector can be transformed into single-valued neutrosophic numbers by using the corresponding reference standards in Table 2:

$$\tilde{A}_B=(\tilde{a}_{B1},\tilde{a}_{B2},\cdots,\tilde{a}_{Bn}) \tag{11}$$

The vector obtained above represents the reference comparison of the best criterion c_B with other criteria c_j, and $j=1,2,\cdots,n$. In particular, when $i=j$, $a_{Bj}=(0.5,0.5,0.5)$.

Step 4: obtained the Others-to-Worst vector.

This step is the same as step 3. Now, the Others-to-Worst vector is obtained according to definition 4. By comparing the other criteria to the worst criterion, the best pair of other vectors is determined, i.e., for reference comparison a_{ij}, where i denotes the other criteria, j denotes the worst criterion, and i can be equal to j. Then, according to the corresponding reference standards in Table 2 [16], the Best-to-Other vector can be transformed into a single neutrosophic number:

$$\tilde{A}_W=(\tilde{a}_{1W},\tilde{a}_{2W},\cdots,\tilde{a}_{nW}) \tag{12}$$

The vector obtained above represents the reference comparison of the other criteria c_j to the worst criterion c_W, and $j=1,2,\cdots,n$. In particular, when $i=j$, $a_{jW}=(0.5,0.5,0.5)$.

Step 5: obtain the optimal weights.

This part is mainly obtained by solving the established mathematical model, which is established as follows:

$$\begin{aligned} &\min\max\left\{\left|\frac{\tilde{w}_B}{\tilde{w}_j}-\tilde{a}_{Bj}\right|,\left|\frac{\tilde{w}_j}{\tilde{w}_W}-\tilde{a}_{jW}\right|\right\} \\ &s.t.\begin{cases} \sum\limits_{j=1}^{n} s(w_j)=1 \\ 0\le t(w_j)+i(w_j)+f(w_j)\le 3 \\ 0\le t(w_j),i(w_j),f(w_j)\le 1 \\ j=1,2,\cdots n \end{cases} \end{aligned} \tag{13}$$

In the above mathematical model, $\tilde{w}_B,\tilde{w}_j,\tilde{w}_W,\tilde{a}_{Bj}$ and $\tilde{a}_{jW}$ are all single-valued neutrosophic numbers, and they are represented as:

$$\tilde{a}_{jW}=(t(\tilde{a}_{jW}),i(\tilde{a}_{jW}),f(\tilde{a}_{jW})$$

$$\tilde{w}_B=(t(\tilde{w}_B),i(\tilde{w}_B),f(\tilde{w}_B),\tilde{w}_j=(t(\tilde{w}_j),i(\tilde{w}_j),f(\tilde{w}_j),$$

$$\tilde{w}_W=(t(\tilde{w}_W),i(\tilde{w}_W),f(\tilde{w}_W),\tilde{a}_{Bj}=(t(\tilde{a}_{Bj}),i(\tilde{a}_{Bj}),f(\tilde{a}_{Bj}).$$

Meanwhile, it is necessary to transform the weights into precise values, as discussed in the literature [18]. In our study, this is achieved by using the score function of the single-valued neutrosophic numbers.

The above mathematical model can be transformed into the following nonlinearly constrained optimization problem.

$$\min \zeta$$
$$s.t.\begin{cases} \left|\frac{\tilde{w}_B}{\tilde{w}_j}-\tilde{a}_{Bj}\right|\le\zeta \\ \left|\frac{\tilde{w}_j}{\tilde{w}_W}-\tilde{a}_{jW}\right|\le\zeta \\ \sum_j^n s(w_j)=1 \\ 0\le t(w_j)+i(w_j)+f(w_j)\le 3 \\ 0\le t(w_j),i(w_j),f(w_j)\le 1 \\ j=1,2,\cdots n \end{cases} \tag{14}$$

