A New SIQR Model and Residual Power Series Method in Wireless Sensor Networks

Jianke Zhang, Zhuoqing Zhang, Chang Zhou, Xiaojue Ma

Abstract—**The spread of computer virus is an important topic, which has an important impact on network security. By studying the propagation law of viruses on wireless sensor networks, the future trends can be predicted, so that preventive measures can be taken as far as possible to minimize the harms. Based on the existing SIQR model and the transmission characteristics of computer virus, a new fractional SIQR model is established by innovating such a dynamic process that an infected node can automatically recover to a recovery node with the probability of ν. Furthermore, the concrete steps to solve the new model are given by the residual power series method. We apply the steps to solve an example, in which the dynamic process is displayed successfully. Finally, by comparing the two different models, it can be shown that the infective component disappears faster in our new model. The improved SIQR model can better simulate the spread of computer virus in wireless sensor networks.**

Index Terms—**residual power series method, fractional differential equation, virus propagation model, wireless sensor network.**

I. Introduction

THE utilization of sensor networks is expanding due to the ongoing advancements in Internet of Things technologies. Wireless sensor networks consist of the vast majority of tiny and low-power sensor nodes, which are distributed in the area to be monitored or controlled. Wireless sensor networks frequently have poor defense capabilities and are vulnerable to malicious software like worms, viruses, or trojans because of the limited memory capacity of individual nodes and the locational disadvantage. In addition, since sensor networks use wireless communication, they are vulnerable to exploitation by hackers or advertisers who may introduce malicious programs into the wireless sensor networks. Due to the nature of wireless communication, malware attacks can cause nodes to deplete energy rapidly, decrease the overall computing speed of the network, and increase network traffic. So far, there is no antivirus software that can completely detect and remove computer virus transmissions. Therefore, to effectively preventing the spread of computer viruses, it is essential to have a clear understanding of their characteristics and transmission patterns, predicting their development direction, and implementing efficient prevention and removaling strategies. In response to this problem, comparing the similarities between the spread of diseases in populations and the spread of computer viruses in wireless sensor networks, a number of infectious disease models have been applied to model and studied the spread of computer viruses in networks.

Over the past few decades, numerous researchers have conducted studies on mathematical modeling of malware object propagation using various models. Kermark and Mc Kendrick ([1]) studied the classical epidemic model of SIR. They proposed a malicious object propagation model, and estimated the time evolution of infected nodes by considering network parameters in network topology [2]. This method is also applied to email propagation schemes [3]. Pastor Satorras and Vespignani ([4, 5]) studied the effect of network topology on disease transmission. The transmission of the SIS infectious disease scaleless networks model is also considered [6] and [7]. Barthelemy ([8]) found that the spread of computer viruses in scale-free networks follows specific hierarchical dynamics. Loecher and Kadtke ([9]) recognized and extended details of graded dissemination, to improving the predictability of the order of infected nodes. In [10], Mishra and Saini proposed a delayed SEIRS epidemiological model to studying propagation of malicious objects in computer networks and the stability of the free equilibrium point of malicious objects. Next, Mishra and Keshri ([11]) established worm transmission of wireless sensors networks by using the SEIRS-V model. It also examined the relationship between the basic reproductive count and stability the equilibrium point. Then, a predator-prey model ([12]) was proposed to analyzing impact of energy saving during worm attacks in wireless sensor networks, and identifing the stability conditions of various a balance point. Recently, Sellali ([13]) studied a partial computerized epidemiological models virus transmission and discussed the effect free-ordered on the dynamic behavior impact of virus spread. Dong ([14]) used the SIQR model to investigating the unpredictable attack behavior of computer viruses in wireless sensor networks.

The infection of malware objects, such as viruses, worms, or trojans in wireless sensor networks occurs through the transmission of radio waves and signals between wireless sensor nodes. Signal transmission, which encompasses memory and genetic processes, is often heavily influenced by flexible environment, the functional history, texture and nature of the material. Describing these phenomena accurately with integer-order differential equations is a big challeng. However, fractional calculus can offer a completely new mathematical approach. It is well known that fractional-order derivatives and integrals have a nonlocal nature. These

Manuscript received December 15, 2023; revised July 16, 2024. This work is supported by Xi'an Science and Technology Plan Project (22GXFW0124) and the Natural Science Basic Research Program of Shaanxi Province (Program No. 2021JQ-709, 2023-JC-YB-623).

Jianke Zhang is a professor of the School of Science, Xi'an University of Posts and Telecommunications, Xi'an 710121, China. (e-mail: jiankezh@163. com).

Zhuoqing Zhang is a graduate student of the School of Computer College, Xi'an University of Posts and Telecommunications, Xi'an 710121, China. (e-mail: zzqing7127@qq. com).

Chang Zhou is an associate professor of the School of Science, Xi'an University of Posts and Telecommunications, Xi'an 710121, China. (e-mail: maytheday@163. com).

Xiaojue Ma is an associate professor of the School of Science, Xi'an University of Posts and Telecommunications, Xi'an 710121, China. (e-mail: mingkeming@126. com).

derivatives and integrals can represent both past information and distributional effects of any physical system. This demonstrates the powerful compared with ordinary calculus, fractional derivative and integral can represent complex real-world phenomena more accurately and efficiently.

In its long history of development, a large number of studies have proven that fractional calculus offers significant advantages in various disciplines and specialties in the real world. In recent years, with the rapid development integral calculus of fractions, some of the main advantages include the mnemonic properties of fractional derivatives and improved simulation in fractal materials or viscoelastic environments, see [15–22]. With the development of fractional calculus, fractional differential equations and fractional partial differential equations have also been studied extensively, see [23–25]. In the past decades, significant progress has been made in studying fractional differential equations under uncertain conditions. For example, Agarwal ([26]) studied the Cauchy problem of fuzzy differential equation, where the fuzzy fractional differential equation was analyzed without the application of fuzzy fractional derivatives. Next, Allahviranloo ([27]) used fuzzy Riemann-Liouville fractional derivatives to investigating the initial value problem of fuzzy fractional differential equations. In the field of fuzzy partial differential equations, Long studied local and nonlocal problems for fuzzy fractional partial differential equations under Caputo's generalized H-differentials, as referenced in [28, 29]. Recently, multiple papers have explored epidemic modeling and its application to computer networks and WSN. For example, Hassouna [30] used variational iteration method and Euler's method to solving the fractional SIS model. Huo and Zhao ([31]) studied a class of fractional-order SIR models with complex heterogeneous networks. Shingh [32] analyzed a fractional-modified infectious disease model associated with the new Caputo-Fabrizio score derivative. Dubey [33] used the homotopy perturbation transformation method, which given an approximate analytical solution for a class of fractional-order computer virus propagation (CVP) with nonlinear characteristics. Graef [34] builted a fractional infectious disease model to investigating user's adoption and abandonment of online social networks (OSNs). Naim [35] studied a fractional SEIR model with latent infection.

Above all, there are few article proposed the residual power series method to studing the fractional order SIQR model in wireless sensor networks. Shatha [36] used the residual power series method to approximating the numerical solutions of fractional SIR models. The residual power series method is a simple and effective technique that has been applied to numerous equations. Qazza [37] used residual power series method to analyzing linear and nonlinear differential equations of fractional order. Qayyum [38] proposed the Residual Power Series Method (RPSM) for solving wave-type partial differential equations (PDEs). Modanli [39] proposed a residual power series method for solving pseudo-hyperbolic partial differential equations under non-local conditions. Abu [40] proposed the residual power series method to solving the fractional Schrödinger equation. Khalouta [41] used fractional residual power series method to solving the Bratu-type equation.

In this paper, we establish a fractional mathematical model for the computer virus propagation in wireless sensor networks, which with isolated and uncertain initial data using the Caputo fractional derivative. The organizational structure of this paper is as follows. In Section 2, the Caputo fractional derivative is introduced. In addition, the concepts of fractional integrals related to the proposed fractional derivatives are also discussed. In Section 3, a mathematical model of fractional transmission of viruses in wireless sensor networks is established. The proposed model consists of the following four components: susceptibility (S), infection (I), quarantine (Q) and recovery (R). It describes the uncertain dynamic behavior of virus attacks in the network by considering isolation and initial data ambiguity. In Section 4, the steps of the residual power series method for solving the established model are proposed. In Section 5, the application and result analysis are summarized in Section 6.

II. Preliminaries

In this section, firstly, we will introduce the definition of the Caputo fractional derivative in detail. Secondly, the concepts related to the fractional residual power series method will be introduced.

Definition 1([42]). Given a continuous function $f(t)$, and let n be the smallest integer greater than $\alpha(\alpha > 0)$, the Caputo fractional derivative is defined by

$$D^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^x (t-\tau)^{n-\alpha-1} \frac{\mathrm{d}^n f(\tau)}{\mathrm{d}\tau^n} \mathrm{d}\tau, n-1 < \alpha < n, \\ \frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}, \alpha = n \in N. \end{cases} \tag{1}$$

Theorem 1([42]). By the Caputo derivative, we get

$$D^{\alpha} x^{\alpha} = \begin{cases} \frac{\Gamma(q+1)}{\Gamma(q+1-\alpha)} x^{q-\alpha}, \alpha \leq q, \\ 0, \alpha > q. \end{cases} \tag{2}$$

Definition 2([43–45]). The Riemann-Liouville fractional integration operator of $\alpha \leq 0$ is defined as

$$J^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(s)}{(t-s)^{1-\alpha}} ds = \frac{1}{\Gamma(\alpha)} t^{\alpha-1} * f(t), \alpha > 0, t > 0, \\ f(t), \alpha = 0. \end{cases} \tag{3}$$

Where $t^{\alpha-1} * f(t)$ is the convolution of $t^{\alpha-1}$ and $f(t)$.

For Riemann-Liouville fractional integrals, we have

1.$J^{\alpha} t^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\alpha+1)} t^{\alpha+\beta}, \beta > -1$,

2.$J^{\alpha}(\lambda f(t) + \mu g(t)) = \lambda J^{\alpha} f(t) + \mu J^{\alpha} g(t)$.

Where, λ and μ are constants.

For the Caputo fractional derivative, we have

1.$D^{\alpha} J^{\alpha} f(t) = f(t)$,

2.$J^{\alpha} D^{\alpha} f(t) = f(t) - \sum_{i=0}^{n-1} y^{(i)}(0) \frac{t^i}{i!}$,

3.$D^{\alpha} c = 0$,

4.$D^{\alpha}(\lambda f(t) + \mu g(t)) = \lambda D^{\alpha} f(t) + \mu D^{\alpha} g(t)$.

Where, λ, μ and c are constants.

Next, this section introduces the relevant definitions and theorems of fractional power series, which are based on the Caputo fractional derivative definition.

Definition 3([46, 47]). The fractional power series expansion form is

$$\sum_{m=0}^{\infty} c_m (t-t_0)^{m\alpha} = c_0 + c_1(t-t_0)^{\alpha} + c_2(t-t_0)^{2\alpha} + \cdots . \tag{4}$$

Which satisfies the conditions $0 \leq \lceil \alpha \rceil - 1 \leq \alpha \leq \lceil \alpha \rceil, t \geq t_0$, where t is a variable and c_m is a constant, which is known as the coefficients of the series.

Theorem 2([46]). Let f have a fractional power series form at t_0, then
$f(t) = \sum_{m=0}^{\infty} f_m(t) =$

$$\sum_{m=0}^{\infty} c_m (t - t_0)^{m\alpha}, 0 \leq \lceil \alpha \rceil - 1 \leq \alpha \leq \lceil \alpha \rceil, t_0 \leq t < t_0 + R. \tag{5}$$

If $D^{m\alpha} f(t) \in (t_0, t_0 + R), m = 0, 1, 2, \cdots$, then the coefficient c_m of equation (5) can be given by the following equation

$$c_m = \frac{D^{m\alpha} f(t_0)}{\Gamma(m\alpha + 1)}, m = 0, 1, 2, \cdots. \tag{6}$$

Where $D^{m\alpha} = D^{\alpha} \cdot D^{\alpha} \cdots D^{\alpha} (m - times)$, R is the radius of convergence.

According to the convergence of the classical residual power series, there exists a real number $\lambda \in (0, 1)$ such that $\|f_m(t)\| \leq \lambda \|f_{m-1}(t)\|$, where $t \in (t_0, t_0 + R)$.

Definition 4([47]). The power series is of the form $\sum_{m=0}^{\infty} f_m(x)(t - t_0)^{k\alpha} =$

$$f_0(x) + f_1(x)(t - t_0)^{\alpha} + f_2(x)(t - t_0)^{2\alpha} + \cdots. \tag{7}$$

Where t is a variable and f_m is a function of x, which also known as the coefficient of the series. The conditions $0 \leq \lceil \alpha \rceil - 1 < \alpha \leq n$ and $t \geq t_0$ are also satisfied.

III. Virus Propagation Model in Wireless Sensor Networks

In this section, we will examine the virus propagation issue in a fractional-order wireless sensor network. In this network, it is assumed those which all sensor nodes are in one of the following four possible states during this process:

State (S): (S) is composed of nodes those which are not attacked by viruses. Because these nodes are highly sensitive to viruses and vulnerable to virus attacks, we refer to them as susceptible nodes.

State (I): When node (I) has been infected by a virus in the sensor network, it may be infected other nodes in state (S). The sensor nodes in this state are referred to as infectious nodes.

State (Q): (Q) is composed of these all wireless sensor nodes which are isolated from the state (I) in (I). In other words, which are referred to as quarantine nodes.

State (R): The sensor node does not contain a virus, which is immune to it.

At time t, $S(t)$, $I(t)$, $Q(t)$, and $R(t)$ are used to representing the number of susceptible, infectious, quarantine, and recovered sensor nodes.

To fight the virus, we need to understand its dynamic characteristics. The infectious disease transmission model is an excellent way to characterize specific information in human society. By understanding the characteristics of malicious software, we can accurately predict the propagation patterns by establishing a comprehensive mathematical model of computer virus spread.

According to the SIQR model proposed in article [48], this paper makes enhancements to align it more closely with the actual scenario of computer virus propagation in wireless sensor networks. Considering the current situation, some infected nodes can recover on their own after being infected, increasing the likelihood that infected nodes can spontaneously recover to become nodes with a certain probability. Based on the above situation, it is also convenient to enhancing the model in the future and adapting to more variables. The details will be described below, as shown in state (I).

The propagation of computer viruses in wireless sensor networks is a very complex process that exhibits non-locality and memory. In this way, fractional differential equations can be used to describe the problem more accurately and efficiently. In this paper, the Caputo derivative is used to characterize the virus propagation rate in wireless sensor networks. In practical applications, it is often necessary to use fuzzy dynamic system methods to solving virus attack problems due to uncertainties, missing or incomplete information. This method allows us to describing and demonstrating uncertainty in the real world. Moreover, such an explanation can be closer to the source of the actual model and have strong generalization ability.

It is assumed that the sensor nodes in each state (S), (I), (Q), and (R) leave the network at a rate of μ. Then, the SIQR model can be represented as follows:

State (S): Assume sensor nodes beyond wireless sensor network access the network at rate A. Since the probability of this node being infected by a virus is $\lambda I(t)$, the speed of its exit from the network is μ, and the probability of having direct immunity is ω, the rate of change of (S) in this system can be expressed as follows:

$$D^{\alpha} S(t) = A - \lambda S(t) I(t) - \mu S(t) - \omega S(t).$$

State (I): In the virus-infected state (S), the sensing node becomes infected by the virus and is then transmitted to an individual with a probability of $\lambda I(t)$. The infected node exits the network at μ. Each infected node can be isolated independently with a probability of γ. On this basis, with the help of the antiviral program, the infected node transitions to the recovery node at the speed of ν. Thus, an equation for the rate of change of (I) can be given by the following formula:

$$D^{\alpha} I(t) = \lambda S(t) I(t) - \mu I(t) - \gamma I(t) - \nu I(t).$$

State (Q): An infecting node can be isolated as a (Q) state with a probability γ, and may also enter by: (i) individual nodes exiting WSN at μ, (ii) returning to (R) state at η. Next, the equation describing the rate of change (Q) is the following formula:

$$D^{\alpha} Q(t) = \gamma I(t) - \mu Q(t) - \eta Q(t).$$

State (R): The rate of change (R) of the recovery state indicates that: (i) each recovery node leaves the network at a rate of μ, and (ii) each isolated node transitions to the recovery state at a rate of η. (iii) Restore the infected node by a certain proportion of ν using antivirus procedures as described above. (iv) A portion of susceptible nodes are autoimmune, and the probability of this portion is ω. Accordingly, the equation associated with state (R) is:

$$D^{\alpha} R(t) = \omega S(t) + \nu I(t) + \eta Q(t) - \mu R(t).$$

Therefore, the spread of computer viruses can be described by the following fuzzy fractional differential equation.

$$\begin{cases} D^{\alpha}S(t) = A - \lambda S(t)I(t) - \mu S(t) - \omega S(t), \\ D^{\alpha}I(t) = \lambda S(t)I(t) - \mu I(t) - \gamma I(t) - \nu I(t), \\ D^{\alpha}Q(t) = \gamma I(t) - \mu Q(t) - \eta Q(t), \\ D^{\alpha}R(t) = \omega S(t) + \nu I(t) + \eta Q(t) - \mu R(t). \end{cases} \tag{8}$$

The initial conditions are:

$$S(0) = S_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0. \tag{9}$$

The following diagram can be used to illustrating the transmission mode of computer viruses, which is depicted in Figure 1.