In Eq.(14), ζ is also a single-valued neutrosophic number, $\zeta=(t_\zeta,i_\zeta,f_\zeta)$, $0\le t_\zeta,i_\zeta,f_\zeta\le 1, and 0\le t_\zeta+i_\zeta+f_\zeta\le 3$. Continue to apply the transformation transform, the following mathematical programming model is obtained:

$$\min \zeta$$
$$s.t.\begin{cases} \left|\frac{(t(\tilde{w}_B),i(\tilde{w}_B),f(\tilde{w}_B))}{(t(\tilde{w}_j),i(\tilde{w}_j),f(\tilde{w}_j))}-(t(\tilde{a}_{Bj}),i(\tilde{a}_{Bj}),f(\tilde{a}_{Bj}))\right| \\ \le(t_\zeta,i_\zeta,f_\zeta) \\ \left|\frac{(t(\tilde{w}_j),i(\tilde{w}_j),f(\tilde{w}_j))}{(t(\tilde{w}_W),i(\tilde{w}_W),f(\tilde{w}_W)}-(t(\tilde{a}_{jW}),i(\tilde{a}_{jW}),f(\tilde{a}_{jW})\right| \\ \le(t_\zeta,i_\zeta,f_\zeta) \\ \sum_j^n s(w_j)=1 \\ 0\le t(w_j)+i(w_j)+f(w_j)\le 3 \\ 0\le t(w_j),i(w_j),f(w_j)\le 1 \\ j=1,2,\cdots n \end{cases} \tag{15}$$

Due to the lack of definition of subtraction and division operations in the neural sets, the above mathematical model cannot be solved. In this case, the mathematical model above needs to be further transformed, and this is also a problem. After multiple experiments, this study decides to transform all single-valued neutrosophic numbers in the mathematical model into clear values using the score function in definition 4 and then perform the operation. Then, Eq.(15) can be transformed into the following mathematical model:

$$\min \zeta$$
$$s.t.\begin{cases} \left|\frac{s(\tilde{w}_B)}{s(\tilde{w}_j)}-s(\tilde{a}_{Bj})\right|\le s(\zeta) \\ \left|\frac{s(\tilde{w}_j)}{s(\tilde{w}_W)}-s(\tilde{a}_{jW})\right|\le s(\zeta) \\ \sum_j^n s(w_j)=1 \\ 0\le t(w_j)+i(w_j)+f(w_j)\le 3 \\ 0\le t(w_j),i(w_j),f(w_j)\le 1 \\ j=1,2,\cdots n \end{cases} \tag{16}$$

Continue to transform the above mathematical model to obtain:

$$\min \zeta$$
$$s.t.\begin{cases} |s(\tilde{w}_B)-s(\tilde{a}_{Bj})s(\tilde{w}_j)|\le s(\zeta)s(\tilde{w}_j) \\ |s(\tilde{w}_j)-s(\tilde{a}_{jW})s(\tilde{w}_W)|\le s(\zeta)s(\tilde{w}_W) \\ \sum_j^n s(w_j)=1 \\ 0\le t(w_j)+i(w_j)+f(w_j)\le 3 \\ 0\le t(w_j),i(w_j),f(w_j)\le 1 \\ j=1,2,\cdots n \end{cases} \tag{17}$$

The optimal weight can be obtained using lingo software to solve the above mathematical model.

TABLE III
CONSISTENCY INDEX OF SINGLE-VALUED NEUTROSOPHIC NUMBERS

$\tilde{a}_{BW}$	Consistency Ratio
(0.3,0.75,0.7)	1.68
(0.4,0.65,0.6)	1.89
(0.5,0.5,0.5)	2.12
(0.6,0.35,0.4)	2.34
(0.7,0.25,0.3)	2.52
(0.8,0.15,0.2)	2.70
(0.85,0.1,0.15)	2.76
(0.9,0.1,0.1)	2.83
(1,0,0)	3

B. *Consistency ratio based on single-valued neutrosophic numbers*

Consistency ratio is usually employed to represent consistency among decision-makers. The smaller the value, the higher the consistency of the decision-maker. This concept is commonly used in decision analysis, e.g., when constructing a consistently mixed matrix, it is necessary to determine the criteria for participating in the decision, such as cost, benefit, feasibility, etc. Then, decision-makers must compare each criterion in pairs to determine their importance. Therefore, the consistency ratio is an important factor in paired comparisons.