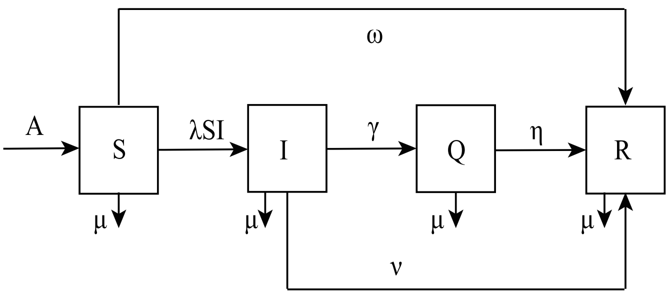

Fig. 1. SIQR model of computer virus propagation

$N(t)$ represents the total number of nodes in the wireless sensor network, where $N(t) = S(t) + I(t) + Q(t) + R(t), \forall t \geq 0$. We will investigate how to assess the impact of a virus attack on this model. The basic number of reproductions is an important indicator, which can be used to describe the contagiousness of an infectious source, such as a virus, worm or Trojan. In the SIQR model, the basic number of reproductions $\Re$ for a system of fractional differential equations (8) is expressed as $\Re = \frac{\lambda A}{\mu(\nu+\gamma+\mu)}$.

IV. Residual Power Series Method for Solving Fractional SIQR Infectious Disease Model

In this section, we will use the residual power series method to solving the fractional SIQR infectious disease model. The calculation steps are described below:

Step 1: Suppose that at $t = 0$, the fractional power series of $S(t), I(t), Q(t)$ and $R(t)$ are

$$\begin{cases} S(t) = \sum_{k=0}^{\infty} \frac{a_k}{\Gamma(k\alpha+1)} \cdot t^{k\alpha}, \\ I(t) = \sum_{k=0}^{\infty} \frac{b_k}{\Gamma(k\alpha+1)} \cdot t^{k\alpha}, \\ Q(t) = \sum_{k=0}^{\infty} \frac{c_k}{\Gamma(k\alpha+1)} \cdot t^{k\alpha}, \\ R(t) = \sum_{k=0}^{\infty} \frac{d_k}{\Gamma(k\alpha+1)} \cdot t^{k\alpha}. \end{cases} \tag{10}$$

Then, $S_n(t), I_n(t), Q_n(t)$ and $R_n(t)$ are used to representing the n-order truncation series of $S(t), I(t), Q(t)$ and $R(t)$ respectively. When $n = 0$, by using the initial condition of the model (9), it can be known that $S(0) = S_0, I(0) = I_0, Q(0) = Q_0$ and $R(0) = R_0$. Therefore, the n-order truncation series of $S(t), I(t), Q(t)$ and $R(t)$ can be written in the following form

$$\begin{cases} S_n(t) = S_0 + \sum_{k=1}^{n} \frac{a_k}{\Gamma(k\alpha+1)} \cdot t^{k\alpha}, \\ I_n(t) = I_0 + \sum_{k=1}^{n} \frac{b_k}{\Gamma(k\alpha+1)} \cdot t^{k\alpha}, \\ Q_n(t) = Q_0 + \sum_{k=1}^{n} \frac{c_k}{\Gamma(k\alpha+1)} \cdot t^{k\alpha}, \\ R_n(t) = R_0 + \sum_{k=1}^{n} \frac{d_k}{\Gamma(k\alpha+1)} \cdot t^{k\alpha}. \end{cases} \tag{11}$$

Step 2: The residual function of model (8) can be agined as

$$\begin{cases} ResS(t) = D^{\alpha}S(t) - A + \lambda S(t)I(t) + \mu S(t) + \omega S(t), \\ ResI(t) = D^{\alpha}I(t) - \lambda S(t)I(t) + \mu I(t) + \gamma I(t) + \nu I(t), \\ ResQ(t) = D^{\alpha}Q(t) - \gamma I(t) + \mu Q(t) + \eta Q(t), \\ ResR(t) = D^{\alpha}R(t) - \omega S(t) - \nu I(t) - \eta Q(t) + \mu R(t). \end{cases} \tag{12}$$

Therefore, the residual functions of order n for $S_n(t), I_n(t), Q_n(t)$ and $R_n(t)$ are

$$\begin{cases} ResS_n(t) = \\ D^{\alpha}S_n(t) - A + \lambda S_n(t)I_n(t) + \mu S_n(t) + \omega S_n(t), \\ ResI_n(t) = \\ D^{\alpha}I_n(t) - \lambda S_n(t)I_n(t) + \mu I_n(t) + \gamma I_n(t) + \nu I_n(t), \\ ResQ_n(t) = \\ D^{\alpha}Q_n(t) - \gamma I_n(t) + \mu Q_n(t) + \eta Q_n(t), \\ ResR_n(t) = \\ D^{\alpha}R_n(t) - \omega S_n(t) - \nu I_n(t) - \eta Q_n(t) + \mu R_n(t). \end{cases} \tag{13}$$

Obviously, when $t \geq 0$, $ResS(t) = ResI(t) = ResQ(t) = ResR(t) = 0$. We can get $\lim_{n\to\infty} ResS_n(t) = ResS(t), \lim_{n\to\infty} ResI_n(t) = ResI(t), \lim_{n\to\infty} ResQ_n(t) = ResQ(t)$ and $\lim_{n\to\infty} ResR_n(t) = ResR(t)$. Since the Caputo derivative of any constant is equal to 0, it follows

$$\begin{cases} D^{(k-1)\alpha}ResS(0) = D^{(k-1)\alpha}ResS_k(0), \\ D^{(k-1)\alpha}ResI(0) = D^{(k-1)\alpha}ResI_k(0), \\ D^{(k-1)\alpha}ResQ(0) = D^{(k-1)\alpha}ResQ_k(0), \\ D^{(k-1)\alpha}ResR(0) = D^{(k-1)\alpha}ResR_k(0). \end{cases} \tag{14}$$

Step 3: To get the coefficients $a_k, b_k, c_k, d_k, k = 1, 2, \cdots, n$, we plug the n order truncation series $S_n(t), I_n(t), Q_n(t)$ and $R_n(t)$ into the equation. Then, the fractional Caputo derivative operators $D^{(n-1)\alpha}$ are applied to calculating $ResS_n(t), ResI_n(t), ResQ_n(t)$ and $ResR_n(t)$, respectively, in the following form

$$\begin{cases} D^{(n-1)\alpha}ResS_n(0) = 0, \\ D^{(n-1)\alpha}ResI_n(0) = 0, \\ D^{(n-1)\alpha}ResQ_n(0) = 0, \\ D^{(n-1)\alpha}ResR_n(0) = 0. \end{cases} \tag{15}$$

Step 4: Solving $a_k, b_k, c_k, d_k, k = 1, 2, \cdots, n$ of the algebraic equation (15), we can get an approximate solution for the residual power series of the fractional order SIQR infectious disease model (8).

Step 5: Repeating the above steps, we can obtain a sufficient number of coefficients. By estimating unknown coefficients in the model, the accuracy is improved.

V. Application and Result Analysis

In this section, an example of the fractional SIQR infectious disease model will be solred by using the residual power series method. By using the Maple program, the numerical simulation demonstrates that the numerical solution of fractional SIQR model (8) and the impact of fractional derivatives virus spreading in wireless sensor networks of different orders. The results are analyzed using graphs and tables.

First, we examine the following SIQR infectious disease model

$$\begin{cases} D^\alpha S(t)=1-0.15S(t)I(t)-0.25S(t)-0.02S(t),\\ D^\alpha I(t)=0.15S(t)I(t)-0.25I(t)-0.35I(t)-0.25I(t),\\ D^\alpha Q(t)=0.35I(t)-0.25Q(t)-0.5Q(t),\\ D^\alpha R(t)=0.02S(t)+0.25I(t)+0.5Q(t)-0.25R(t).\end{cases} \tag{16}$$

The initial condition is $S_0=0.8, I_0=1.4, Q_0=0.6$ and $R_0=0.05$, where α is the order of the fractional derivative under Caputo's definition, and $0<\alpha\le 1$.

Then, the following steps of the residual power series as described in the previous section.

When $n=1$, from Equation (11), the first truncated series is

$$\begin{cases} S_1(t)=0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha,\\ I_1(t)=1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha,\\ Q_1(t)=0.6+\frac{c_1}{\Gamma(\alpha+1)}t^\alpha,\\ R_1(t)=0.05+\frac{d_1}{\Gamma(\alpha+1)}t^\alpha.\end{cases}$$

From equation (13), the first residual function is $ResS_1(t)=D^\alpha(0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha)-1+0.15(0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha)(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)+(0.25+0.02)(0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha)$,
$ResI_1(t)=D^\alpha(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)-0.15(0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha)(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)+(0.25+0.35+0.25)(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)$,
$ResQ_1(t)=D^\alpha(0.6+\frac{c_1}{\Gamma(\alpha+1)}t^\alpha)-0.35(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)+(0.25+0.5)(0.6+\frac{c_1}{\Gamma(\alpha+1)}t^\alpha)$ and
$ResR_1(t)=D^\alpha(0.05+\frac{d_1}{\Gamma(\alpha+1)}t^\alpha)-0.02(0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha)-0.25(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)-0.5(0.6+\frac{c_1}{\Gamma(\alpha+1)}t^\alpha)+0.25(0.05+\frac{d_1}{\Gamma(\alpha+1)}t^\alpha)$.

Then, applying the fractional caputo derivative operator $D^{(n-1)\alpha}$ to calculate the above formula, we obtain:
$D^{(n-1)\alpha}ResS_1(t)=a_1-1+0.15(0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha)(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)+(0.25+0.02)(0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha)$,
$D^{(n-1)\alpha}ResI_1(t)=b_1-0.15(0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha)(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)+(0.25+0.35+0.25)(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)$,
$D^{(n-1)\alpha}ResQ_1(t)=c_1-0.35(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)+(0.25+0.5)(0.6+\frac{c_1}{\Gamma(\alpha+1)}t^\alpha)$ and
$D^{(n-1)\alpha}ResR_1(t)=d_1-0.02(0.8+\frac{a_1}{\Gamma(\alpha+1)}t^\alpha)-0.25(1.4+\frac{b_1}{\Gamma(\alpha+1)}t^\alpha)-0.5(0.6+\frac{c_1}{\Gamma(\alpha+1)}t^\alpha)+0.25(0.05+\frac{d_1}{\Gamma(\alpha+1)}t^\alpha)$.

For $t=0$, by setting the above equation to 0, we solve the equation to get
$a_1=0.6160, b_1=-1.0220, c_1=0.040, d_1=0.6535$,
therefore

$$\begin{cases} S_1(t)=0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha,\\ I_1(t)=1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha,\\ Q_1(t)=0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha,\\ R_1(t)=0.05+\frac{0.6535}{\Gamma(\alpha+1)}t^\alpha.\end{cases}$$

When $n=2$, the truncation series is

$$\begin{cases} S_2(t)=0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha},\\ I_2(t)=1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha},\\ Q_2(t)=0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha+\frac{c_2}{\Gamma(2\alpha+1)}t^{2\alpha},\\ R_2(t)=0.05+\frac{0.6535}{\Gamma(\alpha+1)}t^\alpha+\frac{d_2}{\Gamma(2\alpha+1)}t^{2\alpha}.\end{cases}$$

The residual function is
$ResS_2(t)=D^\alpha(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})-1+0.15(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+(0.25+0.02)(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})$,
$ResI_2(t)=D^\alpha(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.15(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+(0.25+0.35+0.25)(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})$,
$ResQ_2(t)=D^\alpha(0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha+\frac{c_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.35(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+(0.25+0.5)(0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha+\frac{c_2}{\Gamma(2\alpha+1)}t^{2\alpha})$ and
$ResR_2(t)=D^\alpha(0.05+\frac{0.6535}{\Gamma(\alpha+1)}t^\alpha+\frac{d_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.02(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.25(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.5(0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha+\frac{c_2}{\Gamma(2\alpha+1)}t^{2\alpha})+0.25(0.05+\frac{0.6535}{\Gamma(\alpha+1)}t^\alpha+\frac{d_2}{\Gamma(2\alpha+1)}t^{2\alpha})$.

Then, applying the fractional caputo derivative operator $D^{(n-1)\alpha}$ to calculate the above formula, we obtain:
$D^{(n-1)\alpha}ResS_2(t)=D^\alpha(D^\alpha(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})-1+0.15(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+(0.25+0.02)(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha}))=D^\alpha(0.6160+\frac{a_2}{\Gamma(\alpha+1)}t^\alpha-1+0.15(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+(0.25+0.02)(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha}))=a_2+0.15(0.6160+\frac{a_2}{\Gamma(\alpha+1)}t^\alpha)(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+0.15(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})(-1.0220+\frac{b_2}{\Gamma(\alpha+1)}t^\alpha)+(0.25+0.02)(0.6160+\frac{a_2}{\Gamma(\alpha+1)}t^\alpha)$,
$D^{(n-1)\alpha}ResI_2(t)=D^\alpha(D^\alpha(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.15(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+(0.25+0.35+0.25)(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha}))=D^\alpha(-1.0220+\frac{b_2}{\Gamma(\alpha+1)}t^\alpha)-0.15(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+(0.25+0.35+0.25)(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha}))=b_2-0.15(0.6160+\frac{a_2}{\Gamma(\alpha+1)}t^\alpha)(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.15(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})(-1.0220+\frac{b_2}{\Gamma(\alpha+1)}t^\alpha)+(0.25+0.35+0.25)(-1.0220+\frac{b_2}{\Gamma(\alpha+1)}t^\alpha)$,
$D^{(n-1)\alpha}ResQ_2(t)=D^\alpha(D^\alpha(0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha+\frac{c_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.35(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+(0.25+0.5)(0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha+\frac{c_2}{\Gamma(2\alpha+1)}t^{2\alpha}))=D^\alpha(0.040+\frac{c_2}{\Gamma(\alpha+1)}t^\alpha-0.35(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})+(0.25+0.5)(0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha+\frac{c_2}{\Gamma(2\alpha+1)}t^{2\alpha}))=c_2-0.35(-1.0220+\frac{b_2}{\Gamma(\alpha+1)}t^\alpha)+(0.25+0.5)(0.040+\frac{c_2}{\Gamma(\alpha+1)}t^\alpha)$ and
$D^{(n-1)\alpha}ResR_2(t)=D^\alpha(D^\alpha(0.05+\frac{0.6535}{\Gamma(\alpha+1)}t^\alpha+\frac{d_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.02(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.25(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.5(0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha+\frac{c_2}{\Gamma(2\alpha+1)}t^{2\alpha})+0.25(0.05+\frac{0.6535}{\Gamma(\alpha+1)}t^\alpha+\frac{d_2}{\Gamma(2\alpha+1)}t^{2\alpha}))=D^\alpha(0.6535+\frac{d_2}{\Gamma(\alpha+1)}t^\alpha-0.02(0.8+\frac{0.6160}{\Gamma(\alpha+1)}t^\alpha+\frac{a_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.25(1.4+\frac{-1.0220}{\Gamma(\alpha+1)}t^\alpha+\frac{b_2}{\Gamma(2\alpha+1)}t^{2\alpha})-0.5(0.6+\frac{0.040}{\Gamma(\alpha+1)}t^\alpha+\frac{c_2}{\Gamma(2\alpha+1)}t^{2\alpha})+0.25(0.05+\frac{0.6535}{\Gamma(\alpha+1)}t^\alpha+\frac{d_2}{\Gamma(2\alpha+1)}t^{2\alpha}))=d_2-0.02(0.6160+\frac{a_2}{\Gamma(\alpha+1)}t^\alpha)-0.25(-1.0220+\frac{b_2}{\Gamma(\alpha+1)}t^\alpha)-0.5(0.040+\frac{c_2}{\Gamma(\alpha+1)}t^\alpha)+0.25(0.6535+\frac{d_2}{\Gamma(\alpha+1)}t^\alpha)$.