When problems are encountered, there may be a situation where $\tilde{a}_{Bj}\times\tilde{a}_{jW}\neq\tilde{a}_{BW}$, which can lead to inconsistency in the criteria related to paired comparisons. Our study addressed this issue and formulated a formula for calculating the continuity ratio. After analysis, it is found that the main reason for the inconsistency is that the value of $\tilde{a}_{Bj}\times\tilde{a}_{jW}$ is higher or smaller than the value of $\tilde{a}_{BW}$, and when $\tilde{a}_{Bj}=\tilde{a}_{jW}=\tilde{a}_{BW}$, the value of the inequality will reach its maximum. Considering this, in the case of inequality, this study simultaneously subtracts ζ from the left side of the inequality and adds θ to the right side of the inequality. This paper adopts the same method as the reference [18] and also investigates the case where the inequality has a maximum value. The specific equation is as follows:

$$(\tilde{a}_{Bj}-\theta)\times(\tilde{a}_{jW}-\theta)=(\tilde{a}_{BW}+\theta) \tag{18}$$

To maximize the inequality, it is necessary to make $\tilde{a}_{Bj}=\tilde{a}_{jW}=\tilde{a}_{BW}$. Then, the equation is transformed into Eq.(19)

$$(\tilde{a}_{BW}-\theta)\times(\tilde{a}_{BW}-\theta)=(\tilde{a}_{BW}+\theta) \tag{19}$$

Simplify Eq.(19) to obtain:

$$\theta^2-(1+2\tilde{a}_{BW})\theta+(\tilde{a}_{BW}^2-\tilde{a}_{BW})=0 \tag{20}$$

Obviously, $\tilde{a}_{BW}=(t(\tilde{a}_{BW}),i(\tilde{a}_{BW}),f(\tilde{a}_{BW}))$ and $\theta=(t(\theta),i(\theta),f(\zeta))$ are both intermediate numbers, and they cannot be further calculated. Therefore, it is necessary to use the score function in the intermediate set to transform them into clear values before the calculation can be conducted. Then, the equation above is transformed into Eq.(21).

$$s^2(\theta)-(1+2s(\tilde{a}_{BW}))s(\theta)+(s^2(\tilde{a}_{BW})-s(\tilde{a}_{BW}))=0 \tag{21}$$

Now, the consistency index can be calculated very well. For example, take $\tilde{a}_{BW}=(1,0,0)$ and then calculate its score function to obtain $s(\tilde{a}_{BW})=1$, which is then

TABLE IV
LINGUISTIC TERMS FOR PERFERENCES

Criteria	Worst criteria C1
C1	EI
C2	between VSI and AI
C3	AI

substituted into Eq.(21) to obtain $s(\theta)$. By using the above calculation method to calculate all $\tilde{a}_{BW}$ in Table 2, different values of $s(\theta)$ can be obtained, which are referred to as the consistency index, also known as CI. The specific values of CI are listed in Table 3.

The calculation formula for the consistency ratio is similar to the classic BWM calculation method. Due to the use of a scoring function to degenerate single-valued neutrosophic numbers into a crack value, the value obtained is much smaller than imagined. Therefore, when solving the consistency index, a slight change is made to the solving formula to obtain the calculation formula:

$$\mathrm{CR} = \frac{s(\zeta)}{10 * s(\theta)} \tag{22}$$

C. Application of the BWM based on single-Valued neutrosophic numbers

This subsection mainly introduces the practical problem of selecting transportation modes to apply and validate the proposed single-valued neutrosophic numbers. A company hopes to choose the best transportation mode to supply goods to a shopping mall. Meanwhile, the company has the following three critical indicators for the transportation mode.

(1) Flexibility of loading: this indicator is related to transportation efficiency and flexibility. Suppose a small amount of goods to be transported, but there are many types. In this case, container transportation may not be suitable because the container has a fixed size, making it difficult to load goods of various sizes. In contrast, the transportation of bulk goods can be flexibly adjusted based on the type and quantity of the goods.

(2) Accessibility: this indicator considers environmental, safety, and time factors during transportation. For instance, although air transportation has a high-speed, it may not be suitable for goods sensitive to vibration and impact. Meanwhile, railway and road transportation may be more restricted than air transportation in certain regions. Therefore, when choosing a transportation method, one needs to consider the nature of the goods, destination and limiting factors during the transportation process.