Similarly, $t = 0$, setting the above equation to 0 and solving the equation yields:
$a_2 = -0.1730400, b_2 = 0.8754200,$
$c_2 = -0.387700, d_2 = -0.386555,$
therefore

$$\begin{cases} S_2(t) = 0.8 + \frac{0.6160}{\Gamma(\alpha+1)}t^{\alpha} + \frac{-0.1730400}{\Gamma(2\alpha+1)}t^{2\alpha}, \\ I_2(t) = 1.4 + \frac{-1.0220}{\Gamma(\alpha+1)}t^{\alpha} + \frac{0.8754200}{\Gamma(2\alpha+1)}t^{2\alpha}, \\ Q_2(t) = 0.6 + \frac{0.040}{\Gamma(\alpha+1)}t^{\alpha} + \frac{-0.387700}{\Gamma(2\alpha+1)}t^{2\alpha}, \\ R_2(t) = 0.05 + \frac{0.6535}{\Gamma(\alpha+1)}t^{\alpha} + \frac{-0.386555}{\Gamma(2\alpha+1)}t^{2\alpha}. \end{cases}$$

The approximate solution for $n = 5$ is obtained by the same method

$$\begin{cases} S_5(t) = 0.8 + \frac{0.6160}{\Gamma(\alpha+1)}t^{\alpha} + \frac{-0.1730400}{\Gamma(2\alpha+1)}t^{2\alpha} + \frac{-0.0219912000}{\Gamma(3\alpha+1)}t^{3\alpha} \\ \quad + \frac{0.09160317600}{\Gamma(4\alpha+1)}t^{4\alpha} + \frac{-0.1025799482}{\Gamma(5\alpha+1)}t^{5\alpha}, \\ I_5(t) = 1.4 + \frac{-1.0220}{\Gamma(\alpha+1)}t^{\alpha} + \frac{0.8754200}{\Gamma(2\alpha+1)}t^{2\alpha} + \frac{-0.6753950000}{\Gamma(3\alpha+1)}t^{3\alpha} \\ \quad + \frac{0.4884201980}{\Gamma(4\alpha+1)}t^{4\alpha} + \frac{-0.3373100776}{\Gamma(5\alpha+1)}t^{5\alpha}, \\ Q_5(t) = 0.6 + \frac{0.040}{\Gamma(\alpha+1)}t^{\alpha} + \frac{-0.387700}{\Gamma(2\alpha+1)}t^{2\alpha} + \frac{0.597172000}{\Gamma(3\alpha+1)}t^{3\alpha} \\ \quad + \frac{-0.6842672500}{\Gamma(4\alpha+1)}t^{4\alpha} + \frac{0.6841475068}{\Gamma(5\alpha+1)}t^{5\alpha}, \\ R_5(t) = 0.05 + \frac{0.6535}{\Gamma(\alpha+1)}t^{\alpha} + \frac{-0.386555}{\Gamma(2\alpha+1)}t^{2\alpha} + \frac{0.118182950}{\Gamma(3\alpha+1)}t^{3\alpha} \\ \quad + \frac{0.0997516885}{\Gamma(4\alpha+1)}t^{4\alpha} + \frac{-0.2431344341}{\Gamma(5\alpha+1)}t^{5\alpha}. \end{cases}$$

Using the residual error obtained by formula (13), the error is analyzed to verify the accuracy of the residual power series method for solving the fractional SIQR model. Obviously, when $\alpha \in (0, 1]$, the absolute value of the exact solution residuals is 0. The approximate solution is substituted to obtain the residual values for various α values, as illustrated in Tables 1, 2, 3 and 4, and Figures 2, 3, 4 and 5. It can be seen that under different α values, as α approaches 1, the residual decreases, and the approximate solution obtained becomes more accurate. This demonstrates the feasibility of using the residual power series method to solve the new SIQR model proposed in this project.

Figure 6, 7, 8, and 9 depict the approximate solution graphs of susceptible nodes, infectious nodes, isolated nodes, and recovery nodes of the new model with different α values respectively. The same initial data were substituted into the original model and compared with the new model as shown in Figures 10, 11, 12, and 13, the original models are represented by a fork, pentagram, diamond, and square, while the new models are represented by a plus sign, asterisk, triangle, and hollow circle. The above figure fully illustrates the effect of fractional differentiation on the SIQR model. It is evident from the graph that fractional differentiation provides more degrees of freedom than integer differentiation. When the fractional order approaches the integer order, the corresponding intervals $S(t), I(t), Q(t), R(t)$ are very close to the SIQR model.

On this basis, we can calculate the corresponding basic reproduction number $\Re < 1$, which indicates the overall asymptotic stability of the virus-free equilibrium. In fact, it can be seen from Figure 7 that the infected node $I(t)$ will disappear over time. This indicates that the virus has been successfully cleared from the wireless sensor network. In addition, from Figure 11, it can be clearly seen that the infected node $I(t)$ of the new model proposed in this paper disappears earlier, and the recovery of the recovered node $R(t)$ will also be faster, as shown in Figure 13. The susceptible node $S(t)$ is similar, as shown in Figure 10. The quarantined node $Q(t)$, which is more affected by the infected node, follows the same rule, as illustrated in Figure 12. Therefore, the new model proposed in this paper is more suitable for simulating computer virus propagation in wireless sensor networks than the original model.

TABLE I
RESIDUAL VALUE WHEN $\alpha = 0.3$

t	$\lvert ResS(t)\rvert$	$\lvert ResI(t)\rvert$	$\lvert ResQ(t)\rvert$	$\lvert ResR(t)\rvert$
0.1	0.0195	0.0121	0.0150	0.0075
0.2	0.0298	0.0086	0.0425	0.0213
0.3	0.0394	0.0006	0.0780	0.0391
0.4	0.0491	0.0107	0.1201	0.0602
0.5	0.0591	0.0245	0.1679	0.0842
0.6	0.0695	0.0404	0.2207	0.1106
0.7	0.0804	0.0582	0.2781	0.1394
0.8	0.0916	0.0776	0.3397	0.1704
0.9	0.1033	0.0986	0.4054	0.2033
1	0.1154	0.1211	0.4748	0.2381

TABLE II
RESIDUAL VALUE WHEN $\alpha = 0.6$

t	$\lvert ResS(t)\rvert$	$\lvert ResI(t)\rvert$	$\lvert ResQ(t)\rvert$	$\lvert ResR(t)\rvert$
0.1	0.0060	0.0059	0.0001	0.0001
0.2	0.0123	0.0119	0.0008	0.0004
0.3	0.0185	0.0171	0.0028	0.0014
0.4	0.0247	0.0213	0.0067	0.0034
0.5	0.0309	0.0243	0.0131	0.0066
0.6	0.0372	0.0259	0.0227	0.0114
0.7	0.0438	0.0258	0.0361	0.0181
0.8	0.0508	0.0240	0.0539	0.0270
0.9	0.0583	0.0201	0.0767	0.0385
1	0.0665	0.0141	0.1052	0.0527

TABLE III
RESIDUAL VALUE WHEN $\alpha = 0.9$

t	$\lvert ResS(t)\rvert$	$\lvert ResI(t)\rvert$	$\lvert ResQ(t)\rvert$	$\lvert ResR(t)\rvert$
0.1	0.0015	0.0015	0.0000	0.0000
0.2	0.0048	0.0048	0.0000	0.0000
0.3	0.0093	0.0094	0.0001	0.0000
0.4	0.0148	0.0147	0.0002	0.0001
0.5	0.0208	0.0206	0.0005	0.0003
0.6	0.0273	0.0267	0.0012	0.0006
0.7	0.0341	0.0341	0.0024	0.0012
0.8	0.0411	0.0389	0.0044	0.0022
0.9	0.0484	0.0447	0.0075	0.0038
1	0.0559	0.0499	0.0121	0.0060

TABLE IV
RESIDUAL VALUE WHEN $\alpha = 1$

t	$\lvert ResS(t)\rvert$	$\lvert ResI(t)\rvert$	$\lvert ResQ(t)\rvert$	$\lvert ResR(t)\rvert$
0.1	0.0009	0.0009	0.0000	0.0000
0.2	0.0034	0.0034	0.0000	0.0000
0.3	0.0072	0.0072	0.0000	0.0000
0.4	0.0121	0.0120	0.0001	0.0000
0.5	0.0178	0.0177	0.0002	0.0001
0.6	0.0243	0.0241	0.0004	0.0002
0.7	0.0313	0.0309	0.0009	0.0004
0.8	0.0388	0.0379	0.0017	0.0009
0.9	0.0466	0.0450	0.0031	0.0016
1	0.0546	0.0520	0.0053	0.0026

VI. CONCLUSION

In this paper, an enhanced version of the existing SIQR model is proposed based on the characteristics of computer virus propagation. By incorporating the possibility

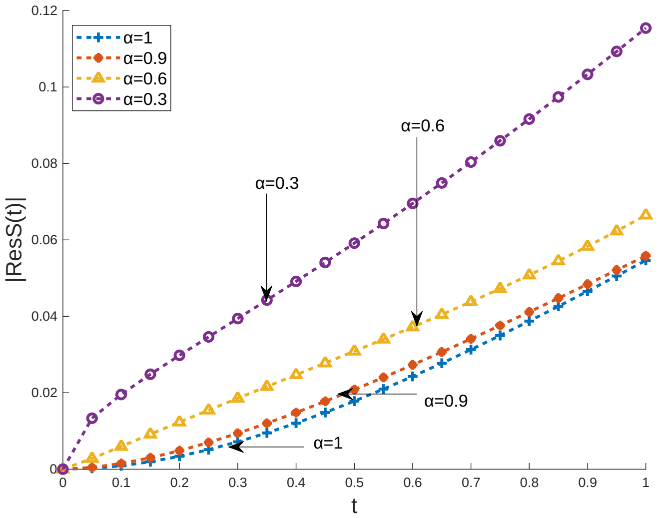

Fig. 2. The residual of S(t) when α takes different values

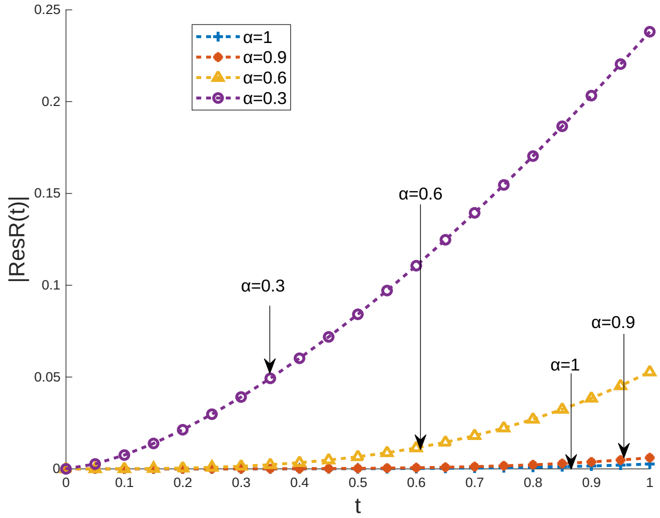

Fig. 5. The residual of R(t) when α takes different values

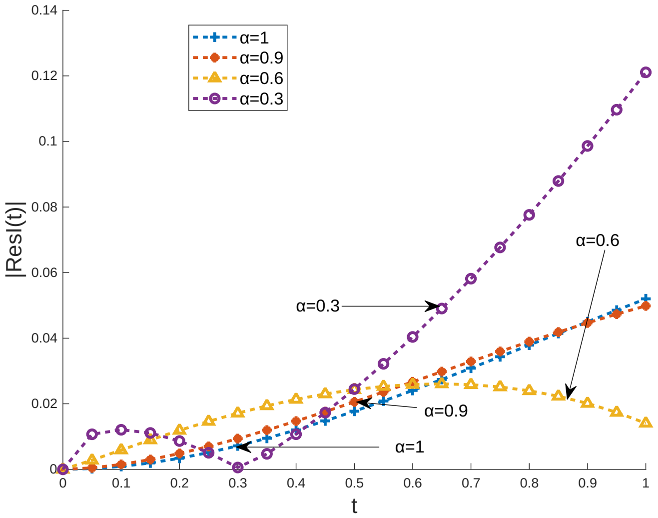

Fig. 3. The residual of I(t) when α takes different values

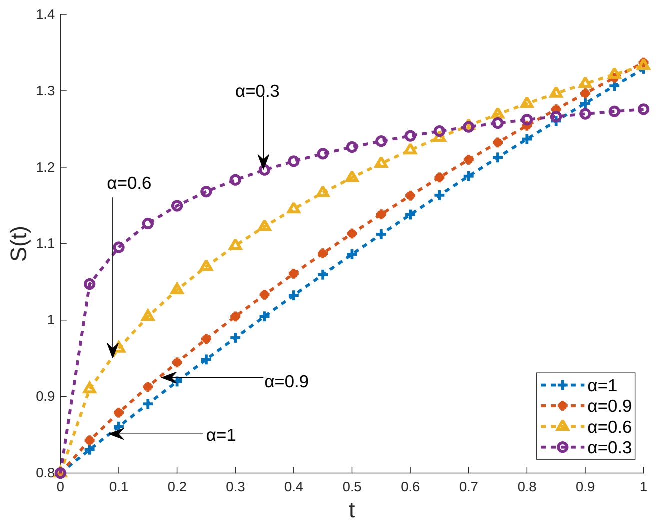

Fig. 6. The value of S(t) under the new model when α takes different values

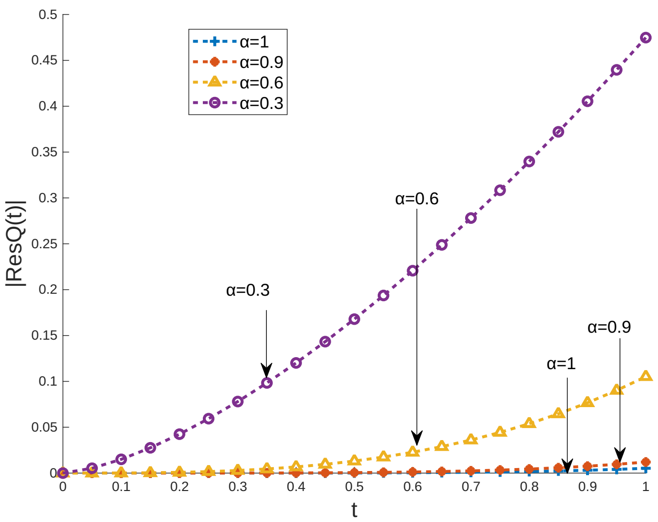

Fig. 4. The residual of Q(t) when α takes different values

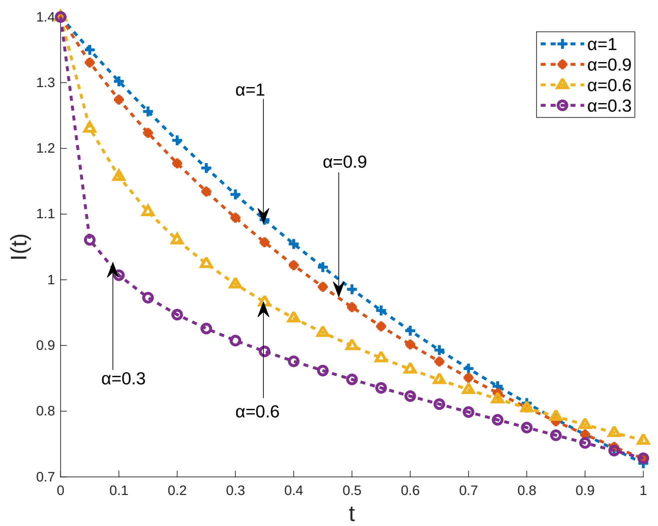

Fig. 7. The value of I(t) under the new model when α takes different values

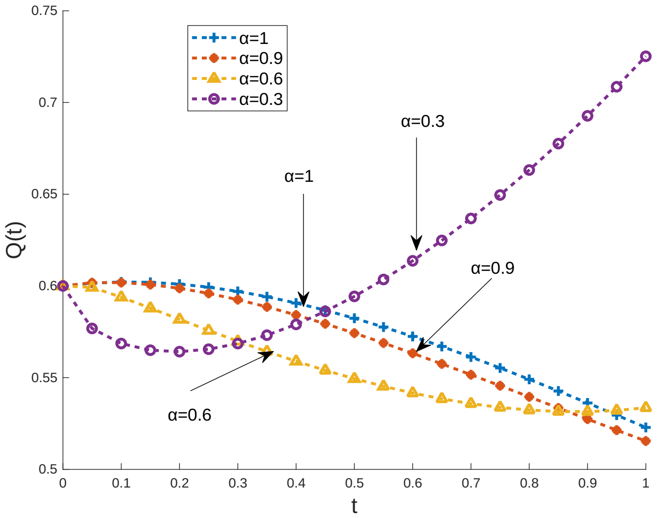

Fig. 8. The value of Q(t) under the new model when α takes different values

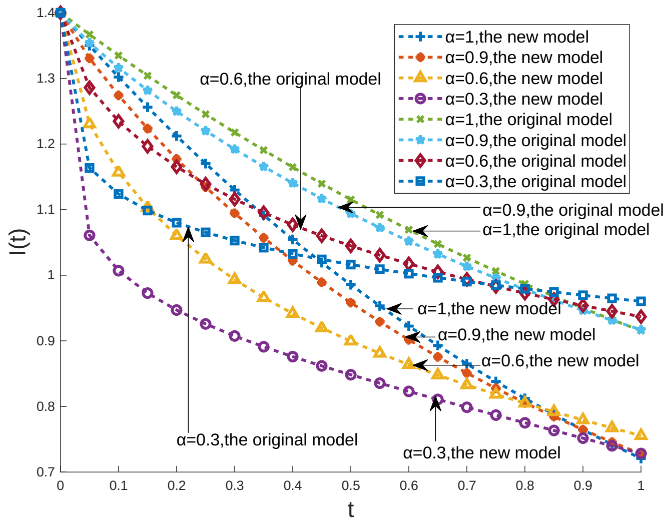

Fig. 11. The value of I(t) under the two models when α takes different values

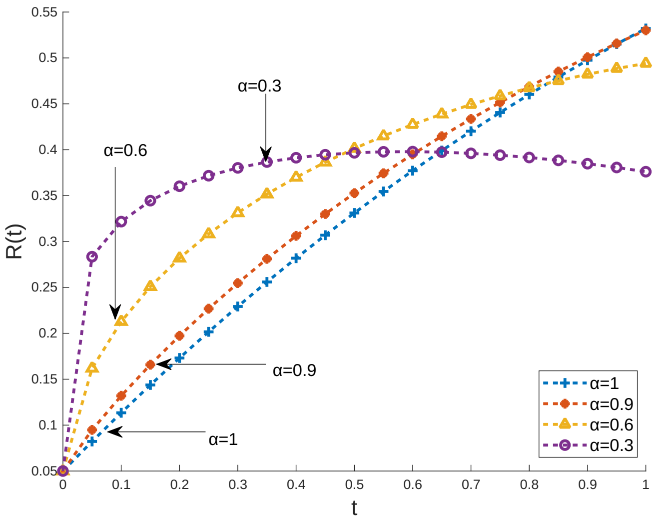

Fig. 9. The value of R(t) under the new model when α takes different values

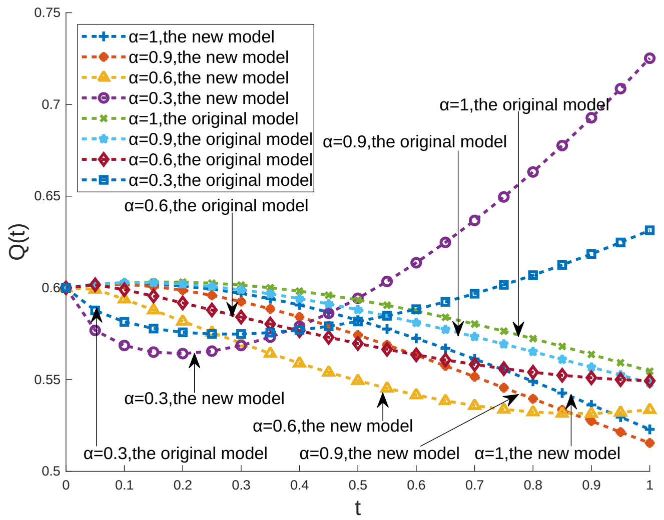

Fig. 12. The value of Q(t) under the two models when α takes different values

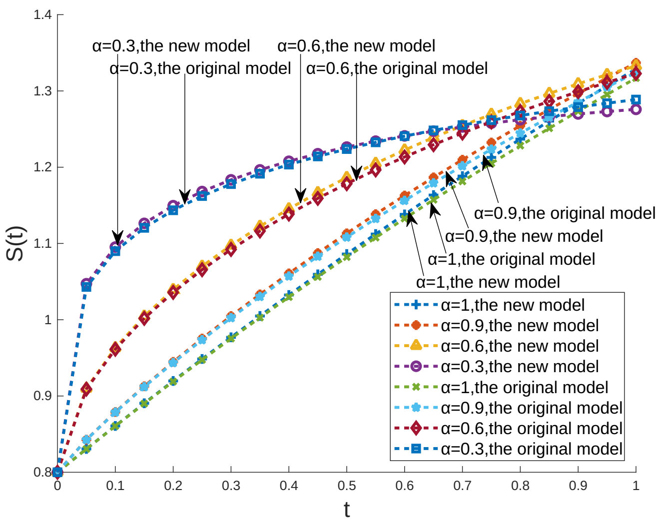