(3) Cost: this indicator is the most intuitive economic factor. Different transportation methods may have different costs, including transportation, loading and unloading and insurance. When choosing a transportation method, it is necessary to consider the value of the goods, transportation distance and time, and various potential risk factors. The above three factors are critical in selecting transportation modes. Rezaei initially proposed this problem, which was then extended to fuzzy environments. Dong et al. [18] also studied this problem. Our study discusses this problem and solves it.

TABLE V
LINGUISTIC TERMS FOR PREFERENCES

criteria	C1	C2	C3
Best criterion C3	AI	STI	EI

Next, the methods introduced in this study are employed to solve this problem.

Step 1: the decision maker determines a set of decisions, which in this case are "Loading flexibility(C1)", "Accessibility(C2)", and "Cost(C3)", is, $\{C1, C2, C3\}$.

Step 2: determine the best and worst criteria. From the perspective of the company, "Cost(C3)" is chosen as the best criterion, whereas "Loading flexibility(C1)" is chosen as the worst criterion.

Step 3: obtain the Best-to-Others vector. Based on Table 4, it can be obtained that the Best-to-others vector is

$$\tilde{A}_B = [\tilde{a}_{B1}, \tilde{a}_{B2}, \tilde{a}_{B3}]$$

where, $\tilde{a}_{B1} = (1, 0, 0)$, $\tilde{a}_{B2} = (0.8, 0.15, 0.2)$, and $\tilde{a}_{B3} = (0.5, 0.5, 0.5)$.

Step 4: obtain the Others-to-Worst vector. From Table 5, the Others-to-Worst vector

$$\tilde{A}_W = [\tilde{a}_{1W}, \tilde{a}_{2W}, \tilde{a}_{3W}]$$

can be obtained. Here, $\tilde{a}_{1W} = (0.5, 0.5, 0.5)$, $\tilde{a}_{2W} = (0.85, 0.1, 0.15)$, $\tilde{a}_{3W} = (1, 0, 0)$.

Step 5: model construction. The specific mathematical model is as follows:

$$\begin{array}{c} \min \zeta \\ s.t. \left\{ \begin{array}{c} |s(\tilde{w}_3) - s(\tilde{a}_{31})s(\tilde{w}_1)| \leq s(\zeta)s(\tilde{w}_1) \\ |s(\tilde{w}_3) - s(\tilde{a}_{32})s(\tilde{w}_2)| \leq s(\zeta)s(\tilde{w}_2) \\ |s(\tilde{w}_3) - s(\tilde{a}_{33})s(\tilde{w}_3)| \leq s(\zeta)s(\tilde{w}_3) \\ |s(\tilde{w}_1) - s(\tilde{a}_{11})s(\tilde{w}_1)| \leq s(\zeta)s(\tilde{w}_1) \\ |s(\tilde{w}_2) - s(\tilde{a}_{21})s(\tilde{w}_1)| \leq s(\zeta)s(\tilde{w}_1) \\ |s(\tilde{w}_3) - s(\tilde{a}_{31})s(\tilde{w}_1)| \leq s(\zeta)s(\tilde{w}_1) \\ s(w_1) + s(w_2) + s(w_3) = 1 \\ 0 \leq t(w_j) + i(w_j) + f(w_j) \leq 3 \\ 0 \leq t(w_j), i(w_j), f(w_j) \leq 1 \\ j = 1, 2, 3. \end{array} \right. \end{array} \tag{23}$$

By substituting the data, the following nonlinear constrained optimization problem can be formulated:

$$\begin{array}{c} \min \zeta \\ s.t. \left\{ \begin{array}{c} s(\tilde{w}_3) - s(\tilde{w}_1) \leq s(\zeta)s(\tilde{w}_1) \\ s(\tilde{w}_3) - s(\tilde{w}_1) \geq -s(\zeta)s(\tilde{w}_1) \\ s(\tilde{w}_3) - 0.82 * s(\tilde{w}_2) \leq s(\zeta)s(\tilde{w}_2) \\ s(\tilde{w}_3) - 0.82 * s(\tilde{w}_2) \geq -s(\zeta)s(\tilde{w}_2) \\ s(\tilde{w}_3) - 0.5 * s(\tilde{w}_3) \leq s(\zeta)s(\tilde{w}_3) \\ s(\tilde{w}_3) - 0.5 * s(\tilde{w}_3) \geq -s(\zeta)s(\tilde{w}_3) \\ s(\tilde{w}_1) - 0.5 * s(\tilde{w}_1) \leq s(\zeta)s(\tilde{w}_1) \\ s(\tilde{w}_1) - 0.5 * s(\tilde{w}_1) \geq -s(\zeta)s(\tilde{w}_1) \\ s(\tilde{w}_2) - 0.72 * s(\tilde{w}_1) \leq s(\zeta)s(\tilde{w}_1) \\ s(\tilde{w}_2) - 0.72 * s(\tilde{w}_1) \geq -s(\zeta)s(\tilde{w}_1) \\ s(\tilde{w}_1) + s(\tilde{w}_2) + s(\tilde{w}_3) = 1 \\ 0 \leq t(w_1) + i(w_1) + f(w_1) \leq 3 \\ 0 \leq t(w_2) + i(w_2) + f(w_2) \leq 3 \\ 0 \leq t(w_3) + i(w_3) + f(w_3) \leq 3 \\ 0 \leq t(w_1), i(w_1), f(w_1) \leq 1 \\ 0 \leq t(w_2), i(w_2), f(w_2) \leq 1 \\ 0 \leq t(w_3), i(w_3), f(w_3) \leq 1 \end{array} \right. \end{array} \tag{24}$$

Using lingo 18.0 software to solve the mathematical model above, clear weight values for three criteria ("Load flexibility," "Accessibility," and "Cost") are obtained as follows:

$$s(\tilde{w}_1) = 0.275; s(\tilde{w}_2) = 0.3125; s(\tilde{w}_3) = 0.4125$$

$$s(\zeta) = 0.5$$

According to the obtained data, the weights of these three criteria ("Load flexibility", "Accessibility" and "Cost") are 0.275, 0.3125 and 0.4125, respectively. In this example, the classic BWM obtains weights of 0.07414, 0.3387 and 0.5899, respectively. Similarly, the three weights calculated using the fuzzy BWM proposed by Guo and Zhao are 0.1431, 0.3496 and 0.5073, respectively. Through a comparative analysis, it can be concluded that although there is a difference in the weights obtained using these three methods, the preference order is the same in terms of the total amount.

Since $a_{BW} = (1, 0, 0)$, the consistency index $CI = 3$ can be obtain from Table 3, and then Eq.(22) is used to calculate the consistency ratio as $CR = \frac{s(\zeta)}{10*s(\theta)} = \frac{0.5}{10\times3} = 0.0167$. The consistency index $CR = 0.058$ is obtained using the classical BWM, and the consistency index $CR = 0.0559$ is obtained using the fuzzy BWM. The CR obtained using our method is smaller than those obtained using the classical and fuzzy BWMs. Therefore, it can be concluded that this BWM based on single-valued neutrosophic numbers demonstrates higher comparative consistency.

V. Conclusion

Considering previous BWMs, such as the classical and fuzzy BWMs, the new BWM proposed in this study is based on single-valued neutrosophic numbers. Therefore, the Best-to-Others vector and the Others-to-Worst vector comprise single-valued neutrosophic numbers. This comprehensively considers the uncertainty and fallacy that decision-makers face when handling real problems. Moreover, in reference [7], the linguistic terms of preference and their corresponding triangular neutrosophic numbers are transformed into single-valued neutrosophic numbers, thereby establishing the connection between the classical BWM and single-valued neutrosophic numbers. In the transformation, since division and subtraction operations are not defined for neutrosophic numbers, the single-valued neutrosophic numbers are transformed into precise values for calculation using the scoring function. The formula is modified because applying the same consistency ratio calculation formula will lead to a too-large result. The consistency ratio obtained in this study is smaller than those obtained by the classical and fuzzy BWMs. Therefore, our proposed BWM based on single-valued neutrosophic numbers is more effective in fuzzy environments.

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