Fig. 10. The value of S(t) under the two models when α takes different values

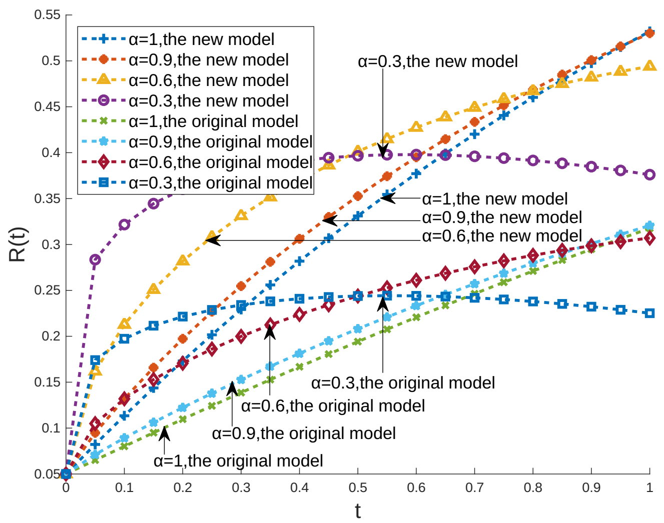

Fig. 13. The value of R(t) under the two models when α takes different values

of infected nodes recovering spontaneously with a certain probability, a new model was developed and solved using the residual power series method. By comparing the results of the original model under the same conditions, it is concluded that the infected component of the new model disappears earlier, the effect is better, and it is more suitable for simulating the spread of computer viruses in wireless sensor networks.

REFERENCES

[1] Kermack W O, McKendrick A G. Contributions to the mathematical theory of epidemics. II.—The problem of endemicity[J]. *Proceedings of the Royal Society of London*. Series A, containing papers of a mathematical and physical character, 1932, 138(834): 55-83.

[2] Mishra B K, Pandey S K. Dynamic model of worms with vertical transmission in computer network[J]. *Applied Mathematics and Computation*, 2011, 217(21): 8438-8446.

[3] Newman M E J, Forrest S, Balthrop J. Email networks and the spread of computer viruses[J]. *Physical Review E*, 2002, 66(3): 035101.

[4] Pastor-Satorras R, Vespignani A. Epidemic spreading in scale-free networks[J]. *Physical Review Letters*, 2001, 86(14): 3200.

[5] Pastor-Satorras R, Vespignani A. Epidemic dynamics in finite size scale-free networks[J]. *Physical Review E*, 2002, 65(3): 035108.

[6] Boguná M, Pastor-Satorras R, Vespignani A. Absence of epidemic threshold in scale-free networks with degree correlations[J]. *Physical Review Letters*, 2003, 90(2): 028701.

[7] Zhu G, Fu X, Chen G. Global attractivity of a network-based epidemic SIS model with nonlinear infectivity[J]. *Communications in Nonlinear Science and Numerical Simulation*, 2012, 17(6): 2588-2594.

[8] Barthélemy M, Barrat A, Pastor-Satorras R, et al. Velocity and hierarchical spread of epidemic outbreaks in scale-free networks[J]. *Physical Review Letters*, 2004, 92(17): 178701.

[9] Loecher M, Kadtke J. Enhanced predictability of hierarchical propagation in complex networks[J]. *Physics Letters A*, 2007, 366(6): 535-539.

[10] Mishra B K, Saini D K. SEIRS epidemic model with delay for transmission of malicious objects in computer network[J]. *Applied Mathematics and Computation*, 2007, 188(2): 1476-1482.

[11] Mishra B K, Keshri N. Mathematical model on the transmission of worms in wireless sensor network[J]. *Applied Mathematical Modelling*, 2013, 37(6): 4103-4111.

[12] Mishra B K, Keshri N. Stability analysis of a predator–prey model in wireless sensor network[J]. *International Journal of Computer Mathematics*, 2014, 91(5): 928-943.

[13] Sellali M, Abdeddaim S, Betka A, et al. Fuzzy-Super twisting control implementation of battery/super capacitor for electric vehicles[J]. *ISA Transactions*, 2019, 95: 243-253.

[14] Dong N P, Long H V, Giang N L. The fuzzy fractional SIQR model of computer virus propagation in wireless sensor network using Caputo Atangana–Baleanu derivatives[J]. *Fuzzy Sets and Systems*, 2022, 429: 28-59.

[15] Graef J R, Kong L, Ledoan A, et al. Stability analysis of a fractional online social network model[J]. *Mathematics and Computers in Simulation*, 2020, 178: 625-645.

[16] Dong N P, Long H V, Khastan A. Optimal control of a fractional order model for granular SEIR epidemic with uncertainty[J]. *Communications in Nonlinear Science and Numerical simulation*, 2020, 88: 105312.

[17] Vargas-De-León C. Volterra-type Lyapunov functions for fractional-order epidemic systems[J]. *Communications in Nonlinear Science and Numerical Simulation*, 2015, 24(1-3): 75-85.

[18] Long H V, Dong N P. An extension of Krasnoselskii's fixed point theorem and its application to nonlocal problems for implicit fractional differential systems with uncertainty[J]. *Journal of Fixed Point Theory and Applications*, 2018, 20: 1-27.

[19] Son N T K, Dong N P, Long H V, et al. Linear quadratic regulator problem governed by granular neutrosophic fractional differential equations[J]. *ISA Transactions*, 2020, 97: 296-316.

[20] Son N T K, Dong N P, Son L H, et al. Complete controllability for a class of fractional evolution equations with uncertainty[J]. *Evolution Equations & Control Theory*, 2022, 11(1).

[21] Son N T K, Thao H T P, Dong N P, et al. Fractional calculus of linear correlated fuzzy-valued functions related to Fréchet differentiability[J]. *Fuzzy Sets and Systems*, 2021, 419: 35-66.

[22] Alkahtani B S T. Chua's circuit model with Atangana–Baleanu derivative with fractional order[J]. *Chaos, Solitons & Fractals*, 2016, 89: 547-551.

[23] Atangana A, Gómez-Aguilar J F. Fractional derivatives with no-index law property: application to chaos and statistics[J]. *Chaos, Solitons & Fractals*, 2018, 114: 516-535.

[24] Atangana A, Gómez-Aguilar J F. Decolonisation of fractional calculus rules: breaking commutativity and associativity to capture more natural phenomena[J]. *The European Physical Journal Plus*, 2018, 133: 1-22.

[25] Atangana A, Hammouch Z. Fractional calculus with power law: The cradle of our ancestors∗[J]. *The European Physical Journal Plus*, 2019, 134(9): 429.

[26] Agarwal R P, Lakshmikantham V, Nieto J J. On the concept of solution for fractional differential equations with uncertainty[J]. *Nonlinear Analysis: Theory, Methods & Applications*, 2010, 72(6): 2859-2862.

[27] Allahviranloo T, Salahshour S, Abbasbandy S. Explicit solutions of fractional differential equations with uncertainty[J]. *Soft Computing*, 2012, 16: 297-302.

[28] Long H V, Son N T K, Tam H T T. The solvability of fuzzy fractional partial differential equations under Caputo gH-differentiability[J]. *Fuzzy Sets and Systems*, 2017, 309: 35–63.

[29] Long H V, Nieto J J, Son N T K. New approach for studying nonlocal problems related to differential systems and partial differential equations in generalized fuzzy metric spaces[J]. *Fuzzy Sets and Systems*, 2018, 331: 26-46.

[30] Hassouna M, Ouhadan A, El Kinani E H. On the solution of fractional order SIS epidemic model[J]. *Chaos, Solitons & Fractals*, 2018, 117: 168-174.

[31] Huo J, Zhao H. Dynamical analysis of a fractional SIR model with birth and death on heterogeneous complex networks[J]. *Physica A: Statistical Mechanics and its Applications*, 2016, 448: 41-56.

[32] Singh J, Kumar D, Hammouch Z, et al. A fractional epidemiological model for computer viruses pertaining to a new fractional derivative[J]. *Applied Mathematics and Computation*, 2018, 316: 504-515.

[33] Dubey V P, Kumar R, Kumar D.A hybrid analytical scheme for the numerical computation of time fractional computer virus propagation model and its stability analysis[J]. *Chaos, Solitons & Fractals*, 2020, 133: 109626.

[34] Graef J R, Kong L, Ledoan A, et al. Stability analysis of a fractional online social network model[J]. *Mathematics and Computers in Simulation*, 2020, 178: 625-645.

[35] Naim M, Lahmidi F, Namir A, et al. Dynamics of an fractional SEIR epidemic model with infectivity in latent period and general nonlinear incidence rate[J]. *Chaos, Solitons & Fractals*, 2021, 152: 111456.

[36] Hasan S, Al-Zoubi A, Freihet A, et al. Solution of fractional SIR epidemic model using residual power series method[J]. *Applied*

Mathematics and Information Sciences, 2019, 13(2): 153-161.

[37] Qazza A, Burqan A, Saadeh R. Application of ARA-residual power series method in solving systems of fractional differential equations[J]. *Mathematical Problems in Engineering*, 2022, 2022.

[38] Qayyum M, Khan A. Residual Power Series Method for Wave Type Equations[C]. *2021 International Conference on Applied and Engineering Mathematics (ICAEM)*. IEEE, 2021: 77-80.

[39] Modanli M, Abdulazeez S T, Husien A M. A residual power series method for solving pseudo hyperbolic partial differential equations with nonlocal conditions[J]. *Numerical Methods for Partial Differential Equations*, 2021, 37(3): 2235-2243.

[40] Abu Arqub O. Application of residual power series method for the solution of time-fractional Schrödinger equations in one-dimensional space[J]. *Fundamenta Informaticae*, 2019, 166(2): 87-110.

[41] Ghazanfari B, Sepahvandzadeh A. Solution of the Fractional Bratu-type Equation Via Fractional Residual Power Series Method[J]. *Journal of Mathematics and Computer Science*, 2014, 8: 236-244.

[42] Podlubny I. An introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications[J]. *Math. Sci. Eng*, 1999, 198: 340.

[43] Torge M, Bottlender R, StraußA, et al. An introduction to the fractional calculus and fractional differential equations [M]. *Wiley*, 1993: 1-10.

[44] Kilbas A A, Srivastava H M, Trujillo J J. Theory and applications of fractional differential equations[M]. *Elsevier*, 2006.

[45] Almeida R, Torres D F M. Calculus of variations with fractional derivatives and fractional integrals[J]. *Applied Mathematics Letters*, 2009, 22(12): 1816-1820.

[46] El-Ajou A, Arqub O A, Zhour Z A, et al. New Results on Fractional Power Series: Theory and Applications[J]. *Entropy*, 2013, 15(12): 5305-5323.

[47] El-Ajou A, Arqub O A, Momani S. Approximate analytical solution of the nonlinear fractional KdVCBurgers equation: A new iterative algorithm[J]. *Journal of Computational Physics*, 2014, 293: 81-95.

[48] Ahmed N, Raza A, Rafiq M, et al. Numerical and bifurcation analysis of SIQR model[J]. *Chaos, Solitons and Fractals*, 2021, 150: 111133.